

Disjoint Segments have Convex Partitions with 2-Edge Connected Dual Graphs

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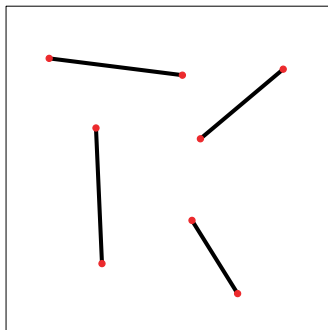
Canadian Conference on Computational Geometry, 2007

The Problem

Does every finite set of disjoint segments in the plane have a convex partition such that the dual graph can be decomposed into two spanning trees?

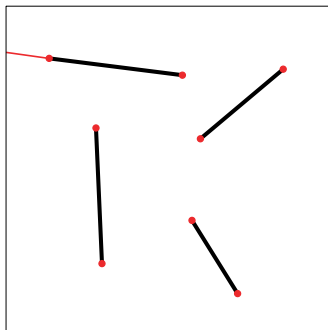
Convex Partition of Disjoint Segments

Given a set of n disjoint segments in the plane, create a convex partition by extending the segment endpoints one-by-one in the order given by some permutation π .



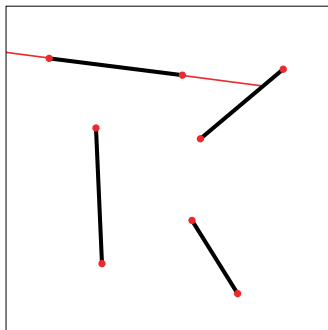
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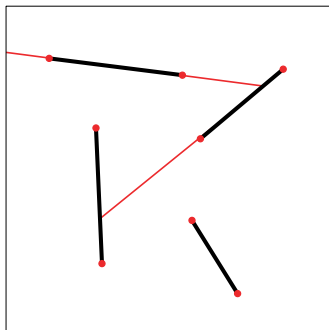
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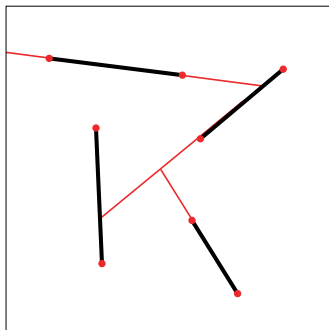
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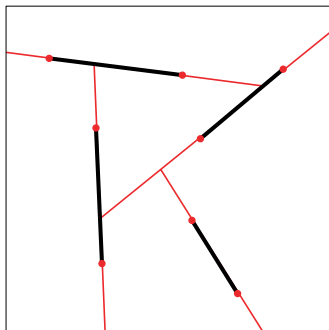
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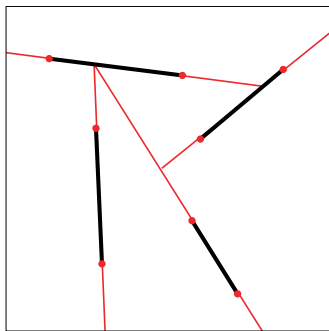
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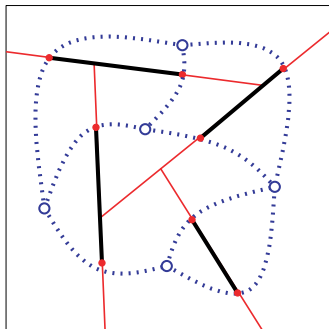
Convex Partition of Disjoint Segments

In general convex partition depends upon the order π in which the extensions are drawn.



Dual Graph of the Convex Partition

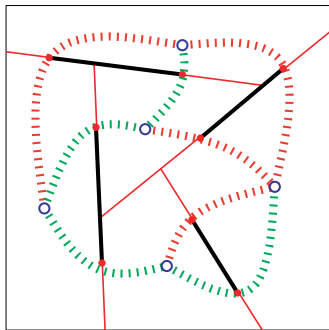
Each convex face in the partition corresponds to a vertex in the dual graph. Every segment endpoint corresponds to an edge between the two incident face on the opposite sides of the segment.



Decomposing Dual Graph into Two Spanning Trees

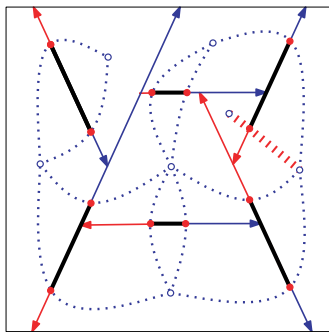
Problem posed by Mikio Kano:

Is there always a dual graph that can be decomposed into two disjoint spanning trees? **Still Open.**



Dual Graph Must be 2-Edge Connected

Example: The dual graph that is not 2-edge connected, and so it cannot be decomposed into two spanning trees.



Summary of Results

- We present an algorithm that for every finite set of disjoint segments computes a convex partition whose dual graph is 2-edge connected.
- We show 2-edge connectivity is not a sufficient condition for decomposition of dual graph into two disjoint spanning trees.

Partition by Induction

Theorem

For any finite set of disjoint line segments in the plane, one can construct a convex partition whose dual graph is 2-edge connected.

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For any finite set S of disjoint line segments in the plane, one can find a nonempty subset $S' \subseteq S$ and construct a convex partition P' of S' such that the dual graph of P' is 2-edge connected and every segment of $S \setminus S'$ lies in the interior of a face of P' .

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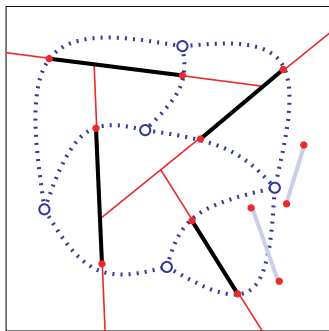
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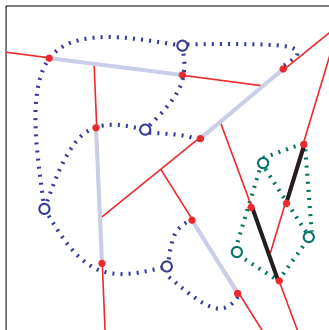
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Example: A non-empty subset of segments with 2-edge connected dual graph. Every other segment lies completely in the interior.



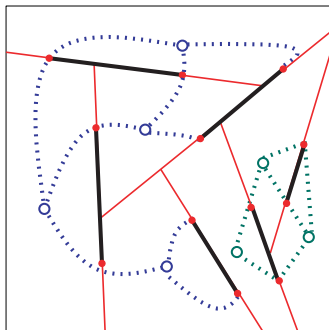
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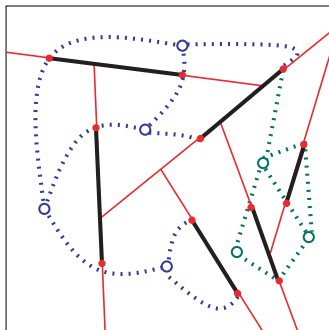
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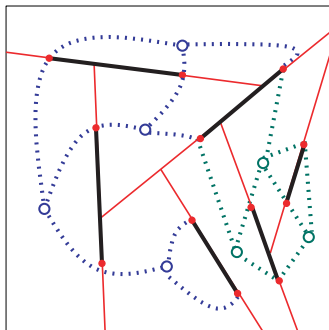
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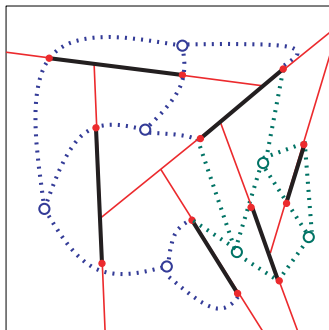
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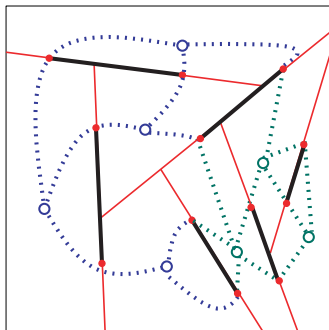
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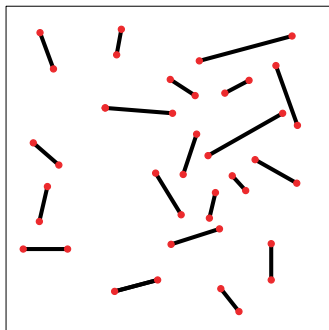
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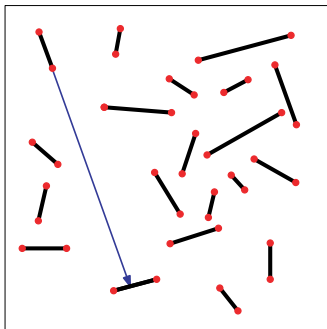
Partition Algorithm

Intuitively, we grow a *separator*, which consists of input segments and some of their extensions. We stop when every segment in the separator has been extended in both directions. At that time, each face of the separator is convex.



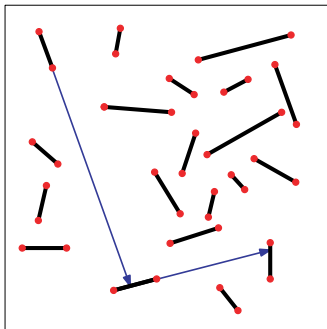
Partition Algorithm

Pick a segment with one endpoint on the convex hull of the segment endpoints, and extend it into a ray \vec{r} beyond the other endpoint until it hit another segment or infinity.



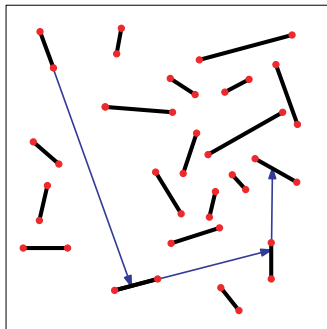
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If the ray \vec{r} hits a new segment, extend the new segment's endpoint that is in counterclockwise position to the ray \vec{r} .



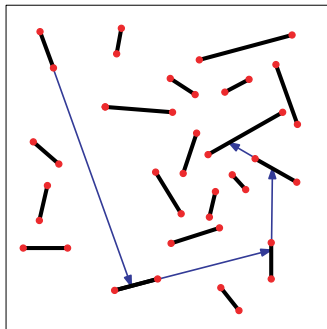
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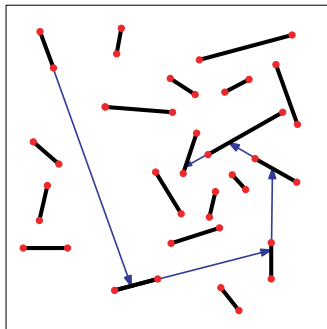
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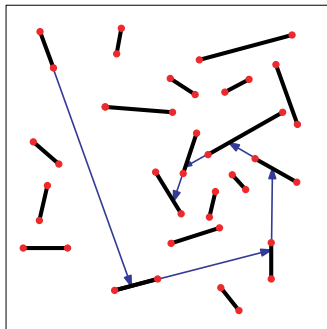
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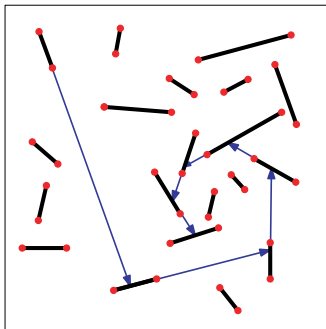
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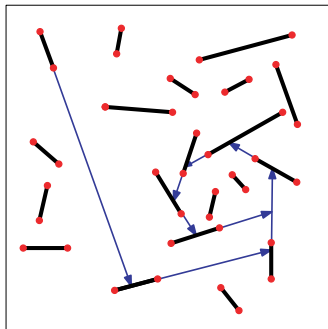
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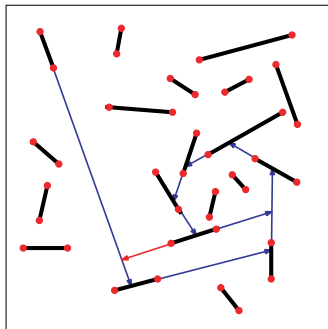
Partition Algorithm

If an extension hits the separator or infinity, then we next consider a segment in the separator that has been extended in only one direction, and now extend it in the opposite direction.



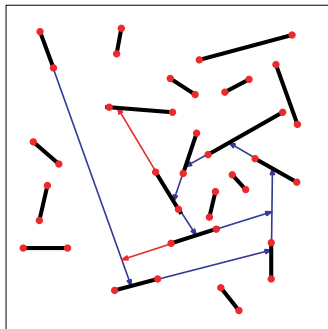
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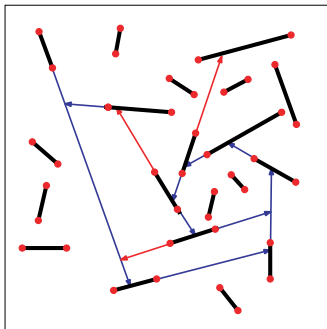
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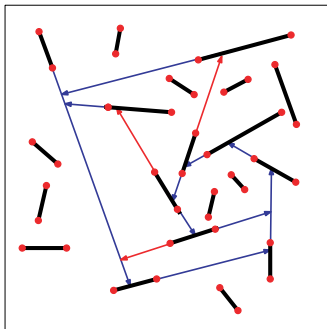
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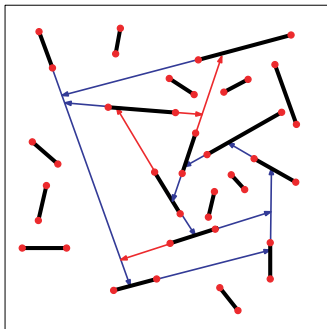
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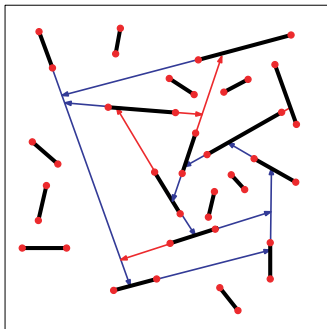
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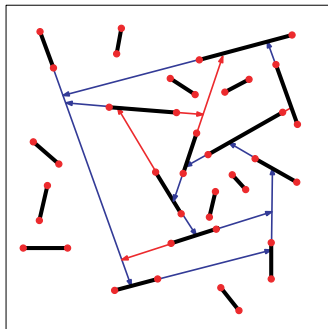
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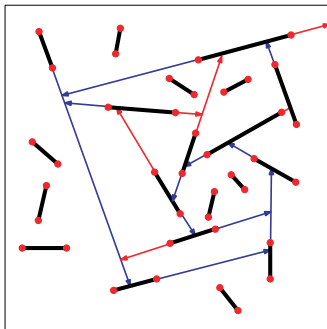
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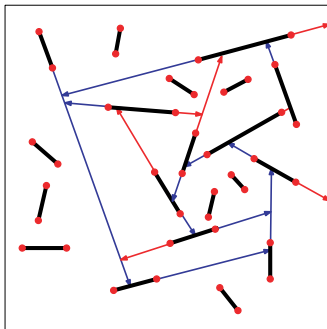
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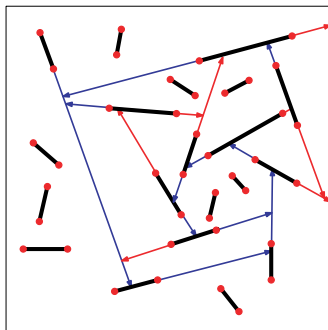
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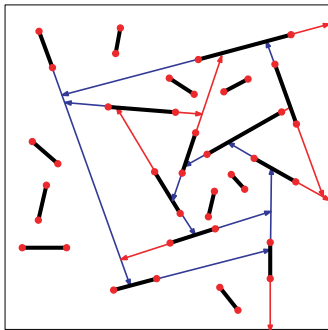
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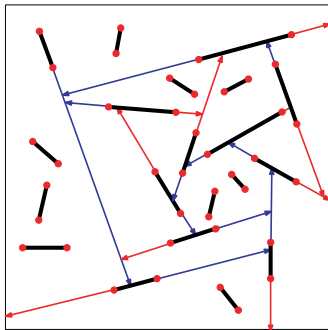
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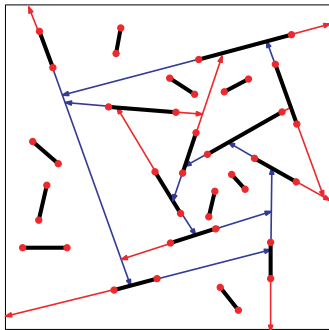
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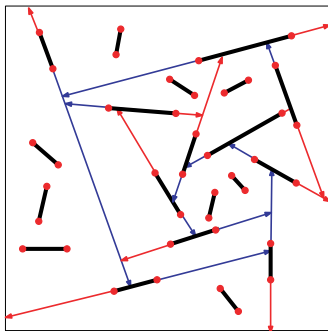
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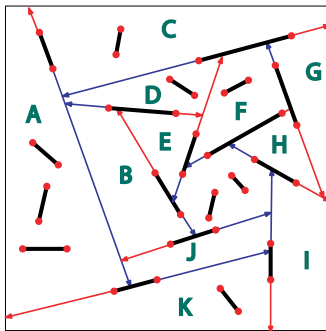
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For every R in the partition, if ∂R contains both clockwise and counterclockwise extensions, then ∂R contains an entire input segment.



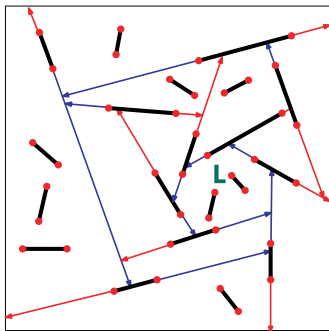
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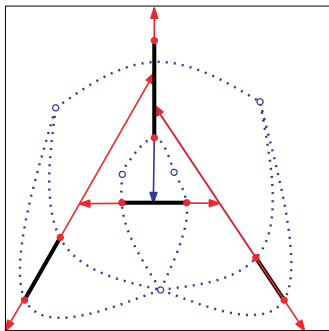
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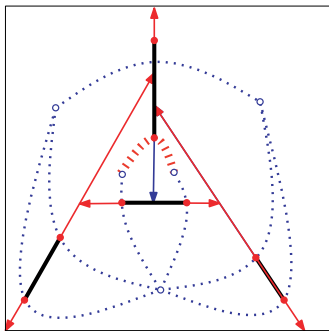
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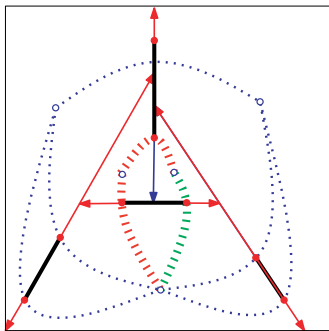
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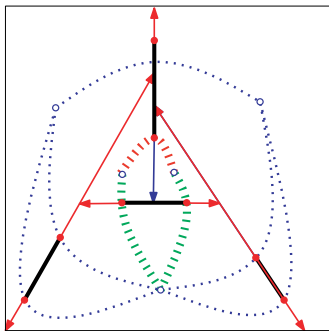
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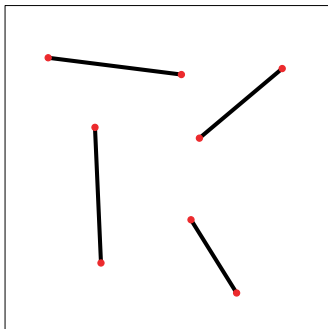
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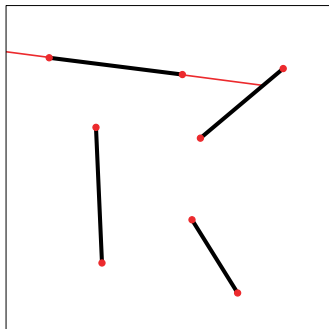
Open Problems

Is there a permutation of every finite set of disjoint segments such that extending the both endpoints of the segment at the same time, the dual graph of the resulting convex partition is 2-edge connected?



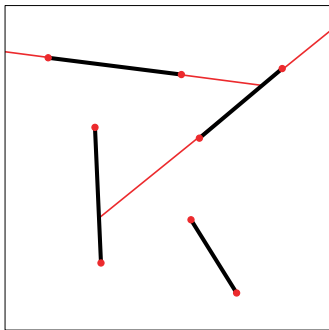
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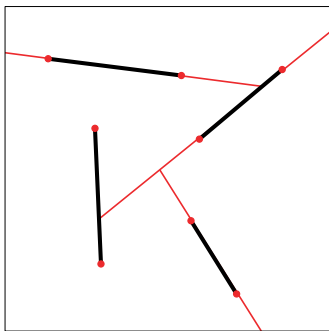
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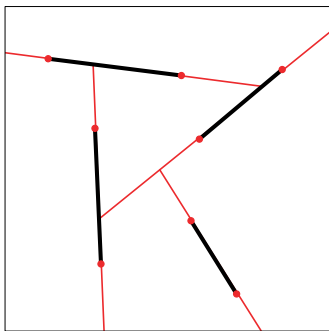
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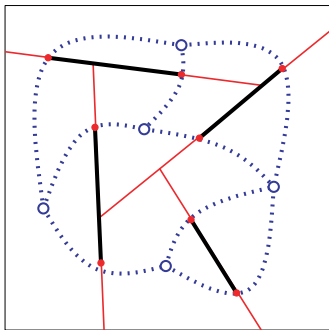
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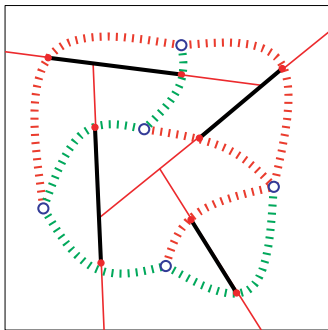
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Questions?

Thank you for listening.