An Inference Mechanism for Point-Interval Logic

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Formal Logic

- Logic is the “art of reasoning”.
- Logic is used to make inferences based on the available information.
- Formal logic makes inferences based purely on the form of the content, without any understanding of the meaning of the content.
- Reasoning based just on the form is important because this means computers can do it.

Temporal Logic

We need a logic to reason about temporal information so as to:

- characterize time-sensitive attributes of a domain to be modeled.
- do temporal analysis of a domain, which will help us in developing a better understanding of the relationship between domain entities.
- identify inconsistencies and anomalies.

Applications

- Forensics: Temporal information is the facts (may be partial) about some incident that has already occured.
- Planning and Scheduling: Temporal information is the constraints/spefication that need to imposed on the desired schedule.

Point-Interval Logic

Point-Interval Logic is a temporal logic.

- It is a tractable subclass of Allen's interval logic.
- Point-Interval Logic (PIL) is a Pointisable logic.
- It combines qualitative and quantitive temporal information.

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Point Graphs

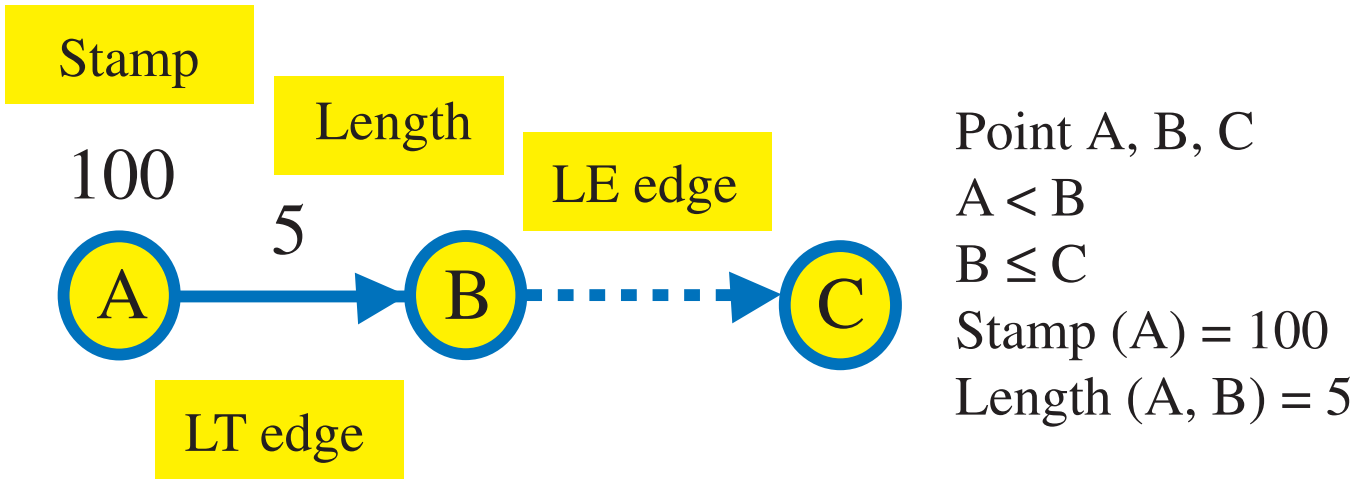
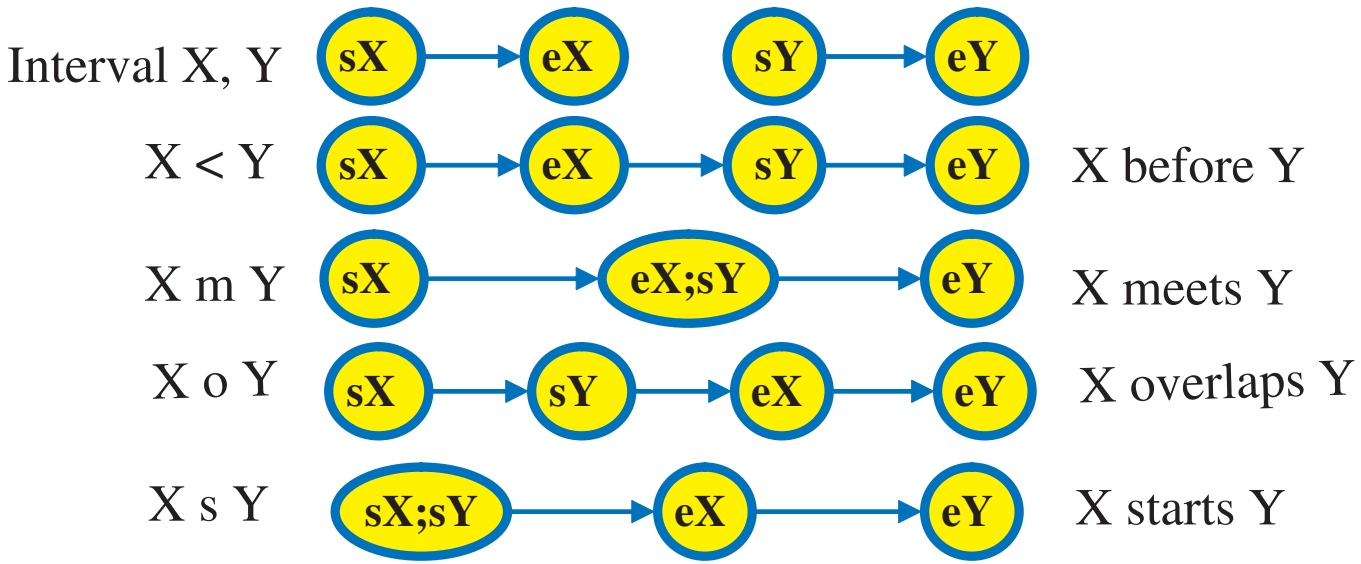
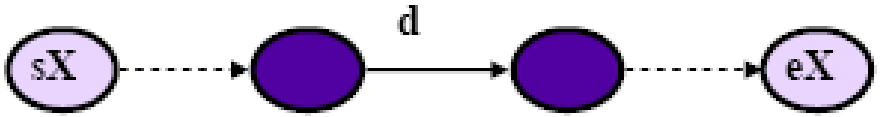
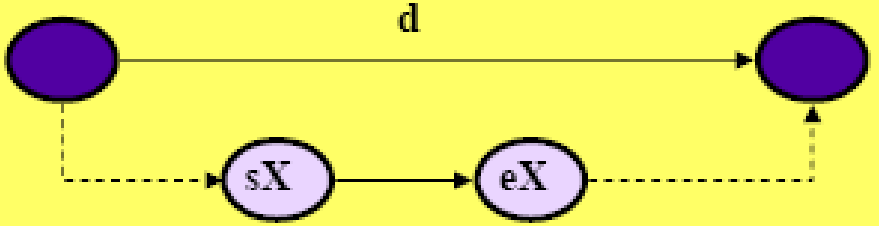


Figure: Point Graph for a Set of PIL Statements

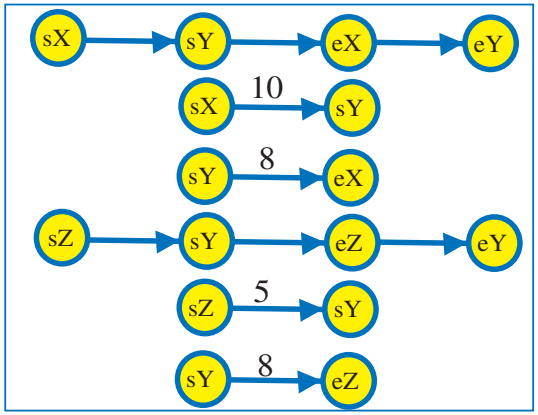
Qualitative Constraints



Quantitative Constraints

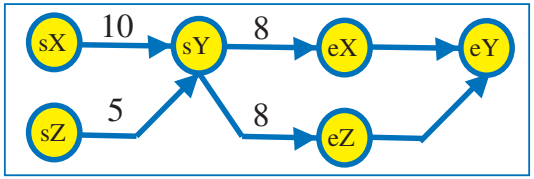
Stamp [P] = t	
Length [X] = d	
Stamp [P] ≥ t	
Stamp [P] ≤ t	
Length [X] ≥ d	
Length [X] ≤ d	

Point Graph Construction



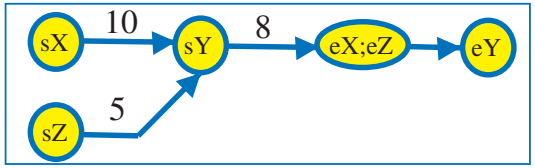
Interval X, Y, Z
X o Y
Length (sX, sY) = 10
Length (sY, eX) = 8
Z o Y
Length (sZ, sY) = 5
Length (sY, eZ) = 8

a) Convert each PIL statement into a Point Graph



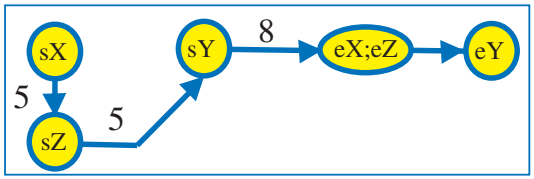
Unify the nodes with same labels.

b) Unify individual Point Graphs into a single Point Graph



Fold branch and join nodes. Before we fold the Point Graph, we must check it for consistency.

c) Fold the branch nodes (outdegree > 1) in the Point Graph



Once we have a folded and consistent Point Graph, we can use it to draw inferences.

d) Fold the join nodes (indegree > 1) in the Point Graph

Basic Temporal Queries

An inference mechanism for Point-Interval logic should be able to answer following basic queries:

- Relationship Query
- Stamp Query
- Length Query

Problem with Previous Mechanism

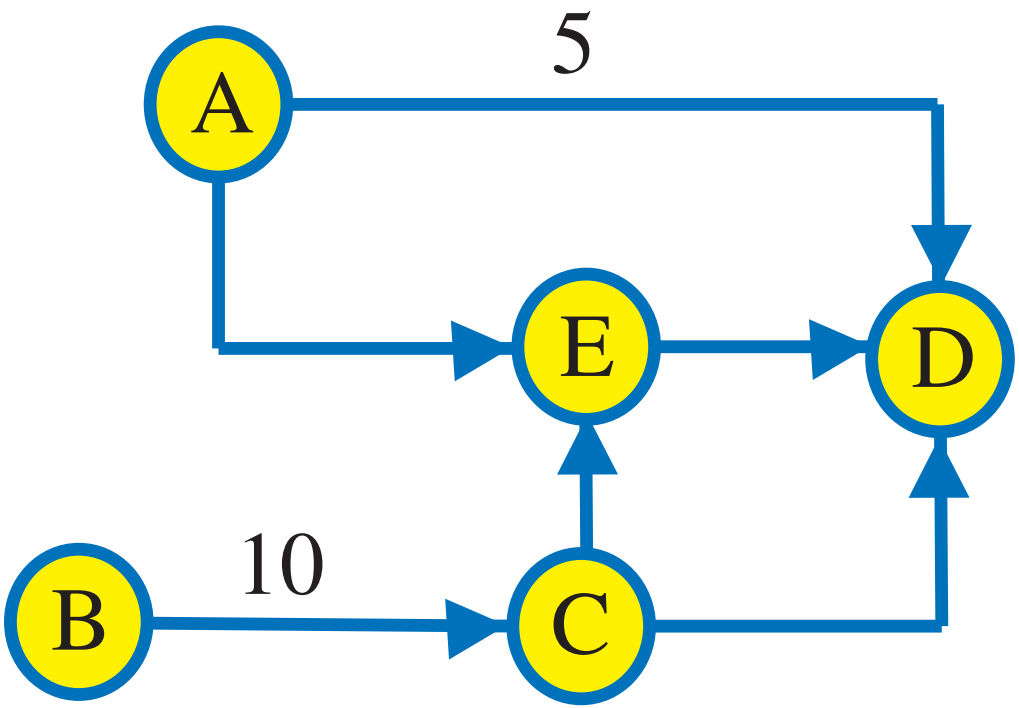


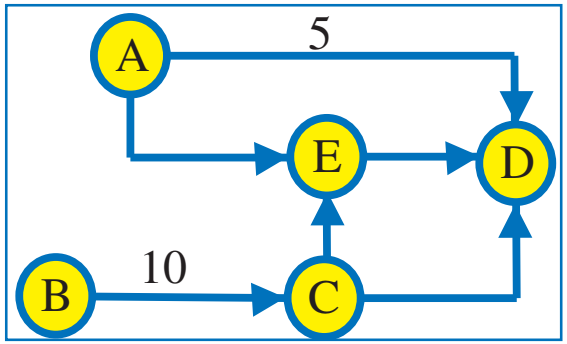
Figure: Illustration of Incompleteness

Relationship Query

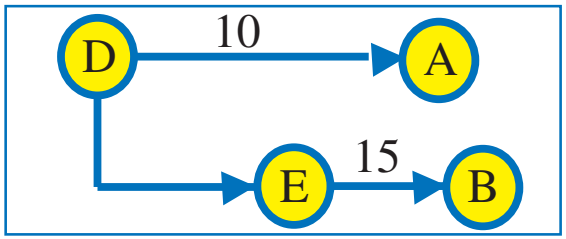
We only need to infer relationship between points.

(sX, sY)	(sX, eY)	(eX, sY)	(eX, eY)	(X, Y)
$<$	$<$	$<$	$<$	$\{<\}$
$<$	$<$	$=$	$<$	$\{m\}$
$<$	$<$	$>$	$<$	$\{o\}$
$<$	$<$	$>$	$=$	$\{f_i\}$
$<$	$<$	$>$	$>$	$\{d_i\}$
$<$	$<$	$>$	$?$	$\{o, f_i\}$
$<$	$<$	$>$	$?$	$\{d_i, f_i\}$
$<$	$<$	$>$	$?$	$\{o, d_i, f_i\}$

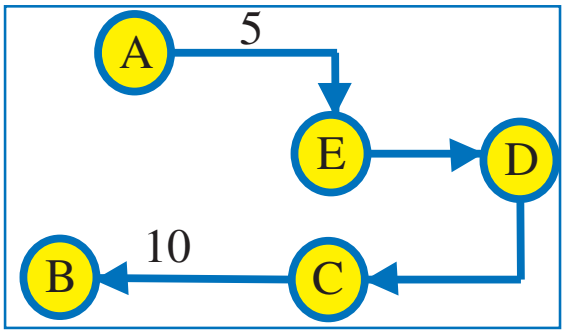
Relationship Query



The longest path from B to D has length at least 10 units, while the longest path from A to D is exactly 5 units. Thus we conclude that B < A.



The longest path (backward) from A to D is exactly 10 units. While the longest path from B to D is at least 15 units. Thus we conclude that A < B.



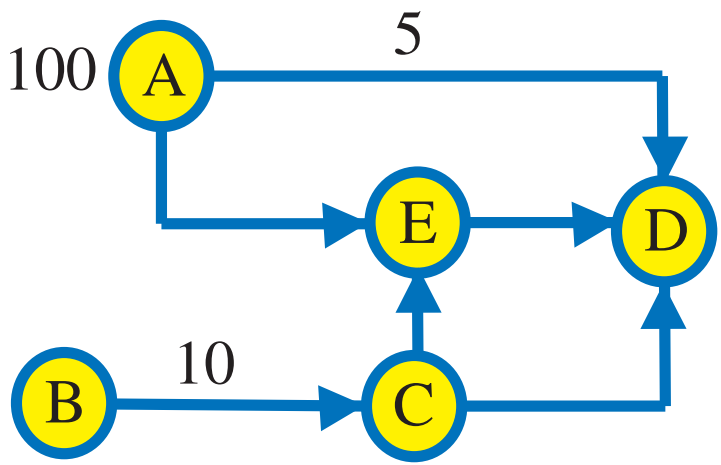
The longest path from A to D has length at least 5 units, while the longest path (backward) from B to D is at least 10 units. Thus we conclude that A < B and greatest lower bound on path length is 15.

Figure: Inferring Relationship Between Points A and B

Algorithm 1 queryRelation (p, q)

Find the longest paths from p and q to every reachable node in the directed graph.
Find the longest paths from p and q to every reachable node in the directed graph obtained by reversing the direction of every edge.
Store at each node in the graph the information about the length and direction (forward/reverse) of the longest paths.
$pathLength_{glb}(p, q) \leftarrow 0$
for all nodes v in the graph **do**
  if forwardPath(p, v) and backwardPath(q, v) **then**
    $relation(p, q) \leftarrow$ ' $<$ '
    $pathLength_{lb}(p, q) \leftarrow pathLength_{glb}(p, v) + pathLength_{glb}(v, q)$
  else if forwardPath(p, v) and forwardPath(q, v) **then**
    if $pathLength_{glb}(p, v) > pathLength_{glb}(q, v)$ **then**
      $relation(p, q) \leftarrow$ ' $<$ '
      $pathLength_{lb}(p, q) \leftarrow pathLength_{glb}(p, v) - pathLength_{glb}(q, v)$
    end if
  end if
end for

Stamp Query



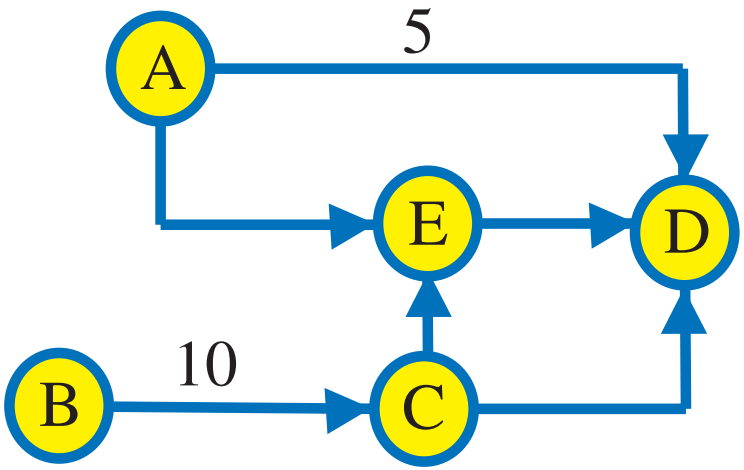
Greatest lower bound on
the length of the path from
B to A = 5
Stamp(B) cannot be inferred.
but Stamp(B) is less than 95

Figure: Inferring Stamp of the Point B

Algorithm 3 queryStamp (p)

Find the nodes s and t with smallest and largest time stamp respectively using breadth first search.
$Stamp(p) \leftarrow ?, Stamp_{glb}(p) \leftarrow ?, Stamp_{lub}(p) \leftarrow ?$
queryRelation (p, s)
if queryRelation returns $p > s$ **then**
  $Stamp_{glb}(p) \leftarrow Stamp(s) + pathLength_{glb}(s, p)$
  if $pathLength_{glb}(s, p)$ is exact **then**
    $Stamp(p) \leftarrow Stamp_{glb}(p)$
    return
  end if
else
  $Stamp_{glb}(p) \leftarrow Stamp(s) + pathLength_{glb}(s, p)$
  return
end if
queryRelation (p, t)
if queryRelation returns $p < t$ **then**
  $Stamp_{lub}(p) \leftarrow Stamp(t) - pathLength_{glb}(p, t)$
end if

Length Query



E and D are the two nodes reachable from both A and B. Based on paths to E we cannot infer anything. But node D allows us to conclude that length of path from B to A is at least 5 units.

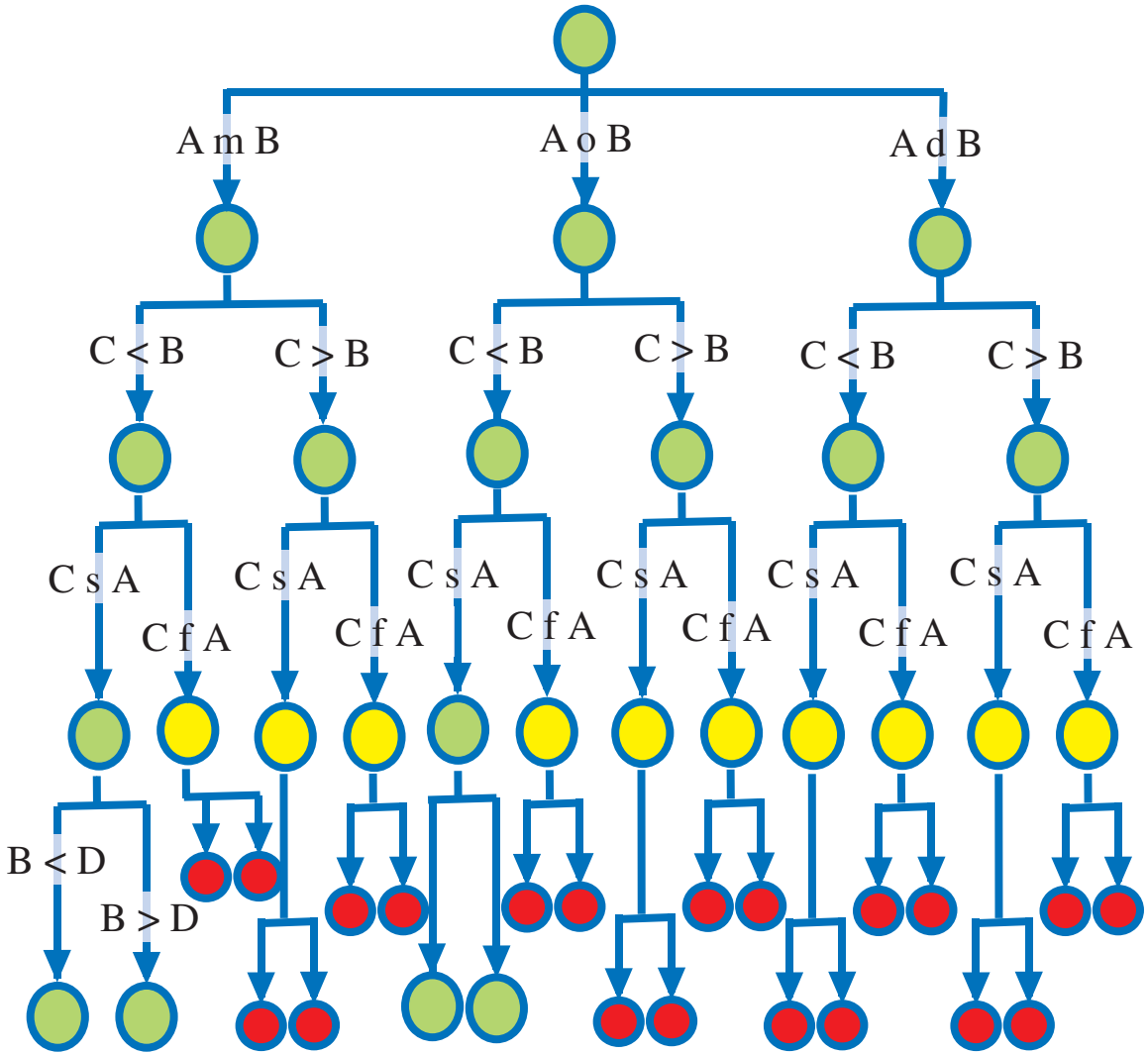
Figure: Inferring Length of the Interval [B, A]

Generalized Point-Interval Logic

Interval A, B, C
(A m B) or (A o B) or (A d B)
(C $<$ B) or (C $>$ B) (disjointedness constraint)
(C s A) or (C f A)
(A $<$ D) or (A $>$ D)

Table: An Instance of GPIL

Search Space



CMI Algorithm

Algorithm 4 CMI (S, i)

{The algorithm is invoked by calling CMI $(S, 1)$, and returns true if the instance is satisfiable}
$(X, Y) \leftarrow$ Variables in s_i
$R_1 \leftarrow$ Relations in s_i
$R_2 \leftarrow$ queryRelation (X, Y)
sort $(R1 \cap R2)$
{sort according to desired heuristics}
for all relation $r_j \epsilon (R1 \cap R2)$ **do**
  addStatement(X, r_j, Y)
  if $i = n$ **then**
    return true
  else if CMI $(S, i + 1)$ **then**
    return true
  else
    deleteStatement (X, r_j, Y)
  end if
end for
return false

Questions?

Thank you for listening.