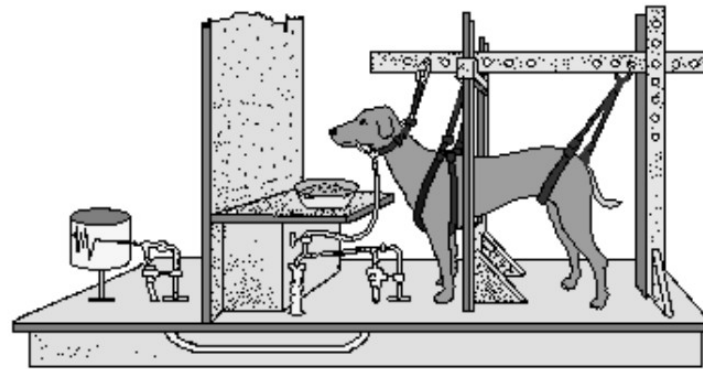
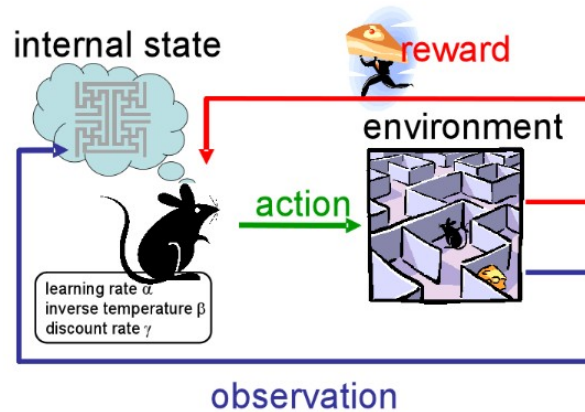
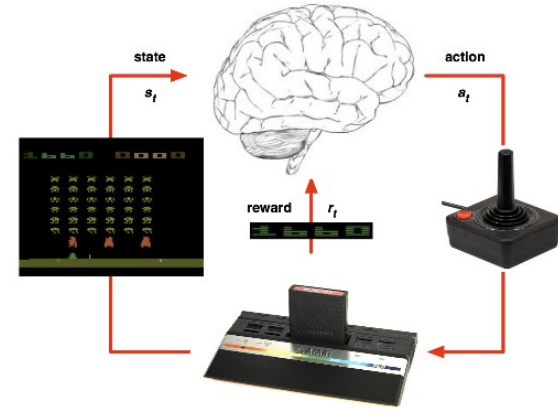
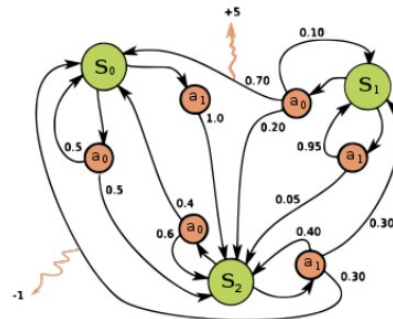
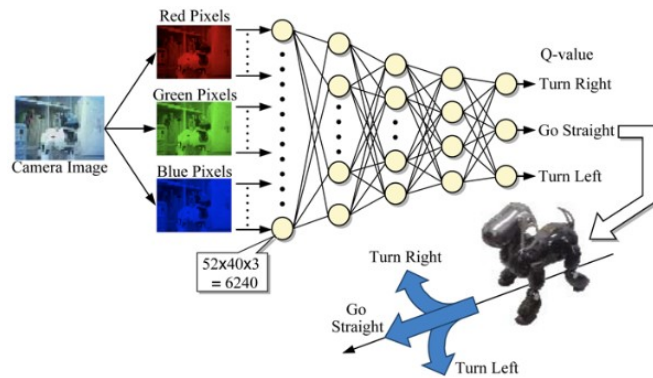


COMP 138: Reinforcement Learning



Instructor: Jivko Sinapov

Webpage: https://www.eecs.tufts.edu/~jsinapov/teaching/comp150_RL_Fall2021/

Today

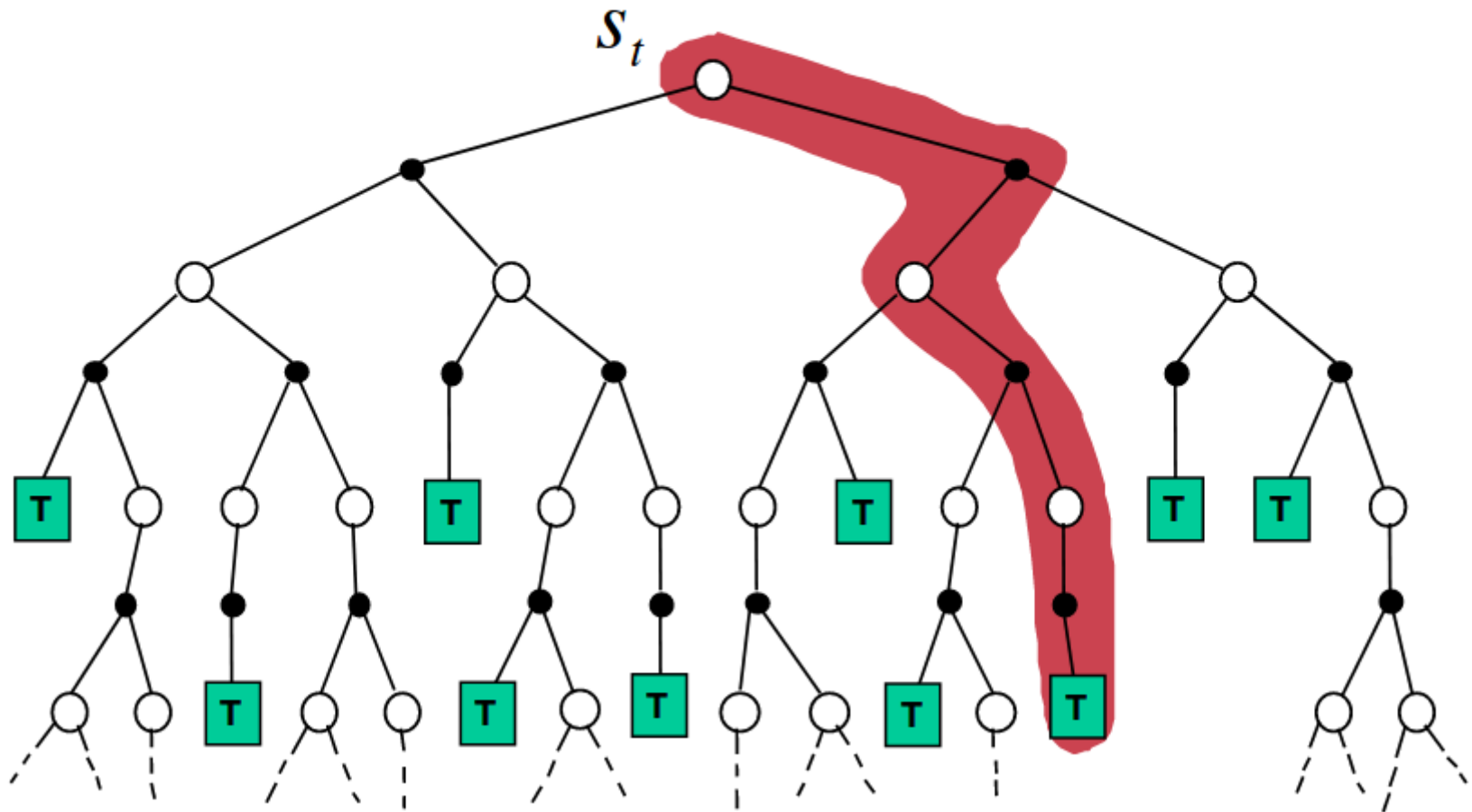
- Introduction to Eligibility Traces
- Project Breakout

Reading Assignment

- Chapter 13: Policy Gradient
- A research article of your choice

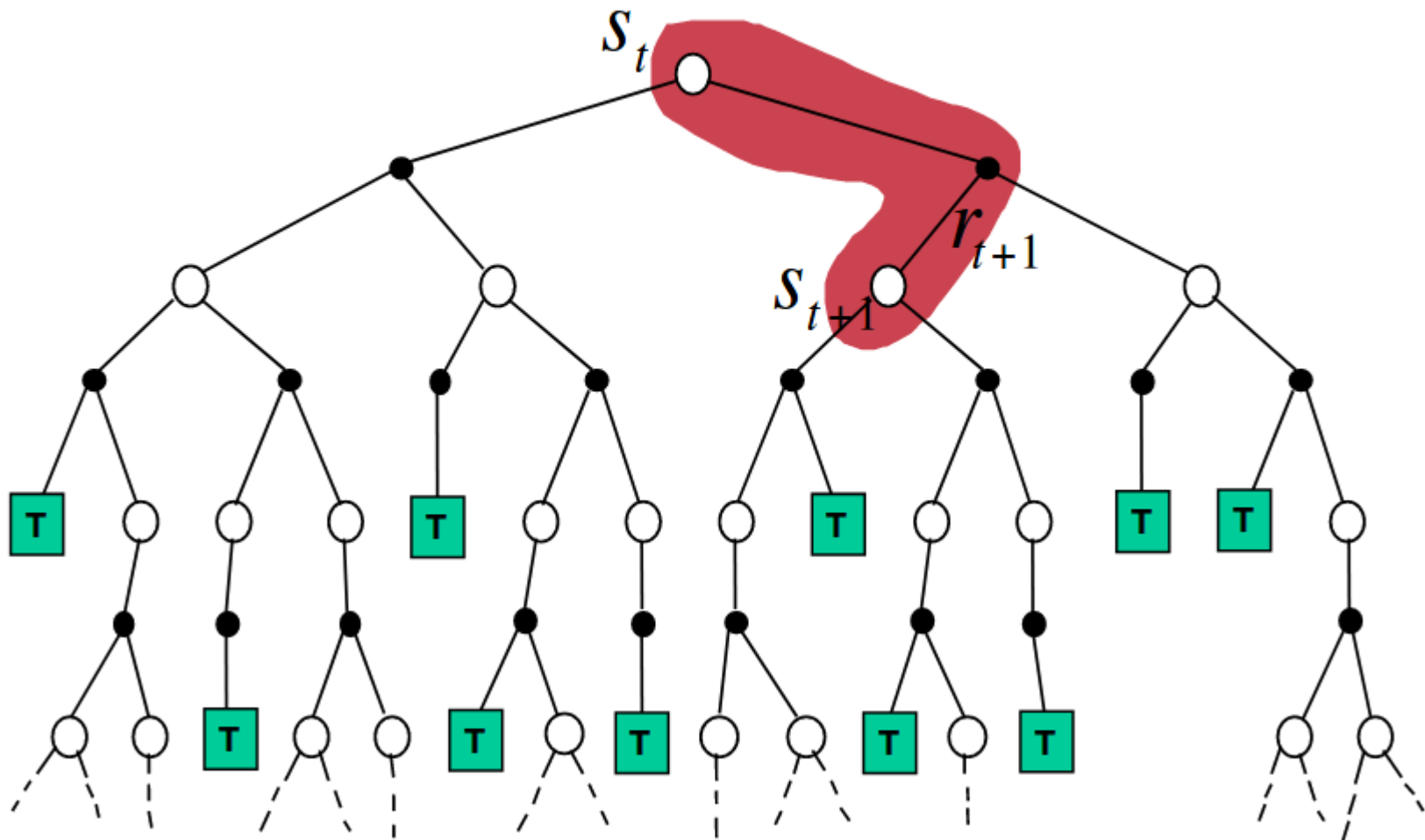
MC Backup

$$V(S_t) \leftarrow V(S_t) + \alpha (G_t - V(S_t))$$



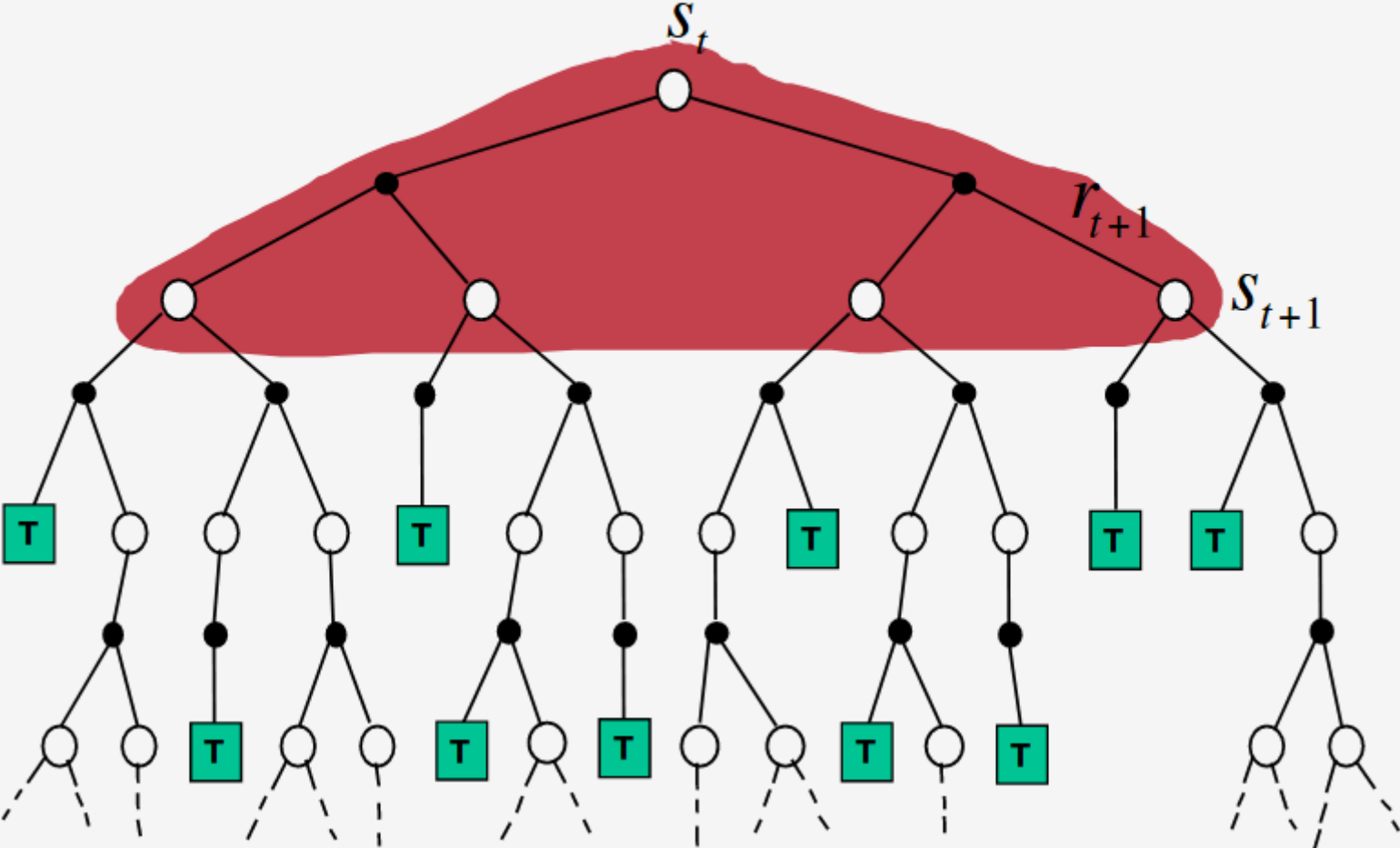
Temporal Difference Backup

$$V(S_t) \leftarrow V(S_t) + \alpha (R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$$



Dynamic Programming Backup

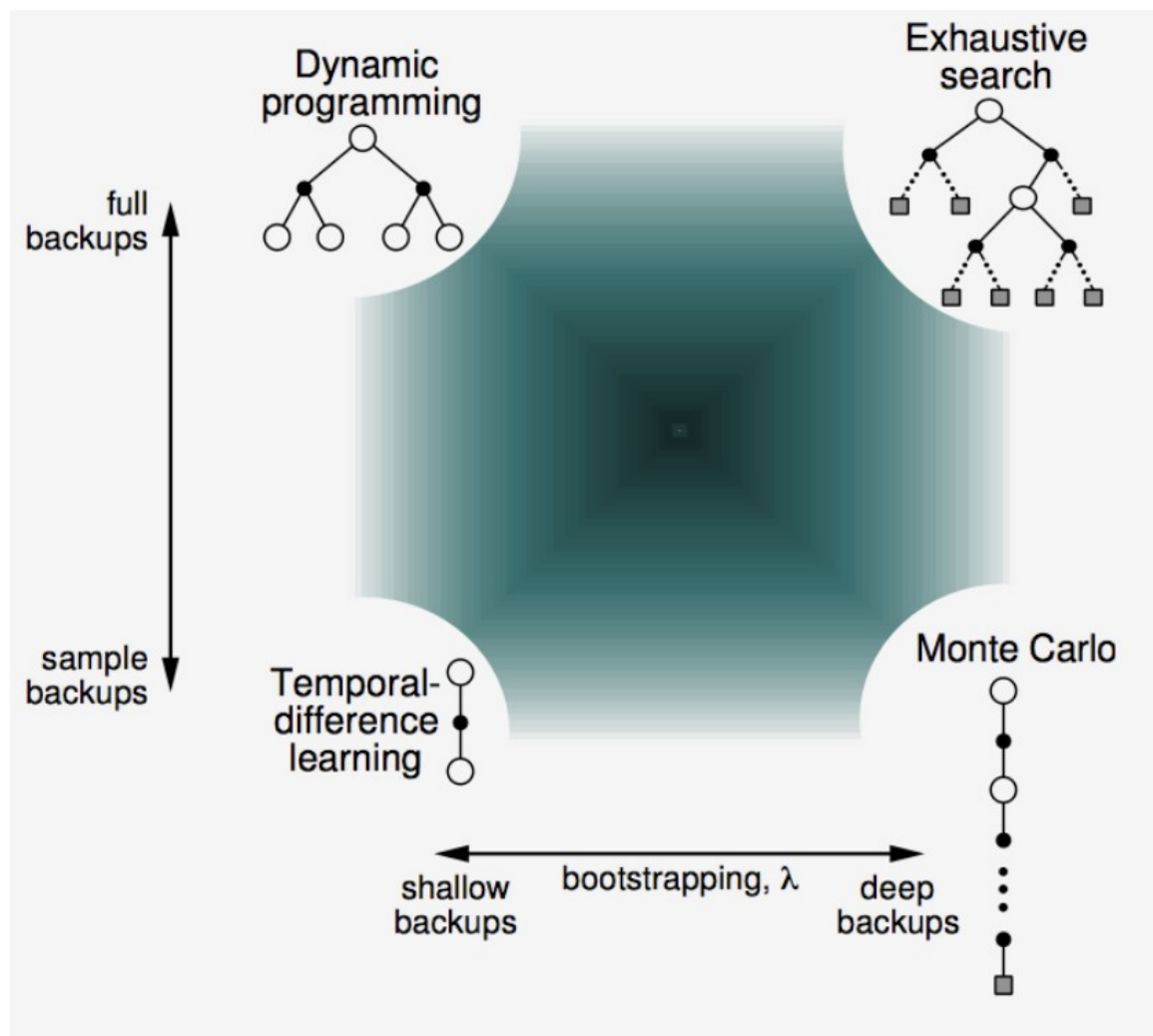
$$V(S_t) \leftarrow \mathbb{E}_\pi [R_{t+1} + \gamma V(S_{t+1})]$$



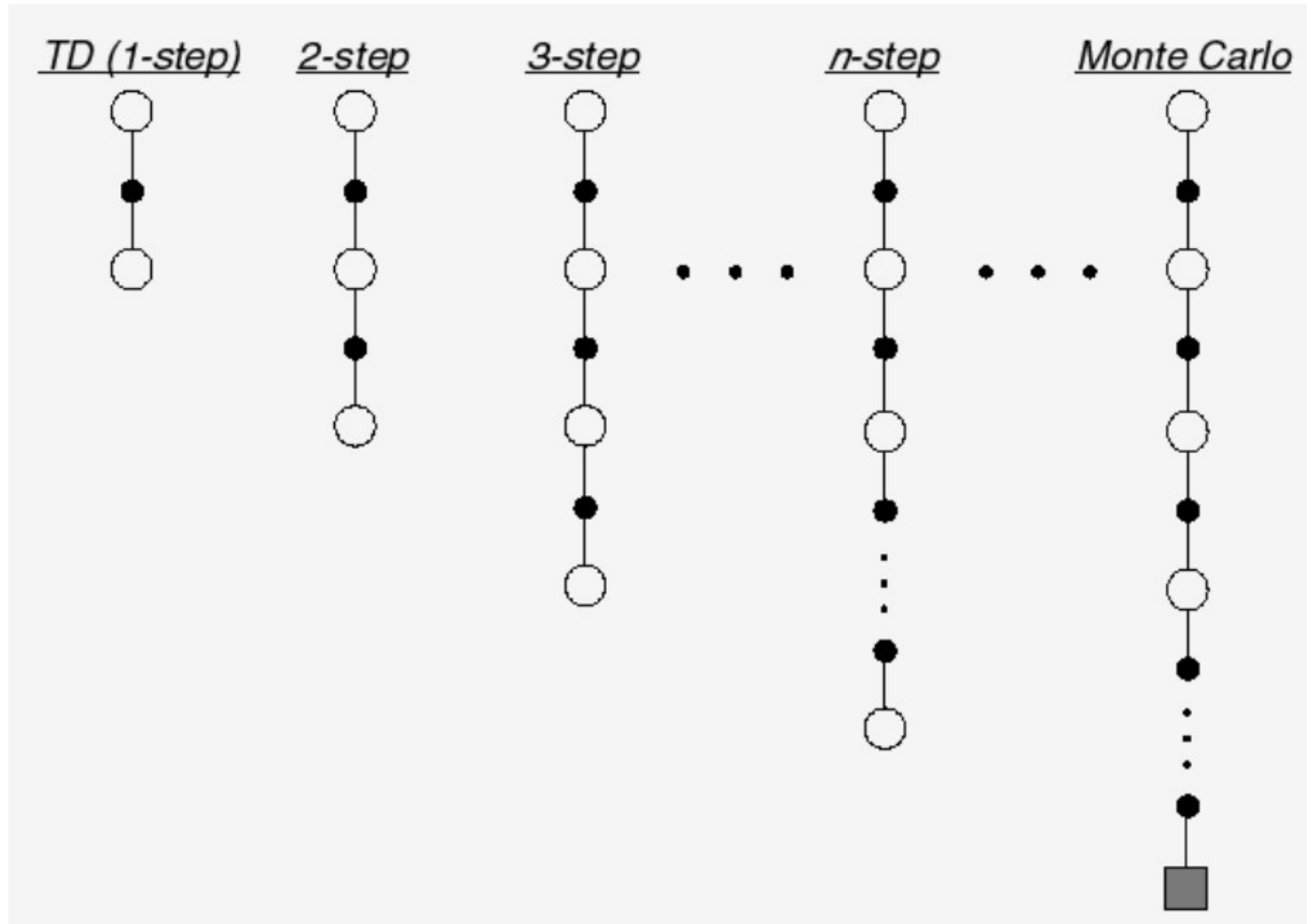
Bootstrapping vs Sampling

- Which of these methods bootstraps? Which samples?

Unified View of RL



n-Step Prediction



n-Step Return

n-Step Return Definition:

$$G_t^{(n)} = R_{t+1} + \gamma R_{t+2} + \dots + \gamma^{n-1} R_{t+n} + \gamma^n V(S_{t+n})$$

n-Step Returns for different n:

$$\begin{array}{ll} n = 1 & (TD) \quad G_t^{(1)} = R_{t+1} + \gamma V(S_{t+1}) \\ n = 2 & \quad \quad G_t^{(2)} = R_{t+1} + \gamma R_{t+2} + \gamma^2 V(S_{t+2}) \\ & \quad \quad \vdots \\ n = \infty & (MC) \quad G_t^{(\infty)} = R_{t+1} + \gamma R_{t+2} + \dots + \gamma^{T-1} R_T \end{array}$$

TD Learning using n-Step Returns:

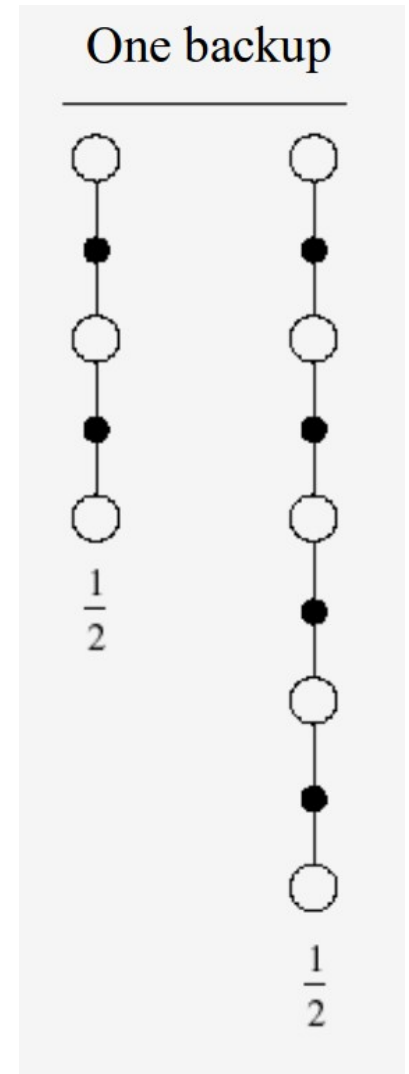
$$V(S_t) \leftarrow V(S_t) + \alpha \left(G_t^{(n)} - V(S_t) \right)$$

Averaging n-Step Returns

- n-Step returns can be averaged
- For example, average of 2-step and 4-step return is:

$$\frac{1}{2} G^{(2)} + \frac{1}{2} G^{(4)}$$

- Can we efficiently combine information from from all time steps?



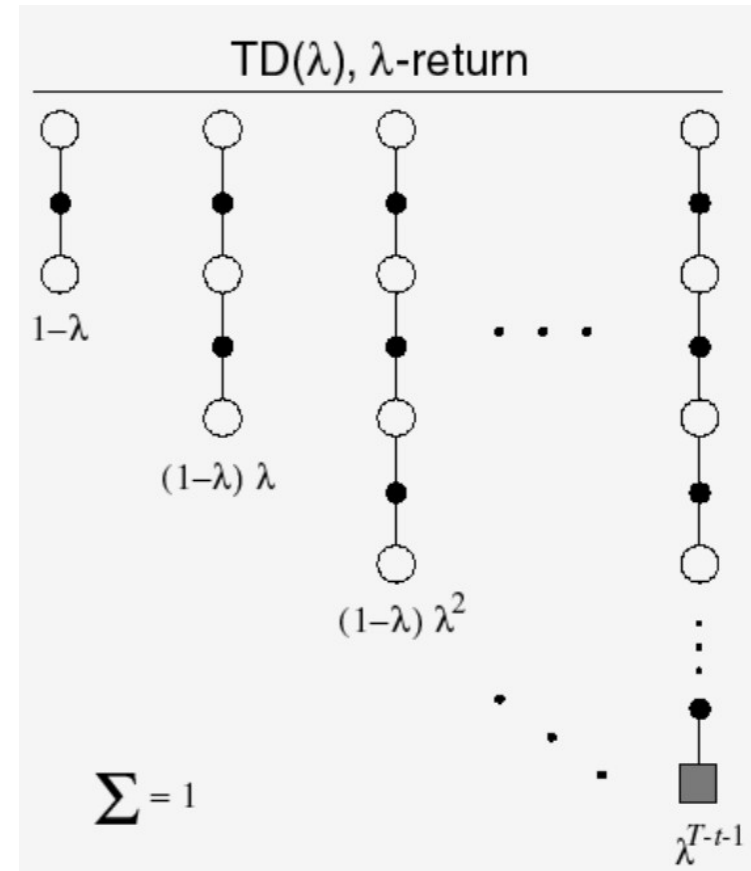
The λ -return

- Main idea: combine all n-step returns

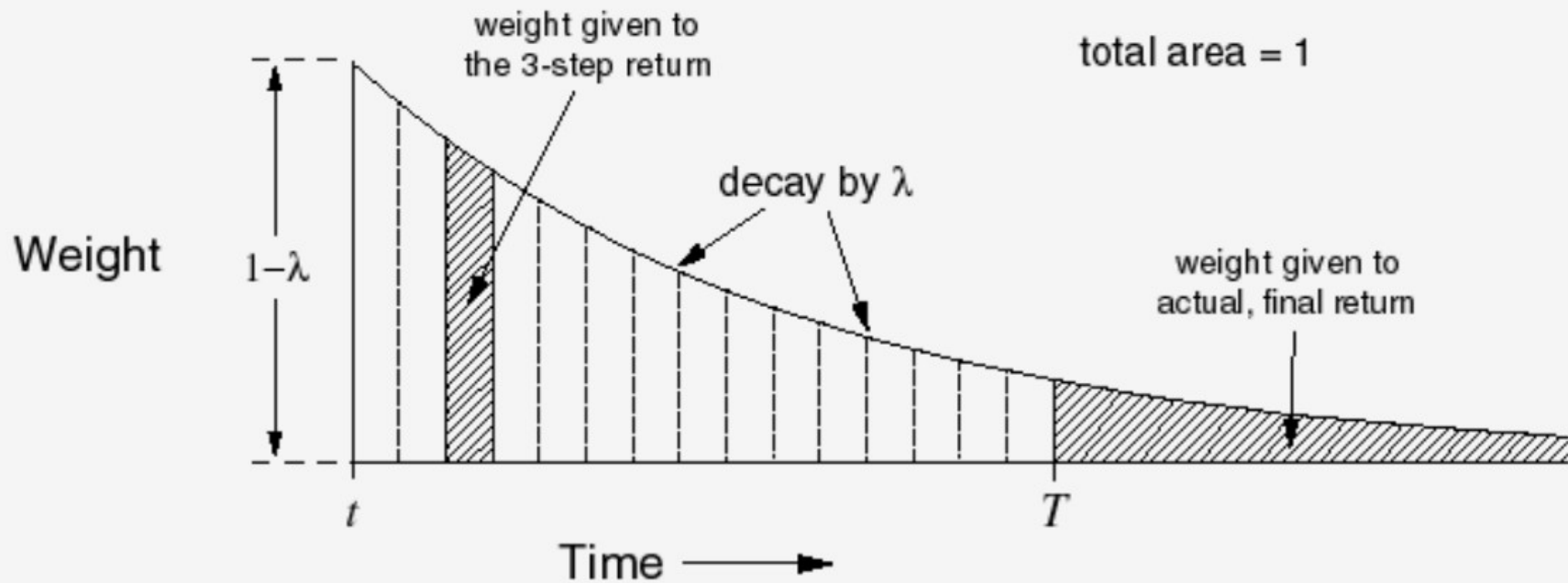
- Definition:
$$G_t^\lambda = (1 - \lambda) \sum_{n=1}^{\infty} \lambda^{n-1} G_t^{(n)}$$

- Update rule of TD(λ):

$$V(S_t) \leftarrow V(S_t) + \alpha \left(G_t^\lambda - V(S_t) \right)$$

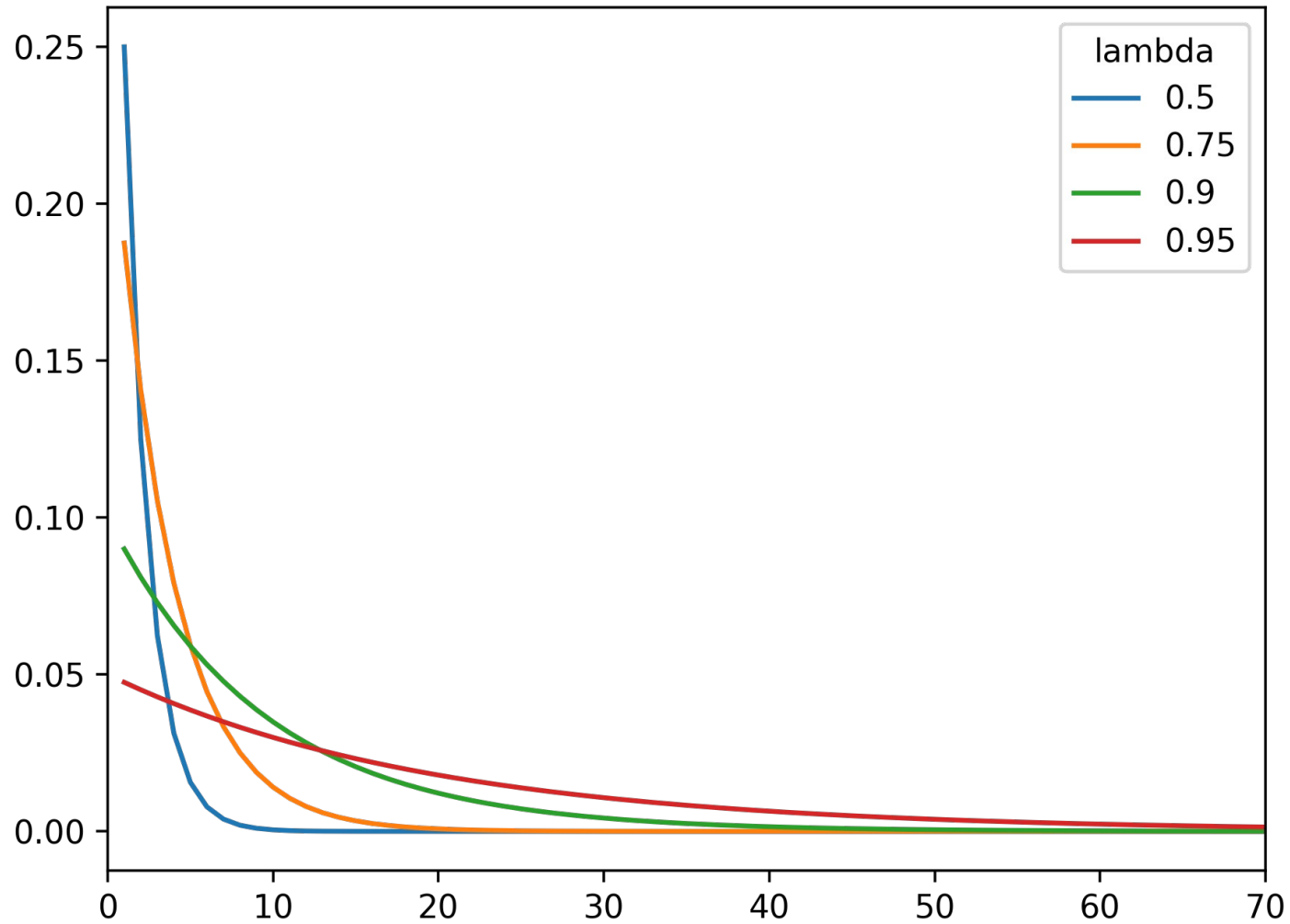


Weighting Function

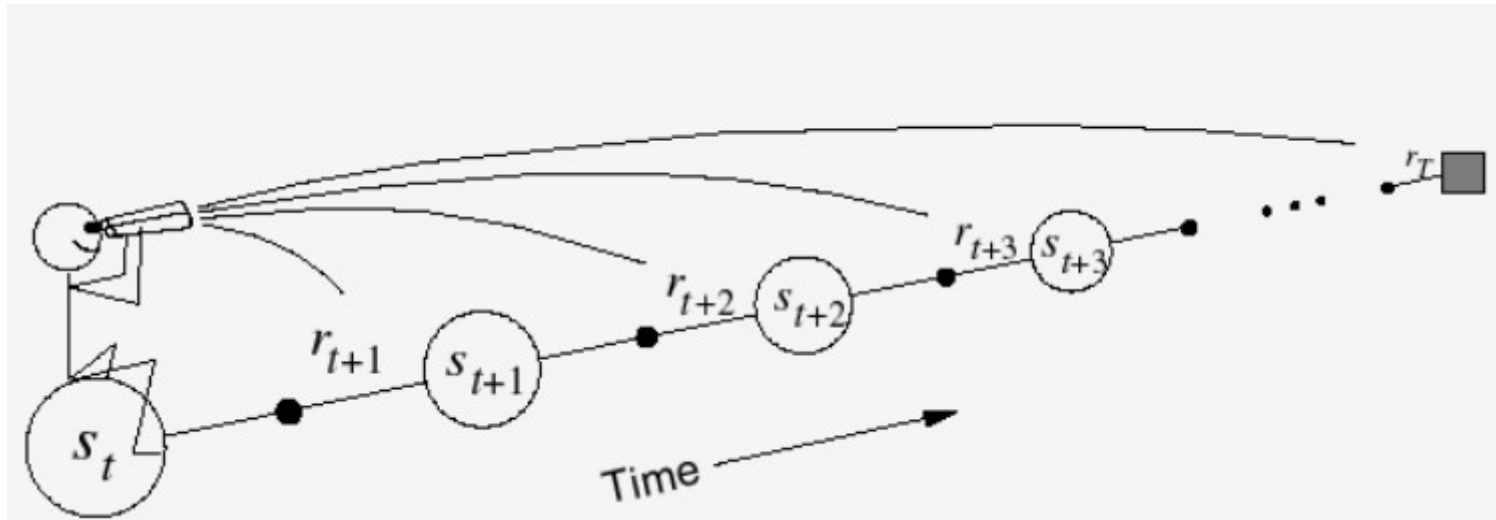


$$G_t^\lambda = (1 - \lambda) \sum_{n=1}^{\infty} \lambda^{n-1} G_t^{(n)}$$

Weighting Function

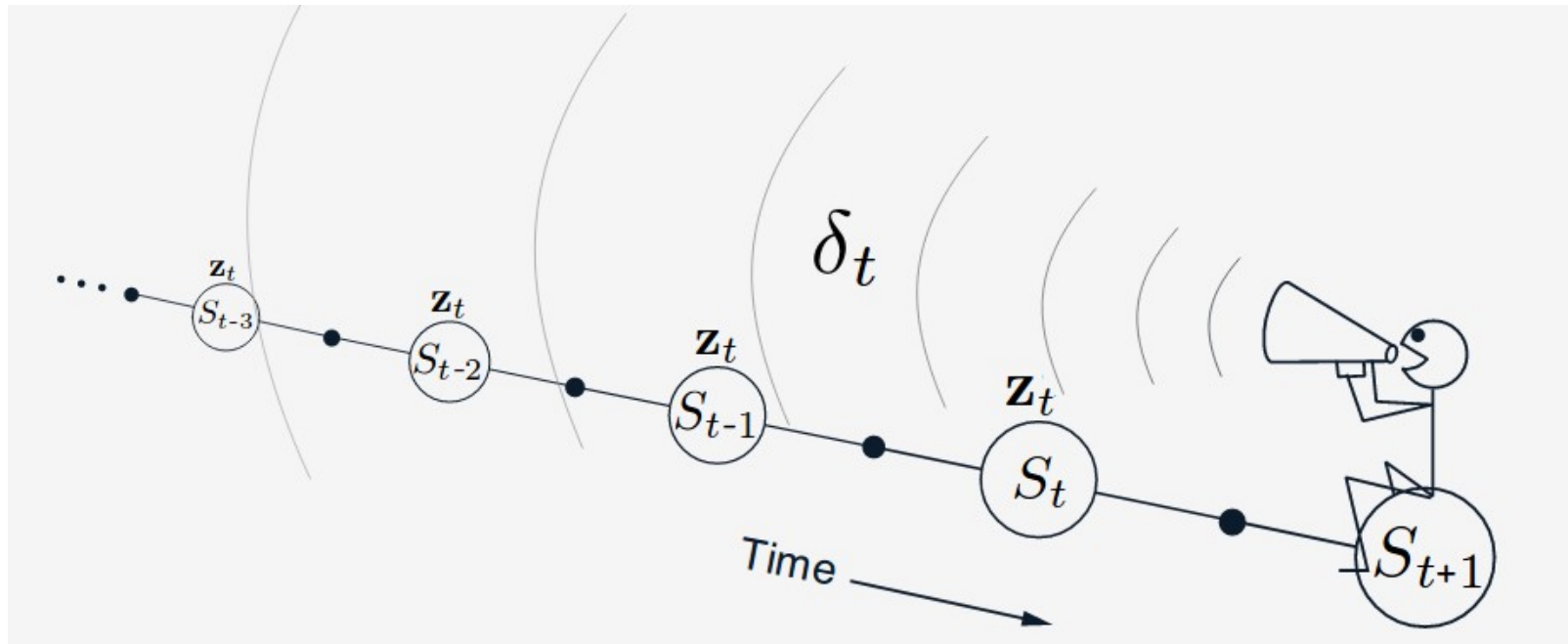


The “forward view”



- Updates value function towards the λ -return
- Looks into the future to compute the return
- Can only be computed from complete episodes

The “backward” view



- Forward view provides theory
- Backward view provides mechanism
- Update, online, after every step from incomplete episodes

The credit assignment problem

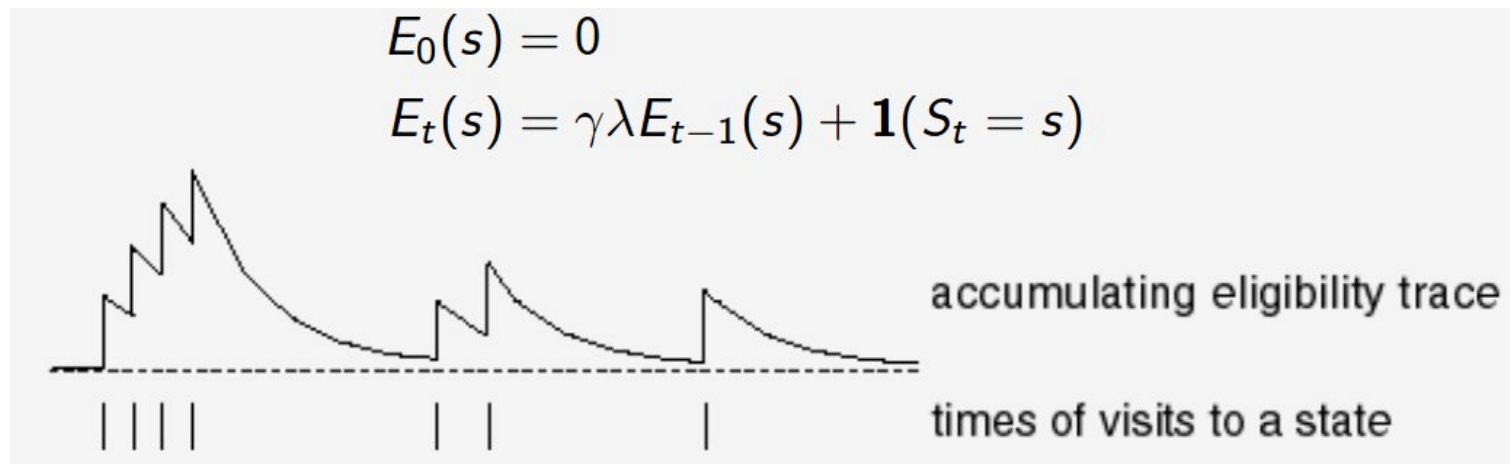
- Did the bell or the light cause the shock?



- Frequency heuristic: give credit to most frequent states
- Recency heuristic: give credit to most recent states

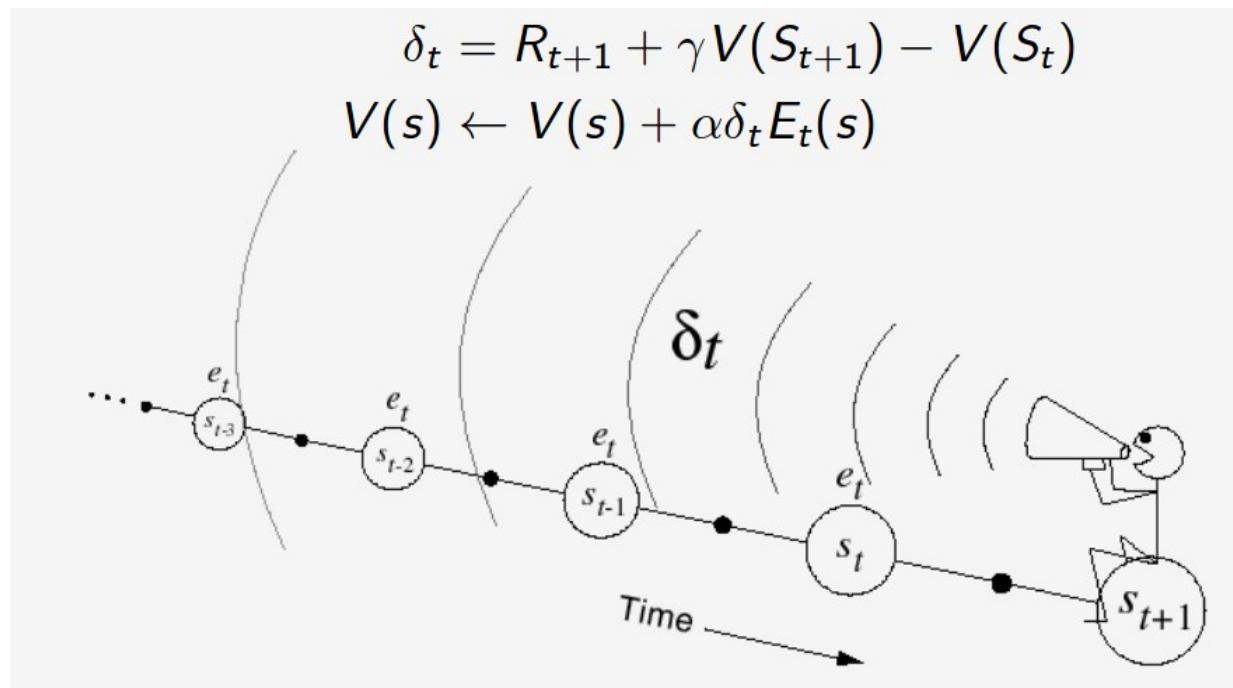
Eligibility Traces

- Eligibility traces combine both heuristics:



Backward view of TD(λ)

- Keep an eligibility trace for every state s
- Update value function for every state in proportion to TD-error and eligibility trace



TD(λ) and TD(0)

- When $\lambda = 0$, only current state is updated:

$$E_t(s) = \mathbf{1}(S_t = s)$$

$$V(s) \leftarrow V(s) + \alpha \delta_t E_t(s)$$

- This is equivalent to the TD(0) update

TD(λ) and MC

- When $\lambda = 1$, credit is deferred until end of episode
- Works with episodic tasks with off-line updates

Online tabular TD(λ)

Initialize $V(s)$ arbitrarily and $e(s) = 0$, for all $s \in S$

Repeat (for each episode) :

Initialize s

Repeat (for each step of episode) :

$a \leftarrow$ action given by π for s

Take action a , observe reward, r , and next state s'

$\delta \leftarrow r + \gamma V(s') - V(s)$

$e(s) \leftarrow e(s) + \delta$

For all s :

$V(s) \leftarrow V(s) + \alpha \delta e(s)$

$e(s) \leftarrow \gamma \lambda e(s)$

$s \leftarrow s'$

Until s is terminal

Sarsa(λ)

Initialize $Q(s,a)$ arbitrarily and $e(s,a) = 0$, for all s,a

Repeat (for each episode) :

Initialize s,a

Repeat (for each step of episode) :

Take action a , observe r,s'

Choose a' from s' using policy derived from Q (e.g. ϵ - greedy)

$$\delta \leftarrow r + \gamma Q(s', a') - Q(s, a)$$

$$e(s,a) \leftarrow e(s,a) + \delta$$

For all s,a :

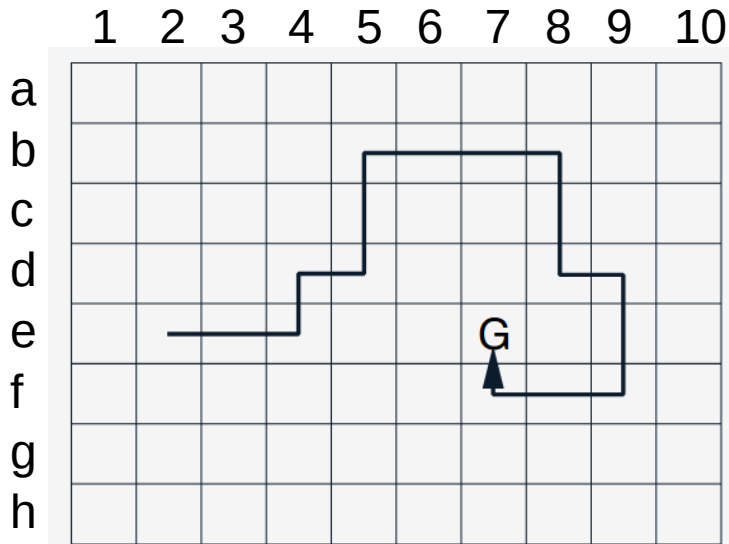
$$Q(s, a) \leftarrow Q(s, a) + \alpha \delta e(s, a)$$

$$e(s, a) \leftarrow \gamma \lambda e(s, a)$$

$$s \leftarrow s'; a \leftarrow a'$$

Until s is terminal

Walk-through



Actions: L,R,U,D

Initialize $Q(s,a)$ arbitrarily and $e(s,a) = 0$, for all s,a

Repeat (for each episode) :

Initialize s, a

Repeat (for each step of episode) :

Take action a , observe r, s'

Choose a' from s' using policy derived from Q (e.g. ? - greedy)

$$\delta \leftarrow r + \gamma Q(s', a') - Q(s, a)$$

$$e(s, a) \leftarrow e(s, a) + 1$$

For all s, a :

$$Q(s, a) \leftarrow Q(s, a) + \alpha \delta e(s, a)$$

$$e(s, a) \leftarrow \gamma \lambda e(s, a)$$

$$s \leftarrow s'; a \leftarrow a'$$

Until s is terminal

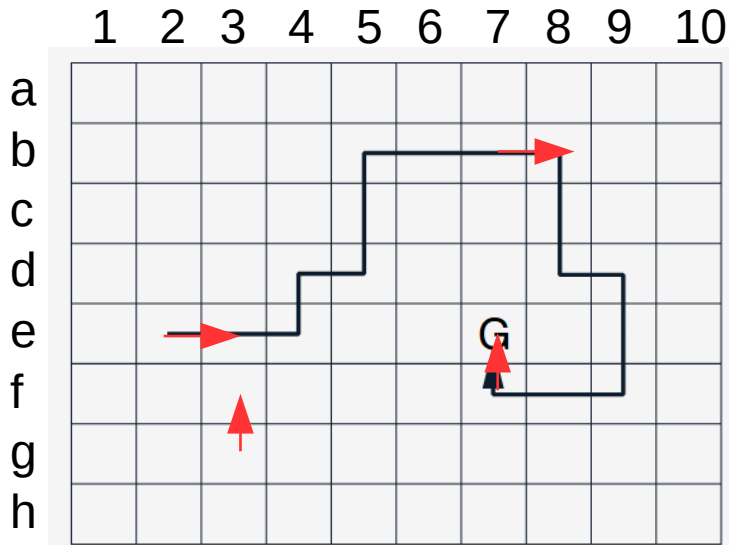
In small groups, compute the updates to $Q(\langle f, 7 \rangle, U)$, $Q(\langle b, 7 \rangle, R)$, $Q(\langle e, 2 \rangle, R)$ and $Q(\langle g, 3 \rangle, U)$ assuming:

Discount factor $\gamma = 0.95$

Goal reward = 100

$\lambda = 0.95$

Walk-through



Actions: L,R,U,D

Initialize $Q(s,a)$ arbitrarily and $e(s,a) = 0$, for all s,a

Repeat (for each episode) :

Initialize s, a

Repeat (for each step of episode) :

Take action a , observe r, s'

Choose a' from s' using policy derived from Q (e.g. ? - greedy)

$$\delta \leftarrow r + \gamma Q(s', a') - Q(s, a)$$

$$e(s, a) \leftarrow e(s, a) + \delta$$

For all s, a :

$$Q(s, a) \leftarrow Q(s, a) + \alpha \delta e(s, a)$$

$$e(s, a) \leftarrow \gamma \lambda e(s, a)$$

$$s \leftarrow s'; a \leftarrow a'$$

Until s is terminal

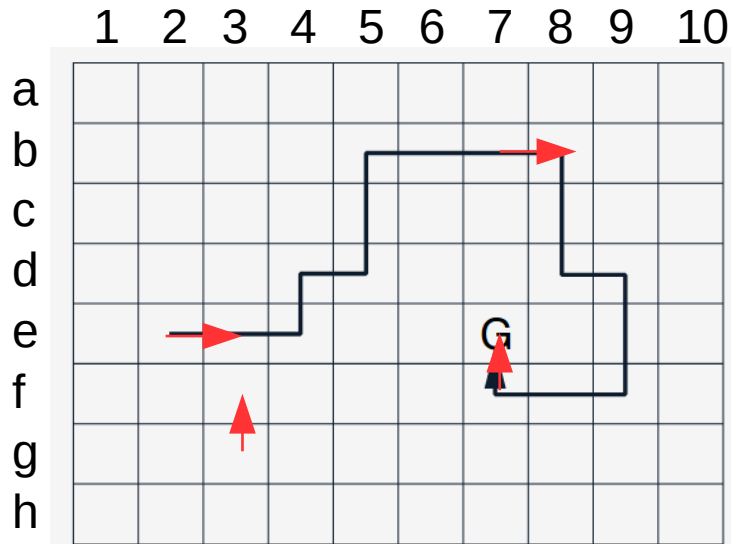
In small groups, compute the updates to $Q(\langle f, 7 \rangle, U)$, $Q(\langle b, 7 \rangle, R)$, $Q(\langle e, 2 \rangle, R)$ and $Q(\langle g, 3 \rangle, U)$ assuming:

Discount factor $\gamma = 0.95$

Goal reward = 100

$\lambda = 0.95$

What did you get?



Actions: L,R,U,D

Initialize $Q(s,a)$ arbitrarily and $e(s,a) = 0$, for all s,a

Repeat (for each episode) :

Initialize s,a

Repeat (for each step of episode) :

Take action a , observe r,s'

Choose a' from s' using policy derived from Q (e.g. ? - greedy)

$$\delta \leftarrow r + \gamma Q(s', a') - Q(s, a)$$

$$e(s,a) \leftarrow e(s,a) + 1$$

For all s,a :

$$Q(s, a) \leftarrow Q(s, a) + \alpha \delta e(s, a)$$

$$e(s, a) \leftarrow \gamma \lambda e(s, a)$$

$$s \leftarrow s'; a \leftarrow a'$$

Until s is terminal

In small groups, compute the updates to $Q(\langle f,7 \rangle, U)$, $Q(\langle b,7 \rangle, R)$, $Q(\langle e,2 \rangle, R)$ and $Q(\langle g,3 \rangle, U)$ assuming:

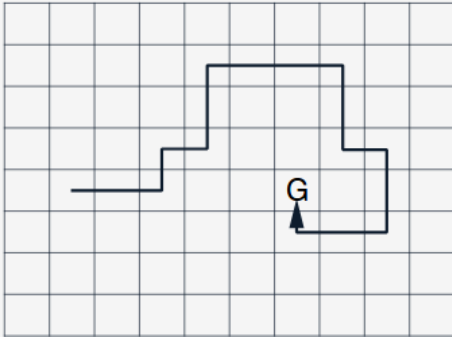
Discount factor $\gamma = 0.95$

Goal reward = 100

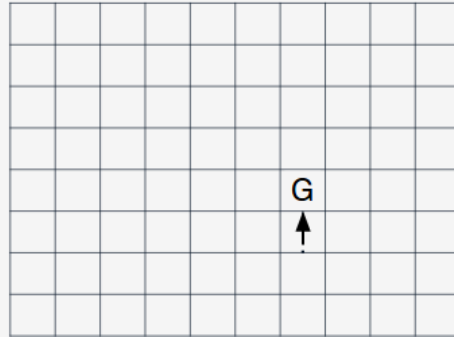
$\lambda = 0.95$

Comparison

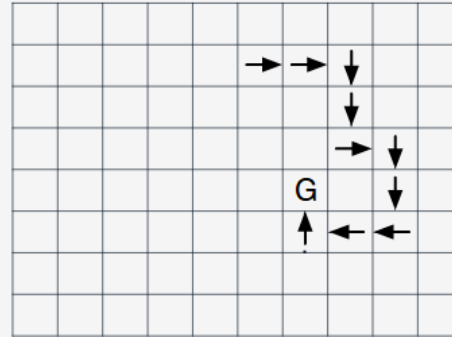
Path taken



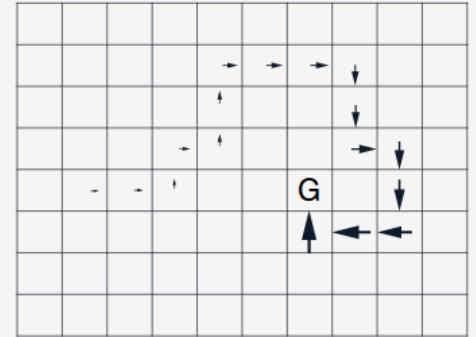
Action values increased by one-step Sarsa



Action values increased by 10-step Sarsa



Action values increased by Sarsa(λ) with $\lambda=0.9$



Project Planning Breakout

- Meet with partner(s) if working in group
- Plan out the activities for this week – make concrete goals that you want to accomplish
- Find research articles relevant to your project
- Write down any questions for me

