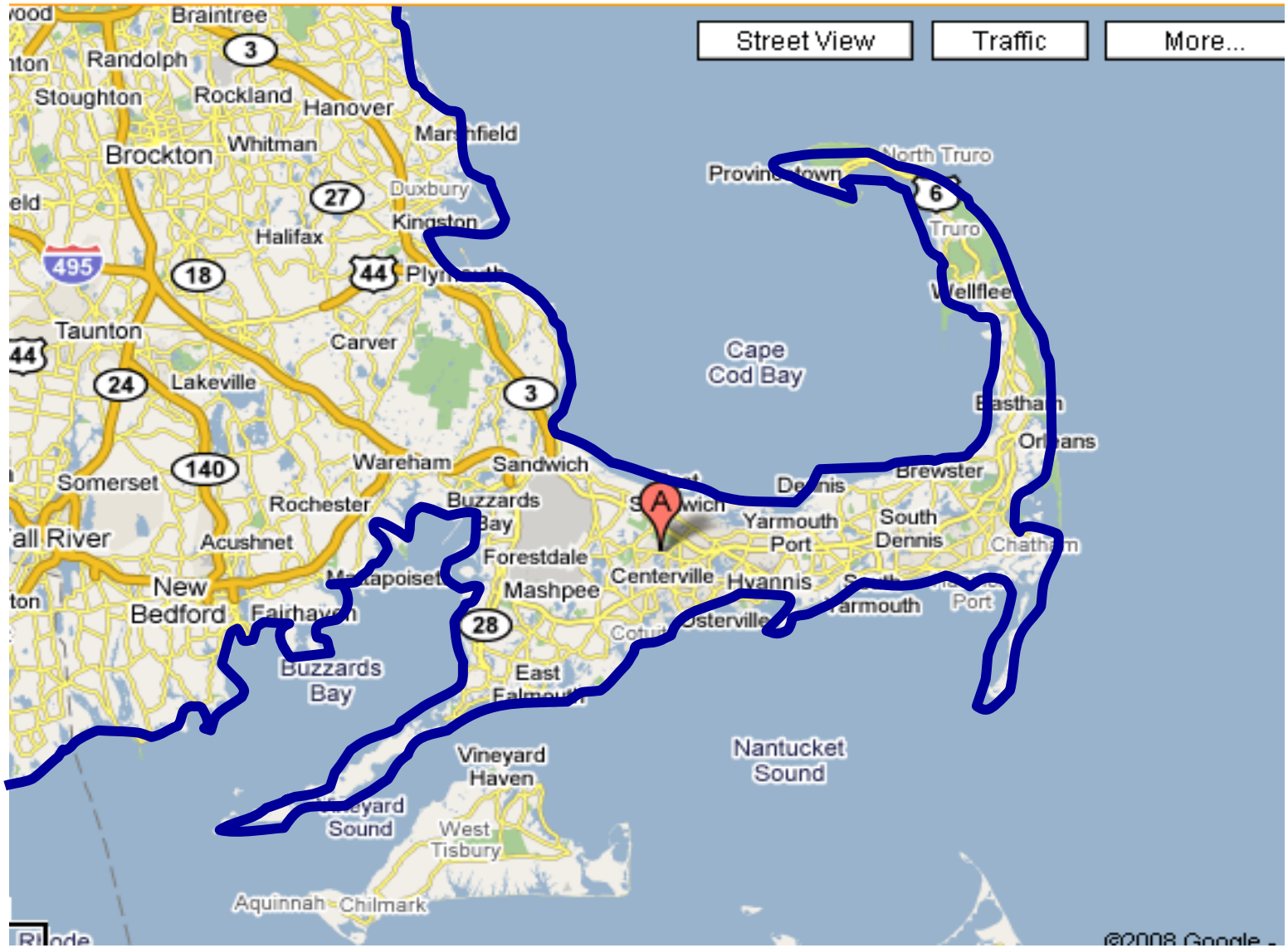
Data Structures for Restricted Triangular Range Searching

13th August, 2008

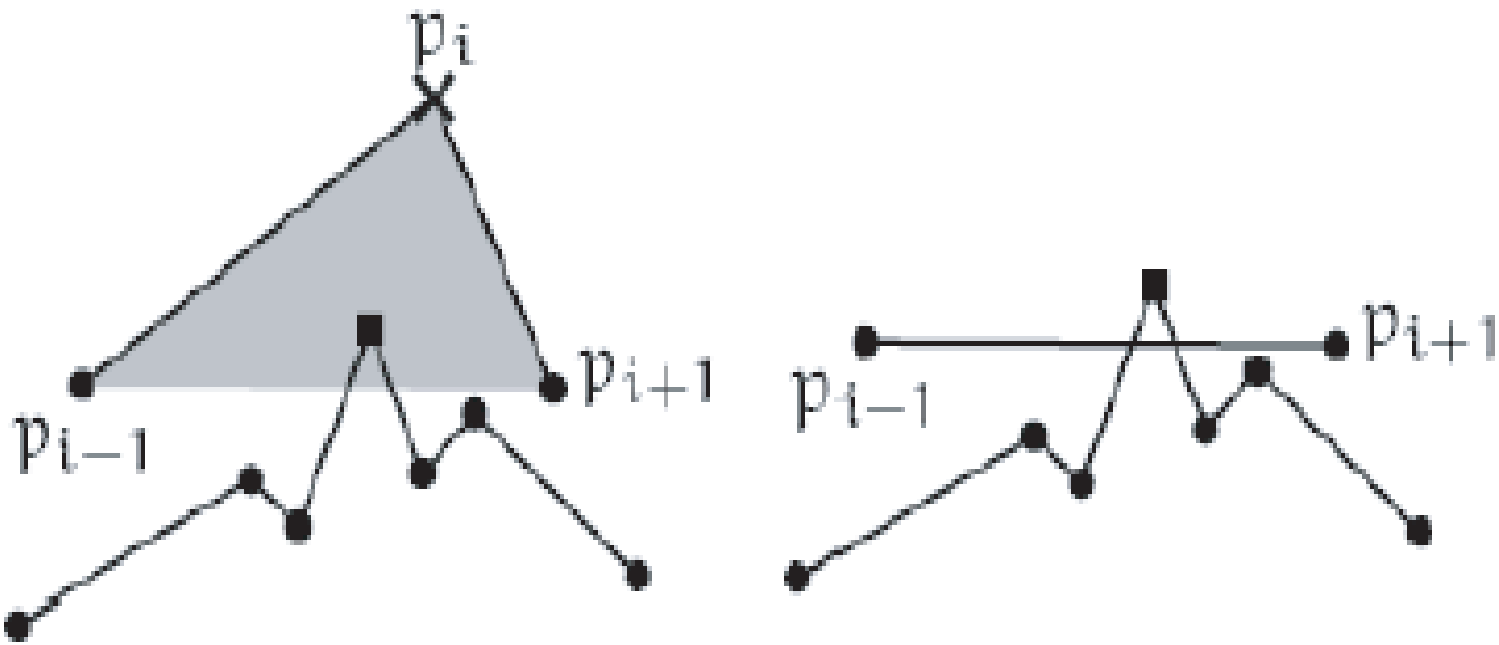
Nadia Benbernou, Mashhood Ishaque, and Diane Souvaine

CCCG 2008, Montreal, Quebec, Aug 13-15.

Coastline (polygonal line) Simplification

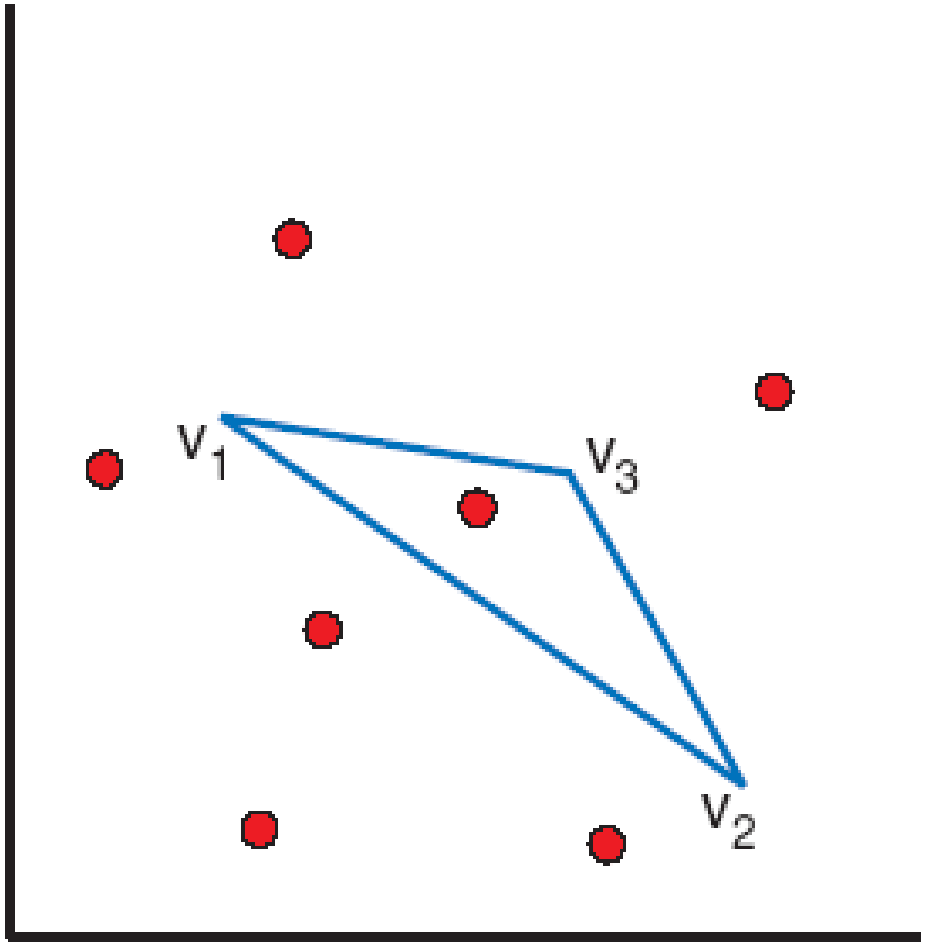


Coastline (polygonal line) Simplification



Falls, C., Liu, Y., Snoeyink, J., Souvaine, D. : Testing Shortcuts to Maintain Simplicity in Subdivision Simplification. CCCG 2005.

Data Structure for Triangular Emptiness?



Data Structure for Triangular Emptiness?

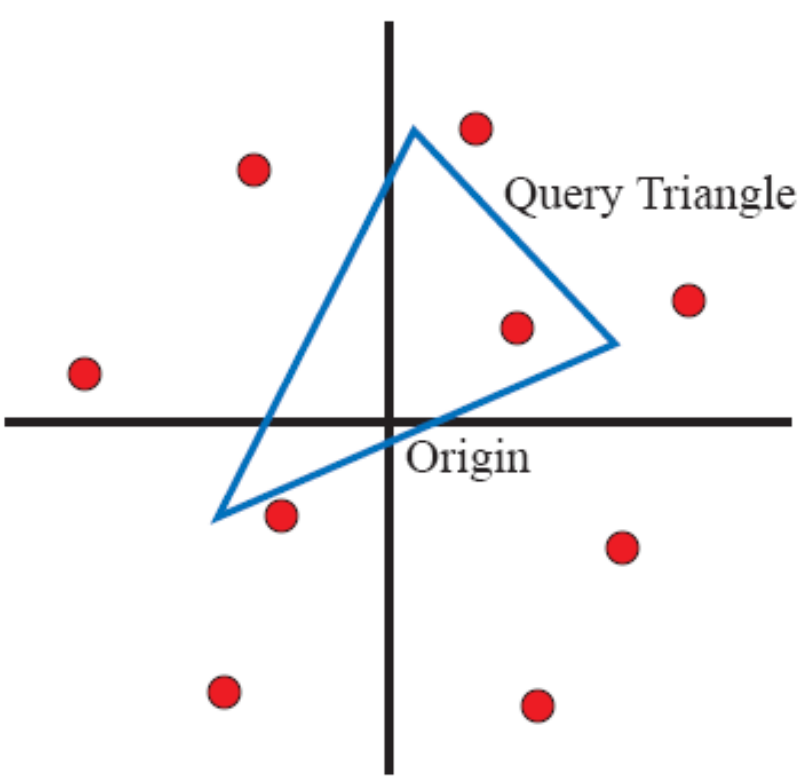
	Emptiness	Reporting	Counting
Halfplane	$\Omega(\lg n)$	$\Omega(\lg n + k)$	$\Omega(n^{1/3})$
Triangular	$\Omega(\lg n)$	$\Omega(n^{1/2-\epsilon} + k)$	$\Omega(n^{1/2})$

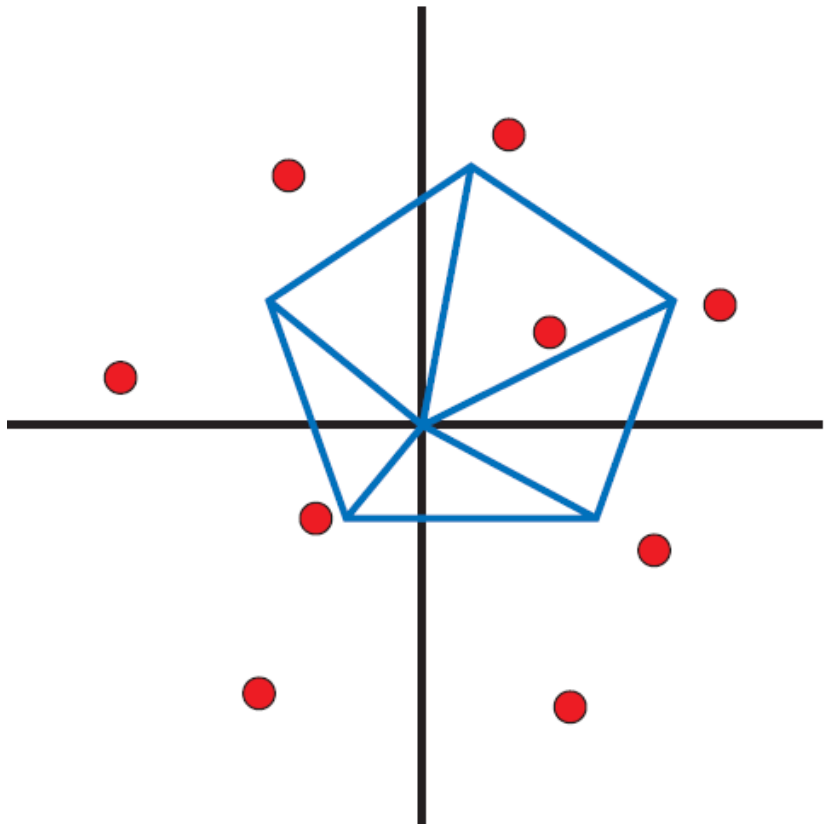
Lower Bounds on query time using near-linear space

	Emptiness	Reporting	Counting
Halfplane	$O(\lg n)$	$O(\lg n + k)$	$O(n^{1/2+\epsilon})$
Triangular	$O(n^{1/2+\epsilon})$	$O(n^{1/2+\epsilon} + k)$	$O(n^{1/2+\epsilon})$

Upper Bounds for query using near-linear space

Restricted Triangular Emptiness

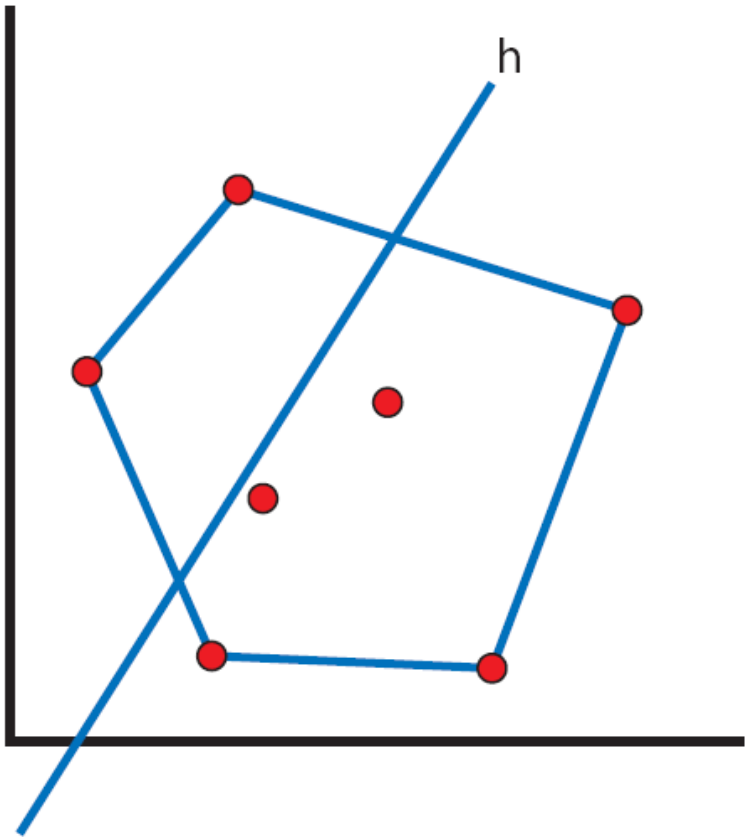


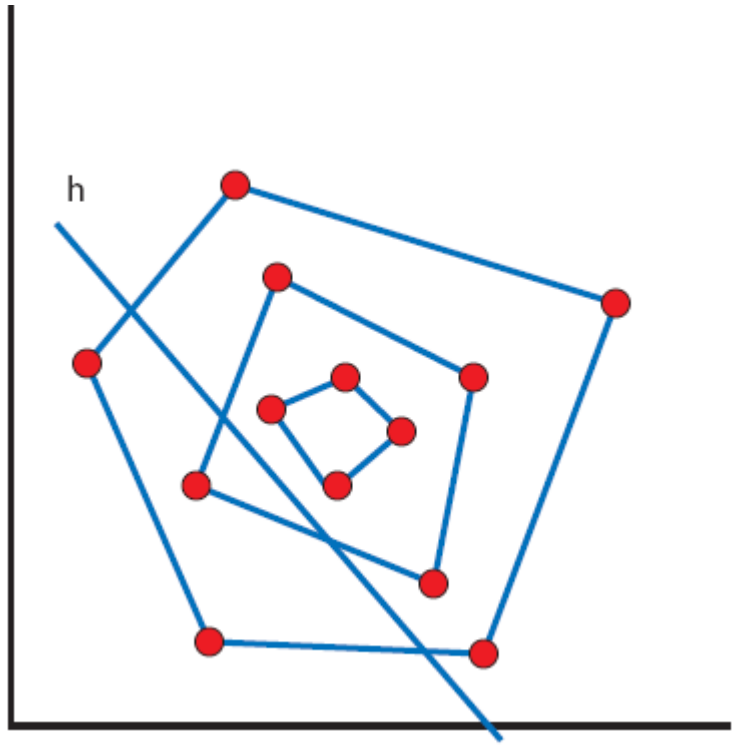


Our Results

1. Restricted triangular queries in $O(\lg^2 n)$ time.
2. Restricted triangular queries in $O(2^{1/\varepsilon} \lg n)$ time.
3. Restricted triangular emptiness queries in $O(\lg n)$ time.
4. Triangular emptiness and reporting queries in polylog time with high probability (vertices of the query triangles come from the given point set).
5. Ray intersection detection and reporting queries among an arrangement of lines in polylog time.
6. Non-orthogonal square emptiness and reporting queries in polylog time.

Halfplane Emptiness and Reporting





Emptiness Query: O(lg n)

Space: O(n)

Reporting Query: O(lg n + k)

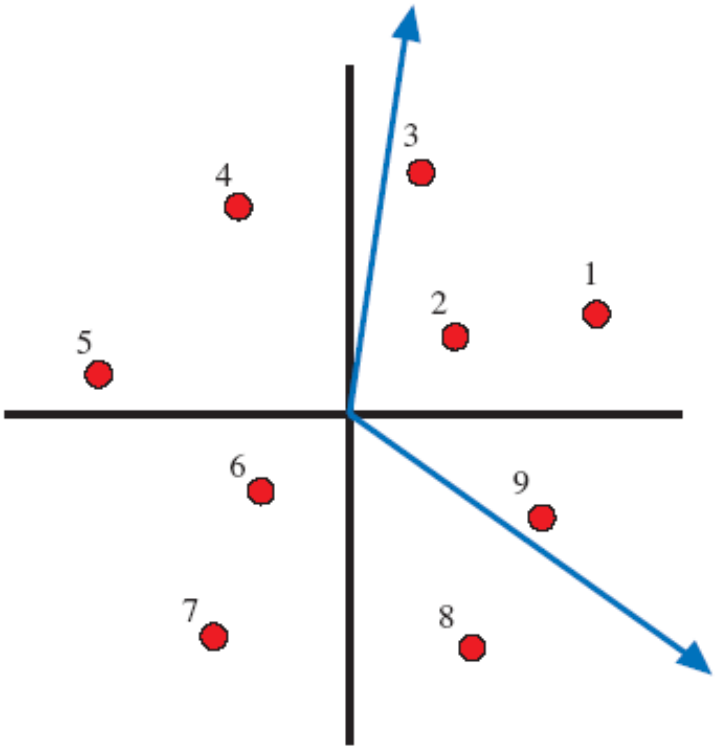
Space: O(n)

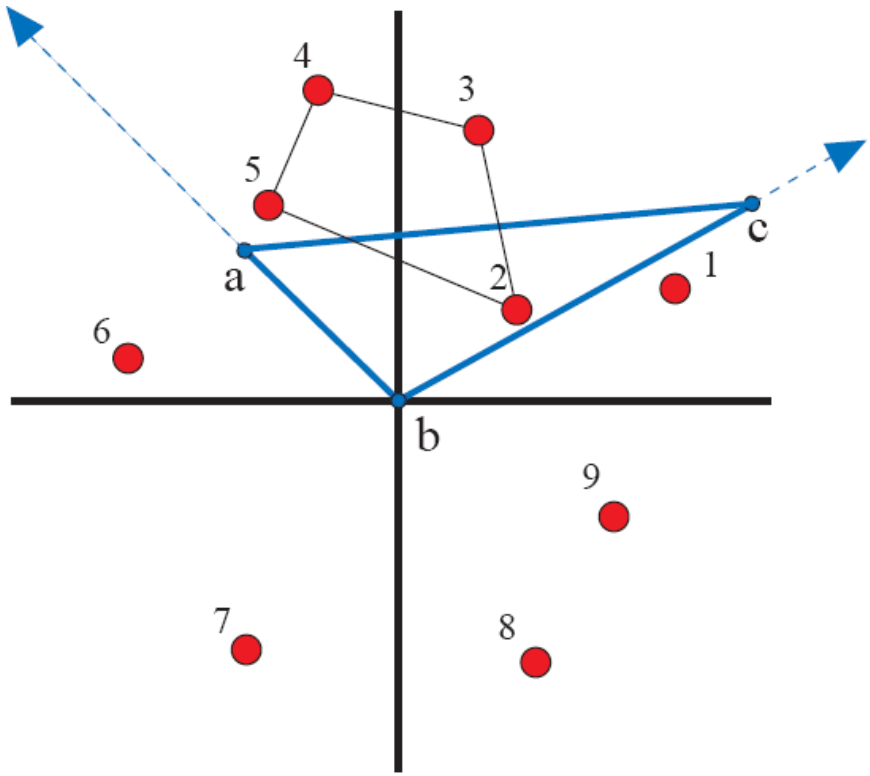
Restricted triangular queries in $O(\lg^2 n)$ time.

- Simple data structure similar to dynamic convex hull structure of Overmars and van Leeuwen.
- Points rotationally sorted around origin. A binary tree storing at each node the (nested) convex hull for the points in the subtree.
- Emptiness query: $O(\lg^2 n)$ time. Space: $O(n)$
- Reporting query: $O(\lg^2 n + k)$ time. Space: $O(n \lg n)$
- The time for emptiness queries can be improved to $O(\lg n)$ using fractional cascading but space become $O(n \lg n)$.

Open Problem: Is it possible to achieve $O(\lg n + k)$ for reporting queries using $O(n \text{ polylog } n)$ space?
The difficulty here is that fractional cascading does not generalize to planar maps.

Restricted triangular queries in $O(2^{1/\varepsilon} \lg n)$ time.

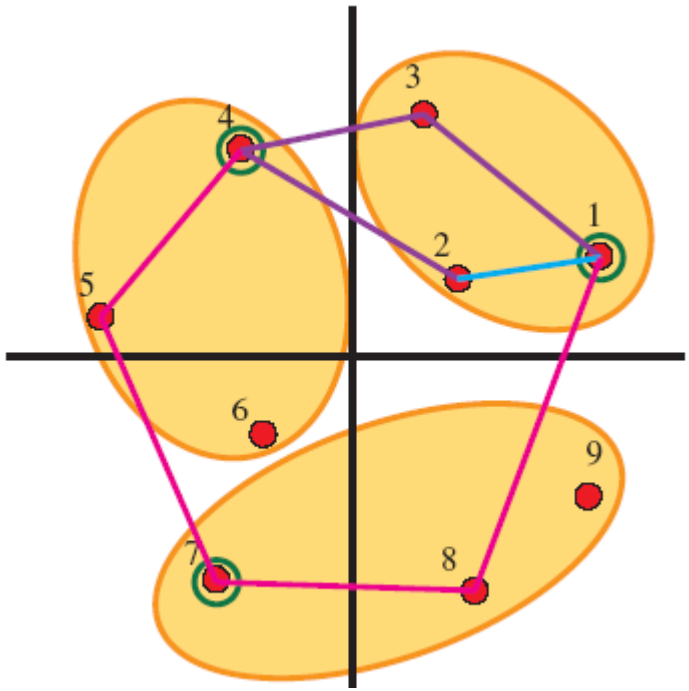




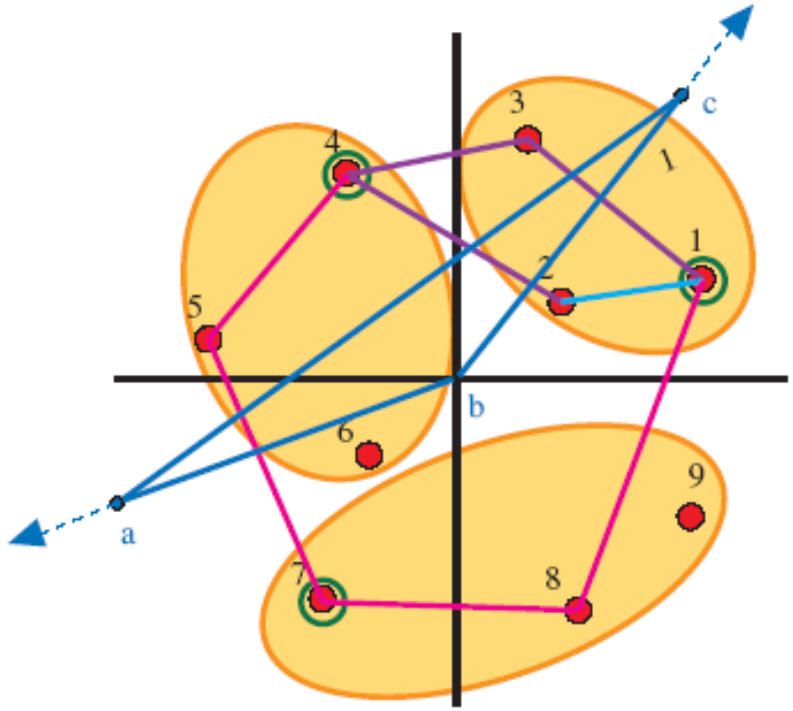
Naïve data structure with $O(n^3)$ space and logarithmic query time.

Restricted triangular queries in $O(2^{1/\varepsilon} \lg n)$ time.

Apply the space-reducing transformation of Aronov *et al.* (2006) to this naïve data structure. Space becomes $O(n^{1+\varepsilon})$ and query time becomes $O(2^{1/\varepsilon} \lg n)$.



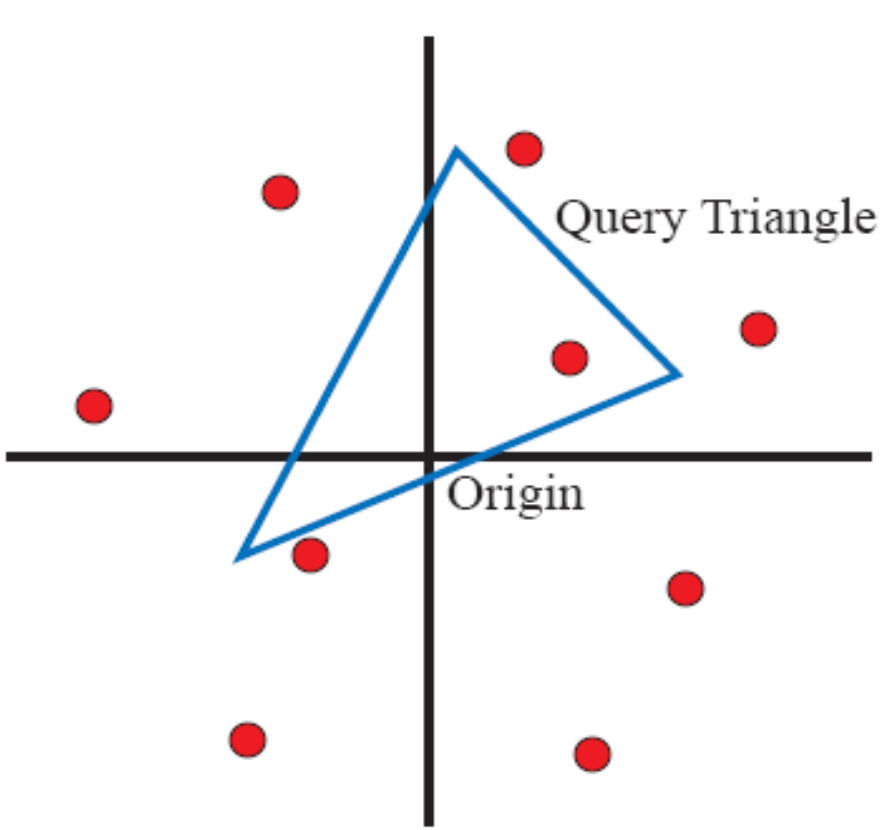
$$M(n) = (n/m + 1)M(m) + O(n^2/m)$$

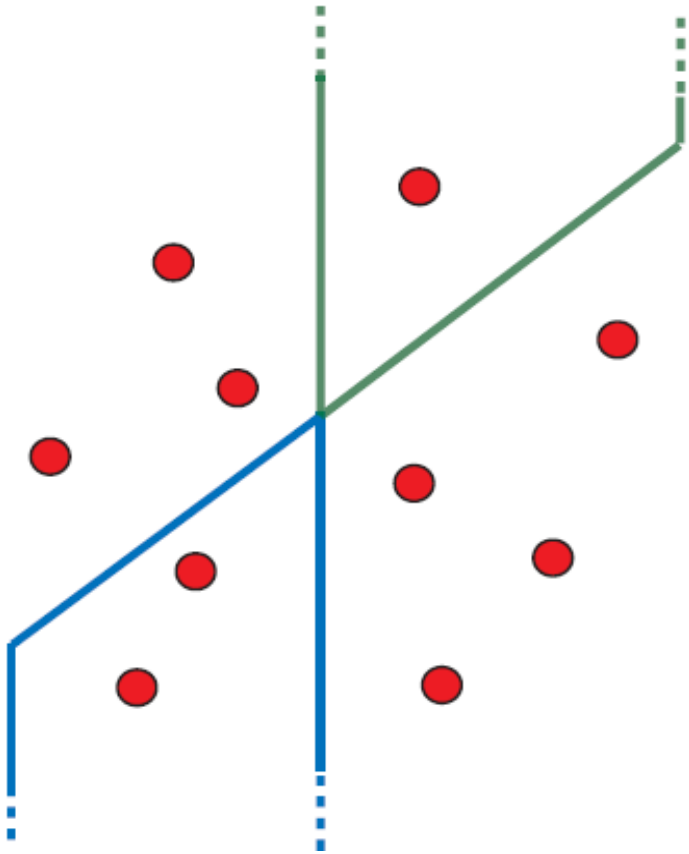


$$Q(n) = 2Q(m) + O(\lg n)$$

B. Aronov, P. Bose, E. Demaine, J. Gudmundsson, J. Iacono, S. Langerman, and M. Smid. Data structures for halfplane proximity queries and incremental Voronoi diagrams. In LATIN 2006.

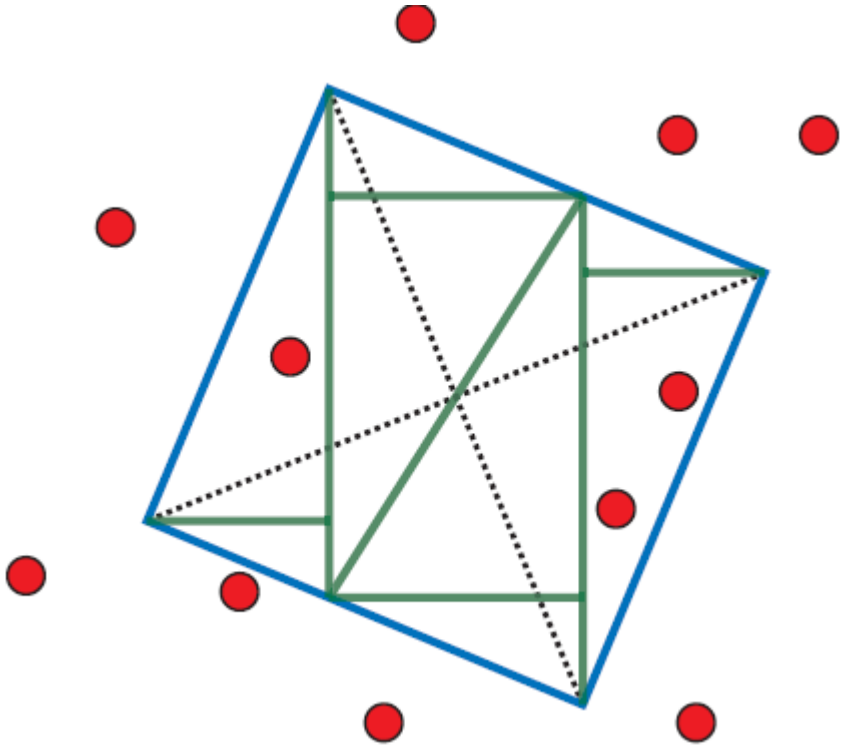
Ray intersection detection and reporting among an arrangement of lines.





Using O(n polylog n) space detection queries can be supported in O(lg n) time and reporting queries in $O(\lg^2 n + k)$ time.

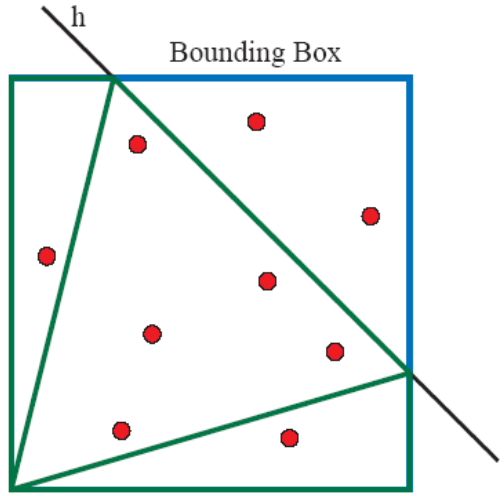
Non-orthogonal square emptiness and reporting queries.



Using O(n polylog n) space non-orthogonal emptiness and reporting queries can be supported in polylog time.

Few Remarks

1. It is not possible to achieve polylog query time for restricted triangular counting queries, using O(n polylog n) space.
2. The fact that the data structures for restricted triangular queries support both emptiness and reporting queries in polylog time indicates that the techniques for restricted case will not extend to general triangular emptiness.



Thanks for Listening!