ABSTRACT

Title of Dissertation: TOWARDS MORE EXPRESSIVE AND USABLE TYPES FOR DYNAMIC LANGUAGES

Milod Kazerounian
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Many popular programming languages, including Ruby, JavaScript, and Python, feature dynamic type systems, in which types are not known until runtime. Dynamic typing provides the programmer with flexibility and allows for rapid program development. In contrast, static type systems, found in languages like C++ and Java, help catch errors early during development, enforce invariants as programs evolve, and provide useful documentation via type annotations. Many researchers have explored combining these contrasting paradigms, seeking to marry the flexibility of dynamic types with the correctness guarantees and documentation of static types.

However, many challenges remain in this pursuit: programmers using dynamic languages may wish to verify more expressive properties than basic type safety; operations for commonly used libraries, such as those for databases and heterogeneous data structures, are difficult to precisely type check; and *type inference*—the process of automatically deducing the types of methods and variables in a program—often produces type annotations that are complex and verbose, and thus less usable for the programmer. To address these issues, I present four pieces of work that aim to

increase the expressiveness and usability of static types for dynamic languages.

First, I present `RTR`, a system that adds *refinement types* to Ruby: basic types extended with expressive predicates. `RTR` uses assume-guarantee reasoning and a novel idea called *just-in-time verification*—in which verification is deferred until runtime—to handle dynamic program features such as mixins and metaprogramming. We found `RTR` was useful for verifying key methods in six Ruby programs.

Second, I present `CompRDL`, a Ruby type system that allows library method type signatures to include *type-level computations* (or *comp types*). Comp types can be used to precisely type check database queries, as well as operations over heterogeneous data structures like arrays and hashes. We used `CompRDL` to type check methods from six Ruby programs, enabling us to check more expressive properties, with fewer manually inserted type casts, than was possible without comp types.

Third, I present `InferDL`, a Ruby type inference system that aims to produce usable type annotations. Because the types inferred by standard, constraint-based inference are often complex and less useful to the programmer, `InferDL` complements constraints with configurable heuristics that aim to produce more usable types. We applied `InferDL` to four Ruby programs with existing type annotations and found that `InferDL` inferred 22% more types that matched the prior annotations compared to standard inference.

Finally, I present `SimTyper`, a system that builds on `InferDL` by using a novel machine learning-based technique called *type equality prediction*. When standard and heuristic inference produce a non-usable type for a position (argument/return/-variable), we use a *deep similarity* network to compare that position to other posi-

tions with usable types. If the network predicts that two positions have the same type, we guess the usable type in place of the non-usable one, and check the guess against constraints to ensure soundness. We evaluated `SimTyper` on eight Ruby programs with prior annotations and found that, compared to standard inference, `SimTyper` finds 69% more types that match programmer-written annotations.

In sum, I claim that `RTR`, `CompRDL`, `InferDL`, and `SimTyper` represent promising steps towards more expressive and usable types for dynamic languages.

TOWARDS MORE EXPRESSIVE AND USABLE TYPES FOR DYNAMIC LANGUAGES

by

Milod Kazerounian

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Advisory Committee:
Professor Jeffrey S. Foster, Chair/Advisor
Professor David Van Horn
Professor Marine Carpuat
Professor Michael Hicks
Professor Donald Yeung

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Chapter 1: Introduction

Dynamic type systems are featured in many popular programming languages including Ruby, Python, and JavaScript. The lack of static type enforcement means programmers can write code that is verbose and flexible, and that takes advantage of highly dynamic features such as metaprogramming. All of this allows for rapid prototyping and development of programs.

Yet there is a clear trade off. Static type systems are useful for catching bugs early in the development process, enforcing invariants, and providing useful documentation in the form of type annotations. These features become all the more beneficial as programs grow in size and are maintained by larger teams of developers. Thus, a growing area of research has examined ways to add static typing to dynamic languages [1, 2, 3, 4, 5, 6, 7]. The goal is to maintain the flexibility of dynamic type systems, while gaining some of the correctness guarantees of static types.

Such hybrid systems—typically deemed *gradual typing* when the system ensures soundness, and *optional typing* otherwise—have been explored for a growing number of languages, including Ruby [4, 5, 8], Racket [3, 9, 10], Python [11, 12, 13], JavaScript [14, 15, 16], and more. TypeScript, a typed superset of JavaScript which includes optional typing, was declared by GitHub to be the 4th most popular lan-

guage in 2020 [17]. Moreover, recent versions of Ruby [18] and Python [19] include syntax for type annotations, which can serve as documentation and can be used in type-based analyses.

However, there are many common features and constructs behind programs written in dynamic languages that are challenging or not possible to capture with prior type systems. Programmers may wish to verify more expressive properties of programs than basic type correctness, such as the functional correctness properties of arithmetic operations. Moreover, the operations for many libraries popular in dynamic languages, such as libraries for database operations and heterogeneous data structures, cannot be precisely type checked using standard type systems. And finally, the process of annotating programs with static types itself can be quite burdensome for the programmer. But the standard solution to this problem—type inference based on constraint solving—often generates types that are verbose, confusing, and less useful to the programmer.

These challenges have motivated the research presented in this dissertation. In particular, I have explored these challenges through the design and implementation of new type systems for the Ruby language. However, the systems and methodologies I have contributed to are generalizable to other dynamic, and in some cases even static, languages.

I present the following thesis:

New mechanisms can be added to conventional type systems to make these systems more expressive and usable. Refinement types can be used to express and enforce stronger properties of programs, and combined with a just-in-time verification

approach, can be flexible enough to handle common dynamic features like mixins and metaprogramming. Type-level computations can express more precise properties for library methods, especially those for databases and heterogeneous data structures. And standard, constraint-based type inference can be complemented with heuristic rules and type equality prediction to produce more usable type annotations.

To substantiate this thesis, I present four novel type systems for Ruby: `RTR`, `CompRDL`, `InferDL`, and `SimTyper`. These four systems build on `RDL`, an existing Ruby type checker which I have contributed to. In the remainder of this chapter, I first provide an overview of `RDL` (§ 1.1), and then I briefly introduce each of the aforementioned novel type systems (§ 1.2).

1.1 `RDL`: A Ruby Type Checker

`RDL` [20] is an existing type and contract checking system for Ruby that I have extensively contributed to. We begin with a brief description of `RDL`, as it is foundational to the remainder of the work in this dissertation. Consider the following example:

```
type '( Integer ) → Integer ', typecheck: :later
def incr_sec(x) if (x==59) then 0 else x+1 end; end
```

On the second line of this example, we define a simple Ruby method `incr_sec`, which takes a value `x` representing a second, and increments it by `1`, wrapping back around to `0` when necessary. Above this method, we have written an `RDL` type annotation, which says that `incr_sec` should take and return an `Integer`. `RDL` will check `incr_sec` against this type and determine that the method is type safe.

Notably, RDL performs type checking by statically analyzing code, but it does so *at runtime*. Indeed, in the example, type is actually a method that we are calling, with a String argument representing the type of the subsequently defined method; in Ruby, methods can be called without enclosing parentheses around the arguments. This String argument will be parsed into RDL's type system and stored in a global table that maintains a program's type environment. We also pass the type method the label :later — in Ruby, strings prefixed by colons are *symbols*, which are interned strings. This label specifies the time that a method should be type checked. When the program subsequently calls RDL.do_typecheck :later (call omitted above), RDL will type check the source code of all methods whose type annotations are labeled :later.

This design allows RDL to support the metaprogramming that is common in Ruby and ubiquitous in Rails. For example, the programmer can perform type checking after metaprogramming code has run, when corresponding type definitions are available. See Ren and Foster [5] for more details.

RDL uses an expressive type language, including nominal, singleton, union, intersection, generic, variable, and structural types. It also includes special precise types for two of Ruby's data structures: *tuple types* for fixed-sized arrays, which give the exact types for each element in an array; and *finite hash types*, which give the names of keys and types of values in a hash. In the coming chapters, we will see how we can piggyback off RDL's expressive type language and unique design to implement refinement type checking, type-level computations, and expressive type inference.

1.2 Novel Type Systems

I now briefly introduce four novel type systems: `RTR` (§ 1.2.1), `CompRDL` (§ 1.2.2), `InferDL` (§ 1.2.3), and `SimTyper` (§ 1.2.4).

1.2.1 `RTR`: Refinement Types for Ruby

While `RDL` is useful for establishing the type safety of a program, its basic types alone cannot verify more expressive properties. To address this, we built `RTR` [21], a system that extends `RDL` with refinement types, which are types extended with expressive logical predicates. For example, recall the method `incr_sec` defined in Section 1.1. Below, we once again define the method, this time annotating it with refinement types:

```
type '( Integer x { 0 ≤ x < 60 }) → Integer r { 0 ≤ r < 60}'
def incr_sec(x) if (x==59) then 0 else x+1 end; end
```

In addition to enforcing basic type safety, the above refinement types enforce bounds on the method input and output: both the input (named `x`), and the output (named `r`) should be in the range [0,60). Effectively, these refinement types encode pre- and postconditions on the method, which are more expressive than basic types alone. Checking such a specification is beyond the scope of traditional type checkers, and would typically have to be done using dynamic checks, which can be tedious and miss program paths. Using `RTR`, we can verify this specification statically for all paths through the method.

`RTR` works by encoding its verification problems into `Rosette`, a solver-aided

host language. RTR is able to achieve verification in the face of some of Ruby's highly dynamic features, such as mixins and metaprogramming, using a combination of assume-guarantee reasoning and a novel approach we call *just-in-time verification*. We formalized RTR by showing a translation from a core, Ruby-like language with refinement types into Rosette. We evaluated RTR by applying it to six Ruby programs and using it to verify a range of functional correctness properties, and we found that RTR can successfully verify key methods in these programs, taking only a few minutes in the worst case to perform verification. We provide a full discussion of RTR in Chapter 2.

1.2.2 CompRDL: Type-Level Computations for Ruby Libraries

Because Ruby is commonly used for web development, Ruby programs frequently interact with a database. However, type checking database queries is a challenging problem. The types of database operations often depend on program values, such as the names of tables or columns being queried, in a way that traditional type systems cannot capture. We found similar issues arise in type checking other Ruby libraries as well, such as those for arrays and hashes.

To address this issue, we developed CompRDL, a type system for Ruby that allows library method type signatures to include *type-level computations* (or comp types for short). Combined with singleton types for table and column names, comp types let us give database query methods type signatures that compute a table's schema to yield very precise type information. Comp types for hash, array, and

string libraries can also increase precision and thereby reduce the need for type casts.

We formalized `CompRDL` using a core, Ruby-like language, and proved its type system sound. Its soundness relies on both a lightweight termination checker and the use of dynamic checks to enforce the non-checked comp types. We evaluated `CompRDL` by writing annotations with type-level computations for several Ruby core libraries and database query APIs. We then used those annotations to type check two popular Ruby libraries and four Ruby on Rails web apps. We found the type annotations were relatively compact and we were able to successfully type check 132 methods across our subject programs. The use of type-level computations allowed us to check more expressive properties, with fewer manually inserted casts, than was possible without type-level computations. In the process, we found three errors that were confirmed by the developers. Chapter 3 gives a full presentation of `CompRDL`.

1.2.3 `InferDL`: Sound, Heuristic Type Annotation Inference for Ruby

The aforementioned type systems are useful systems for ensuring the correctness of programs, but they still require the programmer to manually write type annotations. This process can be burdensome and time-consuming, especially for larger, more complex code bases. The standard solution to this problem is type inference, which traditionally uses constraint solving to catch type errors in a program *without* needing type annotations. We can take this process a step further, not only using constraint solving to catch bugs, but also to generate the most-general

type annotations for program values. However, we have found that in practice, the resulting type annotations are often overly general, resulting in type annotations that are verbose, confusing, and less useful to the programmer.

Building on a prior proposal for heuristic Ruby type inference [22], we developed `InferDL`, a system for generating type annotations that uses heuristic rules to refine the overly general types produced by constraint solving. For example, `InferDL` can use a heuristic rule that guesses that a parameter whose name ends in "count" is an Integer. `InferDL` works by first running standard type inference, then applying heuristics to any positions for which standard type inference produced an overly general type. Heuristic guesses are added as constraints to the type inference problem to ensure they are consistent with the rest of the program and other heuristic guesses; inconsistent guesses are discarded. We formalized `InferDL` in a core type and constraint language. To evaluate `InferDL`, we applied it to four Ruby on Rails apps that had been previously type checked with RDL, and hence had type annotations. We found that, when using heuristics, `InferDL` inferred 22% more types that were as or more precise than the previous annotations, compared to standard type inference without heuristics. We also found one new type error. We further evaluated `InferDL` by applying it to six additional apps, finding five additional type errors. Chapter 4 presents `InferDL` in full.

1.2.4 `SimTyper`: Sound Type Inference for Ruby using Type Equality Prediction

With `InferDL`, we demonstrated that heuristics can be useful for refining constraint-based type inference. However, hard-coded heuristic rules, while useful for their target programs, may generalize poorly to new programs. While in theory a programmer can write new heuristics for each new program, in practice this can be a burdensome and time-consuming process.

To improve on heuristic-based inference, we introduce `SimTyper`, a system that builds on `InferDL` for the purpose of inferring usable annotations. The key novelty of `SimTyper` is *type equality prediction*, a new, machine learning-based technique that predicts when method arguments or returns are likely to have the same type. `SimTyper` finds pairs of positions that are predicted to have the same type yet one has a verbose, overly general solution and the other has a usable solution. It then guesses the two types are equal, keeping the guess if it is consistent with the rest of the program, and discarding it if not. In this way, like `InferDL`, types inferred by `SimTyper` are guaranteed to be sound. Type equality prediction is performed by a *deep similarity* (DeepSim) neural network, which follows the Siamese network architecture and uses `CodeBERT`, a pre-trained model, to embed source tokens into vectors that capture tokens and their contexts. DeepSim is trained on 100,000 pairs of arguments/returns labeled with type similarity information extracted from 371 Ruby programs with manually documented, but not checked, types. We evaluated `SimTyper` on eight Ruby programs and found that, compared to standard type

inference, `SimTyper` finds 69% more types that match programmer-written type information. Moreover, DeepSim can predict rare types that appear neither in the Ruby standard library nor in the training data. Our results show that type equality prediction can help type inference systems effectively produce more usable types.

Chapter 2: Refinement Type for Ruby

This chapter introduces RTR, a refinement type system for Ruby. Refinement types are a popular way to specify and reason about key program properties. RTR is built on top of RDL, and it works by encoding its verification problems into Rosette, a solver-aided host language. RTR is able to handle some of Ruby's highly dynamic features, such as mixins and metaprogramming, through a combination of assume-guarantee reasoning, and a novel approach we call *just-in-time verification*.

2.1 Introduction

Refinement types combine types with logical predicates to encode program invariants [23, 24]. Recall the method incr_sec from § 1.1, which increments a second. We can write the following refinement type specification for this method:

```
type :incr_sec, '( Integer x { 0 ≤ x < 60 }) → Integer r { 0 ≤ r < 60}'
```

With this specification, incr_sec can only be called with integers that are valid seconds (between 0 and 59) and the method will always return valid seconds.

Refinement types were introduced to reason about simple invariants, like safe array indexing [23], but since then they have been successfully used to verify sophisticated properties including termination [25], program equivalence [26], and correct-

ness of cryptographic protocols [27], in various languages (e.g., ML [28], Racket [10], and TypeScript [29]).

We introduce `RTR`, a tool that adds refinement types to `RDL` and verifies them via a translation into Rosette [30], a solver-aided host language. Since Rosette is not object-oriented, `RTR` encodes Ruby objects as Rosette structs that store object fields and an integer identifying the object's class. At method calls, `RTR` uses RDL's type information to statically overestimate the possible callees. When methods with refinement types are called, `RTR` can either translate the callee directly or treat it modularly by asserting the method preconditions and assuming the postcondition, using purity annotations to determine which fields (if any) the method may mutate. (§ 2.2)

In addition to standard object-oriented features, Ruby includes dynamic language features that increase flexibility and expressiveness. In practice, this introduces two key challenges in refinement type verification: *mixins*, which are Ruby code modules that extend other classes without direct inheritance, and *metaprogramming*, in which code is generated on-the-fly during runtime and used later during execution. The latter feature is particularly common in Ruby on Rails, a popular Ruby web development framework.

To meet these challenges, `RTR` uses two key ideas. First, `RTR` incorporates *assume-guarantee checking* [31] to reason about mixins. `RTR` verifies definitions of methods in mixins by assuming refinement type specifications for all undefined, external methods. Then, by dynamically intercepting the call that includes a mixin in a class, `RTR` verifies the appropriate class methods satisfy the assumed refine-

ment types (§ 2.3.1). Second, RTR uses *just-in-time verification* to reason about metaprogramming, following RDL's just-in-time type checking [5]. In this approach, (refinement) types are maintained at run-time, and methods are checked against their types after metaprogramming code has executed but before the methods have been called (§ 2.3.2).

We formalized RTR by showing how to translate λ^{RB}, a core Ruby-like language with refinement types, into λ^{I}, a core verification-oriented language. We then discuss how to map the latter into Rosette, which simply requires encoding λ^{I}'s primitive object construct into Rosette structs and translating some control-flow constructs such as return (§ 2.4).

We evaluated RTR by using it to check a range of functional correctness properties on six Ruby and Rails applications. In total, we verified 31 methods, comprising 271 lines of Ruby, by encoding them as 1,061 lines of Rosette. We needed 73 type annotations. Verification took a total median time (over multiple trials) of 506 seconds (§ 2.5).

Thus, we believe RTR is a promising first step toward verification for Ruby.

2.2 Overview

We begin with an overview of RTR, which extends RDL (§ 1.1) with refinement types. In RTR, program invariants are specified with refinement types (§ 2.2.1) and checked by translation to Rosette (§ 2.2.2). We translate Ruby objects to Rosette structs (§ 2.2.3) and method calls to function calls (§ 2.2.4).

2.2.1 Refinement Type Specifications

Refinement types in RTR are RDL types extended with logical predicates. For example, consider the incr_sec method first introduced in § 1.1. We can tweak the type annotation for this method to include refinement types:

```
type '( Integer x { 0 ≤ x < 60 }) → Integer r { 0 ≤ r < 60}'
def incr_sec(x) if (x==59) then 0 else x+1 end ; end
```

This type indicates the argument and result of incr_sec are integers in the range from 0 to 59. In general, refinements (in curly braces) may be arbitrary Ruby expressions that are treated as booleans, and they should be *pure*, i.e., have no side effects, since effectful predicates make verification either complicated or imprecise [32]. As in RDL, the type annotation, which is a string, is parsed and stored in a global table which maintains the program's type environment.

2.2.2 Verification Using Rosette

RTR checks method specifications by encoding their verification into Rosette [30], a solver-aided host language built on top of Racket. Among other features, Rosette can perform verification by using symbolic execution to generate logical constraints, which are discharged using Z3 [33].

For example, to check incr_sec, RTR creates the equivalent Rosette program:

```
(define (incr_sec x) (if (= x 59) 0 (+ x 1)))
(define-symbolic x_in integer?)
(verify #:assume (assert 0 ≤ x < 60)
#:guarantee (assert (let ([r (incr_sec x)]) 0 ≤ r < 60)))
```

Here x_in is an integer *symbolic constant* representing an unknown, arbitrary integer

argument. `Rosette` symbolic constants can range over the *solvable types* integers, booleans, bitvectors, reals, and uninterpreted functions. We use `Rosette`'s verify function with assumptions and assertions to encode pre- and postconditions, respectively. When this program is run, Rosette searches for an x_in such that the assertion fails. If no such value exists, then the assertion is verified.

2.2.3 Encoding and Reasoning about Objects

We encode Ruby objects in `Rosette` using a *struct type*, i.e., a record. More specifically, we create a struct type object that contains an integer classid identifying the object's class, an integer objectid identifying the object itself, and a field for each instance variable of all objects encountered in the source Ruby program (similar to prior work [34, 35]).

For example, consider a Ruby class Time with three instance variables @sec, @min, and @hour, and a method is_valid that checks all three variables are valid:

```
class Time
  attr_accessor :sec, :min, :hour

  def initialize (s, m, h) @sec = s; @min = m; @hour = h; end

  type '() → bool'
  def is_valid 0 ≤ @sec < 60 ∧ 0 ≤ @min < 60 ∧ 0 ≤ @hour < 24; end
end
```

`RTR` observes three fields in this program, and thus it defines:

```
(struct object ([ classid ][ objectid ]
  [@sec #:mutable] [@min #:mutable] [@hour #:mutable]))
```

Here object includes fields for the class ID, object ID, and the three instance vari-

ables. Note since object's fields are statically defined, our encoding cannot handle dynamically generated instance variables, which we leave as future work.

RTR then translates Ruby field reads or writes as getting or setting, respectively, object's fields in Rosette. For example, suppose we add a method mix to the Time class and specify it is only called with and returns valid times:

```
type :mix, '(Time t1 { t1.is_valid }, Time t2 { t2.is_valid },
             Time t3 { t3.is_valid }) → Time r { r.is_valid }'
def mix(t1,t2,t3) @sec = t1.sec; @min = t2.min; @hour = t3.hour; self; end
```

Initially, type checking fails because the getters' and setters' (e.g., sec and sec=) types are unknown. Thus, we add those types:

```
type :sec,  '()            → Integer i   { i == @sec }'
type :sec=, '(Integer i) → Integer out { i == @sec }'
```

(Note these annotations can be generated automatically using our approach to meta-programming, described in § 2.3.2.) This allows RTR to proceed to the translation stage, which generates the following Rosette function:

```
(define (mix self t1 t2 t3)
        (set-object-@sec! self (sec t1))
        (set-object-@min! self (min t2))
        (set-object-@hour! self (hour t3))
        self)
```

(Asserts, assumes, and verify call omitted.) Here (set-object-x! y w) writes w to the x field of y and the field selectors sec, min, and hour are uninterpreted functions. Note that self turns into an explicit additional argument in the Rosette definition. Rosette then verifies this program, thus verifying the original Ruby mix method.

2.2.4 Method Calls

To translate a Ruby method call e.m(e1, .., en), RTR needs to know the callee, which depends on the runtime type of the receiver e. RTR uses RDL's type information to overapproximate the set of possible receivers. For example, if e has type A in RDL, then RTR translates the above as a call to A.m. If e has a union type, RTR emits Rosette code that branches on the potential types of the receiver using object class IDs and dispatches to the appropriate method in each branch. This is similar to class hierarchy analysis [36], which also uses types to determine the set of possible method receivers and construct a call graph.

Once the method being called is determined, we translate the call into Rosette. As an example, consider a method to_sec that converts a Time object to seconds, after it calls the method incr_sec from § 2.2.1.

```
type '(Time t { t.is_valid }) → Integer r { 0≤r<90060 }'
def to_sec(t) incr_sec(t.sec) + 60 * t.min + 3600 * t.hour; end
```

RTR's translation of to_sec could simply call directly into incr_sec's translation. This is equivalent to inlining incr_sec's code. However, inlining is not always possible or desirable. A method's code may not be available because the method comes from a library, is external to the environment (§ 2.3.1), or has not been defined yet (§ 2.3.2). The method might also contain constructs that are difficult to verify, like diverging loops.

Instead, RTR can model the method call using the programmer provided method specification. To precisely reason with only a method's specification, RTR follows

Dafny [37] and treats pure and impure methods differently.

Pure methods. Pure methods have no side effects and return the same result for the same inputs, satisfying the congruence property $\forall x, y. x = y \Rightarrow m(x) = m(y)$ for a given method m. Thus, pure methods can be encoded using `Rosette`'s uninterpreted functions. The method incr_sec is indeed pure, so we can label it as such:

```
type :incr_sec, '( Integer x { 0≤x<60 }) → Integer r { 0≤r<60 }', :pure
```

With the pure label, the translation of to_sec treats incr_sec as an uninterpreted function. Furthermore, it asserts the precondition $0 \leq x < 60$ and assumes the postcondition $0 \leq r < 60$, which is enough to verify to_sec.

Impure methods. Most Ruby methods have side effects and thus are not pure. For example, consider incr_min, a mutator method that adds a minute to a Time:

```
type '(Time t { t.is_valid ∧ t.min < 59 }) → Time r { r.is_valid }',
       modifies: { t: @min, t: @sec }
def incr_min(t)
   if t.sec<59 then t.sec=incr_sec(t.sec) else t.min+=1; t.sec=0 end
   return t
end
```

A translated call to incr_min generates a fresh symbolic value as the method's output and assumes the method's postcondition on that value. Because the method may have side effects, the modifies label is used to list all fields of inputs which may be modified by the method. Here, a translated call to incr_min will *havoc* (set to fresh symbolic values) t's @min and @sec fields.

We leave support for other kinds of modifications (e.g., global variables, tran-

sitively reachable fields), as well as enforcing the pure and modifies labels, as future work.

2.3 Just-In-Time Verification

Next, we show how RTR handles code with dynamic bindings via mixins (§ 2.3.1) and metaprogramming (§ 2.3.2).

2.3.1 Mixins

Ruby implements mixins via its *module* system. A Ruby module is a collection of method definitions that are added to any class that includes the module at runtime. Interestingly, modules may refer to methods that are not defined in the module but will ultimately be defined in the including class. Such incomplete environments pose a challenge for verification.

Consider the following method that has been extracted and simplified from the *Money* library, used in our evaluations (§ 2.5).

```
module Arithmetic
  type '( Integer x)→ Float r { r == x/value }'
  def div_by_val(x) x/value; end
end
```

The module method div_by_val divides its input x by value. RTR's specification for / requires that value cannot be 0.

Notice that value is not defined in Arithmetic. Rather, it must be defined wherever Arithmetic is included. Therefore, to proceed with verification in RTR, the programmer must provide an annotation for value:

```
type :value, '() → Float v { 0 < v }', :pure
```

Using this annotation, RTR can verify div_by_value. Then when Arithmetic is included in another class, RTR verifies value's refinement type. For example, consider the following code:

```
class Money
  include Arithmetic
  def value()
    if (@val > 0) then (return @val) else (return 0.01) end
  end
end
```

RTR dynamically intercepts the call to include and then applies the type annotations for methods not defined in the included module, in this case verifying value against the annotation in Arithmetic. Thus, RTR follows an assume-guarantee paradigm [31]: it assumes value's annotation while verifying div_by_val and then guarantees the annotation once value is defined.

2.3.2 Metaprogramming

Metaprogramming in Ruby allows new methods to be created and existing methods to be redefined on the fly, posing a challenge for verification. RTR addresses this challenge using just-in-time checking [5], in which, in addition to code, method annotations can also be generated dynamically.

We illustrate the just-in-time approach using an example from *Boxroom*, a Rails app for managing and sharing files in a web browser (§ 2.5). The app defines a class UserFile that is a Rails *model* corresponding to a database table:

```
class UserFile < ActiveRecord::Base
```

```
    belongs_to :folder
    ...
    type '(Folder target) → Bool b { folder == target }'
    def move(target) folder = target; save!; end
    ...
end
```

Here calling belongs_to tells Rails that every UserFile is associated with a folder (another model). The move method updates the associated folder of a UserFile and saves the result to the database. We annotate move to specify that the UserFile's new folder should be the same as move's argument.

This method and its annotation are seemingly simple, but there is a problem. To verify move, RTR needs an annotation for the folder= method, which is not statically defined. Rather, it is dynamically generated by the call to belongs_to.

To solve this problem in RTR, we instrument belongs_to to generate type annotations for the setter (and getter) method, as follows:

```
module ActiveRecord::Associations::ClassMethods
    pre(:belongs_to) do |*args|
        name = args[0].to_s
        cname = name.camelize
        type "#{name}" , "() → #{cname} c", :pure
        type "#{name}=", "(#{cname} i) →#{cname} o { #{name} == i }"
        true
    end
end
```

We call pre, an RDL method, to define a precondition *code block* (i.e., an anonymous function) which will be executed on each call to belongs_to. First, the block sets name and cname to be the string version of the first argument passed to belongs_to and its camelized representation, respectively. Then, we create types for the name and name= methods. Finally, we return true so the contract will succeed. In our

example, this code generates the following two type annotations upon the call to belongs_to:

```
type 'folder', '() → Folder c', :pure
type 'folder=', '(Folder i) → Folder o { folder == i }'
```

These annotations will be generated when belongs_to is invoked with the :folder argument, which happens exactly after the UserFile class is loaded. Thus, not only will the call to belongs_to generate getter and setter methods, but it also generates useful annotations for these methods. With these annotations, verification of the initial move method succeeds.

2.4 From Ruby to Rosette

In this section, we formally describe our verifier and the translation from Ruby to `Rosette`. We start (§ 2.4.1) by defining λ^{RB}, a Ruby subset that is extended with refinement type specifications. We give (§ 2.4.2) a translation from λ^{RB} to an intermediate language λ^{I}, and then (§ 2.4.3) we discuss how λ^{I} maps to a `Rosette` program. Finally (§ 2.4.5), we use this translation to construct a verifier for Ruby.

2.4.1 Core Ruby λ^{RB} and Intermediate Representation λ^{I}

2.4.1.1 λ^{RB}

Figure 2.1 defines λ^{RB}, a core Ruby-like language with refinement types. *Constants* consist of `nil`, booleans, and integers. *Expressions* include constants, variables, assignment, conditionals, sequences, and the reserved variable `self`, which

$$
\begin{array}{rrcl}
\textbf{\textit{Constants}} & c & ::= & \texttt{nil} \mid \texttt{true} \mid \texttt{false} \mid 0, 1, -1, \ldots \\
\textbf{\textit{Expressions}} & e & ::= & c \mid x \mid x{:=}e \mid \texttt{if}\ e\ \texttt{then}\ e\ \texttt{else}\ e \mid e\ ;\ e \\
 & & \mid & \texttt{self} \mid f \mid f{:=}e \mid e.m(\overline{e}) \mid A.\texttt{new} \mid \texttt{return}(e) \\
\textbf{\textit{Refined Types}} & t & ::= & \{x : A \mid e\ \} \\
\textbf{\textit{Program}} & P & ::= & \cdot \mid d, P \mid a, P \\
\textbf{\textit{Definition}} & d & ::= & \texttt{def}\ A.m(\overline{t})::t; l = e \\
\textbf{\textit{Annotation}} & a & ::= & A.m :: (\overline{t}) \to t\ ;\ l \qquad \text{with } l \neq \texttt{exact} \\
\textbf{\textit{Labels}} & l & ::= & \texttt{exact} \mid \texttt{pure} \mid \texttt{modifies}[\overline{x.f}]
\end{array}
$$

$x \in$ var ids, $f \in$ field ids, $m \in$ meth ids, $A \in$ class ids

Figure 2.1: Syntax of the Ruby Subset λ^{RB}.

refers to a method's receiver. Also included are references to an instance variable f and instance variable assignment; we note that in actual Ruby, field names are preceded by a "@". Finally, expressions include method calls, constructor calls $A.\texttt{new}$ which create a new instance of class A, and return statements.

Refined types $\{x : A \mid e\ \}$ refine the basic type A with the predicate e. The basic type A is used to represent both user defined and built-in classes including nil, booleans, integers, floats, etc. The refinement e is a *pure, boolean valued* expression that may refer to the refinement variable x. In the interest of greater simplicity for the translation, we require that `self` *does not* appear in refinements e; however, extending the translation to handle this is natural, and our implementation allows for it. Sometimes we simplify the trivially refined type $\{x : A \mid \texttt{true}\ \}$ to just A.

A *program* is a sequence of method definitions and type annotations over methods. A method definition $\texttt{def}\ A.m(\{x_1 : A_1 \mid e_1\ \}, \ldots, \{x_n : A_n \mid e_n\ \})::t; l = e$ defines the method $A.m$ with arguments $x_1, \ldots, x_n$ and body e. The type specification of the definition is a dependent function type: each argument binder x_i can appear

inside the arguments' refinements types e_j for all $1 \leq j \leq n$, and can also appear in the refinement of the result type t. A method type annotation $A.m :: (\overline{t}) \rightarrow t \ ; \ l$ binds the method named $A.m$ with the dependent function type $(\overline{t}) \rightarrow t$. λ^{RB} includes both method annotations and method definitions because annotations are used when a method's code is not available, e.g., in the cases of library methods, mixins, or metaprogramming.

A *label* l can appear in both method definitions and annotations to direct the method's translation into `Rosette` as described in § 2.2.4. The label `exact` states that a called method will be exactly translated by using the translation of the body of the method. Since method type annotations do not have a body, they cannot be assigned the `exact` label. The `pure` label indicates that a method is pure and thus can be translated using an uninterpreted function. Finally, the `modifies`$[\overline{x.f}]$ label is used when a method is impure, i.e., it may modify its inputs. As we saw earlier, the list $\overline{x.f}$ captures all the argument fields which the method may modify.

2.4.1.2 λ^I.

Figure 2.2 defines λ^I, a core verification-oriented language that easily translates to `Rosette`. λ^{RB} methods map to λ^I functions, and λ^{RB} objects map to a special `object` struct type. λ^I provides primitives for creating, altering, and referencing instances of this type. *Values* in λ^I consist of *constants* c (defined identically to λ^{RB}) and `object`$(i_1,\ i_2,\ [f_1\ w_1] \ldots [f_n\ w_n])$, an instantiation of an `object` type with class ID i_1, object ID i_2, and where each field f_i of the `object` is bound to value

$$
\begin{array}{rrcl}
\textbf{\textit{Values}} & w & ::= & c \mid \texttt{object}(i,\ i,\ \overline{[f\ w]}) \\
\textbf{\textit{Expressions}} & u & ::= & w \mid x \mid x{:=}u \mid \texttt{if}\ u\ \texttt{then}\ u\ \texttt{else}\ u \mid u\ ;\ u \\
 & & \mid & \texttt{let}\ (\overline{[x\ u]})\ \texttt{in}\ u \mid x(\overline{u}) \mid \texttt{assert}(u) \\
 & & \mid & \texttt{assume}(u) \mid \texttt{return}(u) \mid \texttt{havoc}(x.f) \mid x.f := u \mid x.f \\
\textbf{\textit{Program}} & Q & ::= & \cdot \mid d, Q \mid v, Q \\
\textbf{\textit{Definition}} & d & ::= & \texttt{define}\ x(\overline{x}) = u \mid \texttt{define-sym}(x,\ A) \\
\textbf{\textit{Verification Query}} & v & ::= & \texttt{verify}(\overline{u} \Rightarrow u)
\end{array}
$$

$x \in$ var ids, $f \in$ field ids, $A \in$ types, $i \in$ integers

Figure 2.2: Syntax of the Intermediate Language λ^I.

w_i. *Expressions* include `let` bindings ($\texttt{let}\ (\overline{[x_i\ u_i]})\ \texttt{in}\ u$) where each x_i may appear free in u_j if $i < j$. They also include function calls, `assert`, `assume`, and `return` statements, as well as $\texttt{havoc}(x.f)$, which mutates x's field f to a fresh symbolic value. Finally, they include field assignment $x.f := u$ and field reads $x.f$.

A *program* is a series of definitions and verification queries. A *definition* is a function definition or a symbolic definition $\texttt{define-sym}(x,\ A)$, which binds x to either a fresh symbolic value if A is a solvable type (e.g., boolean, integer; see § 2.2.2) or a new `object` with symbolic fields defined depending on the type of A. Finally, a verification query $\texttt{verify}(\overline{u} \Rightarrow u)$ checks the validity of u assuming $\overline{u}$.

2.4.2 From λ^{RB} to λ^I

Figures 2.3 and 2.4 define the translation function $e \rightsquigarrow u$ that maps expressions and programs from λ^{RB} to λ^I.

Expression Translation $\boxed{e \rightsquigarrow u}$

$$\frac{}{c \rightsquigarrow c}\ \text{T-Const} \qquad \frac{}{x \rightsquigarrow x}\ \text{T-Var} \qquad \frac{e_1 \rightsquigarrow u_1 \quad e_2 \rightsquigarrow u_2}{e_1\ ;\ e_2 \rightsquigarrow}\ \text{T-Seq}$$

$$\frac{e_1 \rightsquigarrow u_1 \quad e_2 \rightsquigarrow u_2 \quad e_3 \rightsquigarrow u_3}{\texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3 \rightsquigarrow \texttt{if}\ u_1\ \texttt{then}\ u_2\ \texttt{else}\ u_3}\ \text{T-If} \qquad \frac{}{\texttt{self} \rightsquigarrow \mathit{self}}\ \text{T-Self}$$

$$\frac{e \rightsquigarrow u}{x{:=}e \rightsquigarrow x{:=}u}\ \text{T-VarAssn} \qquad \frac{e \rightsquigarrow u}{\texttt{return}(e) \rightsquigarrow \texttt{return}(u)}\ \text{T-Ret}$$

$$\frac{f \in \mathcal{F}}{f \rightsquigarrow \mathit{self}.f}\ \text{T-Inst} \qquad \frac{f \in \mathcal{F} \quad e \rightsquigarrow u}{f{:=}e \rightsquigarrow \mathit{self}.f := u}\ \text{T-InstAssn}$$

$$\frac{\texttt{classId}(A) = i_c \quad \texttt{freshID}(i_o) \quad f_i \in \mathcal{F}}{A.\texttt{new} \rightsquigarrow \texttt{object}(i_c,\ i_o,\ [f_1\ \texttt{nil}] \ldots [f_{|\mathcal{F}|}\ \texttt{nil}])}\ \text{T-New}$$

$$\frac{\begin{array}{c}\texttt{typeOf}(e_F) = A \quad \texttt{exact} = \texttt{labelOf}(A.m) \\ A\_m \in \mathcal{M} \qquad e_F \rightsquigarrow u_F \qquad e_i \rightsquigarrow u_i\end{array}}{e_F.m(\overline{e}) \rightsquigarrow A\_m(u_F, \overline{u})}\ \text{T-Exact}$$

$$\frac{\begin{array}{c}\texttt{typeOf}(e_F) = A \qquad \texttt{pure} = \texttt{labelOf}(A.m) \\ A\_m \in \mathcal{U} \qquad \texttt{freshVar}(x, r) \\ \texttt{specOf}(A.m) = (\{x : A_x \mid e_x\}) \rightarrow \{r : A_r \mid e_r\} \\ e_F \rightsquigarrow u_F \qquad e \rightsquigarrow u \qquad e_x \rightsquigarrow u_x \qquad e_r \rightsquigarrow u_r\end{array}}{e_F.m(e) \rightsquigarrow\ \texttt{let}\ ([x\ u][r\ A\_m(u_F, a)])\ \texttt{in}\ \texttt{assert}(u_x)\ ;\ \texttt{assume}(u_r)\ ;\ r}\ \text{T-Pure1}$$

$$\frac{\begin{array}{c}\texttt{typeOf}(e_F) = A \qquad \texttt{modifies}[p] = \texttt{labelOf}(A.m) \\ \texttt{specOf}(A.m) = (\{x : A_x \mid e_x\}) \rightarrow \{r : A_r \mid e_r\} \\ h_x = \{u.f \mid f \in \mathcal{F},\ x.f \in p\} \quad h_F = \{u_F.f \mid f \in \mathcal{F},\ \texttt{self}.f \in p\} \\ \texttt{freshVar}(x, r) \qquad e_F \rightsquigarrow u_F \qquad e \rightsquigarrow u \qquad e_x \rightsquigarrow u_x \qquad e_r \rightsquigarrow u_r\end{array}}{e_F.m(e) \rightsquigarrow \begin{array}{l}\texttt{let}\ ([x\ u])\ \texttt{in}\ \texttt{define-sym}(r,\ A_r); \\ \texttt{assert}(u_x)\ ;\ \texttt{havoc}(h_F \cup h_x)\ ;\ \texttt{assume}(u_r)\ ;\ r\end{array}}\ \text{T-Impure1}$$

Figure 2.3: Expression translation from λ^{RB} to λ^{I}. For simplicity rules T-Pure1 and T-Impure1 assume single argument methods.

Program Translation $\boxed{P \rightsquigarrow Q}$

$$\frac{}{\cdot \rightsquigarrow \cdot}\ \textsc{T-Emp} \qquad \frac{P \rightsquigarrow Q}{A.m :: (x_1{:}t_1, \ldots, x_n{:}t_n) \to t\ ;\ l, P \rightsquigarrow Q}\ \textsc{T-Ann}$$

$$\frac{\begin{array}{c} t_i = \{x_i : A_{x_i} \mid e_{x_i}\ \} \qquad t = \{r : A_r \mid e_r\ \} \\ e \rightsquigarrow u \quad e_{x_i} \rightsquigarrow u_{x_i} \quad e_r \rightsquigarrow u_r \quad P \rightsquigarrow Q \quad 1 \le i \le n \end{array}}{\begin{array}{l} \texttt{def}\ A.m(t_1, \ldots, t_n){::}t; l = e, P \rightsquigarrow \\ \end{array} \begin{array}{l} \texttt{define}\ A\_m(self, x_1, \ldots, x_n) = u; \\ \texttt{define-sym}(self,\ A); \\ \texttt{define-sym}(x_i,\ A_{x_i}); \\ \texttt{verify}(u_{x_1}, \ldots, u_{x_n} \Rightarrow u_r)\ ;\ Q \end{array}}\ \textsc{T-Def}$$

Figure 2.4: Program translation from λ^{RB} to λ^{I}.

Global States. The translation uses sets $\mathcal{M}, \mathcal{U}$, and $\mathcal{F}$, to ensure all the methods, uninterpreted functions, and fields are well-defined in the generated λ^I term:

$$\mathcal{M} ::= A_1.m_1, \ldots, A_n.m_n \qquad \mathcal{U} ::= A_1.m_1, \ldots, A_n.m_n \qquad \mathcal{F} ::= f_1, \ldots, f_n$$

In the translation rules, we use the standard set operations $x \in \mathcal{X}$ and $\mid \mathcal{X} \mid$ to check membership and size of the set $\mathcal{X}$. Thus, the translation relation is defined over these sets: $\mathcal{M}, \mathcal{U}, \mathcal{F} \vdash e \rightsquigarrow u$. Since the rules do not modify these environments, in Figures 2.3 and 2.4 we simplify the rules to $e \rightsquigarrow u$. Note that even though the rules "guess" these environments by making assumptions about which elements are members of the sets, in an algorithmic definition the rules can be used to construct the sets.

Expressions. Figure 2.3 defines the translation function for expressions. The rules T-Const and T-Var are identity while the rules T-If, T-Seq, T-Ret, and T-VarAssn are trivially inductively defined. The rule T-Self translates `self` into

the special *variable* named *self* in λ^I. The *self* variable is always in scope, since each λ^{RB} method translates to a λ^I function with an explicit first argument named *self*. The rules T-Inst and T-InstAssn translate a reference from and an assignment to the instance variable f, to a read from and write to, respectively, the field f of the variable *self*. Moreover, both the rules assume the field f to be in global field state $\mathcal{F}$. The rule T-New translates from a constructor call A.`new` to an `object` instance. The `classId`(A) function in the premise of this rule returns the class ID of A. The `freshID`(i_o) predicate ensures the new `object` instance has a fresh object ID. Each field of the new `object`, $f_1, \ldots, f_{|\mathcal{F}|}$, is initially bound to `nil`.

Method Calls. To translate the λ^{RB} method call $e_F.m(\overline{e})$, we first use the function `typeOf`(e_F) to type e_F via `RDL` type checking 1.1. If e_F is of type A, we split cases of the method call translation based on the value of `labelOf`($A.m$), the label specified in the annotation of $A.m$ (as informally described in § 2.2.4).

The rule T-Exact is used when the label is `exact`. The receiver e_F is translated to u_F which becomes the first (i.e., the *self*) argument of the function call to A_m. Moreover, $A.m$ is assumed to be in the global method name set $\mathcal{M}$ since it belongs to the transitive closure of the translation.

We note that for the sake of clarity, in the T-Pure1 and T-Impure1 rules, we assume that the method $A.m$ takes just one argument; the rules can be extended in a natural way to account for more arguments. The rule T-Pure1 is used when the label is `pure`. In this case, the call is translated as an invocation to the uninterpreted function A_m, so $A.m$ should be in the global set of uninterpreted

functions $\mathcal{U}$. The specification `specOf`$(A.m)$ of the method is also enforced. Let $(\{x : A_x \mid e_x\}) \to \{r : A_r \mid e_r\}$ be the specification. We assume that the binders in the specification are α-renamed so that the binders x and r are fresh. We use x and r to bind the argument and the result, respectively, to ensure, via A-normal form conversion [38], that they will be evaluated exactly once, even though x and r may appear many times in the refinements. To enforce the specification, we assert the method's precondition e_x and assume the postcondition e_r.

If a method is labeled with `modifies`$[p]$ then the rule T-Impure1 is applied. We locally define a new symbolic object as the return value, and we `havoc` the fields of all arguments (including *self*) specified in the `modifies` label, thereby assigning these fields to new symbolic values. Since we do not translate the called method at all, no global state assumptions are made.

Programs. Finally, we use the translation relation to translate programs from λ^{RB} to λ^{I}, i.e., $P \rightsquigarrow Q$. This is shown in Figure 2.4. The rule T-Ann discards type annotations. The rule T-Def translates a method definition for $A.m$ to the function definition A_m that takes the additional first argument *self*. The rule also considers the declared type of $A.m$ and instantiates a symbolic value for every input argument. Finally, all refinements from the inputs and output of the method type are translated and the derived verification query is made.

2.4.3 From λ^I to `Rosette`

We write $Q \twoheadrightarrow R$ to encode the translation of the λ^I program Q to the `Rosette` program R. This translation is straightforward, since λ^I consists of `Rosette` extended with some macros to encode Ruby-verification specific operators, like `define-sym` and `return`. In fact, in the implementation of the translation (§ 2.5), we used Racket's macro expansion system to achieve this final transformation.

Handling objects. λ^I contains multiple constructs for defining and altering objects, which are expanded in Rosette to perform the associated operations over `object` structs. The expressions `object`$(i_c,\ i_o,\ \overline{[f\ w]})$ and `havoc`$(x.f)$, and the definition `define-sym`$(x,\ A)$, all described in § 2.4.1, are expanded to perform the corresponding operations over values of the `object` struct type.

Control Flow. Macro expansion is used to translate `return` and `assume` statements, and exceptions into `Rosette`, since those forms are not built-in to the language. To encode `return`, we expand every function definition in λ^I to keep track of a local variable `ret`, which is initialized to a special `undefined` value and returned at the end of the function. We transform every statement `return`(e) to update the value of `ret` to e. Then, we expand every expression u in a function to `unless-done`(u), which checks the value of `ret`, proceeding with u if `ret` is `undefined` or skipping u if there is a return value.

We used the encoding of `return` to encode more operators. For example, `assume` is encoded in `Rosette` as a macro that returns a special `fail` value when

assumptions do not hold. The verification query then needs to be updated with the condition that `fail` is not returned. A similar expansion is used to encode and propagate exceptions.

2.4.4 Primitive Types

λ^{RB} provides constructs for functions, assignments, control flow, etc, but does not provide the theories required to encode interesting verification properties that, for example, reason about booleans and numbers. On the other hand, `Rosette` is a verification oriented language with special support for common theories over built-in datatypes, including booleans, numeric types, and vectors. To bridge this gap, we encode certain Ruby expressions, such as constants c in λ^{RB}, into `Rosette`'s corresponding built-in datatypes.

Equality and Booleans. To precisely reason about equality, we encode Ruby's `==` method over arbitrary objects using the object class' $==$ method if one is defined. If the class inherits this method from Ruby's top class, BasicObject, then we encode $==$ using `Rosette`'s equality operator `equal?` to check equality of object IDs. We encode Ruby's booleans and operations over them as `Rosette`'s respective booleans and their operators.

Integers and Floats. By default, we encode Ruby's infinite-precision `Integer` and `Float` objects as `Rosette`'s built-in infinite-precision `integer` and `real` datatypes, respectively. The infinite-precision encoding is efficient and precise, but it may re-

sult in undecidable queries involving non-linear arithmetic or loops. To perform (bounded) verification in such cases, we provide, via a configuration flag, the option of encoding Ruby's integers as `Rosette`'s built-in finite sized bitvectors.

Arrays. Finally, we provide a special encoding for Ruby's arrays, which are commonly used both for storing arbitrarily large random-access data and to represent mixed-type tuples, stacks, queues, etc. We encode Ruby's arrays as a `Rosette` struct composed of a fixed-size vector and an integer representing the current size of the Ruby array. Because we used fixed-size vectors, we can only perform bounded verification over arrays. On the other hand, we avoid the need for loop invariants for iterators and reasoning over array operations can be more efficient.

2.4.5 Verification of λ^{RB}

We define a verification algorithm RTR^{λ} that, given a λ^{RB} program P, checks if all the definitions satisfy their specifications. The pseudo-code for this algorithm is shown below:

```
def RTR^λ(P)
  (F, U, M) := guess(P)
  for (f ∈ F): add field f to object struct
  for (u ∈ U): define uninterpreted function u
  P ⇝ Q ↠ R
  return if (valid(R)) then SAFE else UNSAFE
end
```

First, we guess the proper translation environments. In practice (as discussed in § 2.4.2), we use the translation of P to generate the minimum environments for which translation of P succeeds. We define an `object` struct in Rosette containing

one field for each member of $\mathcal{F}$, and we define an uninterpreted function for each method in $\mathcal{U}$. Next, we translate P to a λ^I program Q via $P \rightsquigarrow Q$ (§ 2.4.2) and Q to a the `Rosette` program R, via $Q \twoheadrightarrow R$ (§ 2.4.3). Finally, we run the `Rosette` program R. The initial program P is *safe*, i.e., no refinement type specifications are violated, if and only if the `Rosette` program R is *valid*, i.e., all the `verify` queries are valid.

We conclude this section with a discussion of the RTR$^\lambda$ verifier.

RTR$^\lambda$ is Partial. There exist expressions of λ^{RB} that fail to translate into a λ^I expression. The translation requires at each method call $e_F.m(\overline{e})$ that the receiver has a class type A. There are two cases where this requirement fails: (1) e_F has a union type or (2) type checking fails and so e_F has no type. In our implementation (§ 2.5), we extend the translation to handle the first two cases. Handling for (1) is outlined in § 2.2.4. Case (2) can be caused by either a type error in the program or a lack of typing information for the type checker. Translation cannot proceed in either case.

RTR$^\lambda$ may Diverge. The translation to `Rosette` always terminates. All translation rules are inductively defined: they only recurse on syntactically smaller expressions or programs. Also, since the input program is finite, the minimum global environments required for translation are also finite. Finally, all the helper functions (including the type checking `typeOf`$(\cdot)$) do terminate.

Yet, verification may diverge, as the execution of the `Rosette` program may

diverge. Specifications can encode arbitrary expressions, thus it is possible to encode undecidable verification queries. Consider, for instance, the following contrived `Rosette` program in which we attempt to verify an assertion over a recursive method:

```
(define (rec x) (rec x))
(define-symbolic b boolean?)
(verify (rec b))
```

`Rosette` attempts to symbolically evaluate this program, and thus diverges.

RTR$^\lambda$ is Incomplete. Verification is incomplete and its precision relies on the precision of the specifications. For instance, if a pure method $A.m$ is marked as impure, the verifier will not prove the congruence axiom.

RTR$^\lambda$ is Sound. If the verifier decides that the input program is safe, then all definitions satisfy their specifications, assuming that (1) all the refinements are pure boolean expressions and (2) all the labels are sound (i.e., methods match the specifications implied by the labels). The assumption (1) is required since verification under diverging (let alone effectful) specifications is difficult [32]. The assumption (2) is required since our translation encodes pure methods as uninterpreted functions, while for the impure methods it havocs only the unprotected arguments.

2.5 Evaluation

We implemented the Ruby refinement type checker `RTR`[1] by extending `RDL` with refinement types. Table 2.1 summarizes the evaluation of `RTR`.

[1]Code available at: `https://github.com/mckaz/vmcai-rdl`

Benchmarks. We evaluate RTR on six popular Ruby libraries:

- *Money* [39] performs currency conversions over monetary quantities and relies on mixin methods,
- *BusinessTime* [40] performs time calculations in business hours and days,
- *Unitwise* [41] performs various unit conversions,
- *Geokit* [42] performs calculations over locations on Earth,
- *Boxroom* [43] is a Rails app for sharing files in a web browser and uses metaprogramming, and
- *Matrix* [44] is a Ruby standard library for matrix operations.

For verification, we forked the original Ruby libraries and provided manually written method specifications in the form of refinement types. The forked repositories are publicly available [45]. Experiments were conducted on a machine with a 3 GHz Intel Core i7 processor and 16 GB of memory.

We chose these libraries because they combine Ruby-specific features challenging for verification, like metaprogramming and mixins, with arithmetic-heavy operations. In all libraries we verify both (1) *functional correctness of arithmetic operations* (e.g., no division-by-zero, the absolute value of a number should not be negative) and (2) *data-specific arithmetic invariants* (e.g., integers representing months should always be in the range from `1` to `12` and a `data` value added to an aggregate should always fall between maintained `@min` and `@max` fields). In the *Matrix* library, we verify a matrix multiplication method, checking that multiplying a

matrix with r rows by a matrix with c columns yields a matrix of size $r \times c$. Note this method makes extensive use of array operations, since matrices are implemented as an array of arrays.

Quantitative Evaluation. Table 2.1 summarizes our evaluation quantitatively. For each application, we list every verified `Method`. In our experiment, we focused on methods with interesting arithmetic properties.

The `Ruby LoC` column gives the size of the verified Ruby program. This metric includes the lines of all methods and annotations that were used to verify the method in question. For each verified method, `RTR` generates a separate Rosette program. We give the sizes of these resulting programs in the `Rosette LoC` column. Unsurprisingly, the LoC of the `Rosette` program increases with the size of the source Ruby program.

We present the median (`Time(s)`) and semi-interquartile range (`SIQR`) of the `Verification Time` required to verify all methods for an application over 11 runs. For each verified method, the `SIQR` was at most 2% of the verification time, indicating relatively little variance in the verification time. Overall, verification was fast, as might be expected for relatively small methods. The one exception was matrix multiplication. In this case, the slowdown was due to the extensive use of array operations mentioned above. We bounded array size (see § 2.4.4) at 10 for the evaluations. For symbolic arrays, this means `Rosette` must reason about every possible size of an array up to 10. This burden is exacerbated by matrix multiplication's use of two symbolic two-dimensional arrays.

Method	Ru-LoC	Ro-LoC	Spec	Verification Time	
				Time(s)	SIQR
Money					
Money::Arithmetic#-@	7	29	4	5.69	0.14
Money::Arithmetic#eql?	11	40	3	5.74	0.03
Money::Arithmetic#positive?	5	24	3	5.40	0.01
Money::Arithmetic#negative?	5	24	2	5.42	0.01
Money::Arithmetic#abs	5	30	4	5.49	0.01
Money::Arithmetic#zero?	5	26	2	5.38	0.02
Money::Arithmetic#nonzero?	5	24	2	5.43	0.03
App Total	**43**	**197**	**10**	**38.56**	**0.25**
BusinessTime					
ParsedTime#-	10	58	8	6.28	0.02
BusinessHours#initialize	5	26	2	5.36	0.04
BusinessHours#non_negative_hours?	5	26	2	5.4	0.01
Date#week	7	32	2	5.53	0.01
Date#quarter	5	28	2	5.47	0.00
Date#fiscal_month_offset	5	25	2	5.41	0.02
Date#fiscal_year_week	7	33	2	5.53	0.03
Date#fiscal_year_month	12	35	3	5.65	0.02
Date#fiscal_year_quarter	9	42	2	5.72	0.03
Date#fiscal_year	11	32	4	5.81	0.03
App Total	**76**	**337**	**24**	**56.15**	**0.20**
Unitwise					
Unitwise::Functional.to_cel	4	25	2	5.42	0.03
Unitwise::Functional.from_cel	4	25	2	5.44	0.03
Unitwise::Functional.to_degf	4	22	1	5.41	0.01
Unitwise::Functional.from_degf	4	27	2	5.44	0.02
Unitwise::Functional.to_degre	4	27	2	5.44	0.01
Unitwise::Functional.from_degre	4	27	2	5.42	0.01
App Total	**24**	**153**	**6**	**32.55**	**0.11**
Geokit					
Geokit::Bounds#center	7	31	4	5.4	0.02
Geokit::Bounds#crosses_meridian?	7	35	6	5.59	0.12
Geokit::Bounds#==	9	60	5	5.97	0.13
Geokit::GeoLoc#province	5	26	2	5.52	0.11
Geokit::GeoLoc#success?	5	26	2	5.51	0.05
Geokit::Polygon#contains?	26	68	10	10.8	0.07
App Total	**59**	**246**	**21**	**38.80**	**0.50**
Boxroom					
UserFile#move	12	34	3	5.57	0.05
Matrix					
Matrix.*	57	94	9	334.35	3.99
Total	**271**	**1061**	**73**	**505.98**	**5.10**

Table 2.1: `Method` gives the class and name of the method verified. `Ru-LoC` and `Ro-LoC` give number of LoC for a Ruby method and the translated `Rosette` program. `Spec` is the number of method and variable type annotations we had to write. `Verification Time` is the median and semi-interquartile range of the time in seconds over 11 runs. `App Total` rows list the totals for an app, without double counting the same specs.

Finally, Table 2.1 lists the number of type `specifications` required to verify each method. These are comprised of method type annotations, including the refinement type annotations for the verified methods themselves, and variable type annotations for instance variables. Note that we do not quantify the number of type annotations used for Ruby's core and standard libraries, since these are included in `RDL`.

We observe that there is significant variation in the number of annotations required for each application. For example, *Unitwise* required 6 annotations to verify 6 methods, while *Geokit* required 21 annotations for 6 methods. The differences are due to code variations: To verify a method, the programmer needs to give a refinement type for the method plus a type for each instance variable used by the method and for each (non-standard/core library) method called by the method.

Case Study. Next we illustrate the `RTR` verification process by presenting the exact steps required to specify and check the properties of a method from an existing Ruby library. For this example, we chose to verify the « method of the *Aggregate* library [46], a Ruby library for aggregating and performing statistical computations over some numeric data. The method « takes one input, data, and adds it to the aggregate by updating (1) the minimum @min and maximum @max of the aggregate, (2) the count @count, sum @sum, and sum of squares @sum2 of the aggregate, and finally (3) the correct bucket in @buckets.

```
def «(data)
  if 0 == @count
    @min = data ; @max = data
  else
```

```
    @max = data if data > @max ; @min = data if data < @min
  end
  @count += 1 ; @sum += data ; @sum2 += (data * data)
  @buckets[to_index(data)] += 1 unless outlier?(data)
end
```

We specify functional correctness of the method « by providing a refinement type specification that declares that after the method is executed, the input data will fall between @min and @max.

```
type :«, '( Integer data) → Integer { @min≤data≤@max }', verify: :bind
```

Here, the symbol :bind is an arbitrary label. To verify the specification, we load the library and call the verifier with this label:

```
rdl_do_verify :bind
```

RTR proceeds with verification in three steps:

- first use `RDL` to type check the basic types of the method,
- then translate the method to `Rosette` (using the translation of § 2.4), and
- finally run the `Rosette` program to check the validity of the specification.

Initially, verification fails in the first step with the error

```
error: no type for instance variable '@count'
```

To fix this error, the user needs to provide the correct types for the instance variables using the following type annotations.

```
var_type :@count, 'Integer'
var_type :@min, :@max, :@sum, :@sum2, 'Float'
var_type :@buckets, 'Array<Integer>'
```

The « method also calls two methods that are not from Ruby's standard and core libraries: to_index, which takes a numeric input and determines the index of the bucket the input falls in, and outlier?, which determines if the given data is an outlier based on provided specifications from the programmer. These methods are challenging to verify. For example, the to_index method makes use of non-linear arithmetic in the form of logarithms, and it includes a loop. Yet, neither of the calls to_index or outlier? should affect verification of the specification of «. So, it suffices to provide type annotations with a `pure` label, indicating we want to use uninterpreted functions to represent them:

```
type :outlier?, '(Float i) → Bool b', :pure
type :to_index, '(Float i) → Integer out', :pure
```

Given these annotations, the verifier has enough information to prove the postcondition on «, and it will return the following message to the user:

```
Aggregate instance method « is safe.
```

When verification fails, an unsafe message is provided, combined with a counterexample consisting of bindings to symbolic values that causes the postcondition to fail. For instance, if the programmer *incorrectly* specified that data is less than the @min, i.e.,

```
type :«, '(Integer data) → Integer { data < @min }'
```

Then `RTR` would return the following message:

```
Aggregate instance method « is unsafe.
Counterexample: (model [real_data 0][real_@min 0] ...)
```

This gives a binding to symbolic values in the translated `Rosette` program which

would cause the specification to fail. We only show the bindings relevant to the specification here: when real_data and real_@min, the symbolic values corresponding to data and @min respectively, are both 0, the specification fails.

2.6 Related Work

Verification for Ruby on Rails. Several prior systems can verify properties of Rails apps. *Space* [47] detects security bugs in Rails apps by using symbolic execution to generate a model of data exposures in the app and reporting a bug if the model does not match common access control patterns. Bocić and Bultan propose *symbolic model extraction* [48], which extracts models from Rails apps at runtime, to handle metaprogramming. The generated models are then used to verify data integrity and access control properties. *Rubicon* [49] allows programmers to write specifications using a domain-specific language that looks similar to Rails tests, but with the ability to quantify over objects, and then checks such specifications with bounded verification. *Rubyx* [50] likewise allows programmers to write their own specifications over Rails apps and uses symbolic execution to verify these specifications.

In contrast to RTR, all of these tools are specific to Rails and do not apply to general Ruby programs, and the first two systems do not allow programmers to specify their own properties to be verified.

Rosette. Rosette has been used to help establish the security and reliability of several real-world software systems. Pernsteiner et al. [51] use Rosette to build a

verifier to study the safety of the software on a radiotherapy machine. *Bagpipe* [52] builds a verifier using Rosette to analyze the routing protocols used by Internet Service Providers (ISPs). These results show that Rosette can be applied in a variety of domains.

Types For Dynamic Languages. There have been a number of efforts to bring type systems to dynamic languages including Python [11, 12], Racket [3, 9], and JavaScript [16, 53, 54], among others. However, these systems do not support refinement types.

Some systems have been developed to introduce refinement types to scripting and dynamic languages. *Refined TypeScript* (RSC) [29] introduces refinement types to TypeScript [55, 56], a superset of JavaScript that includes optional static typing. RSC uses the framework of Liquid Types [57] to achieve refinement inference. Refinement types have been introduced [10] to Typed Racket as well. As far as we are aware, these systems do not support mixins or metaprogramming.

General Purpose Verification. Dafny [37] is an object-oriented language with built-in constructs for high-level specification and verification. While it does not explicitly include refinement types, the ability to specify a method's type and pre- and postconditions effectively achieves the same level of expressiveness. Dafny also performs modular verification by using a method's pre- and postconditions and labels indicating its purity or arguments mutated, an approach RTR largely emulates. However, unlike Dafny, `RTR` leaves this modular treatment of methods as an option

for the programmer. Furthermore, unlike RTR, Dafny does not include features such as mixins and metaprogramming.

2.7 Conclusion

We formalized and implemented `RTR`, a refinement type checker for Ruby programs using assume-guarantee reasoning and the just-in-time checking technique of `RDL`. Verification at runtime naturally adjusts standard refinement types to handle Ruby's dynamic features, such as metaprogramming and mixins. To evaluate our technique, we used `RTR` to verify numeric properties on six commonly used Ruby and Ruby on Rails applications, by adding refinement type specifications to the existing method definitions. We found that verifying these methods took a reasonable runtime and annotation burden, and thus we believe `RTR` is a promising first step towards bringing verification to Ruby.

Our work opens new directions for further Ruby verification. One potential area for future work is verification of purity and immutability labels, which are currently trusted by `RTR`. Another area would be refinement type inference by adapting Hindley-Milner and liquid typing [57] to the `RDL` setting, and by exploring whether `Rosette`'s synthesis constructs could be used for refinement inference. There is also potential for extending the expressiveness of `RTR` by adding support for loop invariants and dynamically defined instance variables, among other Ruby constructs. Finally, as Ruby is commonly used in the Ruby on Rails framework, it would be interesting extend `RTR` with modeling for web-specific constructs such as access con-

trol protocols and database operations to further support verification in the domain of web applications.

Chapter 3: Type-Level Computations for Ruby Libraries

The previous chapter introduced `RTR`, a system that extends `RDL` with refinement types. While `RTR` is useful for enforcing strong correctness properties of programs, there are still many commonly used constructs in Ruby programs that are out of `RDL`'s reach, even for basic type checking. For example, database queries, as well as operations over heterogeneous data structures such as arrays and hashes, can be difficult to tame with static types. In this chapter, we introduce `CompRDL`, a system that adds *type-level computations* to `RDL`. This greatly increases the expressiveness of `RDL`'s types, and it allows us to type check many common operations that were previously out of reach.

3.1 Introduction

While there is a large body of research on adding static typing to dynamic languages [3, 4, 5, 8, 9, 11, 12, 16, 53, 54], existing systems have limited support for the case when *types* depend on *values*. Yet this case occurs surprisingly often, especially in Ruby libraries. For example, consider the following database query, written for a hypothetical Rails app:

```
Person.joins(:apartments).where({name: 'Alice', age: 30, apartments: {bedrooms: 2}})
```

This query uses the *ActiveRecord* DSL to join two database tables, people[1] and apartments, and then filter on the values of various columns (name, age, bedrooms) in the result.

We would like to type check such code, e.g., to ensure the columns exist and the values being matched are of the right types. But we face an important problem: what type signature do we give joins? Its return type—which should describe the joined table—depends on the value of its argument. Moreover, for n tables, there are n^2 ways to join two of them, n^3 ways to join three of them, etc. Enumerating all these combinations is impractical.

To address this problem, in this chapter we introduce `CompRDL`, which extends `RDL` to include method types with *type-level computations*, henceforth referred to as *comp types*. More specifically, in `CompRDL` we can annotate library methods with type signatures in which Ruby expressions can appear as types. During type checking, those expressions are evaluated to produce the actual type signature, and then typing proceeds as usual. For example, for the call to Person.joins, by using a singleton type for :apartments, a type-level computation can look up the database schemas for the receiver and argument and then construct an appropriate return type.[2]

Moreover, the same type signature can work for any model class and any combination of joins. And, because `CompRDL` allows arbitrary computation in types, `CompRDL` type signatures have access to the full, highly dynamic Ruby environment.

[1] Rails knows the plural of person is people.

[2] The use of type-level computations and singleton types could be considered dependent typing, but as our type system is much more restricted we introduce new terminology to avoid confusion (see § 3.2.4 for discussion).

This allows us to provide very precise types for the large set of Rails database query methods. It also lets us give precise types to methods of *finite hash types* (heterogeneous hashes), *tuple types* (heterogeneous arrays), and *const string types* (immutable strings), which can help eliminate type casts that would otherwise be required.

Note that in all these cases, we apply comp types to library methods whose bodies we do not type check, in part to avoid complex, potentially undecidable reasoning about whether a method body matches a comp type, but more practically because those library methods are either implemented in native code (hashes, arrays, strings) or are complex (database queries). This design choice makes `CompRDL` a particularly practical system which we can apply to real-world programs. To maintain soundness, we insert dynamic checks to ensure that these methods abide by their computed types at runtime. (§ 2.2 gives an overview of typing in `CompRDL`.)

We introduce λ^C, a core, object-oriented language that formalizes `CompRDL` type checking. In λ^C, library methods can be declared with signatures of the form $(a{<:}e_1/A_1) \to e_2/A_2$, where A_1 and A_2 are the conventional (likely overapproximate) argument and return types of the method. The precise argument and return types are determined by evaluating e_1 and e_2, respectively, and that evaluation may refer to the type of the receiver and the type a of the argument. λ^C also performs type checking on e_1 and e_2, to ensure they do not go wrong. To avoid potential infinite recursion, λ^C does not use type-level computations during this type checking process, instead using the conventional types for library methods. Finally, λ^C includes a rewriting step to insert dynamic checks to ensure library methods abide by their

computed types. We prove λ^C's type system is sound. (See § 2.4 for our formalism.)

We implemented `CompRDL` on top of `RDL` (§ 1.1). Since `CompRDL` can include type-level computation that relies on mutable values, `CompRDL` inserts additional runtime checks to ensure such computations evaluate to the same result at method call time as they did at type checking time. Additionally, `CompRDL` uses a lightweight analysis to check that type-level computations (and thus type checking) terminate. The termination analysis uses purity effects to check that calls that invoke iterator methods—the main source of looping in Ruby, in our experience—do not mutate the receiver, which could introduce non-termination. Finally, we found that several kinds of comp types we developed needed to include weak type updates to handle mutation in Ruby programs. (§ 3.4 describes our implementation in more detail.)

We evaluated `CompRDL` by first using it to write type annotations for 482 Ruby core library methods and 104 Rails database query methods. We found that by using helper methods, we could write very precise type annotations for all 586 methods with just a few lines of code on average. Then, we used those annotations to type check 132 methods across two Ruby APIs and four Ruby on Rails web apps. We were able to successfully type check all these methods in approximately 15 seconds total. In doing so, we also found two type errors and a documentation error, which we confirmed with the developers. We also found that, with comp types, type checking these benchmarks required 4.75× fewer type cast annotations compared to standard types, demonstrating comp types' increased precision. (§ 3.5 contains the results of our evaluation.)

Our results suggest that using type-level computations provides a powerful,

practical, and precise way to statically type check code written in dynamic languages.

3.2 Overview

We begin with a demonstration of comp types, by showing how they can be used to check the type correctness of Rails database queries (§ 3.2.1). We then show how comp types can reduce the need for type casts (§ 3.2.2) and how they can be used to type check some queries that incorporate raw SQL (§ 3.2.3). We conclude with a brief discussion (§ 3.2.4), including its use of dynamic checks, its lightweight termination checker, and its ability to encode constant folding.

3.2.1 Typing Ruby Database Queries

While `RDL`'s type system is powerful enough to type check Rails apps in general, it is actually very imprecise when reasoning about database (DB) queries. For example, consider Figure 3.1, which shows some code from the *Discourse* app. Among others, this app uses two tables, users and emails, whose schemas are shown on lines 2 and 3. Each user has an id, a username, and a flag indicating whether the account was *staged*. Such staged accounts were created automatically by *Discourse* and can be claimed by the email address owner. An email has an id, the email address, and the user_id of the user who owns the email address.

Next in the figure, we show code for the class User, which is a *model*, i.e., instances of the class correspond to rows in the users table. This class has one

```
1  # Table Schema
2      # users: { id: Integer, username: String, staged: bool }
3      # emails: { id: Integer, email: String, user_id: Integer }
4
5  class User < ActiveRecord::Base
6      type "(String, String) → %bool", typecheck: :model
7      def self.available? (name, email)
8          return false if reserved?(name)
9          return true if !User.exists? ({username: name})
10         # staged user accounts can be claimed
11         return User.joins(:emails).exists? ({staged: true, username: name, emails: {
               email: email }})
12     end
13 end
```

Figure 3.1: *Discourse* code that uses *ActiveRecord* to query a database.

```
1  type Table, :exists?, "(«schema_type(tself)») →Boolean"
2  type Table, :joins, "(t<:Symbol) →
3            «if t.is_a?(Singleton)
4            then Generic.new(Table, schema_type(tself).merge({t.val ⇒schema_type(t)}))
5            else Nominal.new(Table)
6            end »"
7
8  def schema_type(t)
9      if t.is_a?(Generic) ∧ (t.base == Table) # Table<T>
10         return t.param # return T
11     elsif t.is_a?(Singleton)    # Class or :symbol
12         table_name = t.val # get the class/symbol vale
13         table_type = RDL.db_schema[table_name]
14         return table_type.param
15     else # will only be reached for the nominal type Table
16         return ... # returns Hash<Symbol, Object>
17     end
18 end
```

Figure 3.2: Comp type annotations for query methods.

method, available?, which returns a boolean indicating whether the username and email address passed as arguments are available. The method first checks whether the username was already reserved (line 8, note the postfix if). If not, it uses the database query method exists? to see if the username was already taken (line 9). (Note that in Ruby, {a: b} is a hash that maps the symbol :a, which is suffixed with a colon when used as a key, to the value b.) Otherwise, line 11 uses a more complex query to check whether an account was staged. More specifically, this code joins the users and emails table and then looks for a match across the joined tables.

We would like to type check the exists? calls in this code to ensure they are type correct, meaning that the columns they refer to exist and the values being matched are of the right type. The call on line 9 is easy to check, as RDL can type the receiver User as having an exists? method that takes a particular *finite hash type* {c1: t1, ..., cn: tn} as an argument, where the ci are *singleton types* for symbols naming the columns, and the ti are the corresponding column types.

Unfortunately, the exists? call on line 11 is another story. Notice that this query calls exists? on the result of User.joins(:emails). Thus, to give exists? a type with the right column information, we need to have that information reflected in the return type of joins. Unfortunately, there is no reasonable way to do this in RDL, because the set of columns in the table returned by joins depends on both the receiver and the *value* of the argument. We could in theory overload joins with different return types depending on the argument type—e.g., we could say that User.joins returns a certain type when the argument has singleton type :emails. However, we would need to generate such signatures for every possible way of joining two tables

together, three tables together, etc., which quickly blows up. Thus, currently, RDL types this particular exists? call as taking a Hash<Symbol, Object>, which would allow type-incorrect arguments.

Comp types for DB Queries. To address this problem, CompRDL allows method type signatures to include computations that can, on-the-fly, determine the method's type. Figure 3.2 gives comp type signatures for exists? and joins. It also shows the definition of a helper method, schema_type, that is called from the comp types. The comp types also make use of a new generic type Table<T> to type a DB table whose columns are described by T, which should be a finite hash type.

Line 1 gives the type of exists?. Its argument is a comp type, which is a Ruby expression, delimited by «·», that evaluates to a standard type. When type checking a call to exists? (including those in the body of available?), CompRDL runs the comp type code to yield a standard type, and then proceeds with type checking as usual with that type.

In this case, to compute the argument type for exists?, we call the helper method schema_type with tself, which is a reserved variable naming the type of the receiver. The schema_type method has a few different behaviors depending on its argument. When given a type Table<T>, it returns T, i.e., the finite hash type describing the columns. When given a singleton type representing a class or a symbol, it uses another helper method RDL.db_schema (not shown) to look up the corresponding table's schema and return an appropriate finite hash type. Given any other type, schema_type falls back to returning the type Hash<Symbol, Object>.

This type signature already allows us to type check the exists? call on line 9. On this line, the receiver has the singleton type for the User class, so schema_type will use the second arm of the conditional and look up the schema for User in the DB.

Line 2 shows the comp type signature for joins. The signature's input type binds t to the actual argument type, and requires it to be a subtype of Symbol. For example, for the call on line 11, t will be bound to the singleton type for :emails. The return comp type can then refer to t. Here, if t is a singleton type, joins returns a new Table type that merges the schemas of the receiver and the argument tables using schema_type. Otherwise, it falls back to producing a Table with no schema information. Thus, the joins call on line 11 returns type

Table<{staged:%bool, username:String, id: Integer,

emails: {email:String, user_id: Integer }}>

That is, the type reflects the schemas of both the users and emails tables. Given this type, we can now type check the exists? call on line 11 precisely. On this line, the receiver has the table type given above, so when called by exists? the helper schema_type will use the first arm of the conditional and return the Table column types, ensuring the query is type checked precisely.

Though we have only shown types for two query methods in the figure, we note that comp types are easily extensible to other kinds of queries. Indeed, we have applied them to 104 methods across two DB query frameworks (§ 3.5). Furthermore, we can also use comp types to encode sophisticated invariants. For example, in

```
1 type Hash, :[], "(k) → v"
2 type Array, :first, "() → a"
3 type :page, "() → { info: Array<String>, title: String }''
4
5 type "() → String"
6 def image_url()
7       page[:info].first     # can't type check
8     # Fix: RDL.type_cast( page[:info], "Array<String>") .first
9 end
```

Figure 3.3: Type Casts in a Method.

Rails, database tables can only be joined if the corresponding classes have a declared *association*. We can write a comp type for joins that enforces this. (We omitted this in Figure 3.2 for brevity.)

Finally, we note that while we include a "fallback" case that allows comp types to default to less precise types when necessary, in practice this is rarely necessary for DB queries. That is, parameters that are important for type checking, such as the name of tables being queried or joined, or the names of columns be queried, are almost always provided statically in the code.

3.2.2 Avoiding Casts using Comp Types

In addition to letting us find type errors in code we could not previously type check precisely enough, the increased precision of comp types can also help eliminate type casts.

For example, consider the code in Figure 3.3. The first line gives the type signature for a method of Hash, which is parameterized by a key type k and a value type v (declarations of the parameters not shown). The specific method is

Hash#[],[3] which, given a key, returns the corresponding value. Notably, the form x[k] is desugared to x.[](k), and thus hash lookup, array index, and so forth are methods rather than built-in language constructs.

The second line similarly gives a type for Array#first, which returns the first element of the array. Here type variable a is the array's contents type (declaration also not shown). The third line gives a type for a method page of the current class, which takes no arguments and returns a hash in which :info is mapped to an Array<String> and :title is mapped to a String.

Now consider type checking the image_url method defined at the bottom of the figure. This code is extracted and simplified from a *Wikipedia* client library used in our experiments (§ 3.5). Here, since page is a no-argument method, it can be invoked without any parentheses. We then invoke Hash#[] on the result.

Unfortunately, at this point type checking loses precision. The problem is that whenever a method is invoked on a finite hash type {c1: t1, ..., cn: tn}, RDL (retroactively) gives up tracking the type precisely and promotes it to Hash<Symbol, t1 or...or tn> [20]. In this case, page's return type is promoted to Hash<Symbol, Array<String> or String>.

Now the type checker gets stuck. It reasons that first could be invoked on an array or a string, but first is defined only for the former and not the latter. The only currently available fix is to insert a type cast, as shown in the comment on line 8.

One possible solution would be to add special-case support for [] on finite hash types. However, this is only one of 54 methods of Hash, which is a lot of behavior

[3]Here we use the Ruby idiom that A#m refers to the instance method m of class A.

to special-case. Moreover, Ruby programs can *monkey patch* any class, including Hash, to change library methods' behaviors. This makes building special support for those methods inelegant and potentially brittle since the programmer would have no way to adjust the typing of those methods.

In CompRDL, we can solve this problem with a comp type annotation. More specifically, we can give Hash#[] the following type:

```
type Hash, :[], "(t<:Object) →
    «if tself.is_a? (FiniteHash) ∧ t.is_a?(Singleton)
    then tself.elts[t.val]
    else tself.value_type end»"
```

This comp type specifies that if the receiver has a finite hash type and the key has a singleton type, then Hash#[] returns the type corresponding to the key, otherwise it returns a value type covering all possible values (computed by value_type, definition not shown).

Notice that this signature allows image_url to type check without any additional casts. The same idea can be applied to many other Hash methods to give them more precise types.

Tuple Types. In addition to finite hash types, RDL has a special *tuple type* to model heterogeneous Arrays. As with finite hash types, RDL does not special-case the Array methods for tuples, since there are 124 of them. This leads to a loss of precision when invoking methods on values with tuple types. However, analogously to finite hash tables, comp types can be used to recover precision. As examples, the Array#first method can be given a comp type which returns the type of the first

element of a tuple, and the comp type for Array#[] has essentially the same logic as Hash#[].

Const String Types. As another example, Ruby strings are mutable, hence RDL does not give them singleton types. (In contrast, Ruby symbols are immutable.) This is problematic, because types might depend on string values. In particular, in the next section we explore reasoning about string values during type checking raw SQL queries.

Using comp types, we can assign singleton types to strings wherever possible. We introduce a new *const string* type representing strings that are never written to. CompRDL treats const strings as singletons, and methods on String are given comp types that perform precise operations on const strings and fall back to the String type as needed. We discuss handling mutation for const strings, finite hashes, and tuples in Section 3.4.

3.2.3 SQL Type Checking

As we saw in Figure 3.1, *ActiveRecord* uses a DSL that makes it easier to construct queries inside of Ruby. However, sometimes programmers need to include raw SQL in their queries, either to access a feature not supported by the DSL or to improve performance compared to the DSL-generated query.

Figure 3.4 gives one such example, extracted and simplified from *Discourse*, one of our subject programs. Here there are three relevant tables: posts, which stores posted messages; topics, which stores the topics of posts; and topic_allowed_groups,

```
1  # Table Schema
2     # posts table { id: Integer, topic_id: Integer, ... }
3     # topics table { id: Integer, title: String, ... }
4     # topic_allowed_groups table { group_id: Integer, topic_id: Integer }
5
6  # Query with SQL strings
7  Post.includes ( :topic )
8           .where(' topics.title  IN (SELECT topic_id FROM topic_allowed_groups WHERE
                'group_id' = ?)', self.id)
9
10 type Table, :where, "(t <: «if t.is_a?(ConstString)
11                            then sql_typecheck( tself, t)
12                             else schema_type(tself)
13                            end ») → « tself »"
```

Figure 3.4: Type Checking SQL Strings in *Discourse*.

which is used to limit the topics allowed by certain user groups.

Line 7 shows a query that includes raw SQL. First, the posts and topics tables are joined via the includes method. (This method does eager loading whereas joins does lazy loading.) Then where filters the resulting table based on some conditions. In this case, the conditions involve a nested SQL query, which cannot be expressed except using raw SQL that will be inserted into the final generated query.

This example also shows another feature: any ?'s that appear in raw SQL are replaced by additional arguments to where. In this case, the ? will be replaced by self.id.

We would like to extend type checking to also reason about the raw SQL strings in queries, since they may have errors. In this particular example, we have injected a bug. The inner SELECT returns a set of integers, but topics.title is a string, and it is a type error to search for a string in an integer set.

To find this bug, we developed a simple type checker for a subset of SQL, and

we wrote a comp type for where that invokes it as shown on line 10. In particular, if the type of the argument to where, here referred to by t, is a const string, then we type check that string as raw SQL, and otherwise we compute the valid parameters of where using the schema_type method from Figure 3.2. The result of where has the same type as the receiver.

The sql_typecheck method (not shown) takes the receiver type, which will be a Table with a type parameter describing the schema, and the SQL string. One challenge that arises in type checking the SQL string is that it is actually only a fragment of a query, which therefore cannot be directly parsed using a standard SQL parser. We solve this problem by creating a complete, but artificial, SQL query into which we inject the fragment. This query is never run, but it is syntactically correct so it can be parsed. Then, we replace any ?'s with placeholder AST nodes that store the types of the corresponding arguments.

For example, the raw SQL in Figure 3.4 gets translated to the following SQL query:

```
SELECT * FROM `posts` INNER JOIN `topics`
ON a.id = b.a_id
WHERE topics.title IN (SELECT topic_id FROM topic_allowed_groups WHERE `group_id
    ` = [Integer])
```

Notice the table names (posts, topics) occur on the first line and the ? has been replaced by a placeholder indicating the type Integer of the argument. Also note that the column names to join on (which are arbitrary here) are ignored by our type checker, which currently only looks for errors in the where clause.

Once we have a query that can be parsed, we can type check it using the DB

schema. In this case, the type mismatch between topics.title and the inner query will be reported.

In § 3.2.1, comp types were evaluated to produce a normal type signature. However, we use comp types in a slightly different way for checking SQL strings. The sql_typecheck method will itself perform type checking and provide a detailed message when an error is found. If no error is found, sql_typecheck will simply return the type String, allowing type checking to proceed.

3.2.4 Discussion

Now that we have seen `CompRDL` in some detail, we can discuss several parts of its design.

Dynamic Checks. In type systems with type-level computations, or more generally dependent type systems, comparing two types for equality is often undecidable, since it requires checking if computations are equivalent.

To avoid this problem, `CompRDL` only uses comp types for methods which themselves are not type checked. For example, Hash#[] is implemented in native code, and we have not attempted to type check *ActiveRecord*'s joins method, which is part of a very complex system.

As a result, type checking in `CompRDL` is decidable. Comp types are only used to type check method calls, meaning we will always have access to the types of the receiver and arguments in a method call. Additionally, in all cases we have encountered in practice, the types of the receiver and arguments are ground types (meaning

they do not contain type variables). Thus, comp types can be fully evaluated to non-comp types before proceeding to type checking.

For soundness, since we do not type check the bodies of comp type-annotated methods, `CompRDL` inserts dynamic checks at calls to such methods to ensure they match their computed types. For example, in Figure 3.3, `CompRDL` inserts a check that `page[:info]` returns an `Array`. This follows the approach of gradual [58] and hybrid [59] typing, in which dynamic checks guard statically unchecked code.

We should also note that although our focus is on applying comp types to libraries, they can be applied to any method at the cost of dynamic checks for that method rather than static checks. For example, they could be applied to a user-defined library wrapper.

Termination. A second issue for the decidability of comp types is that type-level computations could potentially not terminate. To avoid this possibility, we implement a termination checker for comp types. At a high level, `CompRDL` ensures termination by checking that iterators used by type-level code do not mutate their receivers and by forbidding type-level code from using looping constructs. We also assume there are no recursive method calls in type-level code. We discuss termination checking in more detail in § 3.4.

Value Dependency. We note that, unlike dependent types (e.g., Coq [60], Agda [61], F* [62]) where types depend directly on terms, in `CompRDL` types depend on the *types* of terms. For instance, in a comp type `(t<:Object) -> tres` the result type `tres` can

depend on the type `t` of the argument. Yet, since singleton types lift expressions into types, we could still use `CompRDL` to express some value dependencies in types in the style of dependent typing.

Constant Folding. Finally, in `RDL`, integers and floats have singleton types. Thus, we can use comp types to lift some arithmetic computations to the type level. For example, `CompRDL` can assign the expression `1+1` the type `Singleton(2)` instead of `Integer`. This effectively incorporates constant folding into the type checker.

While we did write such comp types for `Integer` and `Float` (see Table 3.1), we found that this precision was not useful, at least in our subject programs. The reason is that `RDL` only assigns singleton types to constants, and typically arithmetic methods are not applied to constant values. Thus, though we have written comp types for the `Integer` and `Float` libraries, we have yet to find a useful application for them in practice. We leave further exploration of this topic to future work.

3.3 Soundness of Comp Types

In this section we formalize `CompRDL` as λ^C, a core object-oriented calculus that includes comp types for library methods. We first define the syntax and semantics of λ^C (§ 3.3.1), and then we formalize type checking (§ 3.3.2). The type checking process includes a rewriting step to insert dynamic checks to ensure library methods satisfy their type signatures. Finally, we prove type soundness (§ 3.3.3). For clarity of presentation, we leave the full formalism and proofs to Appendix A. Here we provide only the key details.

Values	v	::=	`nil` | `true` | `false` | A
Expressions	e	::=	v | x | a | `self` | `tself` | A.`new` | $e;e$ | $e == e$
		|	`if` e `then` e `else` e | $e.m(e)$ | $\lceil A \rceil e.m(e)$
Meth. Types	σ	::=	$A \to A$
Lib. Meth. Types	δ	::=	σ | $(a{<:}e/A) \to e/A$
Programs	P	::=	`def` $A.m(x) : \sigma \ = \ e$ | `lib` $A.m(x) : \delta$ | $P; P$
Type Env.	Γ	::=	$\emptyset$ | $x{:}A$
Dyn. Env.	E	::=	$\emptyset$ | $x{:}v$
Class Table	CT	::=	$\emptyset$ | $A.m{:}\delta, CT$
Method Sets	$\mathcal{U}$	:	user-defined methods
	$\mathcal{L}$	:	library methods

$x, a \in$ var IDs, $m \in$ method IDs, $A \in$ class IDs, $\mathcal{U} \cap \mathcal{L} = \emptyset$

Figure 3.5: Syntax and Relations of λ^C.

3.3.1 Syntax and Semantics

Figure 3.5 gives the syntax of λ^C. *Values* v include `nil`, `true`, and `false`. To support comp types, class IDs A, which are the base types in λ^C, are also values. We assume the set of class IDs includes several built-in classes: *Nil*, the class of `nil`; *Obj*, which is the root superclass; *True* and *False*, which are the classes of `true` and `false`, respectively, as well as their superclass *Bool*; and *Type*, the class of base types A.

Expressions e include values v and variables x and a. By convention, we use the former in regular program expressions and the latter in comp types. The special variable `self` names the receiver of a method call, and the special variable `tself` names the *type* of the receiver in a comp type. New object instances are created with A.`new`. Expressions also include sequences $e;e$, conditionals `if` e `then` e `else` e, and method calls $e.m(e)$, where, to simplify the formalism, methods take one argument. Finally, our type system translates calls to library methods into *checked method calls*

$\lceil A \rceil e.m(e)$, which checks at run-time that the value returned from the call has type A. We assume this form does not appear in the surface syntax.

We assume the classes form a lattice with *Nil* as the bottom and *Obj* as the top. We write the least upper bound of A_1 and A_2 as $A_1 \sqcup A_2$. For simplicity, we assume the lattice correctly models the program's classes, i.e., if $A \leq A'$, then A is a subclass of A' by the usual definition. Lastly, three of the built-in classes, *Nil*, *True*, and *False*, are *singleton types*, i.e., they contain only the values `nil`, `true`, and `false`, respectively. Extending λ^C with support for more kinds of singleton types is straightforward.

Method Types σ are of the form $A' \to A$ where A' and A are the domain and range types, respectively. *Library Method Types* δ are either method types or have the form $(a{<:}e'/A') \to e/A$, where e' and e are expressions that evaluate to types and that can refer to the variables a and `tself`. The base types A' and A provide an upper bound on the respective expression types, i.e., for any a, expressions e' and e should evaluate to subtypes of A' and A, respectively. These upper bounds are used for type checking comp types (§ 3.3.2).

Finally, *programs* are sequences of method definitions and library method declarations.

Dynamic Semantics. The dynamic semantics of λ^C are the small-step semantics of Ren and Foster [5], modified to throw blame (§ 3.3.3) when a checked method call fails. They use *dynamic environments* E, defined in Figure 3.5, which map variables to values. We define the relation $\langle E, e \rangle \Downarrow e'$, meaning the expression e evaluates

to e' under dynamic environment E. The full evaluation rules use a stack as well (§ A), but we omit the stack here for simplicity.

Example. As an example comp type in the formalism, consider type checking the expression `true`. $\wedge$ (`true`), where the $\wedge$ method returns the logical conjunction of the receiver and argument. Standard type checking would assign this expression the type *Bool*. However, with comp types we can do better.

Recall that `true` and `false` are members of the singleton types *True* and *False*. Thus, we can write a comp type for the $\wedge$ method that yields a singleton return type when the arguments are singletons, and *Bool* in the fallback case:

$$
\begin{array}{l}
\texttt{lib}\ \mathit{Bool}.\wedge(x) : (a{<:}\mathit{Bool}/\mathit{Bool}) \to (\\
\quad \texttt{if}\ (\texttt{tself} == \mathit{True}).\wedge(a == \mathit{True})\ \texttt{then}\ \mathit{True}\\
\quad \texttt{else if}\ (\texttt{tself} == \mathit{False}).\vee(a == \mathit{False})\ \texttt{then}\ \mathit{False}\\
\quad \texttt{else}\ \mathit{Bool})/\mathit{Bool}
\end{array}
$$

The first two lines of the condition handle the singleton cases, and the last line is the fallback case.

3.3.2 Type Checking and Rewriting

Figure 3.6 gives a subset of the rules for type checking λ^C and rewriting λ^C to insert dynamic checks at library calls. The remaining rules, which are straightforward, can be found in Appendix A. These rules use two additional definitions from Figure 3.5. *Type environments* Γ map variables to base types, and the *class table* CT

$$\frac{}{\Gamma \vdash_{CT} A \hookrightarrow A : \mathit{Type}} \text{ C-Type}$$

$$\frac{\Gamma \vdash_{CT} e \hookrightarrow e' : A \quad CT(A.m) = A_1 \to A_2 \quad A.m \in \mathcal{U} \quad \Gamma \vdash_{CT} e_x \hookrightarrow e'_x : A_x \quad A_x \leq A_1}{\Gamma \vdash_{CT} e.m(e_x) \hookrightarrow e'.m(e'_x) : A_2} \text{ C-AppUD}$$

$$\frac{\Gamma \vdash_{CT} e \hookrightarrow e' : A \quad CT(A.m) = A_1 \to A_2 \quad A.m \in \mathcal{L} \quad \Gamma \vdash_{CT} e_x \hookrightarrow e'_x : A_x \quad A_x \leq A_1}{\Gamma \vdash_{CT} e.m(e_x) \hookrightarrow \lceil A_2 \rceil e'.m(e'_x) : A_2} \text{ C-AppLib}$$

$$\frac{\begin{array}{c}\Gamma \vdash_{CT} e \hookrightarrow e' : A \quad CT(A.m) = (a{<:}e_{t1}/A_{t1}) \to e_{t2}/A_{t2} \\ A.m \in \mathcal{L} \quad \Gamma \vdash_{CT} e_x \hookrightarrow e'_x : A_x \\ a{:}\mathit{Type}, \texttt{tself}{:}\mathit{Type} \vdash_{[\![CT]\!]} e_{t1} \hookrightarrow e'_{t1} : \mathit{Type} \\ a{:}\mathit{Type}, \texttt{tself}{:}\mathit{Type} \vdash_{[\![CT]\!]} e_{t2} \hookrightarrow e'_{t2} : \mathit{Type} \\ \langle [a \mapsto A_x][\texttt{tself} \mapsto A], e'_{t1} \rangle \Downarrow A_1 \quad A_x \leq A_1 \\ \langle [a \mapsto A_x][\texttt{tself} \mapsto A], e'_{t2} \rangle \Downarrow A_2\end{array}}{\Gamma \vdash_{CT} e.m(e_x) \hookrightarrow \lceil A_2 \rceil e'.m(e'_x) : A_2} \text{ C-App-Comp}$$

Figure 3.6: A subset of the type checking and rewriting rules for λ^C.

maps methods to their type signatures. We omit the construction of class tables, which is standard. We also use disjoint sets $\mathcal{U}$ and $\mathcal{L}$ to refer to the user-defined and library methods, respectively.

The rules in Figure 3.6 prove judgments of the form $\Gamma \vdash_{CT} e \hookrightarrow e' : A$, meaning under type environment Γ and class table CT, source expression e is rewritten to target expression e', which has type A.

Rule (C-Type) is straightforward: any class ID A that is used as a value is rewritten to itself, and it has type *Type*. We include this rule to emphasize that types are values in λ^C.

Rule (C-AppUD) finds the receiver type A, then looks up $A.m$ in the class table. This rule only applies when $A.m$ is user-defined and thus has a (standard)

method type $A_1 \to A_2$. Then, as is standard, the rule checks that the argument's type A_x is a subtype of A_1, and the type of the whole call is A_2. This rule rewrites the subexpressions e and e_x, but it does not itself insert any new checks, since user-defined methods are statically checked against their type signatures (rule not shown).

Rule (C-AppLib) is similar to Rule (C-AppUD), except it applies when the callee is a library method. In this case, the rule inserts a check to ensure that, at run-time, the library method abides by its specified type.

Rule (C-App-Comp) is the crux of λ^C's type checking system. It applies at a call to a library method $A.m$ that uses a type-level computation, i.e., with a type signature $(a{<:}e_{t1}/A_{t1}) \to e_{t2}/A_{t2}$. The rule first type checks and rewrites e_{t1} and e_{t2} to ensure they will evaluate to a type (i.e., have type *Type*). These expressions may refer to a and `tself`, which themselves have type *Type*. The rule then *evaluates* the rewritten e_{t1} and e_{t2} using the dynamic semantics mentioned above to yield types A_1 and A_2, respectively. Finally, the rule ensures that the argument e_x has a subtype of A_1; sets the return type of the whole call to A_2; and inserts a dynamic check that the call returns an A_2 at runtime. For instance, the earlier example of the use of logical conjunction would be rewritten to $\lceil \mathit{True} \rceil \texttt{true}.\wedge(\texttt{true})$.

There is one additional subtlety in Rule (C-App-Comp). Recall the example above that gives a type to *Bool*.$\wedge$. Notice that the type-level computation itself uses *Bool*.$\wedge$. This could potentially lead to infinite recursion, where calling *Bool*.$\wedge$ requires checking that *Bool*.$\wedge$ produces a type, which requires recursively checking that *Bool*.$\wedge$ produces a type etc.

To avoid this problem, we introduce a function $[\![CT]\!]$ that rewrites class table CT to drop all annotations with type-level expressions. More precisely, any comp type $(a{<:}e_1/A_1) \to e_2/A_2$ is rewritten to $A_1 \to A_2$. Then type checking type-level computations in the premise of (C-App-Comp) is done under the rewritten class table.

Note that, while this prevents the type checking rules from infinitely recursing, it does not prevent type-level expressions from themselves diverging. In λ^C, we assume this does not happen, but in our implementation, we include a simple termination checker that is effective in practice (§ 3.4).

3.3.3 Properties of λ^C.

Finally, we prove type soundness for λ^C. For brevity, we provide only the high-level description of the proof. The details can be found in Appendix A.

Blame. The type system of λ^C does not prevent null-pointer errors, i.e., `nil` has no methods yet we allow it to appear wherever any other type of object is expected. We encode such errors as *blame*. We also reduce to *blame* when a dynamic check of the form $\lceil A' \rceil A.m(v)$ fails.

Program Checking and *CT*. In the Appendix A we provide type checking rules not just for λ^C expressions but also for programs P. These rules are where we actually check user-defined methods against their types. We also define a notion of *validity* for a class table CT with respect to P, which enforces that CT's types

for methods and fields match the declared types in P, and that appropriate subtyping relationships hold among subclasses. Given a well typed program P, it is straightforward to construct a valid CT.

Type Checking Rules. In addition to the type checking and rewriting rules of Figure 3.6, we define a separate judgment $\Gamma \vdash_{CT} e : A$ that is identical to $\Gamma \vdash_{CT} e \hookrightarrow e : A$ except it omits the rewriting step, i.e., only performs type checking.

We can then prove soundness of the judgment $\Gamma \vdash_{CT} e : A$ using preservation and progress, and finally prove soundness of the type checking and rewriting rules as a corollary:

Theorem 1 (Soundness)**.** *For any expressions e and e', type A, class table CT, and program P such that CT is valid with respect to P, if* $\emptyset \vdash_{CT} e \hookrightarrow e' : A$ *then* e' *either reduces to a value, reduces to blame, or does not terminate.*

3.4 Implementation

We implemented `CompRDL` as an extension to `RDL`, a type checking system for Ruby [5, 8, 20, 63]. In total, `CompRDL` comprises approximately 1,170 lines of code added to `RDL`.

`RDL`'s design made it straightforward to add comp types, since types are already runtime values (§ 1.1). We extended `RDL` so that, when type checking method calls, type-level computations are first type checked to ensure they produce a value of type *Type* and then are executed to produce concrete types, which are then used

in subsequent type checking. Comp types use `RDL`'s contract mechanism to insert dynamic checks for comp types.

Heap Mutation. For simplicity, λ^C does not include a heap. By contrast, `CompRDL` allows arbitrary Ruby code to appear in comp types. This allows great flexibility, but it means such code might depend on mutable state that could change between type checking and the execution of a method call. For example, in Figure 3.2, type-level code uses the global table RDL.db_schema. If, after type checking the method available?, the program (pathologically) changed the schema of User to drop the username column, then available? would fail at runtime even though it had type checked. The dynamic checks discussed in § 3.2 and § 3.3 are insufficient to catch this issue, because they only check a method call against the initial result of evaluating a comp type; they do not consider that the same comp type might yield a new result at runtime.

To address this issue, `CompRDL` extends dynamic checks to ensure types remain the same between type checking and execution. If a method call is type checked using a comp type, then prior to that call at runtime, `CompRDL` will reevaluate that same comp type on the same inputs. If it evaluates to a different type, `CompRDL` will raise an exception to signal a potential type error. An alternative approach would be to re-check the method under the new type.

Of course, the evaluation of a comp type may itself alter mutable state. Currently, `CompRDL` assumes that comp type specifications are correct, including any mutable computations they may perform. If a comp type does have any erroneous

```
1  type :m1, ..., terminates: :+
2  type :m2, ..., terminates: :+
3  type :m3, ..., terminates: :-
4
5  type Array, :map, ..., terminates: :blockdep
6  type Array, :push, ..., pure: :-
7
8  def m1()
9    m2() # allowed: m2 terminates
10   m3() # not allowed: m3 may not terminate
11   while ... end # not allowed: looping
12
13   array = [1,2,3] # create new array
14   array.map { |val| val+1 } # allowed
15   array.map { |val| array.push(4) }
16   # not allowed: iterator calls impure method push
17 end
```

Figure 3.7: Termination Checking with `CompRDL`.

effects, program execution could fail in an unpredictable manner. Other researchers have proposed safeguards for this issue of effectful contracts by using guarded locations [64] or region based effect systems [65]. We leave incorporating such safeguards for comp types as future work. We note, however, that this issue did not arise in any comp types we used in our experiments.

Termination of Comp Types. A standard property of type checkers is that they terminate. However, because comp types allow arbitrary Ruby code, `CompRDL` could potentially lose this property. To address this issue, `CompRDL` includes a lightweight termination checker for comp types.

Figure 3.7 illustrates the ideas behind termination checking. In `CompRDL`, methods can be annotated with *termination effects* :+, for methods that always terminate (e.g., m1 and m2) and :- for methods that might diverge (e.g., m3).

CompRDL allows terminating methods to call other terminating methods (Line 9) but not potentially non-terminating methods (Line 10). Additionally, terminating methods may not use loops (Line 11). CompRDL assumes that type-level code does not use recursion, and leave checking of recursion to future work.

We believe it is reasonable to forbid the use of built-in loop constructs, and to assume no recursion, because in practice most iteration in Ruby occurs via methods that iterate over a structure. For instance, array.map { block } returns a new array in which the block, a *code block* or lambda, has been applied to each element of array. Since arrays are by definition finite, this call terminates as long as block terminates and does not mutate the array. A similar argument holds other iterators of Array, Hash, etc.

Thus, CompRDL checks termination of iterators as follows. Iterator methods can be annotated with the special termination effect :blockdep (Line 5), indicating the method terminates if its block terminates and is pure. CompRDL also includes *purity effect* annotations indicating whether methods are pure (:+) or impure (:-). A pure method may not write to any instance variable, class variable, or global variable, or call an impure method. CompRDL determines that a :blockdep method terminates as long as its block argument is pure, and otherwise it may diverge. Using this approach, CompRDL will allow Line 14 but reject reject Line 15.

Type Mutations and Weak Updates. Finally, to handle aliasing, our type annotations for Array, Hash, and String need to perform weak updates to type information when tuple, finite hash, and const string types, respectively, are mutated.

For example, consider the following code:

```
a = [1, 'foo']; if...then b = a else...end ; a[0]='one'
```

Here (ignoring singleton types for simplicity), a initially has the type t = [Integer, String], where t is a Ruby object, specifically an instance of RDL's TupleType class. At the join point after the conditional, the type of b will be a union of t and its previous type.

We could potentially forbid the assignment to a[0] because the right-hand side does not have the type Integer. However, this is likely too restrictive in practice. Instead, we would like to mutate t after the write. However, b shares this type. Thus we perform a *weak update*: after the assignment we mutate t to be [Integer ∪ String, String], to handle the cases when a may or may not have been assigned to b.

For soundness, we need to retroactively assume t was always this type. Fortunately, for all tuple, finite hash, and const string types τ, RDL already records all asserted constraints $\tau' \leq \tau$ and $\tau \leq \tau'$ to support promotion of tuples, finite hashes, and const strings to types Array, Hash, and String, respectively [20]. We use this same mechanism to replay previous constraints on these types whenever they are mutated. For example, if previously we had a constraint $\alpha \leq [\mathsf{Integer}, \mathsf{String}]$, and subsequently we mutated the latter type to [Integer∪String, String], we would "replay" the original constraint as $\alpha \leq [\mathsf{Integer} \cup \mathsf{String}, \mathsf{String}]$.

Library	Comp Type Definitions	Ruby LoC	Helper Methods
Ruby Core Library			
Array	114	215	15
Hash	48	247	15
String	114	178	12
Float*	98	12	1
Integer*	108	12	1
Database DSL			
ActiveRecord	77	375	18
Sequel	27	408	22
Total	586	1447	83

*Helper methods for Float and Integer are shared.

Table 3.1: Library methods with comp type definitions.

3.5 Experiments

We evaluated `CompRDL` by writing comp type annotations for a number of Ruby core and third party libraries (§ 3.5.1) and using these types to type check real-world Ruby applications (§ 3.5.2). We discuss the results of type checking these benchmarks, including the type errors we found in the process (§ 3.5.3). In all, we wrote 586 comp type annotations for Ruby library methods, used them to type check 132 methods across six Ruby apps, found three bugs in the process, and used significantly fewer manually inserted type casts than are needed using `RDL`.

3.5.1 Library Types

Table 3.1 details the library type annotations we wrote.

We chose to define comp types for these libraries due to their popularity and because, as discussed in § 3.2, they are amenable to precise typing with comp types. These types were written based on the libraries' documentation as well as manual

testing to ensure type specifications matched associated method semantics.

- *Ruby core libraries:* These are libraries that are written in C and automatically loaded in all Ruby programs. We annotate the methods from the Array, Hash, String, Integer, and Float classes.

- *ActiveRecord:* ActiveRecord is the most used object-relational model (ORM) DSL of the Ruby on Rails web framework. We wrote comp types for ActiveRecord database query methods.

- *Sequel:* Sequel is an alternative database ORM DSL. It offers some more expressive queries than are available in ActiveRecord.

Table 3.1 lists the number of methods for which we defined comp types in each library and the number of Ruby lines of code (LoC) implementing the type computation logic. The LoC count was calculated with sloccount [66] and does not include the line of the type annotation itself.

In developing comp types for these libraries, we discovered that many methods have the same type checking logic. This helped us write comp types for entire libraries using a few common helper methods. In total, we wrote comp type annotations for 586 methods across these libraries, comprising 1447 lines of type-level code and using 83 helper methods. Once written, these comp types can be used to type check as many of the libraries' clients as we would like, making the effort of writing them potentially very worthwhile.

3.5.2 Benchmarks

We evaluated `CompRDL` by type checking methods from two popular Ruby libraries and four Rails web apps:

- *Wikipedia Client* [67] is a Ruby wrapper library for the Wikipedia API.
- *Twitter Gem* [68] is a Ruby wrapper library for the Twitter API.
- *Discourse* [69] is an open-source discussion platform built on Rails. It uses ActiveRecord.
- *Huginn* [70] is a Rails app for setting up agents that monitor the web for events and perform automated tasks in response. It uses ActiveRecord.
- *Code.org* [71] is a Ruby app that powers `code.org`, a site that encourages people, particularly students, to learn programming. It uses a combination of ActiveRecord and Sequel.
- *Journey* [72] is a web application that provides a graphical interface to create surveys and collect responses from participants. It uses a combination of ActiveRecord and Sequel.

We selected these benchmarks because they are popular, well-maintained, and make extensive use of the libraries noted in § 3.5.1. More specifically, the APIs often work with hashes representing JSON objects received over HTTP, and the Rails apps rely heavily on database queries.

Since CompRDL performs type checking, we must provide a type annotation for any method we wish to type check. Our subject programs are very large, and hence annotating all of the programs' methods is infeasible. Instead, we focused on methods for which comp types would be most useful.

In *Wikipedia*, we annotated the entire Page API. To simplify type checking slightly, we changed the code to replace string hash keys with symbols, since RDL's finite hash types do not currently support string keys. In *Twitter*, we annotated all the methods of stream API bindings that made use of methods with comp types.

In *Discourse* and *Huginn*, we chose several larger Rails model classes, such as a User class that represents database rows storing user information. In *Code.org* and *Journey*, we type checked all methods that used Sequel to query the database. Within the selected classes for these four Rails apps, we annotated a subset of the methods that query the database using features that CompRDL supports. The features CompRDL does not currently support include the use of Rails *scopes*, which are essentially macros for queries, and the use of SQL strings for methods other than where.

Finally, because CompRDL performs type checking at runtime (similar to RDL—see § 1.1), we must first load each benchmark before type checking it. We ran the type checker immediately after loading a program and its associated type annotations.

Program	# Meth	LoC	Extra Anns.	Casts	RDL Casts	Time (s) Median ± SIQR	Test Time No Chk (s)	Test Time w/Chk. (s)	Err
API client libraries									
Wiki.	16	47	3	1	13	0.06 ± 0.00	6.3 ± 0.13	6.32 ± 0.11	0
Twitter	3	29	11	3	8	0.02 ± 0.00	0.07 ± 0.00	0.08 ± 0.00	0
Rails Applications									
Disc.	36	261	32	13	22	7.77 ± 0.39	80.24 ± 0.63	81.04 ± 0.34	0
Huginn	7	54	6	3	6	2.46 ± 0.29	4.30 ± 0.21	4.59 ± 0.48	0
Code.org	49	530	53	3	68	0.49 ± 0.01	2.49 ± 0.13	2.74 ± 0.02	1
Journey	21	419	78	14	59	4.12 ± 0.08	4.52 ± 0.22	4.76 ± 0.24	2
Total	132	1340	183	37	176	14.93 ± 0.77	97.93 ± 1.31	99.53 ± 1.20	3

Table 3.2: `CompRDL` type checking results.

3.5.3 Results

Table 3.2 summarizes our type checking results. In the first group of columns, we list the number of type checked methods and the total lines of code (computed with sloccount) of these methods. The third column lists the number of additional annotations we wrote for any global and instance variables referenced in the method, as well as any methods called that were not themselves selected for type checking. The last column in this group lists the number of type casts we added. Many of these type casts were to the result of JSON.parse, which returns a nested Hash/Array data structure depending on its string input. Most of the remaining casts are to refine types after a conditional test; it may be possible to remove these casts by adding support for occurrence typing [10]. We further discuss type casts, in particular the reduced type casting burden afforded by comp types, below.

Increased Type Checking Precision. Recall from § 3.2.2 that comp types can potentially reduce the need for programmer-inserted type casts. The next column reports how many casts were needed using normal `RDL` (i.e., no comp types). As

shown, approximately 4.75× fewer casts were needed when using comp types. This reflects the significantly increased precision afforded by comp types, which greatly reduces the programmer's annotation burden.

Performance. The next group of columns report performance. First we give the type checking time as the median and semi-interquartile range (SIQR) of 11 runs on a 2017 MacBook Pro with a 2.3GHz i5 processor and 8GB RAM. In total, we type checked 132 methods in approximately 15 seconds, which we believe to be reasonable. *Discourse* took most of the total time (8 out of 15 seconds). The reason turned out to be a quirk of *Discourse*'s design: it creates a large number of methods on-the-fly when certain constants are accessed. Type checking accessed those constants, hence the method creation was included in the type checking time.

The next two columns show the performance overhead of the dynamic checks inserted by `CompRDL`. We selected a subset of each app's test suite that directly tested the type checked methods, and ran these tests without ("No Chk") and with ("w/Chk") the dynamic checks. In aggregate (last row), checks add about 1.6% overhead, which is minimal.

Errors Found. Finally, the last column lists the number of errors found in each program. We were somewhat surprised to find any errors in large, well-tested applications. We found three errors. In *Code.org*, the `current_user` method was documented as returning a `User`. We wrote a matching `CompRDL` annotation, and `CompRDL` found that the returned expression—whose typing involved a comp type—has a hash

type instead. We notified the *Code.org* developers, and they acknowledged that this was an error in the method documentation and made a fix.

In *Journey*, `CompRDL` found two errors. First, it found a method that referenced an undefined constant `Field`. We notified the developers, who fixed the bug by changing the constant to `Question::Field`. This bug had arisen due to namespace changes. Second, it found a method that included a call with an argument `{:action => prompt, ...}` which is a hash mapping key `:action` to `prompt`. The value `prompt` is supposed to be a string or symbol, but as it has neither quotes nor begins with a colon, it is actually a call to the `prompt` method, which returns an array. The developers confirmed this bug.

When type checking the aforementioned methods in `RDL` (i.e., without comp types), two out of three of the bugs are hidden by other type errors which are actually false positives. These errors can be removed by adding four type casts, which would then allow us to catch the true errors. With `CompRDL`, however, we do not need any casts to find the errors.

3.6 Related Work

Types For Dynamic Languages. There is a large body of research on adding static type systems to dynamic languages, including Ruby [4, 5, 8], Racket [3, 9], JavaScript [16, 53, 54], and Python [11, 12]. To the best of our knowledge, this research does not use type-level computations.

Dependent typing systems for dynamic languages have been explored as well.

Ou et al. [73] formally model type-level computation along with effects for a dynamic language. Other projects have sought to bring dependent types to existing dynamic languages, primarily in the form of refinement types [28], which are base types that are refined with expressive logical predicates. As discussed in Chapter 2, refinement types have been applied to Ruby [21], Racket [10], and JavaScript [15, 29]. In contrast to `CompRDL`, these systems focus on type checking methods which themselves have dependent types. On the other hand, `CompRDL` uses type-level computations only for non-type checked library methods, allowing us to avoid checking comp types for equality or subtyping (§ 3.2.4). While sacrificing some expressiveness, this makes `CompRDL` especially practical for real-world programs.

Turnstile [74] is a metalanguage, hosted in Racket, for creating typed embedded languages. It lets an embedded DSL author write their DSL's type system using the host language macro system. There is some similarity to `CompRDL`, where comp types manipulate standard `RDL` types. However, `CompRDL` types are not executed as macros (which do not exist in Ruby), but rather in standard Ruby so they have full access to the environment, e.g., so the `joins` type signature can look up the DB schema.

Types For Database Queries. There have been a number of prior efforts to check the type safety of database queries. All of these target statically typed languages, an important distinction from `CompRDL`.

Chlipala [75] presents Ur, a web-specific functional programming language. Ur uses type-level computations over record types [76] to type check programs that con-

struct and run SQL queries. Indeed, CompRDL similarly uses type-level computations over finite hash types (analogous to record types) to type check queries. To the best of our knowledge, Ur focuses on computations over records. In contrast, CompRDL supports arbitrary type-level computations targeting unchecked library methods, making comp types more easily extensible to checking new properties and new libraries. As discussed in § 3.2, for example, comp types can not only compute the schema of a joined table, but also check properties like two joined tables having a declared Rails association. Further, comp types can be usefully applied to many libraries beyond database queries (§ 3.5).

Similar to Ur, Baltopoulos et al. [77] make use of record types over embedded SQL tables. Using SMT-checked refinement types, they can statically verify expressive data integrity constraints, such as the uniqueness of primary keys in a table and the validation of data inserted into a table. In addition to the contrast we draw with Ur regarding extensibility of types, to the best of our knowledge, this work does not include more intricate queries like joins, which are supported in CompRDL.

New Languages for Database Queries. Domain-specific languages have long been used to write programs with correct-by-construction, type safe queries. Leijen and Meijer [78] implement Haskell/DB, an embedded DSL that dynamically generates SQL in Haskell. Karakoidas et al. [79] introduce J%, a Java extension for embedding DSLs into Java in an extensible, type-safe, and syntax-checked way. Fowler and Brady [80] use dependent types in the language Idris to enforce safety protocols associated with common web program features including database queries

written in a DSL.

Language-integrated query is featured in languages like LINQ [81] and Links [82, 83]. This approach allows programmers to write database queries directly within a statically-typed, general purpose language.

In contrast to new DSLs and language-integrated query, our focus is on bringing type safety to an existing language and framework rather than developing a new one.

Dependent Types. Traditional dependent type systems are exemplified by languages such as Coq [60], Agda [61], and F* [62]. These languages provide powerful type systems that allow programmers to prove expressive properties. However, such expressive types may be too heavyweight for a dynamic language like Ruby. As discussed in § 3.2.4, our work has focused on applying a limited form of dependent types, where types depend on argument types and not arbitrary program values, resulting in a system that is practical for real-world Ruby programs.

Haskell allows for light dependent typing using the combination of singleton types [84] and type families [85]. `CompRDL`'s singleton types are similar to Haskell's, i.e., both lift expressions to types, and comp types are analogous to anonymous type families. However, unlike Haskell, `CompRDL` supports runtime evaluation during type checking, and thus does not require user-provided proofs.

Scala supports *path dependent types*, a limited form of type/term dependency in which types can depend on variables, but, as of Scala version 2, does not allow dependency on general terms [86]. This allows for reasoning about database queries.

For example, the Scala library *Slick* [87], much like our approach, allows users to write database queries in a domain specific language (a lifted embedding) and uses the query's AST to type check the query using Scala's path dependent types. Unlike `CompRDL`, Scala's path dependent types do not allow the execution of the full host language during type computations.

3.7 Conclusion

We presented `CompRDL`, a system for adding type signatures with type-level computations, which we refer to as comp types, to Ruby library methods. `CompRDL` makes it possible to write comp types for database queries, enabling us to type check such queries precisely. Comp type signatures can also be used for libraries over heterogeneous hashes and arrays, and to treat strings as immutable when possible. The increased precision of comp types can reduce the need for manually inserted type casts, thereby reducing the programmer's burden when type checking. Since comp type-annotated method bodies are not themselves type checked, `CompRDL` inserts run-time checks to ensure those methods return their computed types. We formalized `CompRDL` as a core language λ^C and proved its type system sound.

We implemented `CompRDL` on top of `RDL`, an existing type system for Ruby. In addition to the features of λ^C, our implementation includes run-time checks to ensure comp types that depend on mutable state yield consistent types. Our implementation also includes a termination checker for type-level code, and the type signatures we developed perform weak updates to type certain mutable methods.

Finally, we used `CompRDL` to write comp types for several Ruby libraries and two database query DSLs. Using these type signatures, we were able to type check six popular Ruby apps and APIs, in the process discovering three errors in our subject programs. We also found that type checking with comp types required 4.75× fewer type casts, due to the increased precision. Thus, we believe that `CompRDL` represents a practical approach to precisely type checking programs written in dynamic languages.

Chapter 4: Sound, Heuristic Type Annotation Inference for Ruby

The previous two chapters showed how to make `RDL` more expressive by introducing refinement types and type-level computations. However, `RDL` still requires the programmer to manually write type annotations for any type checked methods, as well as any variables and methods referenced in type checked code. This process can be tedious and time-consuming, especially when introducing types to large programs.

This chapter introduces `InferDL`, a system for automatically generating type annotations for the methods and variables in Ruby programs. While the traditional approach to this problem is to use constraint-based type inference, this chapter discusses why this approach sometimes fails to generate usable type annotations that reflect a programmer's intent, and explains how `InferDL` complements constraint solving with configurable heuristic rules in order to generate more useful annotations.

Acknowledgments

The core idea of refining constraint-based type inference with heuristic rules in order to produce more usable type annotations belongs to Ren [22]. They also contributed the STRUCT-TO-NOMINAL heuristic rule presented in this chapter. I

reimplemented this work, came up with the central algorithm and the idea of using configurable (rather than baked-in) heuristics, wrote the other 7 heuristic rules presented in this chapter, and conducted the presented evaluations.

4.1 Introduction

While many researchers have explored ways to add static types to dynamic languages, one key challenge in using such retrofitted type systems is finding type annotations for code that was previously untyped. *Type inference*, which aims to type check programs with few or no type annotations, is an obvious solution, and indeed there are several type inference systems for dynamic languages [4, 11, 16, 88]. Beyond type checking, type inference can also be extended to generate type annotations for program values. These annotations provide a useful form of documentation, and can be used in other forms of program analysis such as code completion for IDEs.

However, type inference systems typically aim to find the *most general type* for every position, i.e., the least restrictive possible type. If the type language is rich—particularly if it includes structural types—the most general possible annotations might be large, hard to read, and unnatural for programmers. For example, An et al. [88] describe a type inference system for Ruby that infers that a certain position accepts any object with >, <<, >>, &, and ^ methods. In contrast, a programmer would most likely, and much more concisely, say that position takes an Integer. Moreover, even if we are not interested in producing annotations, large, complex types can lead to difficult-to-understand error messages.

In this chapter we present `InferDL`, a novel Ruby type inference system that aims to infer sound and useful type annotations. More specifically, `InferDL` allows the programmer to specify heuristics for guessing type annotations. For example, one simple but effective heuristic is to guess the type Integer for any variables whose names end with id, num, or count. `InferDL` runs such heuristics for positions for which standard type inference is overly-general (meaning, among others, positions with inferred structural types). Heuristic guesses are added as additional type constraints and checked for consistency with the rest of the program. Only consistent solutions are kept. In this way, `InferDL` maintains soundness while producing a less general, but potentially more useful, solution than standard type inference. (§ 4.2 gives an overview of `InferDL`.)

We describe `InferDL` more formally on a core type and constraint language. We present standard constraint resolution rules, which rewrite a set of constraints into *solved form* from which most general solutions can be extracted. We describe that solution extraction procedure in detail and then show how to incorporate heuristics. (See § 4.3 for our formal description.)

We implemented `InferDL` as an extension to `RDL` (§ 1.1). We modified `RDL` to generate and resolve type constraints, run heuristics, and extract solutions to produce annotations. `InferDL` currently includes eight heuristics: one that replaces structural types with nominal types that match the structure; six that look at variable names, such as the one mentioned above for names ending in id, num, or count; and one that produces precise hash types. We also extended `RDL` with *choice types*, an idea inspired by variational typing [89] that helps type inference work in the

presence of overloaded methods. (§ 4.4 describes the implementation of `InferDL`.)

We evaluated `InferDL` by applying it to four Ruby on Rails apps for which `RDL` type annotations already existed [5, 90]. We note that our results are preliminary, and further work is needed to affirm they generalize beyond our benchmarks. For these apps, `InferDL` inferred 496 type annotations. Of these, 399 exactly matched or were more precise than the programmer-supplied annotations, compared to only 290 such annotations when not using heuristics. `InferDL` also found one previously unknown type error. We also applied `InferDL` to six additional Ruby programs for which we did not have previously written annotations, and `InferDL` found five previously unknown type errors. (§ 4.5 discusses our evaluation.)

We believe that `InferDL` is an effective type inference system and represents a promising approach to generating useful, sound type annotations.

4.2 Overview

We begin by discussing standard type inference, which generates and solves type constraints to yield type annotations. We then discuss why this approach alone can be inadequate and give a high-level overview of how `InferDL` uses heuristics to infer more precise, useful types.

4.2.1 Standard Type Inference

Figure 4.1 shows a code snippet taken from *Discourse*, a Ruby on Rails web app used in our evaluation (§ 4.5). The code defines two methods, normalize_username

```
1  class User < ActiveRecord::Base
2    # α → β
3    def self.normalize_username(name)
4      name.unicode_normalize.downcase if name.present?
5    end
6    # γ → δ
7    def self.find_by_name(name)
8      find_by(name_lower: normalize_username(name))
9    end
10 end
```

Constraints Generated
(1) $\alpha \leq [\mathsf{unicode\_normalize} : \bot \to \epsilon]$
(2) $\alpha \leq [\mathsf{present?} : \bot \to \zeta]$
(3) $\epsilon \leq [\mathsf{downcase} : \bot \to \eta]$
(4) $\eta \leq \beta$
(5) $\gamma \leq \alpha$
(6) $\mathsf{User} \leq \delta$
Resolved Constraints
(7) $\gamma \leq [\mathsf{unicode\_normalize} : \bot \to \epsilon]$
(8) $\gamma \leq [\mathsf{present?} : \bot \to \zeta]$

Figure 4.1: Method from the Discourse app, and the resulting constraints generated during inference.

and find_by_name, in the class User. Because User is a subclass of ActiveRecord::Base, it is a Rails *model*, meaning instances of the class represent rows of a database table.

Suppose we wish to infer types for these two methods using the standard, constraint-based approach. We first generate a *type variable* for the method argument and return types, as shown in the comments on lines 2 and 6, e.g., normalize_username takes a value of type α and returns type β. Then we analyze the method body, generating *constraints* of the form $x \leq y$, indicating that x must be a subtype of y. In this case we also say x is a *lower bound* on y and y is an *upper bound* on x.

The top portion of the table in Figure 4.1 shows the constraints generated from this example. Constraint (1) arises from the call name.unicode_normalize[1]. In this constraint, the *structural type* [unicode_normalize : $\bot \to \epsilon$] represents an object with a unicode_normalize method that takes no argument (here written $\bot$) and returns ϵ, a fresh type variable generated at the call. Hence, by standard subtyping rules, α must be a type that contains at least this method with appropriate argument and return types. Constraint (2) is similar.

Constraint (3) arises from calling downcase on the result of unicode_normalize. Constraint (4) arises because the result of the call to downcase is returned. Note that normalize_username may also return nil (if the conditional guard is false), but nil is a subtype of all other types in `InferDL`, so we omit this constraint here. Finally, constraint (5) arises from the call to normalize_username on line 8, and constraint (6) arises because User's find_by method returns a User (as indicated by find_by's type annotation, omitted here).

After generating constraints, `InferDL` performs *constraint resolution*, which applies a series of rewriting rules to the constraints. For example, one resolution rule is transitive closure: If $a \leq b$ and $b \leq c$ then we add constraint $a \leq c$ (see § 4.3 for a complete list of constraint resolution rules). In our example, constraint resolution generates the two new constraints (7) and (8).

During constraint resolution, if `InferDL` generates any invalid constraints, the program is untypable, and `InferDL` signals a type error. Otherwise, if constraint resolution terminates without finding any inconsistencies, then the program is ty-

[1]In Ruby, the parentheses in a method call are optional.

pable.

Solution Extraction. Many traditional type inference approaches have the singular goal of uncovering type errors, and hence they stop after propagating constraints. Since our goal is to also infer type annotations, we must go a step further by extracting a solution for all type variables from the constraints. The standard approach is to compute a *most general solution*. For a method type, this means computing the *least solution* for its return and the *greatest solution* for its arguments, which are the solutions that are least constraining on the method's callers.

Fortunately, after constraint resolution, the constraints are in *solved form* [91], which means that to extract a variable's solution we need only look at its lower and upper bounds. More specifically, for a return, we compute the union of its lower bounds, ignoring variables (since any transitive constraints from them have been propagated by resolution), and for an argument, we compute the intersection of its upper bounds. Thus, in our example, the solution for α and γ is $[\mathsf{unicode\_normalize} : \bot \to \epsilon, \mathsf{present?} : \bot \to \zeta]$, i.e., an object that has those methods, and the solution for δ is User.

However, notice there are some problems with producing type annotations using this approach. First, the solution for α and γ is in fact not fully expanded. Using the same approach, we could recursively compute a solution to ϵ to get the following solution for α and γ: $[\mathsf{unicode\_normalize} : \bot \to [\mathsf{downcase} : \bot \to \eta], \mathsf{present?} : \bot \to \zeta]$. However, such nested structural types are difficult to read and comprehend, and worse, in the presence of recursion, the type may not be expressible in finite form

without additional syntax.

Second, notice that η is the most general solution for β, and there is no most general solution for η and ζ that we can write down as ground terms (i.e., terms with no type variables). That is, in fact we cannot always ignore type variables in solutions, because they are needed to express relationships among different parts of the solution (here, η is the return type of `downcase` and ζ is the return type of `present?`). This makes understanding the most general solution even more complex and difficult.

4.2.2 Type Inference with Heuristics

`InferDL` aims to infer more useful, readable, and understandable type annotations by extending standard inference with heuristics that guess nominal types, or small unions of nominal types, as solutions. For example, so far δ has a nominal type as a solution, and we would like the same thing for other type variables. To ensure type annotations are consistent, `InferDL` adds any solutions found by heuristics to the constraints and runs constraint resolution afterward; if the result is a type error, the heuristic choice is rejected.

Using Heuristics. We illustrate the use of heuristics on our example with one particular rule, STRUCT-TO-NOMINAL, defined (in English) as follows:

> When an argument type variable's upper bounds include structural types, search all classes to see which have the methods in those types. If there are ten or fewer such classes, guess the union of these classes as the type

variable's solution.

Note that this rule matches by method name only and not by method type. We chose ten as a cutoff because in our experience, larger unions are less useful than the original structural type. In our running example, STRUCT-TO-NOMINAL can be applied to α and γ. It turns out that String is the only class that defines both unicode_normalize and present?, so the nominal type String would be our heuristic guess.

To ensure this guess is sound, we add String as a solution for variables α and γ to our constraints. More specifically, we add the *solution constraint* $\alpha = \mathsf{String}$ (and similarly for γ) to the constraints, where $a = b$ is shorthand for the pair of constraints $a \leq b$ and $b \leq a$. We then resolve these new constraints, which in this case does not lead to any inconsistency, so we accept String as the solution.

Moreover, the additional constraints on α and γ in turn yield better solutions elsewhere because:

$\Rightarrow$ $\mathsf{String} \leq \alpha$ is added as a constraint. Transitively propagating to α's upper bounds yields...

$\Rightarrow$ $\mathsf{String} \leq [\mathsf{unicode\_normalize} : \bot \to \epsilon]$. To check this constraint, we look up String's unicode_normalize method type and generate the constraint...

$\Rightarrow$ $\bot \to \mathsf{String} \leq \bot \to \epsilon$. Propagating to the methods' return types yields...

$\Rightarrow$ $\mathsf{String} \leq \epsilon$. Transitively propagating through ϵ yields...

$\Rightarrow$ $\mathsf{String} \leq [\mathsf{downcase} : \bot \to \eta]$. Looking up String's type for downcase, we get

the constraint...

$\Rightarrow \bot \to \mathsf{String} \leq \bot \to \eta$, and propagating to return types yields...

$\Rightarrow \mathsf{String} \leq \eta$. Finally, propagating to η's upper bound yields...

$\Rightarrow \mathsf{String} \leq \beta$

Thus, now β has nominal type String as a solution. Putting this together with the (most general) solution for δ and the (heuristic) solutions for α and γ, `InferDL` has now inferred fully nominal type annotations for our example:

normalize_username: String → String

find_by_name: String → User

Implementing Heuristics. in `InferDL`, heuristics are not baked-in. Rather, they can be created by the programmer, allowing heuristics to be adapted if needed to the target program. As an example, consider the following method, taken from the Rails app *Journey* and slightly simplified:

```
def self.find_answer(response, question)
  where(response: response.id, question: question.id).first
end
```

We wish to infer the type of find_answer. However, notice that only id is called on each argument. In Rails, the method id is typically defined for all model classes, e.g., in *Journey*, 48 different classes include an id method. This means STRUCT-TO-NOMINAL will fail to infer a precise annotation for the arguments in this case.

Instead, we develop another heuristic that takes advantage of a common practice: Ruby programmers often name a variable after the class of the value it will hold, especially for Rails models. Indeed, the arguments response and question are intended to take instances of the model classes Response and Question, respectively. We define a new heuristic IS_MODEL that guesses types based on this convention:

```
RDL::Heuristic.add :is_model { |var|
  if (var.base_name.camelize.is_rails_model?)
  then var.base_name.to_type end }
```

To define this heuristic, we call RDL::Heuristic.add, passing the name of the heuristic, in this case :is_model, and a *code block*. Code blocks are Ruby's version of anonymous functions or lambdas. The RDL::Heuristic.add method expects a code block that takes a single argument, which is the type variable whose solution the heuristic should guess. Note that in `InferDL`, which is built on `RDL`, types are actual values we can compute with; we discuss this in greater detail in § 4.4. The code block returns either nil, if there is no guess, or the guessed type.

The IS_MODEL heuristic consists of a single if statement. The guard calls, in order, var.base_name, to return the name of the variable as a String; the Rails method camelize to camel-case this string; and finally is_rails_model?, a method we defined (code omitted) to determine if there exists a Rails model with the same name as the receiver. If this last condition is true, the code block calls to_type (code omitted) to return the nominal type for the model class. Otherwise, by standard Ruby semantics the conditional will return nil.

During type inference, `InferDL` runs all heuristics for each type variable, accepting the solution from the first heuristic that produces a consistent type. For

$$
\begin{array}{lrcl}
\textit{Types} & \tau & ::= & \alpha \mid A \mid [m : \tau_m] \mid \\
 & & & \tau \cup \tau \mid \tau \cap \tau \mid \bot \mid \top \\
\textit{Method Types} & \tau_m & ::= & \tau \to \tau \\
\textit{Constraints} & C & ::= & \tau \leq \tau \mid C \cup C
\end{array}
$$

$A \in$ class IDs, $m \in$ meth IDs

Figure 4.2: Core types and constraints.

find_answer, the IS_MODEL heuristic produces appropriate nominal types for the arguments, which then become the final type annotations for those positions.

4.3 Constraints, Solutions, and Heuristics

In this section, we describe `InferDL` more formally. For brevity, we do not define a core language, nor do we describe constraint generation in detail. Rather, we focus on the language of types and constraints, constraint resolution, solution extraction, and heuristics.

Figure 4.2 formally defines a core subset of the types and constraints in `InferDL`. Types τ include type variables α and nominal types A, which is the set of class IDs. Structural types $[m : \tau_m]$ name a method m and its corresponding method type τ_m. For brevity, structural types can only comprise a single method, and method types may only take a single argument. Types also include union types $\tau \cup \tau$, intersection types $\tau \cap \tau$, the bottom type $\bot$, and the top type $\top$. Constraints C consist of subtyping constraints $\tau_1 \leq \tau_2$ and unions of constraints $C_1 \cup C_2$, which allow us to build up sets of constraints.

Generating Constraints. Constraint generation is a straightforward modification of standard type checking in which, instead of type rules checking constraints, we view them as *generating* constraints. For example, the rule for typing a method call is

$$\frac{\Gamma \vdash e_1 : \tau_{rec} \qquad \Gamma \vdash e_2 : \tau_{arg} \qquad \tau_{rec} \leq [m : \tau_{arg} \to \tau_{ret}] \qquad \tau_{ret} \text{ is fresh}}{\Gamma \vdash e_1.m(e_2) : \tau_{ret}}$$

This rule types a method call $e_1.m(e_2)$ (where each e is an expression) in type environment Γ (a map from local variables to types), yielding type τ_{ret}. To apply this rule, we recursively type the receiver and the argument, yielding types τ_{rec} and τ_{arg}, respectively. We then generate a constraint $\tau_{rec} \leq [m : \tau_{arg} \to \tau_{ret}]$, where τ_{ret} is a fresh type variable. Then, the return type of the method call has type τ_{ret}. By convention, we assume any constraint in the premise of a rule is automatically added to a global set of constraints C.

Full details of type checking rules for a core Ruby language can be found in section 3.3, or in Ren and Foster [5] or Kazerounian et al. [90], all of which formalize Ruby in a core language and provide type checking rules. As with the above example, those rules can be turned into inference rules by viewing them as generating constraints and adjusting them as needed to make all constraints explicit, e.g., in the rule above, we specify the type of e_1 as τ_{rec} and write an explicit constraint on τ_{rec}, rather than implicitly constraining e_1's type to have a particular shape in the rule.

$$
\begin{array}{ll}
(1) & C \cup \tau_1 \leq \alpha \cup \alpha \leq \tau_2 \Rightarrow C \cup \tau_1 \leq \alpha \cup \alpha \leq \tau_2 \cup \tau_1 \leq \tau_2 \\
(2) & C \cup A \leq A' \Rightarrow C \quad \text{if } A = A' \text{ or } A \text{ is a subclass of } A' \\
(3) & C \cup A \leq A' \Rightarrow \mathit{error} \quad \text{if } A \neq A' \text{ and } A \text{ is not a subclass of } A' \\
(4) & C \cup A \leq [m : \tau_1 \to \tau_2] \Rightarrow C \cup \tau_1 \leq \tau_1' \cup \tau_2' \leq \tau_2 \\
 & \qquad\qquad \text{if } A \text{ has method } m \text{ with type } \tau_1' \to \tau_2' \\
(5) & C \cup A \leq [m : \tau_1 \to \tau_2] \Rightarrow \mathit{error} \quad \text{if } A \text{ has no method } m \\
(6) & C \cup (\tau_1 \cup \tau_2 \leq \tau_3) \Rightarrow C \cup \tau_1 \leq \tau_3 \cup \tau_2 \leq \tau_3 \\
(7) & C \cup (\tau_1 \leq \tau_2 \cap \tau_3) \Rightarrow C \cup \tau_1 \leq \tau_3 \cup \tau_2 \leq \tau_3 \\
(8) & C \cup \bot \leq \tau \Rightarrow C \\
(9) & C \cup \tau \leq \top \Rightarrow C
\end{array}
$$

Figure 4.3: Standard constraint resolution rules.

Resolving Constraints. Figure 4.3 gives standard constraint resolution rules. Each rule has the form $C \Rightarrow C'$, meaning a set of constraints matching C can be rewritten to C'. The rules are applied exhaustively until they either yield *error* or no additional constraints can be generated.

Rule (1) adds transitive constraints, as discussed earlier. Rule (2) eliminates a constraint among two nominal types as long as the subtyping is valid. On the other hand, Rule (3) produces *error* if there is an inconsistent constraint among nominal types. Rule (4) handles constraints of the form $A \leq [m : \tau_1 \to \tau_2]$. In this case, if A has a method m of some type $\tau_1' \to \tau_2'$, we erase the constraint and add two new constraints on the argument and return types. If A does not have a method m, Rule (5) yields *error*. Finally, Rules (6) and (7) simplify unions on the left and intersections on the right of a constraint, respectively, and Rules (8) and (9) eliminate constraints with $\bot$ on the left and $\top$ of the right, respectively.

4.3.1 Solution Extraction

Recall from § 4.2.1 that after generating and resolving constraints, the next step in standard type inference is to extract solutions for type variables. More precisely, standard type inference produces solutions using the following procedure:

```
procedure STANDARD_SOLUTION(C, α)
    if (α represents arg) then
        sol = ⊤
        for each constraint α ≤ τ ∈ C do
            sol = sol ∩ τ
    else                                          ▷ α represents return
        sol = ⊥
        for each constraint τ ≤ α ∈ C do
            sol = sol ∪ τ
    return sol
```

As discussed earlier, to derive the most general solution, for each return position we compute the union of its lower bounds, and for each argument position we compute the intersection of its upper bounds.

`InferDL` uses the same procedure as a subroutine to its heuristic inference algorithm, described next.

Heuristics. Formally, we can model `InferDL`'s heuristics as a set of additional constraint resolution rules beyond those in Figure 4.3. For example, we can express

STRUCT-TO-NOMINAL as the following constraint rewriting rule:

$$\text{STRUCT-TO-NOMINAL}(C, \alpha) =$$

$$C' \cup \alpha \leq [m_1 : ...] \cup \ldots \cup \alpha \leq [m_n : ...] \;\Rightarrow$$

$$C' \cup (\alpha = (A_1 \cup \ldots \cup A_k))$$

if $k \leq 10$ and $A_1, ..., A_k$ are all classes with $m_1 ... m_n$.

This rule applies to a type variable α that has one or more structural type upper bounds. If there are at most 10 classes $A_1 \ldots A_k$ matching those structural types, then we replace the structural constraints with a solution $A_1 \cup \ldots \cup A_k$ for α. Recall from § 4.2 that a *solution constraint* of the form $\tau_1 = \tau_2$ is shorthand for the two constraints $\tau_1 \leq \tau_2$ and $\tau_2 \leq \tau_1$.

Given a set H of heuristic rules, `InferDL` uses the following procedure to try each rule until some rule succeeds or all rules fail:

```
procedure HEURISTIC_SOLUTION(H, C, α)
    for h ∈ H do
        C', sol = h(C, α)
        C' = resolve(C')
        if C' ≠ error and sol ≠ nil then
            return C', sol
    return C, nil
```

Here we abuse notation slightly and assume that heuristic rules return a pair C', sol, where C' is the new set of constraints after running the rule, and sol is the solution found for α. If h does not match C, then h returns the pair $\mathsf{nil}, \mathsf{nil}$. When h does return a set of constraints C' and a solution, we perform constraint resolution on C' to propagate the new solution and detect any inconsistencies in the set of

constraints. This is done by calling the RESOLVE procedure, which simply invokes the constraint resolution rules of Figure 4.3. If the resulting resolved constraint set C' is valid, then we return C' and *sol*. If all heuristics run without finding a valid solution, we return the original constraints C and *nil*.

Notice that an important consequence of this formulation is that the order in which heuristics are run matters, since only the first guessed solution will be returned. Future work could examine how to overcome this reliance on ordering and handle the case that heuristics return differing solutions.

Extracting Solutions. The next step is to combine standard and heuristic solution extraction, doing the latter when a standard solution is overly-general. Thus, we need to define what *overly-general* means. Based on our prior experience, we believe that, in most cases, nominal types are far easier for programmers to understand and use than structural types because they are smaller and simpler. The developers of Sorbet [92], another Ruby type checker, feel the same way—they have found that structural types can be less intuitive and more difficult to read when used in error messages [93].

Generalizing this insight, we define an *overly-general* solution as any non-nominal type, that is, any type of the form α, $\tau \cup \tau$, $\tau \cap \tau$, $[m : \tau_m]$, $\bot$, and $\top$. Our motivation for treating union and intersection types as overly-general is the same as for structural types: they make types bigger and more complex. We treat the top and bottom types as overly-general since they are only ever used as solutions when a type variable has no constraints, and we consider type variables overly-general

since they represent unknown types. We do note that, in general, there is no single correct definition of overly-general, and we leave exploring alternative definitions to future work.

Next, we can provide the pseudocode for solution extraction of a type variable:

```
procedure EXTRACT_SOLUTION(H, C, α)
    sol = STANDARD_SOLUTION(C, α)
    if overly_general(sol) then
        C', sol_h = HEURISTIC_SOLUTION(H, C, α)
        if sol_h ≠ nil then
            sol = sol_h
    else
        C' = resolve(C ∪ (α = sol))
    return C', sol
```

This procedure first extracts a standard solution for the given α and C. If the resulting solution is overly-general, it calls HEURISTIC_SOLUTION to possibly yield a better solution for α. We only use the new solution if it is non-nil (note that, with the definition of HEURISTIC_SOLUTION, the set of constraints will be unchanged in the event that the heuristic solution is nil). Otherwise, if the standard solution was not too general, we add the new solution to the set of constraints and perform constraint resolution. We perform constraint resolution again because, just like heuristic solutions can lead to other solutions, so too can standard solutions.

Finally, as shown in § 4.2.2, one extracted solution may lead to the discovery of other solutions. Thus, we continue to extract solutions for type variables until no new constraints are generated. More precisely, the following procedure takes H, C, and a set of all type variables $\mathcal{V}$ as input, and extracts solutions until no new constraints are generated.

procedure EXTRACT_ALL_SOLUTIONS(H, C, $\mathcal{V}$)
  $solutions\_map = \{\ \}$
  repeat
    for each $\alpha \in \mathcal{V}$ **do**
      C, sol = EXTRACT_SOLUTION(H, C, α)
      $solutions\_map[\alpha] = sol$
  until no new constraints are added to C
  return $solutions\_map$

4.4 Implementation

`InferDL` is built on top of the Ruby type checker `RDL` [20]. An overview of `RDL` can be found in § 1.1. In this section, we briefly describe how we extended `RDL` to perform type inference, and some of the other implementation challenges in `InferDL`.

Adding Standard Inference. `InferDL` extends `RDL` so that inferred methods are specified with a call to RDL.infer:

```
RDL.infer User, 'self.normalize_username', time: :later
```

Then, when RDL.do_infer :later is called, `InferDL` runs type inference to produce type annotations for any method associated with :later.

`RDL` already included type variables to support parametric polymorphism. `InferDL` extends type variables to store constraints as a list of upper and lower bounds on each type variable. Then, to perform constraint generation, `InferDL` modifies `RDL`'s type checker so that, whenever two types are checked for subtyping, and at least one of the types is a type variable, we store the subtyping constraint. After constraint generation, `InferDL` performs constraint resolution and solution

extraction, as explained in § 4.3.

Heuristics. Though heuristics are not baked-in to `InferDL` and are thus configurable, we have written eight heuristics that we found useful in practice, listed below. Recall from § 4.3.1 that heuristics are applied in a specified order. We list heuristics in the order in which they are applied.

- IS_MODEL: See § 4.2.2 for a description. This rule is only used for Rails apps.
- IS_PLURALIZED_MODEL: If a variable name is the pluralized version of the name of model X, then guess solution Array<X> ∪ ActiveRecord_Relation<X>. Note that ActiveRecord_Relation is a data structure provided by Rails that extends common array operations with some database queries. This rule is only used for Rails apps.
- STRUCT-TO-NOMINAL: See § 4.2.2 or § 4.3.1.
- INT_NAMES: If a variable name ends with id, count, or num, guess solution Integer.
- INT_ARRAY_NAME: If a variable name ends with ids, counts, or nums, guess solution Array<Integer>.
- PREDICATE_METHOD: If a method name ends with ?, guess solution %bool (`RDL`'s boolean type) for the method return type.
- STRING_NAME: If a variable name ends with name, guess solution String.

- HASH_ACCESS: Like all other values, hashes are objects in Ruby, not built-in constructs. As such, they are accessed using the method [], and written to using the method []=. This rule states that, if all of an argument type variable's upper bounds are structural types consisting of the methods [] and []=, and all of the keys given to these methods are symbols, then guess the solution that is the finite hash type consisting of the keys and the unions of corresponding assigned values for these methods. For example, if an argument type variable α had constraints $\alpha \leq$ [:[]= : :id $\to$ Integer], $\alpha \leq$ [:[]= : :id $\to$ String], and $\alpha \leq$ [:[]= : :name $\to$ String], then the solution for α would be the finite hash type { id: String $\cup$ Integer, name: String }. This type says that α is a hash mapping the symbol :id to a String or Integer, and :name to a String.

As discussed in § 4.3.1, `InferDL` treats type variables, union, intersection, structural, and bottom and top types as overly-general. In our implementation, we also treat Object and nil (which were omitted from the formalism for brevity but are almost the same as the top and bottom types, respectively) as overly-general. In addition to nominal types, `RDL` also includes several additional kinds of types that we treat as sufficiently precise: generic types (which are parameterized nominal types), finite hash and tuple types (which are more precise versions of Hash and Array types), and singleton types (such a type has only one value as an inhabitant).

Choice Types. Ruby methods often have intersection types, which pose a challenge for type inference. Consider the Array indexing method [], which has the following type in `RDL`:

$$(\mathsf{Integer}) \to \mathsf{t} \;\cap\; (\mathsf{Range<Integer>}) \to \mathsf{Array<t>}$$

Here, t is the type parameter for the Array class. When given an Integer index, [] returns a single element, and when given a Range<Integer> corresponding to multiple indexes, [] returns the subarray of elements at those indexes. Now consider the following contrived code snippet:

```
def foo(x) arr = [1,2,3]; return arr[x] + 1; end
```

Suppose that `InferDL` assigns x the type variable α. Then, when analyzing the call arr[x], we encounter a problem: During constraint generation, we do not know α's solution. One choice would be to assume both arms of the intersection are possible. However, then the result of the method call would have type Integer $\cup$ Array<Integer>, which leads to a type error when analyzing the larger expression arr[x] + 1, since we cannot add an Array to an Integer.

To address this issue, we introduce *choice types*, a type system feature loosely inspired by variational type checking [89]. A choice type, written $\mathsf{Choice}_i(\tau_1, \ldots, \tau_n)$, represents a choice among the types τ_j. Each choice type also has a label i. During inference, if one τ_j of a choice type would result in a type error, then arm j is eliminated from all choice types with the same index i.

In the example above, the call to arr[x] would result in the constraint

(1) $\alpha \leq \mathsf{Choice}_1(\mathsf{Integer}, \mathsf{Range<Integer>})$

because `InferDL` reasons that it has a choice between the two input types of Array's [] method. Additionally, the return type of arr[x] would be

(2) $\mathsf{Choice}_1(\mathsf{Integer}, \mathsf{Array<Integer>})$

representing both possible returns. Both choice types have the label 1, indicating that they are decided together. Then, when type checking the call arr[x] + 1, InferDL would recognize that the Array<Integer> arm of type (2) results in a type error, and it would eliminate that arm from both (2) and (1). Effectively, this would retroactively make the return type of arr[x] be the sole type Integer, and it would allow us to infer α's solution as the sole type Integer. If InferDL ever eliminates all arms of a choice type, it raises a type error.

Library Types. RDL comes with type annotations for Ruby's core and standard libraries, as well as for common Rails methods and methods from *Sequel*, a popular framework for database queries. However, it is common for Ruby programs to make extensive use of other third-party libraries as well. Typically, a type checker would require type annotations for any such methods used in the subject program. But writing these type annotations is burdensome and often requires knowledge of the library's implementation. This task is all the more tedious in the context of type inference, where the programmer aims to infer type annotations, not write them.

InferDL's approach to library types avoids this issue. During constraint generation, if InferDL encounters a call to a method that both lacks a type annotation and is not itself the target of inference, InferDL finds the method definition to determine the method's arity. InferDL then creates a type signature for the method with fresh type variables for the return and each argument. If no method definition is found, InferDL raises a type error.

This approach is similar to type inference for a method, except we do not

generate constraints from the method body. Thus, type inference might be unsound, producing a solution that would be impossible if we knew the library method's implementation. However, in practice, we found this approach was essential for allowing us to apply inference to each new benchmark, and it also helped us discover a previously unknown bug in one of our benchmarks (§ 4.5.1).

Variable Types. In addition to method types, `InferDL` can infer type annotations for the three non-local kinds of Ruby variables: global, class, and instance variables. Recall that STANDARD_SOLUTION (§ 4.3.1), used as a subroutine in `InferDL`, aims to infer most general types for methods. However, observe that variables are both read from and written to, i.e., for a field `@x`, there is conceptually a getter of type $\bot \to \alpha$ and a setter of type $\alpha \to \bot$. Notice that the α appears both co- and contravariantly. Hence, unlike method arguments and returns, it is not the case that a least or greatest solution will always be most general.

Instead, to extend STANDARD_SOLUTION to variables, we take an ad-hoc approach: we take the intersection of the upper bounds on a variable's type when it has upper bounds and, if not, we take the union of its lower bounds. We found this approach works reasonably well in practice, and we apply the same heuristic rules, with the same definition of overly-general, as for method types.

4.5 Evaluation

We evaluated `InferDL` on four Ruby on Rails web apps:

- *Journey* [72] is a web app that provides a graphical interface to create surveys

and collect responses from participants.

- *Discourse* [69] is an open-source discussion platform built on Rails.
- *Code.org* [71] is a Rails app that powers `code.org`, a programming education website.
- *Talks* [94] is a Rails app written by one of the authors for sharing talk announcements.

We chose these apps because they have all been used as type checking benchmarks in prior work [5, 90]. Thus, we could use the previously written type annotations for these apps as "gold standards" to compare against. We inferred type annotations for all methods and global, class, and instance variables for which type annotations existed in prior work. This includes both methods (and the variables they used) that were type checked in prior work and those that were not checked, but were annotated to assist in the type checking of other methods.

The only additional type annotations used were for the special Rails params hash, which contains values that come from a user's browser. The params hash always maps symbols to various types of objects. Without annotations, `InferDL` typically infers that params has type Hash<K, V>, where V was the union of all observed value types for the hash. This effectively treats all values from the hash as belonging to the same type, which causes false positive type errors during inference. Rather than add type casts for these cases, we instead used the type annotations for params from the prior type checking work [5, 90]. In the future, we plan to incorporate special handling of the Rails params hash to avoid this issue.

Program	Num Meths	Meth Typs	Var Typs	Total Typs	Type Casts	Time (s) Median ± SIQR
						heur / *std*
Journey	23	33	26	59	1	1.68 ± 0.05 / 1.01 ± 0.09
Discourse	43	77	0	77	0	7.70 ± 0.64 / 0.59 ± 0.04
code.org	74	152	12	164	4	20.1 ± 0.22 / 5.01 ± 0.10
Talks	110	149	47	196	8	2.43 ± 0.04 / 2.08 ± 0.15
Total	250	411	85	496	13	31.91 ± 0.95 / 8.68 ± 0.39

Table 4.1: Type inference: number of targets and time performance.

Below, we discuss the results of our evaluation. We note that our results are preliminary, and further work is needed to affirm they generalize beyond our benchmarks, in particular for detecting type errors in real-world programs.

4.5.1 Results

Table 4.1 summarizes some our type inference results. The first column gives the number of methods we inferred types for, totaling 250 methods across the four apps. The subsequent group of three columns counts the number of types we inferred. The first of these columns, Meth Typs, counts the number of method argument and return types inferred. We count each argument and return type separately so that we can more precisely evaluate `InferDL`'s performance. The second of these columns, Var Typs, shows the number of global, class, and instance variable types inferred. Finally, the Total Typs column counts the total number of types inferred, i.e., Meth Typs + Var Typs.

The next column shows the number of type casts we had to write to run `InferDL` on each app, without which `InferDL` would raise false positive type errors. Almost all of these type casts were needed when handling heterogeneous data structures like arrays and hashes, because there are cases where `InferDL` cannot

Program	Correct Meths	Correct Vars	Correct Total	Heuristic Uses
	heur / *std*	*heur* / *std*	*heur* / *std*	STN / Name / Hash
Journey	33 / 30	19 / 13	52 / 43	0 / 13 / 0
Discourse	61 / 47	0 / 0	61 / 47	3 / 42 / 1
code.org	111 / 60	10 / 10	121 / 70	0 / 80 / 5
Talks	127 / 102	38 / 28	165 / 130	7 / 58 / 3
Total	332 / 239	67 / 51	399 / 290	10 / 193 / 9

Table 4.2: Type inference: assessing inferred types.

determine the type of a value accessed from one of these data structures.

The subsequent column reports `InferDL`'s running time on a 2014 MacBook Pro with a 3GHz i7 processor and 16GB RAM. We give the time as the median and semi-interquartile range (SIQR) of 11 runs. For comparison, we provide `InferDL`'s runtime when using the heuristics presented in § 4.4 (shown under "*heur*"), and when not using any heuristics (shown under "*std*"). In total, `InferDL` took 31.91s to run on all benchmarks when using heuristics, with an SIQR of just 0.95s, indicating little variance across runs. By comparison, when not using any heuristics, `InferDL` took 8.68s to run on all benchmarks. Upon closer examination, we found that approximately 75% of `InferDL`'s runtime when using heuristics was spent on just one rule, STRUCT-TO-NOMINAL. The rule involves searching through the space of all existing classes, and for each one, searching through the names of all its methods. This can be quite expensive for larger programs. We found we could achieve speedups by caching search results, and by building a mapping from method names to the classes that implement them in advanced of running the rule. Nevertheless, this remains an expensive operation.

Table 4.2 reports the remainder of our inference results. In particular, it

counts the number of types `InferDL` inferred correctly—the same as or more precise than the original type annotation—both with and without the use of heuristics. To determine whether inference results were correct, we automatically counted those cases where an inferred type matched the original annotation exactly, and we used manual inspection when they differed. For example, if the annotation for a type was `%any` (`RDL`'s top type), and `InferDL` inferred `Integer`, we would count this as a more specific type. In our experience, we didn't find any case where `InferDL` predicted a more specific type that was not an accurate reflection of programmer intent.

The first of these columns gives the number of method argument and return types correctly inferred for heuristic and standard inference. For example, `InferDL` correctly inferred 332 out of 411 total argument and return types for all apps when using heuristics, compared to just 239 correct types when performing standard inference. The next column gives the number of variable types correctly inferred, and finally, the Correct Total column gives the number of total types inferred correctly. As shown, the use of heuristics enables `InferDL` to infer about 22% more correct type annotations, a significant improvement. We found this percentage was fairly consistent across the benchmarks, indicating that the heuristics we used were not specific to one app, but rather captured some more common, general properties. We also found this improvement was approximately the same for types of global, class, and instance variables, and types of method inputs/outputs, indicating our approach to variables (discussed in § 4.4) is effective.

Note that inferred types which do not fall under the "Correct Types" column are not necessarily "incorrect"—typically, these types are simply more general than

the original, programmer-written annotation. For example, across the apps there were a number of cases where `InferDL` inferred the type Array<α> for some type variable α, when the programmer's annotation was a variable-free type (e.g., Array<String>).

In our subjective experience, many types `InferDL` failed to infer (with or without heuristics) were for arrays and hashes. This is largely because `RDL` treats Array and Hash types as invariant in their type parameters. This means, e.g., the constraint Hash<String, Integer> $\leq$ Hash<String, Object> is invalid, since the type parameters are not equivalent. This leads to many potentially correct types being rejected due to the conservatism of type invariance. We are interested in exploring better approaches to type inference for heterogeneous data structures as future work.

Finally, the last column shows the number of times a heuristic successfully found a type for each app, that is, the heuristic's guess was actually used as a solution. For brevity, we present the counts for all of the six name-based heuristics (IS_MODEL, IS_PLURALIZED_MODEL, INT_NAMES, INT_ARRAY_NAME, PREDICATE_METHOD, and STRING_NAME) under a single column Name, while the STN column gives the count for STRUCT-TO-NOMINAL, and the Hash column for HASH_ACCESS. It is clear that the name-based rules were by far the most useful heuristics for inferring types, comprising a total of 193 of the successful heuristic applications. Of those 193 applications, 103 were of the PREDICATE_METHOD rule. Overall, this suggests that variable and method names are a strong indicator of intended types.

Error Caught. In the process of inferring types, we discovered a previously unknown bug in the *Journey* app. This was particularly surprising because the method it was found in was *already type checked* in prior work [90]. The bug existed in the following code, which creates and saves a new person:

```
begin
 invitee = IllyanClient::Person.new (:person⇒{:email⇒email})
 invitee.save
 ...
rescue
 logger.error  "Error during  invite. "
 ...
end
```

The bug arises because there is no save method for the IllyanClient::Person class, so the call invitee.save always raises an error. Moreover, because this error exists within a begin...rescue clause, the bug will never be directly seen at runtime since control will always pass to the rescue clause. The bug could potentially have been detected via manual programmer inspection of the error log, though it never was. We confirmed this bug with the *Journey* developer. This bug was not caught by type checking in prior work because the programmer who wrote type annotations in that work wrongly assumed that the IllyanClient::Person#save method did exist. Thanks to `InferDL`'s handling of library types (§ 4.4), the same mistake was not made here.

4.5.2 Case Studies

To further evaluate `InferDL`, we applied it to five additional Ruby libraries and one additional Ruby app:

Program	Num Meths	Meth Typs	Var Typs	Total Typs	Type Casts	Time (s) Median ± SIQR	Heuristic Uses
						heur / *std*	STN/Name/Hash
AM	148	275	62	337	5	1.71 ± 0.06 /0.80 ± 0.07	29 / 103 / 0
Diff-LCS	80	187	40	227	23	2.65 ± 0.02 /2.38 ± 0.06	20 / 14 / 0
MM	79	166	13	179	7	0.57 ± 0.15 /0.26 ± 0.01	4 / 10 / 0
Optcarrot	430	763	367	1130	48	38.9 ± 2.64 /78.6 ± 17.4	204 / 22 / 1
Sidekiq	344	623	96	719	12	3.28 ± 0.56 /2.12 ± 0.13	37 / 63 / 3
TZInfo	251	511	57	568	9	3.15 ± 0.25 /5.06 ± 0.07	118 / 42 / 0
Total	1332	2525	635	3160	104	50.22 ± 3.67/89.2 ± 17.8	412 / 254 / 4

Table 4.3: Case Study Inference Results. Program name "AM" is short for Active Merchant, and "MM" is short for MiniMagick

- *Active Merchant* [95], a payment abstraction library.
- *Diff-LCS* [96] a library for generating difference sets between Ruby sequences.
- *MiniMagick* [97], an image processing library.
- *Optcarrot* [98], a Nintendo Entertainment System (NES) emulator implemented in Ruby and intended as a benchmark for runtime performance evaluation.
- *Sidekiq* [99], a background job handler library.
- *TZInfo* [100], a time management library.

Because we do not have gold standard type annotations for these programs, we refer to these experiments as case studies. With the exception of Optcarrot, we picked these programs because they are all highly popular, well-maintained, and well-tested. We chose Optcarrot because of its intended use as a Ruby benchmark and because, as an emulator, it relies heavily on binary arithmetic, which distinguishes it from other Ruby programs we looked at.

We ran inference for all methods defined in these programs, excluding methods that use features not supported by `RDL`; the most common unsupported feature was mixin methods. We also excluded methods defined in the *Active Merchant* payment gateways, a set of 215 distinct payment gateways comprising over 60,000 lines of code. Running `InferDL` for this many lines of code would have required a significant manual effort to add type casts to circumvent false positive errors, so we decided to leave them out of our case study. We discuss the issue of type casts further below.

Using `InferDL`. It would be tedious and time-consuming to call `InferDL`'s infer method (§ 4.4) on every method in our subject programs. Instead, we used `InferDL`'s infer_file and infer_path methods, which take a file or path, respectively, as an argument and then call infer on every method statically defined in that file or path. We called these methods for all code in a program's `lib/` directory, which by convention holds the program's implementation (and excludes testing code, code for handling dependencies, etc.).

The first time `InferDL` runs on a new subject program, it often reports type errors. We manually inspected and addressed each type error, iterating until none remained. Overall, the errors found by `InferDL` fell into three categories:

- True errors resulting from bugs in the program. We discuss these below.
- False positives due to `InferDL`'s conservatism. We inserted appropriate type casts to suppress these type errors. We discuss type casts below.
- Errors resulting from features unsupported by `InferDL`. As mentioned earlier,

we exclude such methods from future rounds of inference.

As an aside, we note that currently, it can sometimes be difficult to find the underlying cause of a type error reported by `InferDL`. If an invalid constraint is generated during resolution, `InferDL` reports the invalid constraint and the line number origins of the left- and right-hand sides of the constraint. But often these constraints were generated through a series of propagations resulting from many different places in the code, so their origins do not always reveal the underlying cause. In the future, we hope to incorporate ideas from prior work on diagnosing type inference errors [101, 102, 103].

Results. Table 4.3 contains the results of running `InferDL` on the case study apps. This table includes the same columns as Tables 4.1 and 4.2, excluding the "Correct" columns. In total, we inferred types for 1,332 methods constituting 2,525 individual arguments and returns, and 635 global, class, or instance variables, for a total of 3,160 individual types.

We wrote 104 total type casts to run inference for these programs, or approximately one type cast for every 30 types we inferred. In addition to the need for type casts when accessing values from heterogeneous data structures (discussed in § 4.5.1), we encountered many cases where type casts were necessary for path-sensitive typing. For example, consider the code snippet in Figure 4.4, simplified from the *TZInfo* library.

This snippet refers to the instance variable @transitions, which has type Array<TimezoneTransition>. On line 2, we enter a loop for values of i=index down

```
1 index = @transitions.length
2 index.downto(0) do |i|
3  start_transition  = i > 0 ?  @transitions[i  - 1] :  nil
4 end_transition =  @transitions[i]
5  offset  =  start_transition  ?   start_transition.offset   :   end_transition.previous_offset
6  ...
7 end
```

Figure 4.4: A code snippet from the *TZInfo* library.

to i=0. On line 3, we use Ruby's *ternary operator* to conditionally assign the variable start_transition to either a TimezoneTransition or to nil. Then, on line 5, we use the ternary operator again, this time with the variable start_transition as our condition. In Ruby, the value nil is falsey. Thus, on line 5, we only evaluate the expression start_transition.offset if start_transition is non-nil. This call is safe, because TimezoneTransition has a method offset defined.

However, `InferDL` does not know that start_transition is non-nil because it has limited support for path-sensitive typing. It will conservatively reason that start_transition may be nil on line 5 and thus raise a type error. To avoid this issue, we insert the following type cast for the call to offset:

```
RDL.type_cast(start_transition, TimezoneTransition).offset
```

This notifies `InferDL` that start_transition is a TimezoneTransition when offset is called. Such path-sensitive logic enables libraries to be maximally flexible for clients. We leave handling these cases without type casts to future work.

The next column of Table 4.3 reports the median time and SIQR taken across 11 runs of `InferDL`, when using vs. not using heuristics. Interestingly, on these apps `InferDL` actually took *less* total time when using heuristics compared to not

using them. This was attributable to two apps in particular, *Optcarrot* and *TZInfo*, that took 2× and 1.6× as long, respectively, when not using heuristics. This can occur due to the way that `InferDL` performs type inference. As shown in § 4.3.1, `InferDL` will repeatedly perform constraint resolution and solution extraction until no new constraints are generated. In some cases, heuristics may lead to solutions for type variables earlier on in this process, thereby helping to reach a constraint set fixpoint sooner. For instance, *Optcarrot* performed just 6 rounds of solution extraction when using heuristics, compared with 15 rounds of solution extraction when not using heuristics; for *TZInfo*, the numbers were 3 and 9, respectively.

Finally, we report the number of successful applications of heuristics for inferring types. Notably, the STRUCT-TO-NOMINAL heuristic is far more useful for our case study programs than for the Rails apps in Table 4.2. It was used 412 times when running inference for 3,160 total types in our case studies, versus just 10 times for 496 total types for the Rails apps. This disparity is at least partly attributable to the order in which heuristics are run (§ 4.4). STRUCT-TO-NOMINAL is applied after the rules IS_MODEL and IS_PLURALIZED_MODEL. But the latter two rules are only used for Rails apps, meaning STRUCT-TO-NOMINAL is applied third for Rails apps, and first for non-Rails apps. We tried re-running `InferDL` on the Rails apps with STRUCT-TO-NOMINAL ordered first, and found it was applied 28 times for the Rails apps, which at least partly closes the gap with non-Rails apps.

We also more closely examined the uses of STRUCT-TO-NOMINAL across all 10 programs in § 4.5.1 and § 4.5.2, and we found that approximately 14% of the time the heuristic produced a union type, while the remainder of the time it produced

just a single, nominal type. The usefulness of the produced union types varied. Sometimes, the unions were quite sensible. For example, in the *TZInfo* program, STRUCT-TO-NOMINAL produced the type TZInfo::Timestamp ∪ Time as the solution for a number of variables. Both the TZInfo::Timestamp and Time classes represent time values, and many of *TZInfo*'s methods are implemented to handle objects from both of these classes, so this is a sensible solution. In other cases, we found that STRUCT-TO-NOMINAL produced unions of unrelated classes that happened to have some same-named methods, thereby producing a solution that is less useful.

Name-based heuristics were also useful for our case studies, having been applied 254 times to infer types. However, this clearly comprises a far smaller proportion of uses than for the Rails apps. This may be because Rails emphasizes the principle convention over configuration, making names more important than in regular Ruby programs. Moreover, libraries are very domain-specific, and the names used in these programs reflect their domain. For example, *TZInfo* features many variables with names like time, datetime, timezone, etc. It is challenging to write general-purpose heuristics that can capture such domain-specific naming.

Finally, note that the HASH_ACCESS rule was used only four times across the programs in Table 4.3, and only nine total times across the programs in Table 4.2. This is partly attributable to the invariance of hashes (as discussed in § 4.5.1). Though the rule was applied few times in practice, we still believe it was useful in the cases it was used for converting structural type solutions to a more readable finite hash type. For example, for one type variable in the *code.org* app, `InferDL` used the HASH_ACCESS rule to infer the finite hash type solution { id: Integer, email:

String, gender: α } (some key/value pairs omitted for brevity), rather than a much larger and more difficult to read intersection of structural types.

Errors Found. `InferDL` found five previously unknown bugs in the case study programs, all of which were confirmed with the developers:

- In *Active Merchant*, `InferDL` caught a reference to an undefined constant Billing::Integrations.

- In *Sidekiq*, `InferDL` caught a reference to an undefined identifier `e` inside a rescue clause that was as follows:

```
rescue Exception ⇒ ex
...
raise e
end
```

  The notation rescue Exception => ex catches exceptions of type Exception and binds the specific Ruby exception object to ex. This rescue clause was meant to perform some error handling (elided with the ... above) and then raise the original error, but it erroneously referred to an undefined `e` rather than ex.

- In *Sidekiq*, `InferDL` caught a reference to an undefined constant UNKNOWN.

- In *Diff-LCS*, `InferDL` caught two different calls to an undefined method Diff::LCS.YieldingCallb

- In *Diff-LCS*, `InferDL` caught a reference to an undefined constant Text::Format.

We do note that given the nature of the above bugs (undefined methods, variables, and constants), it is possible that they could be found through alternative

Program	Choice Type Uses	Unknown Types
Journey	0	30
Discourse	1	33
code.org	3	20
Talks	2	56
Active Merchant	1	69
Diff-LCS	124	14
MiniMagick	1	33
Optcarrot	154	73
Sidekiq	7	82
TZInfo	14	41
Total	307	451

Table 4.4: Testing Implementation Choices.

analyses. Nevertheless, `InferDL`'s ability to catch these errors in popular and well-tested libraries indicates it is useful not only for generating type annotations, but also for catching type errors.

4.5.3 Testing Implementation Choices

Finally, in § 4.4 we discussed two novel design features of `InferDL`: the use of choice types for resolving calls to overloaded methods and `InferDL`'s handling of calls to library methods for which we do not have types. To evaluate these choices, we provide some relevant data in Table 4.4 collected from all 10 of the programs discussed in § 4.5.1 and § 4.5.2.

For each program, the first column gives the number of choice types used while running `InferDL` on the program. As discussed in § 4.4, a choice type is used when type checking a call to an overloaded method, when `InferDL` is not able to determine which type of the method to use. They can help avoid false positive type errors (and thus reduce the need for type casts), and to infer more precise types. In

total, we used 307 choice types across all benchmarks. The vast majority of these uses were in just two programs, *Diff-LCS* and *Optcarrot*. This is likely because these programs rely heavily on array manipulation, and therefore make frequent use of the Array accessing method `[]`, which requires choice types to resolve its overloaded method type (see § 4.4 for an example). Thus, we found that the need for choice types commonly arises in programs, but they are especially useful for programs that make frequent use of overloaded methods.

The next column gives the number of uses of "Unknown Types"—this is the name we give to the method types composed entirely of type variables that we generate for library methods and other methods for which we do not have a type. In total, we used 451 unknown types across all programs. Without our approach of generating unknown types, we would have had to write a type annotation in every one of these cases so that `InferDL` could type check the programs. Thus, we believe `InferDL`'s approach to handling calls to methods without a type is effective for reducing the programmer's annotation burden.

4.6 Related Work

Researchers have been studying type inference for many decades. Traditionally, the problem is posed as follows: given a program without type annotations, can we determine the most general type for each expression in the program, and rule out any type errors? The problem was first formulated and solved by Curry and Feys [104] for the simply typed lambda calculus. Perhaps most famously, Hindley [105],

Milner [106], and Damas and Milner [107] developed an approach known today as Hindley-Milner-Damas type inference. Its central algorithm, Algorithm W, works by generating constraints on type variables, then resolving those constraints through a process known as unification.

Researchers have extended type inference to subtyping systems [1, 6, 91, 108] as well, and this has enabled the development of practical type inference systems for dynamic languages such as Python [11], JavaScript [14, 16], and Ruby [4]. As discussed earlier, these systems are often aimed at catching type errors rather than displaying the results of type inference to the user, and in our experience, the types inferred by such systems can be quite hard to understand. In contrast, `InferDL` aims to infer usable types.

Furr et al. [4] present DRuby, a static type inference system for Ruby, which features an expressive type language including intersection, union, optional, and structural types. While DRuby also focuses exclusively on finding type errors in programs, many of the type system features it includes are part of `RDL` [20], on which `InferDL` is built.

Finally, there are a number of probabilistic type inference systems which aim to infer usable type annotations. We direct the reader to Section 5.6 for a discussion of these systems.

4.7 Conclusion

We presented `InferDL`, a novel type inference system for Ruby. In addition to uncovering type errors, `InferDL` aims to produce useful type annotations for methods and variables. Because the constraint-based approach to type inference often results in types that are overly-general, `InferDL` incorporates heuristics that guess a solution for type variables that better matches what a programmer would write. `InferDL` enforces the correctness of heuristic guesses by checking them against existing constraints. Moreover, heuristics are not baked-in to `InferDL` but rather provided as code blocks, making `InferDL` highly configurable.

We formalized the type and constraint language of `InferDL` and provided the rules and procedures for resolving type constraints, producing standard type solutions, and using heuristics to produce more useful, sound type annotations. We implemented `InferDL` on top of `RDL`, an existing Ruby type checker which we extended with support for constraint generation, heuristics, and choice types to handle overloaded methods. We also discussed the eight heuristics we found useful in applying `InferDL` to programs.

Finally, we evaluated `InferDL` by applying it to four Rails apps for which we already had type annotations. We found that, without using heuristics, we were able to correctly infer about 58% of all type annotations for these apps, and when using heuristics we were able to infer 80% of annotations. We also applied `InferDL` to six additional case study Ruby programs. Across the Rails apps and the case study apps, `InferDL` discovered six previously unknown bugs. Thus, we believe that

`InferDL` is an effective type inference system and represents a promising approach to generating useful, correct type annotations.

Chapter 5: `SimTyper`: Sound Type Inference for Ruby using Type Equality Prediction

The last chapter introduced `InferDL`, a type annotation inference system that complements constraint solving with heuristic rules. While the evaluations of `InferDL` showed that heuristics were useful for inferring more annotations that match what a programmer would write, there remains room for improvement. First, there were still methods and variables in the benchmarks for which `InferDL` failed to infer matching type annotations. Second, seven out of eight of the heuristic rules used by `InferDL` were written while applying it to the benchmarks. In this chapter, we show that, while these heuristics perform well on their initial target programs, they sometimes generalize poorly to new programs. While we could write new heuristic rules for each new target program, this process can be time-consuming.

In this chapter we introduce `SimTyper`, a type inference system that builds on `InferDL` by adding *type equality prediction*, a novel machine learning-based approach to inferring useful type annotations. We evaluated `SimTyper` on eight Ruby programs and found that, compared to standard constraint-based inference, `SimTyper` finds 69% more types that match programmer-written annotations. Moreover, we found that when using type equality prediction (without the use of heuristics),

`SimTyper` inferred 19% more matching type annotations than `InferDL`. When running exclusively on benchmarks that were not included in the previous chapter (and thus had not been incorporated in the development of `InferDL`'s heuristics), that number improved to 42%.

5.1 Introduction

Many researchers have explored ways to add static types to dynamic languages, aiming to provide the benefits of static typing while preserving the flexibility of the language [1, 2, 3, 5, 6, 13, 53, 58, 90]. In this setting, type inference [4, 11, 14, 16, 109] is potentially very attractive, as it can catch type errors without requiring users of these retrofitted type systems to provide many new type annotations. Typically, traditional static type inference (see § 5.6 for a discussion of machine learning-based type inference systems) aims to infer *most general types*, so as not to reject any program that would be statically typable. Unfortunately, as demonstrated in Chapter 4, in the presence of complex type features such as subtyping, structural types, and union types—which are often needed to type existing dynamic language code—most general types can be verbose, confusing, and difficult to understand. For example, when typing a Ruby method that performs arithmetic computations on an argument `x`, the most general type of `x` might be a *structural type* like *any object with +, -, *, and / methods*. In contrast, a programmer would much more likely write the shorter, simpler, and more understandable *nominal type* Numeric, giving up some generality for the sake of usability.

In this chapter, we introduce `SimTyper`, a type inference system that aims to infer usable types for Ruby. `SimTyper` is built on top of `InferDL` (Chapter 4), a type inference system that combines constraint solving with manually written heuristics whose guesses are checked against the constraints, to ensure soundness [110]. `SimTyper` uses the same basic infrastructure, but adds a novel *deep similarity* (DeepSim) network that performs *type equality prediction*. More precisely, DeepSim predicts type similarity scores among method arguments. If, after standard type inference, an argument has an *overly general* type (e.g., a structural type) and DeepSim predicts that argument is similar to another argument with a *usable* type (e.g., a nominal type), `SimTyper` guesses the overly general type can be replaced by the usable type. If that guess is consistent with the rest of the constraints, it is kept. Otherwise it is discarded, and further guesses are made up to some bound. Thus, even though it is probabilistic, `SimTyper` is still guaranteed to be sound. `SimTyper` applies the same idea to method returns and to instance, class, and global variables. (§ 5.2 shows how `SimTyper` integrates standard type inference, heuristics, and type equality prediction.)

We describe `SimTyper`'s algorithm on a core language of types and constraints. `SimTyper` begins by running the standard, constraint-based type inference algorithm which, at a high level, generates constraints among types; applies constraint resolution to check that the constraints are consistent; and then extracts solutions for the type variables. Next, for each type variable α with an overly general solution, `SimTyper` finds the type variable β that is most similar to α and has a usable type solution. `SimTyper` then adds a constraint $\alpha = \tau$, where τ is β's solution, and runs

constraint resolution again. If the constraints are still consistent, α's solution is set to τ. If not, `SimTyper` retracts the $\alpha = \tau$ constraint and tries the next most similar usable type, and so on. This guessing-and-backtracking approach was first proposed for `InferDL`, and `SimTyper` uses the same machinery, enabling `SimTyper` to infer types with both DeepSim-based predictions and guesses based on `InferDL` heuristics. (§ 5.3 describes `SimTyper`'s inference algorithm.)

The DeepSim network itself takes as input the tokenized source code of two methods and the positions within that code of the arguments or method return sites of interest. The network then uses `CodeBERT`, a transformer-based pre-trained code embedding model [111], to transform each token at the given positions into a fixed-dimensional vector that captures both the token and its surrounding context. DeepSim then averages those vectors to produce one vector for each input. These vectors are then passed through a trained similarity function to predict whether they are similar or dissimilar. The network itself is trained on 371 Ruby programs that include `YARD` [112] documentation. We extract type information from `YARD` (which is not checked against code and hence might be noisy) to create a training data set with 100,000 pairs labeled as either similar (for two positions with the same `YARD` types) or dissimilar. (§ 5.4 describes the DeepSim network in detail.)

We evaluated `SimTyper` by applying it to eight Ruby programs with type information: four web apps written in Ruby on Rails (a popular web development framework) that `InferDL` was previously evaluated on, and four popular Ruby libraries with `YARD` documentation. We then compared the types inferred by `SimTyper` to existing, programmer-written annotations. Following prior work [113], we count

the number of cases where the the inferred type matches the existing type, as well as the number of *matches up to parameter*—generic types where the base matches but the parameter is different (e.g., inferring Array<Integer> when the original was Array<String> would fall in this category). We found that, by combining constraint solving, `InferDL` heuristics, and type equality prediction from DeepSim, `SimTyper` generated 66% more type annotations that matched programmer-written types compared to using constraint solving alone. If we include matches up to parameter, this number improves to 69%. Moreover, `SimTyper` inferred 16% more matching type annotations when using DeepSim alone than when using heuristics alone. Including matches up to parameter improves this to 19%. DeepSim was also able to correctly predict *rare* types, including 16 types that did not appear in either the standard Ruby library or the training data. This number is the same whether or not we consider matches up to parameter. (§ 5.5 describes these results and several other experiments in more detail.)

In summary, we believe that by incorporating type equality prediction with DeepSim, `SimTyper` takes an important step forward towards type inference systems that produce more usable types.

5.2 Overview

We begin by illustrating how `SimTyper` is used to infer a type annotation for the method shown in Figure 5.1a, which is extracted and simplified from *TZInfo*, one of the benchmarks in our experiments (§ 5.5). The method, Timestamp.create,

```
1  class Timestamp
2    # Assigned method type: (α, β, γ) → δ
3    def self.create (year, month, day)
4      raise ArgumentError, 'year must be an Integer' unless year.kind_of?(Integer)
5      raise ArgumentError, 'month must be an Integer' unless month.kind_of?(Integer)
6      raise ArgumentError, 'day must be an Integer' unless day.kind_of?(Integer)
7
8      after_february = month > 2
9      year = year - 1 unless after_february
10     era = year / 400 # eras are 400 year periods
11     day_of_year = day + (153 * (month + ...
12      ... # additional computation
13     value = ... * 24 * 60 * 60 # seconds since unix time
14
15     new(value)
16   end
17 end
```

(a) A method from the TZInfo library.

(1) $\alpha \leq$ [kind_of? : Class $\rightarrow \epsilon$]	(6) $\alpha \leq$ [/ : Number $\rightarrow \kappa$]
(2) $\beta \leq$ [kind_of? : Class $\rightarrow \zeta$]	(7) $\gamma \leq$ [+ : Number $\rightarrow \lambda$]
(3) $\gamma \leq$ [kind_of? : Class $\rightarrow \eta$]	(8) $\beta \leq$ [+ : Number $\rightarrow \mu$]
(4) $\beta \leq$ [> : Number $\rightarrow \theta$]	(9) Timestamp $\leq \delta$
(5) $\alpha \leq$ [- : Number $\rightarrow \iota$]	

(b) Constraints generated on type variables.

Figure 5.1: Generating type constraints in *TZInfo*.

takes three integers representing a year, month, and day corresponding to a date, and returns a new Timestamp encoding that date as the number of seconds since the start of the Unix Epoch (midnight on 01/01/1970).

The first three lines of the method check that all parameters are integers.[1] The subsequent lines (lines 8–13) perform the computation. The details of the computation [114] are not important, but notice the parameters appear in various places in somewhat complex arithmetic expressions. The method returns the value of the expression on the last line, which constructs a new Timestamp (code omitted here) with the computed number of seconds.

5.2.1 Standard Type Inference

The first step of `SimTyper`, inherited from `InferDL`, is to perform standard, constraint-based type inference [4]. We refer the reader to Chapter 4 for a more detailed treatment of standard type inference; here, we quickly go over the details. We begin by assigning a type variables α, β, and γ to the parameters (in that order) and δ to the return type. `SimTyper` then analyzes the method body to generate constraints on these type variables. Figure 5.1b shows several of the constraints generated for this example. Constraint (1), from line 4, states that α, the type of year, must define a method kind_of? that takes a Class and returns a fresh unknown type ϵ. Or, more formally, α is a subtype of the structural type [kind_of? : Class $\rightarrow$ ϵ]. Constraints (2) and (3) are similar.

[1] Amusingly, we could in theory create a heuristic (§ 5.2.2) that uses this coding pattern to guess the argument types. That heuristic, however, would have to be hand-written and would be specific to this pattern. One strength of `SimTyper` is that it can discover useful types even without manually creating heuristics.

Constraint (4), from line 8, states that month must define a > method that takes a Number (for simplicity, all Ruby numeric objects are typed as Number in `SimTyper`) and returns an unknown type. Note that in Ruby, binary arithmetic operations are actually method calls on the left-hand argument, e.g., month > 2 is syntactic sugar for month.>(2). Constraints (5)–(8) are similar, with the first two arising from lines 9 and 10, respectively, and the last two from line 11. Finally, constraint (9) arises from returning the newly created Timestamp on line 15.

Next, `SimTyper` performs constraint resolution, which applies a set of constraint rewriting rules until reaching saturation. In this particular case, because Timestamp.create is relatively simple and is considered in isolation, resolution does not change the set of constraints.

After constraint resolution, the constraints are in solved form [91], meaning `SimTyper` can read off a type variable's *most general solution*—any other solution would be more restrictive—by looking at its immediate bounds. As seen in Chapter 4, for (contravariant) method arguments, we compute the *greatest solution* by intersecting all of the argument type's upper bounds, excluding type variables. For (covariant) method returns, we compute the *least solution* by unioning its non-variable lower bounds. We leave type variables out of solutions because their bounds will have already been transitively propagated during resolution.

For the constraints in Figure 5.1b, `SimTyper` finds the following solutions:

$$
\begin{aligned}
\alpha &= [\mathsf{kind\_of?} : \mathsf{Class} \to \epsilon,\ \mathsf{-} : \mathsf{Number} \to \iota,\ \mathsf{/} : \mathsf{Number} \to \kappa] \\
\beta &= [\mathsf{kind\_of?} : \mathsf{Class} \to \zeta,\ \mathsf{>} : \mathsf{Number} \to \theta,\ \mathsf{+} : \mathit{Number} \to \mu] \\
\gamma &= [\mathsf{kind\_of?} : \mathsf{Class} \to \eta,\ \mathsf{+} : \mathsf{Number} \to \lambda] \\
\delta &= \mathsf{Timestamp}
\end{aligned}
$$

Out of these four solutions, the only one that matches developer-provided documentation (not shown) is δ, which has a simple, easy-to-understand nominal type. In contrast, the solutions for α, β, and γ, while very precise, are also complex, verbose, and hard to read. Moreover, they are not even fully expanded, since they contain type variables—and adding solutions for the nested variables would only make the types more complex. Standard type inference with subtyping and structural types often produces such difficult-to-use types.

5.2.2 Heuristic Type Inference

To address this problem, `SimTyper` builds on an approach pioneered by `InferDL` and applies a series of heuristic rules that aim to guess more useful types, typically nominal or generic types, for positions for which standard type inference produces *overly general* types, like those for α, β, and γ above. More precisely, any type that is not one of the following is considered overly general: nominal types, generic types, finite hash and tuple types (which are more precise types for hashes and arrays, described in detail in Chapter 3), singleton types (which represent just a single value),

and the boolean type.

In this case, `SimTyper` applies the STRUCT-TO-NOMINAL rule (inherited from `InferDL`), which converts structural types to nominal types; see Chapter 4.2 for a full definition of this rule. When `SimTyper` and `InferDL` apply STRUCT-TO-NOMINAL to our running example, the heuristic guesses that the solution for α (corresponding to the parameter year) is Number, because the only existing classes that define methods kind_of?, -, and / are Ruby's numeric classes. This guess is consistent with the other constraints, so `SimTyper` and `InferDL` would both set $\alpha = \mathsf{Number}$ as the solution.

However, STRUCT-TO-NOMINAL fails to infer a new solution for β, because more than ten classes define the set of methods {kind_of?, >, +}—e.g., possible classes include String, Set, Time, and others—and similarly for γ. Thus, while heuristics are effective, there is still room for improvement. Moreover, while `InferDL` allows users to write new heuristics that apply specifically to their programs, doing so requires a lot of care and insight, and heuristics may not be portable across programs. For example, `InferDL` also includes a heuristic INT_NAMES that, among others, guesses that an argument named id has type Number. However, while this is an excellent guess for Rails code, in other codebases, e.g., in the the Stripe codebase, ids are generally Strings [93].

5.2.3 Predicting Type Equalities

To address the limitations of heuristics, `SimTyper` builds on `InferDL`'s approach by additionally using a machine learning-based approach to guess types.

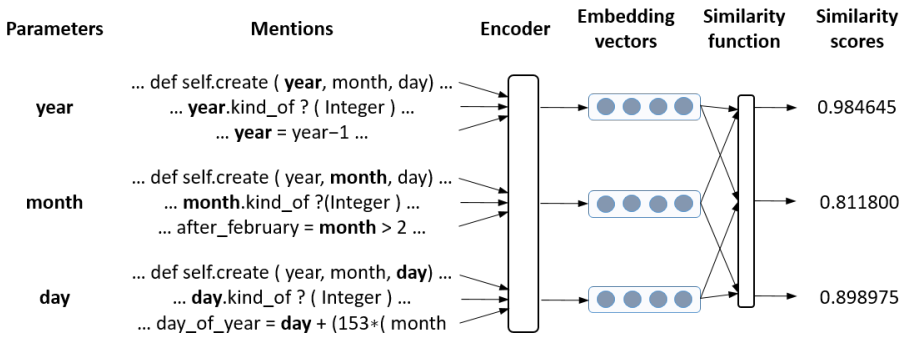


Figure 5.2: An illustration of how DeepSim calculates the pairwise similarity scores for the set of input parameters {year, month, day} for the method from Figure 5.1a.

These guesses can either be used in place of or in addition to guesses from hand-written heuristics. Like `InferDL`, these guesses are checked for consistency with the underlying constraints to ensure soundness, and any inconsistent guesses are discarded.

More specifically, `SimTyper` uses a novel network based on the Siamese network architecture [115] which we call a *deep similarity* (DeepSim) network. DeepSim is a deep neural network model that we use to guess when two positions have equal types. For example, notice that year, month, and day are closely related words in English. Moreover, in Timestamp.create, they are used in similar contexts: First they appear in nearly-identical dynamic type checks (lines 4–6) and then in arithmetic expressions (lines 8–13). Thus, the DeepSim network guesses that all three variables have equal types—and since `SimTyper` previously determined that year has type Number, using DeepSim it will guess the same type for the other two arguments.

Guessing Types with the DeepSim Neural Network. DeepSim is based on the Siamese network architecture [115]. The network takes two inputs, which are

the source code for two methods with position markers indicating mentions of the relevant parameter/return that is being compared. The inputs are first run through identical encoders. The encoder uses a state-of-the-art model trained on programming and natural languages [111] to produce a fixed-dimension *contextualized vector representation* for the input. This is a numeric vector that encodes not only the name of an identifier (e.g., the name of an argument), but also the code context in which it occurs. The encoded vectors are such that parameters with similar names (e.g., "day" and "month") and contexts (e.g., basic arithmetic expressions) will be encoded as vectors that are close together. Contextualized embedding models have recently been shown to achieve state-of-the-art performance on a range of natural language processing [116] and programming language [111, 117] tasks.

The encoded inputs are then run through a trained similarity function, which produces a *similarity score* between 0 and 1, indicating the network's belief that two inputs have the same (1) or different (0) types. For example, Figure 5.2 illustrates how DeepSim calculates the pairwise similarity scores for the set of input parameters {year, month, day} for the method from Figure 5.1a. More details on the encoder and the similarity function are in Section 5.4.

`SimTyper` applies the DeepSim network after standard inference and hand-written heuristics have been run. `SimTyper` considers all remaining overly general type positions and uses DeepSim to compare them to all positions for which a "usable" (i.e., not overly general) solution was found. After eliminating positions with scores below 0.5—which indicates the network believes those positions have different types—`SimTyper` guesses type equivalence with the highest scoring position. If that

guess is consistent with the constraints, it is accepted. If that guess is inconsistent, `SimTyper` continues guessing equivalence with the next highest scoring position, and so on for the top N scores (§ 5.5 evaluates choices for N).

For example, picking up from the heuristic guesses in § 5.2.2, `SimTyper` asks DeepSim for the expected type similarity among year, which has a usable solution at this point, and month and day, which have overly general solutions. As shown in Figure 5.2, the network has a very high degree of confidence that year and month have the same type, so `SimTyper` guesses that $\beta = \alpha =$ Number. This guess is consistent, and so it is accepted. Next, day is predicted to be most likely similar to month, so `SimTyper` guesses $\gamma = \beta =$ Number, which is also accepted. Thus, after applying standard type inference, hand-written heuristics, and the DeepSim network, `SimTyper` has successfully inferred the type (Number, Number, Number) $\rightarrow$ Timestamp for Timestamp.create, which matches the hand-written documentation.

Cascading Type Predictions. Similar to heuristic guesses in `InferDL`, guesses made via DeepSim can cascade through the constraints, leading to further usable solutions. For example, consider the code snippet in Figure 5.3 extracted and simplified from *code.org*, one of our benchmarks (§ 5.5). This code defines a class method initial that, given a String called name, returns the first non-whitespace character in name as an upper-case letter.

`SimTyper` assigns initial the type $(\alpha) \rightarrow \beta$ and generates the constraints shown in the right of Figure 5.3. Constraints (1), (2), and (3) result from the calls to strip, [], and upcase, respectively, and constraint (4) results from the return. Using

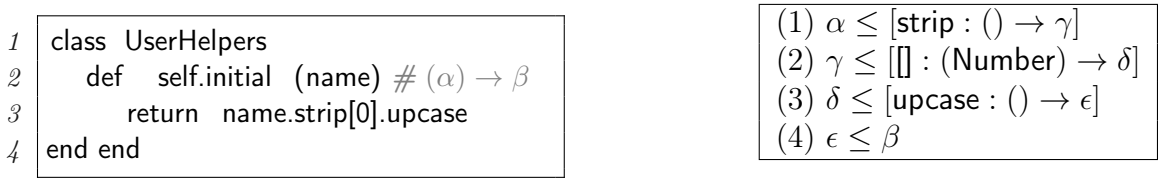


Figure 5.3: A method defined in the *code.org* app and the resultingconstraints.

standard type inference, `SimTyper` would generate the type ([strip : () → γ]) → ϵ for this method.

However, DeepSim predicts that name has a similarity score of approximately 0.996 with another parameter, also called name, from a different method (code omitted for brevity). Because the other parameter's type is determined to be String, `SimTyper` guesses String as a solution for the name parameter of self.initial, which is accepted as consistent.

But something interesting happens when String is added as the solution for name. The type String propagates further through the constraints:

1. String = α is added as the solution. Propagating this yields...

2. String ≤ [strip : () → γ]. Looking up the type of String#strip (`SimTyper` includes types for Ruby's core and standard libraries) yields...

3. String ≤ γ. In other words, strip returns a String. Propagating further yields...

4. (in several steps) String ≤ δ, i.e., [0] also returns a String. Propagating further yields...

5. (in several steps) String ≤ ϵ, i.e., upcase returns a String. Propagating to ϵ's upper bound...

6. $\mathsf{String} \leq \beta$.

Then, using the usual rules for computing the solution at a return position, we can set $\beta = \mathsf{String}$ also. Thus, we have found that the method has type $(\mathsf{String}) \to \mathsf{String}$, which matches its documentation. This exemplifies another benefit of `SimTyper`: by integrating DeepSim within the constraint solver, the former can lead the latter to better type solutions, and vice versa.

As an aside, we note that the same cascading effect can happen with heuristics—and in fact, in this case that would occur, as `InferDL` includes the rule STRING_NAME that guesses that arguments called `name` have type `String`. While there are many cases from our benchmarks where DeepSim alone leads to a cascading solution, we present the example of Figure 5.3 due to its relative brevity and simplicity.

Discussion. An alternative approach would be to use machine learning to directly predict types. However, prior work [113, 118, 119], as well as our own dataset (§ 5.4), has found that the distribution of types in programs is Zipfian: a small number of types occur very frequently, while most types occur rarely. Moreover, some types are program-specific and thus will not occur in a training dataset at all. This makes it challenging to train a direct prediction model for the many infrequent types. Moreover, DeepSim can perform one-shot type prediction, in which it predicts the correct type of an argument/return by knowing the type of just one other instance. Finally, DeepSim is tightly integrated with type inference, allowing it to propagate any usable types that standard inference infers.

$$
\begin{array}{lrcl}
\textit{Types} & \tau & ::= & \alpha \mid A \mid [m : \tau \to \tau] \mid \tau \cup \tau \mid \tau \cap \tau \mid \bot \mid \top \\
\textit{Constraints} & C & ::= & \tau \leq \tau \mid C \cup C \\
\textit{Solutions} & S & ::= & \{\alpha_1 \mapsto \tau_1, ..., \alpha_n \mapsto \tau_n\}
\end{array}
$$

$$\alpha \in \text{type vars} \quad A \in \text{class names} \quad m \in \text{meth names}$$

$$\textit{resolve} : C \to C \cup \textit{error} \qquad \text{EXTRACT\_SOLUTION} : H \times C \times \alpha \to C \times \tau$$

$$H \in \text{heuristics}$$

Figure 5.4: Types, constraints, solutions, and inherited functions.

5.3 Type Inference Algorithm

In this section we present `SimTyper`'s algorithm more formally. We first briefly discuss the elements of the algorithm that are inherited from `InferDL` (§ 5.3.1), then build on these to present the type equality prediction algorithm (§ 5.3.2).

5.3.1 Standard and Heuristic Inference

In Section 4.3, we provided a full discussion of `InferDL`'s algorithm, which combines constraint solving and configurable heuristics. Here, we briefly summarize the basic definitions and relevant aspects of this algorithm that are inherited by `SimTyper`.

The top part of Figure 5.4 defines a core language of types, constraints, and solutions. Types include variables α, nominal types A, structural types $[m : \tau \to \tau]$ (which assume a single argument for simplicity), union types, intersection types, the bottom type $\bot$, and the top type $\top$. Constraints C take the form $\tau_1 \leq \tau_2$, meaning τ_1 is a subtype of τ_2. We use union to construct sets of constraints. Finally, a

solution S is a mapping from variables to their types.

The bottom part of Figure 5.4 gives the types for the *resolve* and EXTRACT_SOLUTION functions, which we inherit from `InferDL`. Given a set of constraints, *resolve* performs constraint resolution and returns either a new set of constraints or *error* if the given set of constraints are inconsistent. The EXTRACT_SOLUTION function takes a list of heuristics, a set of constraints, and a type variable α, and uses a combination of constraint solving and heuristics to extract a solution for α; it returns a new set of constraints C which includes the new solution, as well as the solution τ itself. We will use both of these functions in the type equality prediction algorithm defined below, and we refer the reader to section 4.3 for their definitions.

Finally, we assume the existence of two functions. The first is the predicate $og(\tau)$, which returns true when τ is considered overly-general and false otherwise. Following Chapter 4, for this core language of types, $og(\tau)$ is true when τ is a non-nominal type. The second function, $usable\_sols$, returns the set of solutions in a given mapping that are not overly general:

$$usable\_sols(S) = \{\alpha \mapsto \tau \in S \mid \neg og(\tau)\}$$

5.3.2 Type Equality Prediction Algorithm

We can now build on these definitions to introduce the type equality prediction algorithm. We first introduce the function *sim* to represent the DeepSim neural

procedure DEEPSIM_SOL(α, S, C)
  $poss\_sols \leftarrow \{(\tau_i, s) \mid \alpha_i \mapsto \tau_i \in usable\_sols(S),\ s = sim(\alpha, \alpha_i), s \geq 0.5\}$
  $k \leftarrow 1$
  while $k \leq size(poss\_sols) \wedge k \leq N$ **do**
    pick kth highest scoring solution τ_k in $poss\_sols$
    $C_{new} \leftarrow resolve(C \ \cup \ \tau_k \leq \alpha \ \cup \ \alpha \leq \tau_k)$
    if $C_{new} \neq error$ **then**
      return C_{new}, τ_k
    $k \leftarrow k + 1$
  return C, $S[\alpha]$

(a) DEEPSIM_SOL, which uses DeepSim to find a solution for a single type variable.

procedure TYPE_EQ_INFERENCE(H, C, $\mathcal{V}$)
  $S \leftarrow \{\}$
  $first\_round \leftarrow true$
  repeat
    for each $\alpha \in \mathcal{V}$ **do**
      C, $sol \leftarrow$EXTRACT_SOLUTION(H, C, α)
      $S[\alpha] \leftarrow sol$
      if $!first\_round \wedge og(sol)$ **then**
        $C, sol \leftarrow$ DEEPSIM_SOL(α, S, C)
        $S[\alpha] \leftarrow sol$
    $first\_round \leftarrow false$
  until no new constraints are added to C
  return S

(b) `SimTyper`'s overall algorithm.

Figure 5.5: Procedures used in `SimTyper`.

network. Given two type variables α_1 and α_2, $sim(\alpha_1, \alpha_2) \in [0, 1]$ is a similarity score between α_1 and α_2, where scores closer to 1 indicate greater similarity and a score below 0.5 indicates dissimilarity. We also assume there is an N specifying the maximum number of similar variables to try for a solution.

Now we can introduce DEEPSIM_SOL, the procedure for finding a single DeepSim network solution, which we define at the top of Figure 5.5. Given a type variable α, a solution S, and a constraint set C, the function DEEPSIM_SOL returns an updated constraint set C_{new} and a solution for α; if DeepSim did not find any new

solution, then the returned constraints and solution are the same as the original ones provided to the function.

The first line of DEEPSIM_SOL defines $poss\_sols$ to be a set of pairs, where the first element of the pair is a possible type solution, and the second element is the corresponding similarity score. The set is constructed by comparing α with every $\alpha_i \in dom(usable\_sols(S))$, and keeping the corresponding type solution τ_i when its similarity score is above 0.5. Then the function loops, picking the highest scoring solution τ_k in $potential\_sols$. The function then "tests" the solution by equating it to α and running constraint resolution. If this succeeds, we have found a consistent guess, so the function returns the new set of constraints and the found solution τ_k. Otherwise, the loop continues with the next highest score, etc. If we exceed N iterations or explore all the potential matching solutions, then the function returns the original constraint set and the original solution for α in S, since no consistent guess was found.

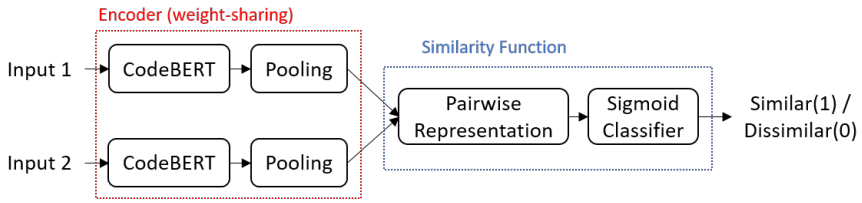
Finally, we present `SimTyper`'s overall algorithm, the procedure TYPE_EQ_INFERENCE defined at the bottom of Figure 5.5. This procedure is very similar to the EXTRACT_ALL_SOLUTIONS procedure used by `InferDL` (§ 4.3), but it adds the use of DEEPSIM_SOL. The procedure takes as input a set of heuristics H, a set of constraints C, and a set of type variables to find solutions for $\mathcal{V}$. It begins by assigning an empty solution map to S, and by assigning the variable $first\_round$, which keeps track of whether we are performing the first round of solution extraction, to *true*. Next, we iterate through each variable in $\mathcal{V}$ and perform standard and heuristic inference by calling EXTRACT_SOLUTION, updating the constraints C and

solutions S as we go. We perform one full round of standard and heuristic inference over all type variables before ever using DeepSim; this gives us more usable solutions to compare against with DeepSim. After one round, we perform inference again, this time calling DEEPSIM_SOL for any type variables which still have overly general solutions. We update C and S with any new solutions. This process repeats until no new constraints are added to C; by looping until we reach a fixpoint in C, we can allow any new solutions found by DeepSim or constraint solving to cascade to new solutions, as discussed in Section 5.2.

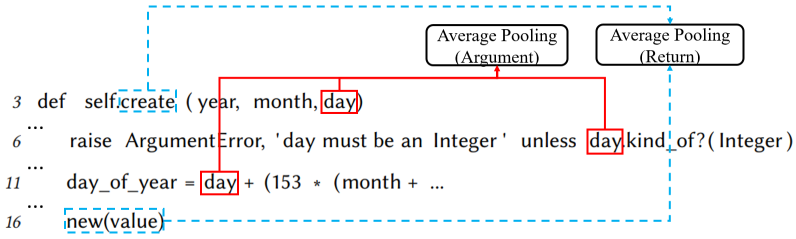
Note that this algorithm is greedy, so the order order in which DeepSim network solutions are generated may matter. In particular, if the network generates incompatible solutions for two different type variables (i.e., the resulting constraints are inconsistent), then the solution that was generated earlier may effectively block the later solution. In our implementation, the order used is effectively arbitrary. Determining a way to pick among incompatible solutions is an interesting avenue for future work.

5.4 Implementing the DeepSim Network

Figure 5.6a shows DeepSim's network architecture. DeepSim encodes a pair of inputs into a pair of fixed-dimensional embedding vectors (of the same dimensionality) via a weight-sharing encoder, and then runs them through a similarity function to predict the likelihood both inputs have the same type. Following the Siamese Network structure [115], the encoder used for both inputs is the same. Next, we



(a) Diagram of the DeepSim network.



(b) Two types of pooling in DeepSim, demonstrated on the method from Figure 5.1a. After converting each position into contextualized vectors via `CodeBERT`, the blue dashed lines show the average pooling for the method return, and the red solid lines show the average pooling for the argument `day`.

Figure 5.6: `SimTyper`'s Deep Similarity (DeepSim) Neural Network.

discuss the network in more detail.

Network Input. For each argument, the network takes as input the tokenized source code for the method containing the argument plus the positions at which the argument appears. More formally, suppose arg_1 and arg_2 are the method arguments to be compared, and that X_i is the tokenized source code of the method in which arg_i appears. Then the input to DeepSim is the two token sequences $X_1 = \ldots, x_1^{i_1}, \ldots, x_1^{i_2}, \ldots, x_1^{i_m}, \ldots$ and $X_2 = \ldots, x_2^{j_1}, \ldots, x_2^{j_2}, \ldots, x_2^{j_n}, \ldots$, where each $x_1^{i_k}$ represents the mention of arg_1 at position i_k in the first sequence, and each $x_2^{j_k}$ represents the mention of arg_2 at position j_k in the second sequence. The input also includes the sequences $i_1, i_2, \ldots, i_m$ and $j_1, j_2, \ldots j_n$, i.e., the indices of the parameters within the source code tokens.

DeepSim uses an analogous approach when comparing method returns: The input is the method's tokenized source code plus the positions of the method name itself and all the returns within its body. We use the method's name because it is often used to describe the return value; § 5.5 includes an evaluation of different approaches to representing returns.

Contextualized Vector Representations. Next, DeepSim encodes each input token sequence into a sequence of *contextualized vector representations* (one vector per token) using `CodeBERT` [111], a transformer-based [120] pre-trained code embedding model.[2] Contextualized vector representations can capture both the English-language meaning of tokens and the surrounding code context. The goal is for tokens with similar meanings (e.g., year and month) and usage (e.g., used inside basic arithmetic expressions) to map to nearby vectors in the vector space.

Continuing the formal notation just above, the output of this layer is the sequences of vectors $CV_1^1, \ldots, CV_1^m$ and $CV_2^1, \ldots, CV_2^n$, where CV_1^k is the contextualized vector representation of token $x_1^{i_k}$ for arg_1, and CV_2^k is the representation of token $x_2^{j_k}$ for arg_2. These vectors are of dimension $d = 768$, the size of vectors produced by `CodeBERT`. Note that we keep only the contextualized vectors for the tokens of the relevant argument/return.

Pooling. Next, the encoder uses a pooling layer to combine the contextualized vector representations into a single, fixed-dimension vector. We use mean pool-

[2] `CodeBERT` was trained on over 8.5 million methods and functions from programs written in Ruby, Java, JavaScript, Go, PHP, and Python.

ing, which is shown to be effective in Siamese Network models for text similarity tasks [121]. For example, as illustrated in Figure 5.6b for our running example, for the argument day, the pooling layer averages the vectors corresponding to its mentions in the method header and method body. For the return of self.create, the pooling layer averages the vectors for the method name in the method header and for the last line of the method, whose value is returned. Interestingly, our experiments suggest mean pooling is important for arguments but not for returns (§ 5.5.5).

Formally, the output of this layer is two fixed-dimensional vectors $V_1 = MeanPool(CV_1^1, \ldots, CV_1^m)$ and $V_2 = MeanPool(CV_2^1, \ldots, CV_2^n)$, where $MeanPool$ averages its vector arguments.

Similarity Function. The subsequent stage, the *similarity function*, produces a similarity score for the encoded inputs (now a pair of fixed-dimension vectors). First, the pair of vectors are joined to form a relational vector representing the pair as well as their interactive features. Then this relational vector is run through a sigmoid function to produce a similarity score in the range (0, 1), where a score closer to 1 indicates the inputs likely have the same type, and a score closer to 0 indicates the inputs likely have different types.

Formally, the similarity function begins by concatenating V_1 and V_2 with the element-wise difference $|V_1 - V_2|$ to generate the pair representation $V = (V_1, V_2, |V_1 - V_2|)$. This approach follows Reimers and Gurevych [121], who show it to be effective in capturing both input features and the interactive features between the pair. We then apply a fully-connected layer with trainable weight matrix $W \in \mathbb{R}^{3d}$ (recall

```ruby
# Multiplies the monetary value with the given number and returns a new
# +Money+ object with this monetary value and the same currency.
# @param [Numeric] value Number to multiply by.
# @return [Money] The resulting money.
def *(value) ... end
```

Figure 5.7: A method * from the *Money* library which has YARD documentation.

$d = 768$ is the dimensionality of the CodeBERT contextualized vectors) and bias term $b \in \mathbb{R}$, and then pass the result through the sigmoid function σ, which squashes values into the valid probability range [0,1] to generate the probability of X_1 and X_2 being similar ($y = 1$):

$$P(y = 1|X_1, X_2) = \sigma(W \cdot V + b) = \frac{1}{1 + e^{-(W \cdot V + b)}}$$

Taking N training pairs with labels 1 or 0 ($y_i = 1$ or 0 for the ith pair), DeepSim is trained with the Adam optimizer [122] by minimizing the binary cross-entropy loss, which is a common choice for binary classification tasks:

$$\mathcal{J} = -\sum_{i=1}^{N}(y_i \cdot log(p(y_i)) + (1 - y_i) \cdot log(1 - p(y_i))$$

5.4.1 Training the DeepSim Network.

`SimTyper` trains two different networks: one for comparing arguments, and one for comparing returns. To train the networks, we need type information for Ruby programs. However, Ruby is dynamically typed and does not include type annotations. Recently, several Ruby type systems have emerged, including `RDL` [20], Sorbet [92], and the new types available with Ruby 3.0. However, to the best of our

knowledge, the number of publicly available programs with type annotations is still very small and therefore is insufficient for training. One idea for generating training data would be run Ruby programs and document the observed runtime method types. Though Strickland et al. [123] found that such types are often specific to a single run of a program and thus may miss possible types, it is possible this approach would be sufficient for a training dataset and it is worth exploring in the future.

Instead, we collect type information from programs that use `YARD`, a popular Ruby documentation tool. Figure 5.7 shows an example of `YARD` documentation for method `*` of the *Money* library, one of our benchmarks. The first two comment lines provide a general description, and the last two lines give structured information including types of the parameter (of type Numeric) and the return (of type Money). Note that this documentation is noisy because it may not be accurate—there is no system that automatically checks `YARD` documentation against code to enforce its correctness—and the standard notation for types in `YARD` is not enforced. Nevertheless, for purposes of training DeepSim, it is still very effective, especially since DeepSim can tolerate some noise.

To build our training dataset, we looked at the top 1000 starred Ruby repositories on Github, and the top 1000 gems on `rubygems.org`, Ruby's central package hosting service. After eliminating overlapping programs and removing programs without `YARD` type data, we were left with 371 Ruby programs, comprising over 285,000 methods with documented types, and over 417,000 individual data points, where each parameter and return type is counted as a separate data point.

However, for training, DeepSim expects pairs of inputs labeled with 1 for pairs

with the same type and 0 otherwise. The set of all possible pairs from our dataset of over 417,000 would be prohibitively large, so we restrict ourselves to a set of 100,000 randomly chosen pairs; the number of pairs was chosen through a tuning process discussed below. Moreover, we restrict training pairs to come from the same program, with the idea that the naming choices and coding patterns ued within a program are more likely to be cohesive than the choices between different programs.

Hyperparameters. We tuned three hyperparameters for the DeepSim network: the number of data points, the number of training epochs, and the learning rate. We considered all data sizes from 25,000 to 200,000 in increments of 25,000, all numbers of epochs from 25 to 200 in increments of 25, and all learning rates in the set {0.001, 0.0005, 0.0001, 0.0005}. We used grid search, training networks using all possible combinations of values for the hyperparameters, then selecting the models that scored the highest accuracy on a validation dataset that was independent from the training data and test benchmarks used in our experiments (§ 5.5). Ultimately, we trained the argument model using 150 epochs and the return model using 100 epochs, and both models were trained with 100,000 data points and a learning rate of 0.001.

Types for Instance, Class, and Global Variables. In addition to predicting type similarity for arguments and returns, `SimTyper` also uses DeepSim to do the same for instance, class, and global variables. However, there are a few changes required. First, because variables can be both read and written, it is not the case

that a greatest or least solution will always be most general. Instead, `SimTyper` follows the approach of Kazerounian et al. [110] (also presented in Chapter 4), which was shown to work well in practice: when the variable's type has upper bounds, `SimTyper` uses the intersection of the type's upper bounds, and otherwise `SimTyper` uses the union of its lower bounds.

Second, for a given variable, pooling averages the vectors corresponding to all uses of that variable. However, unlike arguments and returns, instance, class, and global variables can be accessed in multiple methods. Hence, the encoding step must vectorize all the methods that refer to the variable. Note that although class, global, and (some) instance variables can be accessed outside of methods, `SimTyper` currently does not include such occurrences in its analysis.

Finally, `YARD` does not currently support documentation for instance, class, and global variables. Thus we could not collect training data for them. Instead, we use the network we trained for arguments to answer questions about variables, since we expect arguments may be named similarly to and used in similar contexts to non-local variables. In the future, we are also interested in exploring whether we could use just a single network for returns, arguments, and variables.

5.5 Evaluation

We evaluated `SimTyper` on a range of Ruby benchmarks with existing type information, which we treated as gold standard types we aim to infer. Our benchmarks came from two sources. First, we used the same four Rails web apps that `InferDL`

was evaluated on (Section 4.5). We refer to these as the *InferDL Benchmarks*:

- *code.org* [71] – the `code.org` website app
- *Discourse* [69] – online discussion platform
- *Journey* [72] – site for creating surveys and collecting responses
- *Talks* [94] – site for sharing talk announcements

For these apps, we use `SimTyper` to infer types for all methods and instance, class, and global variables for which manually written type annotations already existed in the `InferDL` study.

Second, we applied `SimTyper` to four popular, well-maintained libraries that have extensive `YARD` documentation that provides types for a majority of their methods. We refer to these as the *YARD Benchmarks*:

- *TZInfo* [100] – library for manipulating timezone data
- *MiniMagick* [97] – wrapper for the ImageMagick image manipulation platform
- *Ronin* [124] – platform for vulnerability research and exploit development
- *Money* [39] – library for currency arithmetic and conversion

We were particularly interested in libraries because, in our experience, they are more likely to have well-documented APIs compared to complete programs like web apps. For these libraries, we use `SimTyper` to generate types for all methods with `YARD` type documentation except those that use features that are not supported by `InferDL`.

Program	# Meths	LoC	# Vars	# Casts
code.org	74	689	11	4
Discourse	43	331	0	0
Journey	23	375	26	1
Talks	110	878	47	8
MiniMagick	40	216	0	2
Money	87	444	0	12
Ronin	226	1628	0	23
TZInfo	241	1292	0	5
Total	**844**	**5853**	**84**	**55**

Table 5.1: Benchmark statistics.

The most common feature that blocked standard type inference was the presence of mixins, which are only partially supported by `InferDL`. Note that we excluded any measurements about instance, class, and global variables for these benchmarks because `YARD` does not include documentation about variable types. Finally, we withheld all the type data for these programs from the datasets we used for training and validation (§ 5.4.1).

Table 5.1 summarizes the benchmarks' statistics. For each benchmark, the table lists the number of methods for which `SimTyper` generates a type annotation that we compare against a gold standard, followed by the number of lines of code comprised by those methods. Additionally, the subsequent column lists the number of non-local (instance, class, and global) variable annotations generated by `SimTyper` that we compare against gold standards. In total, we ran `SimTyper` on 844 methods comprising 5,853 lines of code, and generated types for 84 non-local variables. Note that the number of types for variables for the YARD Benchmarks was 0, since `YARD` does not include documentation for these variables that we can compare against. We additionally note that, while we ran `SimTyper` for more methods in the YARD

Benchmarks, there were some which did not include gold standard types to compare against.

The table also shows the number of type casts required to run standard type inference on the benchmark's methods. The first four benchmarks already came with type casts from `InferDL`; we wrote type casts for the last four benchmarks. In our experience, the most common reasons for needing type casts were to handle path-sensitive typing and to cast a value extracted from heterogeneous data structures like arrays and hashes.

Additionally, we note that there were some method and variable types that `SimTyper` inferred which we did not have a gold standard to compare against. More precisely, all of the YARD Benchmarks included some method types inferred by `SimTyper` without corresponding gold standards, and seven out of eight of all of the benchmarks included at least one instance, class, or global variable type inferred by `SimTyper` without a corresponding gold standard. While we could not compare these inferred types against gold standards, we found that of 418 non-compared argument, return, and variable types, 277 were usable types, 81 were overly-general, and `SimTyper` failed to infer types for 60 of these positions. The full results are shown in Table B.1 in the Appendix.

Evaluation Methodology. We ran `SimTyper` on the above benchmarks under four separate configurations: using constraint solving alone (C), constraint solving and `InferDL`'s built-in set of heuristics (CH), constraint solving and the DeepSim network (CD), and all three approaches together (CHD). We compare the results to

the original type annotations (InferDL Benchmarks) or `YARD` documentation (YARD Benchmarks). To provide finer-grained analysis, we make comparisons on a per-argument, per-return, and (for InferDL Benchmarks) per-variable basis, rather than, e.g., comparing whole method signatures at once. Inspired by prior work [113], we place each comparison in one of three categories:

- *Match.* `SimTyper` inferred a type that exactly matches the gold standard or is a subtype of the gold standard. We also consider a match exact if both the generated and gold standard types are from the set {String, Symbol, String ∪ Symbol }. In Ruby, Symbol is a special kind of interned String, and the two types are often used interchangeably. Lastly, we also treat the types Array and ActiveRecord_Relation (Rails' special array implementation, used for database queries) as interchangeable.

- *Match up to Parameter.* The gold standard type is a generic type, and `SimTyper` inferred the base of the generic type but not the parameter. For example, if the gold standard was Array<String>, then `SimTyper` generating either Array or Array<Integer> would fall in this category. This category provides a notion of partial matches.

- *Different Type.* `SimTyper` inferred a type that was consistent with the constraints—hence it is sound—but differs from the gold standard type in a way that does not fall into the above categories. For example, inferred structural types fall into this category because they very rarely occur in the gold standard types (out of 1,496 gold standard annotations in our benchmarks, just 8 use struc-

tural types). Note that it is possible for programmer-provided and `SimTyper`-inferred types to be incomparable, e.g., below we mention a case when one is Integer and the other is String, and both are sound because they share a common structural supertype.

5.5.1 SimTyper Results

Figure 5.8 shows the types inferred by `SimTyper` under the C, CH, CD, and CHD configurations, categorized as just described. We split the last category, *Different Type*, into structural and non-structural types, to aid the discussion below. These results were collected using the top-3 solutions suggested by DeepSim with a similarity score cutoff of 0.5 (see § 5.3). Below, unless we say otherwise, comparisons of *matching* sum both *Match* and *Match up to Parameter*. Table B.2 in Appendix B presents the same data as the figure.

Looking at the totals (lower right corner of the figure), we see that the CHD configuration outperformed all others in the number of matches to the gold standard. In total, `SimTyper` inferred 1,033 total matching annotations under CHD, compared to 610 under C alone, a 69% increase. This is still a 66% increase if we exclude matches up to parameter. CHD inferred the most matching types not only in total, but also for each individual benchmark, with the exception of *Money*, for which CD performed only slightly better. This suggests that combining constraint solving, heuristics, and the DeepSim network is an effective approach to inferring type annotations that match what a programmer would write.

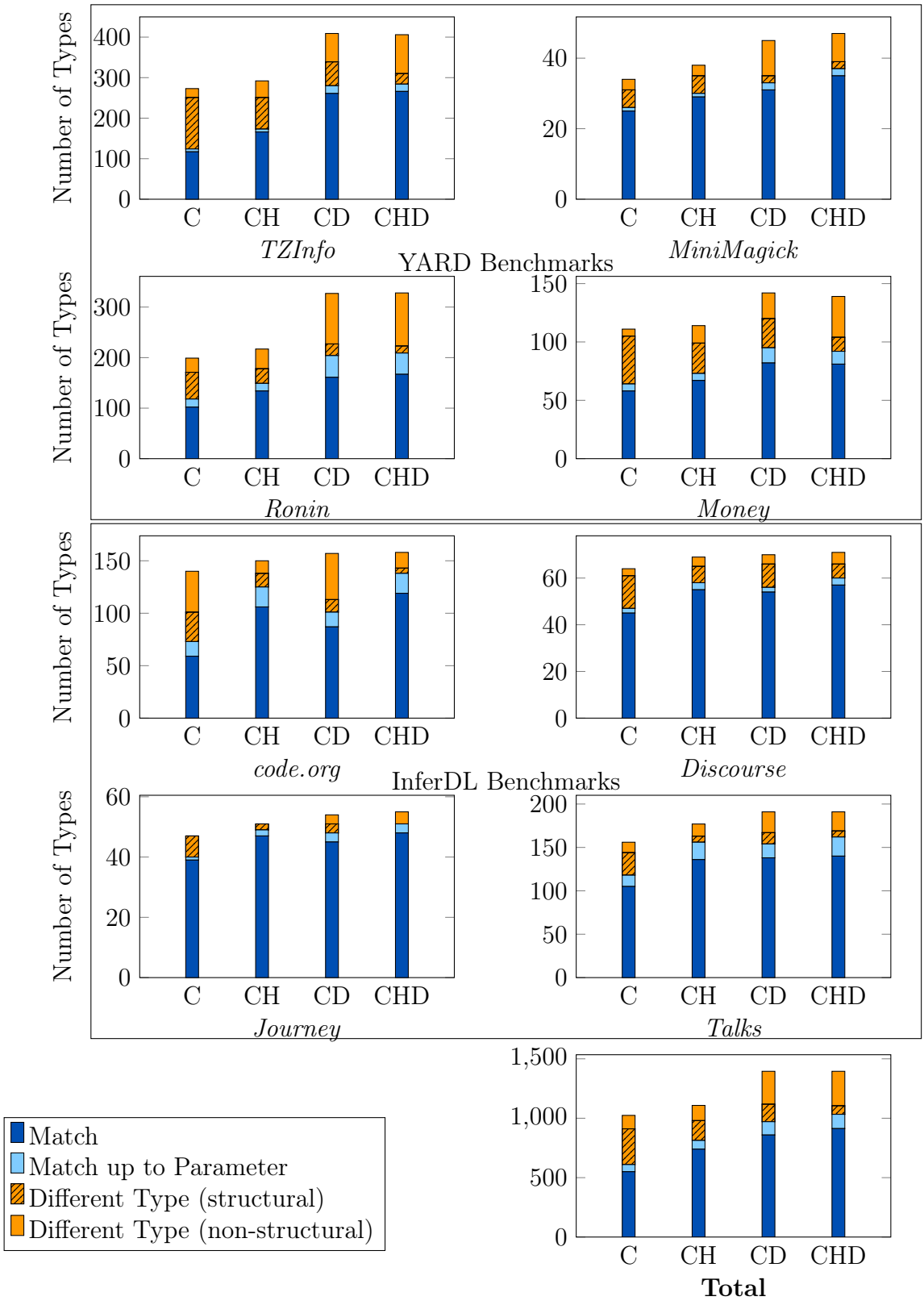


Figure 5.8: Assessing the types inferred by `SimTyper`. We collected results under four configurations: constraint solving (C), constraint solving plus heuristics (CH), constraint solver plus the DeepSim network (CD), and all three (CHD). Results presented here use top-3 thresholding. Note the y-axis is scaled for each benchmark.

Comparing CD to CH, we see that in total, CD inferred 19% more matching types than CH (or 16% excluding matches up to parameter), indicating that DeepSim can outperform hand-written heuristics. Interestingly, while this was true overall, there is a contrast between the YARD Benchmarks and the InferDL Benchmarks. For the YARD Benchmarks, CD infers 44% more matches than CH, but for the InferDL Benchmarks, CD generates 7% fewer matches than CH. The biggest single contributor to the difference is *code.org*, where CD generates 20% fewer matches than CH. We believe the reason for this overall difference is that the heuristic rules of `InferDL` were developed while applying type inference to the InferDL Benchmarks. For example, `InferDL` includes a heuristic INT_NAMES that guesses that arguments ending in _id have type Number and arguments ending in _ids have type Array<Number>. Without this heuristic, in the *code.org* app, most such positions are inferred by standard type inference to be Number ∪ Array<Number>. DeepSim can at best propagate this union type—it has no particular mechanism to refine it—and so it cannot improve on standard type inference in these cases. Moreover, the INT_NAMES heuristic was applied more than 30 times for *code.org*, compared to just 5 times for all other benchmarks combined. Thus, we see the tradeoff between DeepSim and hand-written heuristics: the heuristics perform well on their initial target but do not necessarily generalize to other programs, while DeepSim generalizes well but does not fully cover all uses of heuristics. Thus, we think `SimTyper`'s architecture, which incorporates both approaches, is the right design choice.

Finally, we examined the *Different Types* category in more detail. We note that in the C configuration, structural types constitute the majority of different

types inferred. The one exception is *code.org*, where C infers union types in many positions as discussed just above. As we introduce heuristics and DeepSim, we see a clear trend where the number of structural different types decreases as they are replaced by nominal and generic types (indeed, by design, `InferDL` heuristics and DeepSim do not infer any new structural types), some of which become matches and some of which remain different types. The other category that decreases, not shown explicitly in the figure but included in the table in Appendix B, is positions where C could only infer a variable type, but heuristics or DeepSim could infer a nominal or generic type.

To get more insight into the non-structural different types, we manually examined their occurrence in *Money* under CHD. We found these types fall into roughly two categories. First, in some cases the gold standard type is a union and DeepSim's inferred type was one arm of the union. For example, for one parameter new_currency, the gold standard type is Money::Currency ∪ String ∪ Symbol and the DeepSim inferred type is Money::Currency. Second, sometimes DeepSim infers a type that was unrelated to the gold standard but happened to be consistent with the program. For example, for one parameter named amount, the gold standard type is Number but DeepSim inferred the type String. The latter type is consistent because the only use of amount is to call to_d on it (to convert it to a BigDecimal), and that method is also defined on String. It is interesting future work to try to address both of these cases.

Failed Inference. There are also some arguments, returns, and variables for which `SimTyper` fails to infer any type. Table B.2 in the Appendix presents the specific number of positions for which this was the case under each configuration. Under CHD, across all benchmarks, `SimTyper` fails to infer a type for 6.8% of arguments, returns, and variables. These are cases where there are not enough constraints for standard inference to produce a solution: there are no non-type variable upper bounds (for arguments and variables) or lower bounds (for returns and variables). In other words, for arguments, no method is called on it (otherwise it would at least have a structural type upper bound); and for returns, typically the returned value comes from a method SimTyper does not analyze (e.g., a third-party library), and hence its signature has type variables that are not constrained by its method body.

Additionally, it must be that both heuristics and DeepSim either fail to guess a type, or they guess a type that is inconsistent with existing constraints. Of the 6.8% of positions (arguments/returns/variables) for which SimTyper failed to infer a type under CHD, the DeepSim network guessed a type for about 46% of these positions, but the guess was rejected due to inconsistency with constraints; none of the heuristics guessed a type for any of these positions. Interestingly, in 5 of these positions, the type guessed by the DeepSim network was actually a correct array or hash type, but the guess was rejected because the type checker conservatively uses invariant subtyping for arrays and hashes.

Precision and Recall. Another way of measuring `SimTyper`'s results is in terms of precision and recall. Following Pradel et al. [125], we compute precision as

Config	Precision	Recall
C	59.6%	40.8%
CH	73.4%	54.3%
CD	69.6%	64.9%
CHD	74.1%	69.1%

Table 5.2: Precision and recall of `SimTyper`.

n_{match}/n_{type}, where n_{match} is the number of type matches (including up to parameter) and n_{type} is the total number of positions (arguments, returns, and variables) for which `SimTyper` inferred any type. We compute recall as n_{match}/n_{all}, where n_{all} is the total number of positions for which `SimTyper` attempted to infer a type (whether it did so or not).

From the table, we see that, consistent with the earlier interpretation of the data, CHD achieved the highest precision and recall. Additionally, CH outperformed CD on precision by 3.8%, while CD outperformed CH on recall by 10.6%. This means that heuristics alone are less likely to predict a matching type than DeepSim alone, but when they do predict a type, the type is slightly more likely to be a matching one. C was the worst performing configuration in both precision and recall.

We note that our notion of precision and recall is slightly different from Allamanis et al. [113]. They compute precision as $n_{neutral}/n_{type}$, where $n_{neutral}$ is the number of "neutral" types: predicted types that pass a type checker. By this measurement, under all four configurations, `SimTyper`'s precision would be 100%, since all of its predicted types are consistent with the program's constraints. Moreover, they compute recall as n_{type}/n_{all}, that is, the proportion of the dataset for which any type was inferred. By this metric, the recall for CHD would be 93.2%. Instead,

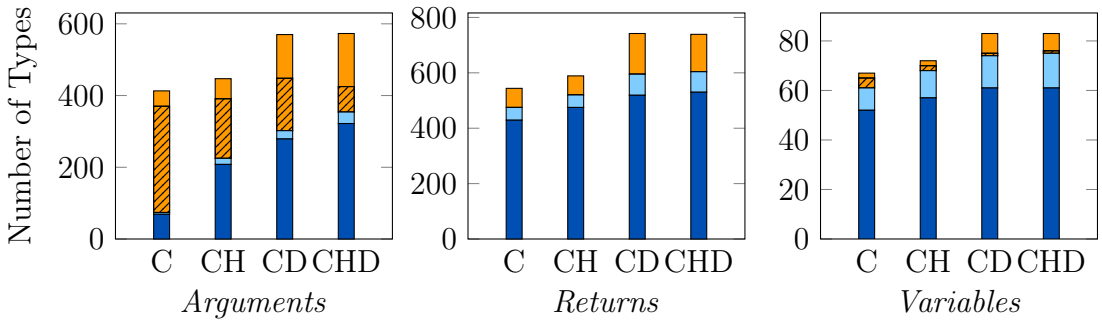


Figure 5.9: Measuring `SimTyper`'s performance for arguments, returns, and instance, class, and global variables across all benchmarks. The plots use the same legend as Figure 5.8. Measurements were taken using top-3 thresholding.

we focus on the number of matches inferred by `SimTyper`, since this measures types that reflect programmer intent.

Arguments vs. Returns vs. Variables. Recall that `SimTyper` uses separate networks for arguments and returns, and uses the argument network for instance, class, and global variables with some small adaptations (§ 5.4.1). Figure 5.9 measures `SimTyper`'s performance separately for these three groups. The data for these plots are included in Appendix B.

The figure shows that DeepSim improved performance the most on arguments. Under CD and CHD, `SimTyper` infers approximately 308% and 57% more matching (including up to parameter) types relative to the C and CH configurations, respectively. This is at least in part because arguments had the most room for improvement. For example, under C, `SimTyper` inferred matches for just 12% of all argument types, while for return and variable types, it inferred 60% and 73% matches, respectively.

However, it is also plausible that DeepSim is best tuned for arguments. First,

Program	Median Time (s) ± SIQR			
	C	CH	CD	CHD
code.org	4.0 ± 0.03	124 ± 0.36	72 ± 1.52	189 ± 0.27
Discourse	1.0 ± 0.10	32 ± 0.09	33 ± 0.33	62 ± 0.23
Journey	0.7 ± 0.06	3 ± 0.11	42 ± 0.31	41 ± 0.40
MiniMagick	0.5 ± 0.02	2 ± 0.02	39 ± 0.66	37 ± 0.37
Money	1.0 ± 0.03	5 ± 0.12	72 ± 1.91	71 ± 1.07
Ronin	2.0 ± 0.04	12 ± 0.21	172 ± 2.23	177 ± 2.22
Talks	2.0 ± 0.10	6 ± 0.21	68 ± 0.80	70 ± 1.35
TZInfo	2.0 ± 0.04	8 ± 0.21	156 ± 0.99	161 ± 0.75
Total	**13.2** ± 0.42	**192** ± 1.33	**654** ± 8.75	**808** ± 6.66

Table 5.3: Running time of `SimTyper` over nine runs.

recall that we could not train a network specifically on variables since we did not have this data (§ 5.4). Second, we found that incorporating return sites into return embeddings does not significantly improve performance (§ 5.5.5). We leave exploring other ways to incorporate method code into DeepSim's predictions to future work.

5.5.2 Performance

Table 5.3 measures the performance of `SimTyper` in performing type inference. We report the median time and semi-interquartile range (SIQR) over nine runs under each configuration. Times were measured on a 2014 MacBook Pro with a 3GHz i7 processor and 16GB RAM. We can see clearly that DeepSim introduces overhead. CHD and CD are approximately 4.2× and 50× slower than CH and C, respectively. Upon closer inspection, we found the biggest bottleneck was running `CodeBERT`; in the future, we plan to explore methods for speeding up this performance, such as alternative methods for batching inputs to `CodeBERT`.

Interestingly, there was just one benchmark, *code.org*, that took longer under

CH than under CD. For *Discourse*, CH and CD performance was nearly equal, and for all other benchmarks, CD took notably longer than CH. The slowdown for *code.org* and *Discourse* occurred primarily due to the STRUCT-TO-NOMINAL heuristic, which involves searching all methods defined for all classes in the program, which is a particularly large search space for these benchmarks.

5.5.3 Comparing DeepSim and Heuristics

Table 5.4 reports how often DeepSim predicted a matching (including up to parameter) type that a heuristic rule also guessed. For each heuristic rule (descriptions of the rules are in Chapter 4), the table lists how many types the heuristic matched in CH followed by how many of those matches DeepSim also predicted in CD.

From the table, we see that DeepSim performed best on types guessed by IS_MODEL and STRING_NAME, two name-based heuristics, predicting 87% of those types. DeepSim also inferred a majority of types for PREDICATE_METHOD, another name-based heuristic. This makes sense as DeepSim's embeddings reflect argument and method names. However, as discussed earlier, DeepSim did poorly on INT_NAMES and INT_ARRAY_NAME, even though they are also name-based. This was primarily due to the aforementioned pattern in *code.org* that DeepSim failed to capture.

Overall, DeepSim predicted 43% of the types guessed by heuristics. We also examined the dual (not shown in the table): of the 267 matching types inferred by

Heuristic Rule	H Matches	DS Matches
STRUCT-TO-NOMINAL	61	17 (28%)
IS_MODEL	21	18 (86%)
IS_PLURALIZED_MODEL	6	2 (33%)
INT_NAMES	32	5 (16%)
INT_ARRAY_NAME	3	0 (0%)
PREDICATE_METHOD	32	21 (66%)
STRING_NAME	18	16 (89%)
HASH_ACCESS	9	0 (0%)
Total	**182**	**79 (43%)**

Table 5.4: DeepSim's ability to predict types also guessed by heuristics. *H Matches* is the number of matching (including up to parameter) types inferred by the heuristic in CH, and *DS Matches* counts the subset of those types also inferred by DeepSim in CD. Measurements with the DeepSim network were taken with the top-3 threshold.

DeepSim under CD, heuristics guessed 80 (about 30%) of them under CH. Because these two sets are largely non-overlapping, these results reinforce that using DeepSim alongside hand-written heuristics is an effective combination.

5.5.4 Predicting Rare Types

One potential benefit of `SimTyper` is that it can infer *rare* types, i.e., those that are relatively less common across programs. Such types could be inferred through standard constraint solving, e.g., in Figure 5.1b, standard type inference found Timestamp as a solution; through applying heuristics, e.g., the STRUCT-TO-NOMINAL heuristic can guess a user-defined type that matches a structural type; and through DeepSim, which could predict a rare type by guessing two positions have the same type. In this section, we measure how often DeepSim can predict rare types, since rare types have historically posed a challenge for machine learning-based type inference (§ 5.6).

Library		Training		Program	
Match incl. param.	All	Match incl. param.	All	Match incl. param.	All
240	356	12	13	16	38

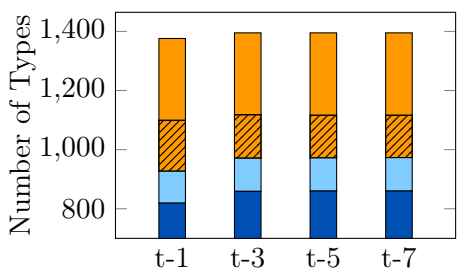
Table 5.5: Numbers of Library, Training, and Program types guessed by DeepSim across all benchmarks, under CD with top-3 cutoff.

Table 5.5 measures three different categories of types inferred specifically by DeepSim. First, the *Library* types are the 78 types in Ruby's standard and core library as well as the core Rails classes. These types are "common" because we expect them to appear frequently in Ruby programs. DeepSim predicted 356 types in this category, of which 240 were matches (67% precision).

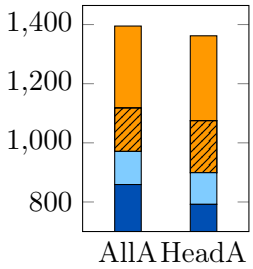
The *Training* types are "rare" types that occur in the training dataset but not in the Library types. In theory we could train a machine learning-based classifier to predict them directly. However, the training data has 9,149 distinct types, yet 71% of all argument and return types in the data are Library types. Hence in practice a classifier would not be very likely to predict non-Library types. From the table, we see that DeepSim predicted 13 Training types, of which 12 were matches (92% precision).

Finally, *Program* types only occur in the benchmarks and not in the standard Ruby libraries or in the training data. Thus, a machine learning-based direct classifier would have no ability to predict these at all. In contrast, DeepSim predicted 38 such types, of which 16 were matches (42% precision).

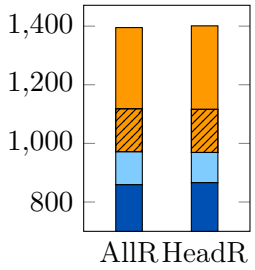
Thus, overall, we can see that while DeepSim often predicts common types (which is expected, since they are common), it can also effectively predict rare types.



(a) Comparing top-1, -3, -5, and -7 thresholding.



(b) Comparing arg embeddings using all arg uses (AllA) vs. just the header (HeadA).



(c) Comparing ret. embeddings, using method names and ret. sites (AllR) vs. names only (HeadR).

Figure 5.10: Evaluating design choices in `SimTyper`. Plots use the same legend as in Figure 5.8 and show data from CD across all benchmarks. Note y-axis starts at 700. Data for all plots appears in Appendix B.

5.5.5 `SimTyper` Design Choices

Finally, we evaluate two design choices in `SimTyper`. First, Figure 5.10a compares top-1, top-3, top-5, and top-7 thresholds for solutions predicted by DeepSim running under CD, across all benchmarks. (We omit heuristics because we are specifically interested in DeepSim here.) The data for these plots is in Appendix B. From the figure, we can see an increase in matches (including up to parameter) from top-1 (927 matches) to top-3 (971 matches). However, the results for top-3, top-5, and top-7 are nearly identical: for each category (match, match up to parameter, etc), the numbers are within two of each other. Thus, we settled on top-3 for our experiments.

Next, we evaluate the use of mean pooling. Figure 5.10b compares pooling the vectors for all uses of an argument (AllA), as in § 5.4, with using just the argument in the method header (HeadA). Figure 5.10c similarly compares pooling method names and return sites (AllR), also as in § 5.4, to using just method names

(NameR). We see that under AllA, `SimTyper` infers 72 more matches than under HeadA, while AllR yields just two more matches than HeadR. This suggests that for arguments, the context from the uses in the method body is important, while for returns the method name alone is likely sufficient.

5.6 Related Work

For a discussion of related work on standard type inference systems, we direct the reader to Section 4.6.

In recent years, researchers have proposed a number of probabilistic type inference systems that aim to address the shortcomings of standard type inference. To the best of our knowledge, `SimTyper` is the first system to apply this style of type inference to Ruby.

JSNice [126] was one of the earliest probabilistic inference systems. JSNice represents JavaScript source as a dependency graph and uses conditional random fields to predict program properties, including type annotations. JSNice is limited to predicting a small set of types seen in training data. DeepTyper [119] trains a bidirectional recurrent neural network (RNN) classifier on JavaScript source code to predict type annotations from over 11,000 types seen in its training data. NL2Type [118] similarly trains an RNN, but exclusively on natural language information (i.e., identifier names and comments) from JavaScript programs. NL2Type predicts type annotations from a set of 1,000 types. Unlike `SimTyper`, none of the above approaches are able to predict types outside their training dataset, nor are

they sound.

TypeWriter [125] trains a neural model to predict types based on identifier names, comments, and source code from Python programs. After ranking the model's predictions, TypeWriter uses a gradual type checker to rule out any inconsistent predictions, similarly to `SimTyper`'s use of constraints to rule out inconsistent predictions from DeepSim. However, unlike `SimTyper`, all of TypeWriter's types come from the neural model, whereas `SimTyper` uses standard type inference to produce an initial set of solutions, and to propagate DeepSim's solutions. Moreover, TypeWriter is restricted to predicting types from its training dataset, while `SimTyper` is not (§ 5.5.4).

Typilus [113] uses a graph neural network model to map program values to an embedding in a *type space*. Types are then predicted based on the similarities of embeddings. Typilus also checks predicted types against an optional type checker to rule out invalid types. Because new types can be added to the type space, Typilus, like `SimTyper`, is able to predict rare types. However, such types must be manually added to the type space. In contrast, in `SimTyper`, rare or user-defined types can be inferred by standard type inference or heuristics and then propagated through use of DeepSim. And, like TypeWriter, all types in Typilus come from the neural network model, whereas `SimTyper` starts with standard type inference.

5.7 Conclusion

We presented `SimTyper`, a system that combines standard type inference via constraint solving, manually written heuristics, and type equality prediction via the DeepSim network, in order to generate usable types. `SimTyper` iterates through the overly general type variable solutions remaining after constraint solving and heuristics. For each such type variable α, it finds the usable type τ from the position most likely similar to α, and then guesses $\alpha = \tau$. Guesses that are consistent with the other constraints are kept, and inconsistent guesses are discarded. In this way, even though DeepSim is probabilistic, `SimTyper` itself always makes sound inferences.

The DeepSim network operates by using `CodeBERT` to encode source tokens into a vector space and then pooling vectors that represent occurrences of the same argument or, for returns, the return positions in the code. A pair of encoded inputs is then run through a trained similarity function, which predicts whether those arguments or returns are likely to have the same type. The network is trained on a set of Ruby programs that include manual type documentation.

We evaluated `SimTyper` on eight Ruby benchmarks and found that combining constraint solving, heuristics, and type equality prediction results in inferring significantly more types that match hand-written types, compared to constraint solving alone. Moreover, we found that the DeepSim network can help to infer rare and program-specific types. Our results show that type equality prediction can help type inference systems effectively produce more usable types. Chapter 5 presents `SimTyper` in full.

Chapter 6: Conclusion

In this dissertation, I presented several ideas for making static type systems for dynamic languages more expressive and usable.

First, in Chapter 2, I presented `RTR`, a system that adds refinement types, basic types extended with expressive logical predicates, to the Ruby type checker `RDL`. `RTR` works by encoding its verification problems into `Rosette`, a solver-aided host language. `RTR` is able to verify ruby programs that use highly dynamic features, such as metaprogramming and mixins, using a combination of assume-guarantee reasoning and a novel approach called just-in-time verification, whereby program verification is deferred until runtime. We evaluated `RTR` by using it to verify a range of functional correctness properties in six Ruby programs.

Though `RDL` is useful for type checking, it still has trouble handling some constructs that are common in Ruby programs, such as databases and heterogeneous data structures. To address this, in Chapter 3 I introduced `CompRDL`, a system that extends `RDL` with type-level computations; we call types that include such computations *comp types*. With comp types, we can check more precise properties in Ruby programs, e.g., we can check the correctness of the names of columns and types of values in complex database queries. Additionally, due to the precision

of comp types, fewer manually inserted type casts are needed for operations over heterogeneous data structures such as arrays and hashes. We evaluated `CompRDL` by writing 586 comp type annotations over six Ruby libraries, then using these to type check six Ruby programs. We found that comp types significantly reduced the need for type casts when type checking these programs. Moreover, comp types helped us discover three previously unknown errors.

One difficulty in using `RDL` (as well as `RTR` and `CompRDL`) is that it places the burden of writing type annotations on the programmer. To address this issue, in Chapter 4 I introduced `InferDL`, a system for inferring usable type annotations. While standard, constraint-based type inference generates types that are technically correct, they are often complex, verbose, and overly general. `InferDL` complements constraint-based inference with configurable heuristic rules that guess more usable type annotations, i.e., annotations that precisely capture programmer intent. Heuristic guesses are checked for consistency with a program's constraints, and any inconsistent guesses are discarded, thereby ensuring the soundness of inferred types. We evaluated `InferDL` by writing eight heuristics and running it on four benchmarks and six case study programs. We found that, compared to constraint-based inference, `InferDL` inferred 22% more annotations that were as or more precise than programmer-written annotations, and `InferDL` also found six new type errors in the process.

While `InferDL` helps generate more usable type annotations, there is still room for improvement. While in theory, a programmer could continue writing new heuristics for each new target program, this process is time-consuming, and the heuristics

may not generalize well to new targets. Thus, to improve on `InferDL`, in Chapter 5 I introduced `SimTyper`, a system that builds on `InferDL` by introducing a novel technique called *type equality prediction*. This machine learning-based technique uses a deep similarity (DeepSim) neural network to predict when two arguments, variables, or returns have the same type. This allows `SimTyper` to assign usable types to positions with overly general types. Like `InferDL`, any inferred annotations are checked against existing constraints, and thus are guaranteed to be sound. Moreover, unlike prior work on probabilistic type inference, `SimTyper` is able to infer rare and program-specific type annotations. We evaluated `SimTyper` by applying it to eight Ruby programs, and we found that, when using a combination of constraint solving, heuristic, and the DeepSim network, `SimTyper` finds 69% more types that match programmer-written annotations compared to constraint solving alone. Moreover, we found that the DeepSim network was able to infer more matching annotations than the heuristics of `InferDL`.

High Level Conclusion. Based on the novel ideas and promising results presented in this dissertation, I conclude that the proposed type systems represent an important step forward in bringing more expressive and usable static types to dynamic languages.

6.1 Future Work

There are a number of interesting avenues for future work.

Exploring Choice Types. Chapter 4 introduced choice types, an idea loosely inspired by variational type checking [89]. Choice types are used in `InferDL` and `SimTyper` for the specific purpose of considering multiple possible types for inputs to/outputs from a method with an intersection type. Subsequent constraints in a program allow us to eliminate inconsistent arms of a choice type. Thus, they provide us with a way of considering multiple possible types for a value, and eliminating those which are not possible as our program analysis provides more information.

Though we used choice types for this one specific purpose in `InferDL` and `SimTyper`, it is likely they would prove useful for other purposes. For instance, choice types could effectively represent the types of values received from a heterogeneous data structure, e.g., a value taken from an array containing both integers and strings. As we discussed in Chapter 3, we often need manually written type casts for such values since it is difficult for a type checker to statically determine the value's type, which can lead to false-positive type errors. However, a choice type would allow us to consider multiple possible types for the value, and later eliminate inconsistent types rather than raise a type error. Choice types may also prove useful for other purposes, e.g., for handling path-sensitive typing in cases where a value's type may differ depending on which path through a program is taken. Thus, I believe greater study of choice types for dynamic languages could prove interesting and useful.

Type Casts. One challenge that affects all four of the type systems presented in this dissertation is the need for manually written type casts to suppress false positive type errors. Though `CompRDL` reduced the number of casts needed for many

programs, it did not eliminate this need entirely. The necessity of manually written type casts negatively affects the usability of these type systems, and thus it would be useful to explore ways to reduce this need even further. There are multiple possible routes for this exploration. First, we could explore ways to further increase the precision of these type systems, thereby reducing the occurrence of false positive type errors. The choice type discussion above suggests one way for increasing precision. Another possibility would be to use occurrence typing [3], which could reduce false positive errors resulting from path-sensitive typing; occurrence typing is challenging in a language like Ruby, which provides many possible ways for dynamically checking the type of a value, and thus this could be an interesting subject to study.

A different way to reduce the need for type casts would be to develop a means for inferring type casts. This would necessitate (1) determining which program locations could be responsible for false positive type errors, and (2) generating the appropriate casts for these locations.

Error Messages. One challenge for type inference systems generally, and `InferDL` and `SimTyper` in particular, is that error messages are often difficult to understand. This can occur because type errors are sometimes related to many different locations in a program, and thus choosing which locations to report in an error message is nontrivial. Some prior work has suggested ways to improve error messages in similar cases [101, 102, 103]; we plan to explore these for our systems in the future. It is possible that this would also be helpful for one challenge with type casting presented above: how to determine which program locations result in false positive errors.

Conflicts in Inferred Types. One challenge for both `InferDL` and `SimTyper` is that it is possible for the system to infer two or more type annotations that are inconsistent; in `InferDL`, this can occur when heuristics guess conflicting annotations, and in `SimTyper` it can occur when the DeepSim network guesses conflicting annotations. In both cases, the system takes a greedy approach and uses whichever annotation is guessed first, thereby effectively blocking out subsequent annotations that are inconsistent. In the future, we would like to explore alternative ways for handling such conflicts. One way could be to assign a confidence score to each inferred annotation, and pick those annotations with higher scores. Another approach may be to use backtracking in cases where inconsistencies occur.

Return Embeddings. As discussed in Section 5.5, we found that incorporating return sites into the embeddings representing method returns did not have a significant effect on `SimTyper`'s results for the benchmarks. This suggests that we are not effectively incorporating clues from a method body's code as to the type of the method return. In the future, we plan to explore alternative ways of producing return embeddings.

Appendix A: Soundness of `CompRDL`

This appendix contains the full definitions, static and dynamic semantics, and proof of soundness for λ^C (Chapter 3).

Figure A.1 once again presents the syntax of λ^C, as well as some new auxiliary definitions to be used for defining the semantics and proving soundness. Here we see for the first time object instances $[A]$, which denote an instance of a class A. The dynamic semantics rules are shown in Figure A.2. Most of the rules are standard. The slightly more intricate rule is (E-Context), which takes a step within a subexpression, and it contains premises that differentiate it from (E-AppUD), the other context cases, and ensure that the context C is the largest possible context for purposes of disambiguation. Figure A.3 once again defines the dynamic check insertion rules. Finally, Figure A.4 defines the type checking rules, which are largely a simplification of the check insertion rules; it is for these type checking rules that we prove preservation and progress. We prove soundness of the check insertion rules as a corollary of the soundness of the type checking rules.

$$
\begin{array}{rrcl}
\texttt{values} & v & ::= & \texttt{nil} \mid [A] \mid A \mid \texttt{true} \mid \texttt{false} \\
\texttt{expressions} & e & ::= & v \mid x \mid \texttt{self} \mid \texttt{tself} \mid e; e \\
 & & \mid & A.\texttt{new} \mid \texttt{if } e \texttt{ then } e \texttt{ else } e \mid e == e \\
 & & \mid & e.m(e) \mid \lceil A \rceil e.m(e) \\
\texttt{method types} & \sigma & ::= & A \to A \\
\texttt{TLC method types} & \delta & ::= & \sigma \mid (a{<:}e/A) \to e/A \\
\textit{programs} & P & ::= & \texttt{def } A.m(x) : \sigma \ = \ e \mid \texttt{lib } A.m(x) : \delta \mid P; P
\end{array}
$$

$x \in$ var IDs, $m \in$ meth IDs, $A \in$ class IDs

$$
\begin{array}{rrcl}
\texttt{dyn env} & E & : & \text{var ids} \to \text{values} \\
\texttt{contexts} & C & ::= & \Box \mid C.m(e) \mid v.m(C) \mid \lceil A \rceil C.m(e) \mid \lceil A \rceil v.m(C) \\
 & & \mid & C; e \mid \texttt{if } C \texttt{ then } e \texttt{ else } e \mid C == e \mid v == C \\
\texttt{stack} & S & ::= & \cdot \mid (E, C) :: S \\
\texttt{type stack} & TS & ::= & \cdot \mid (\Gamma[A], A) :: TS \\
\texttt{typ env} & \Gamma, \Delta & : & \text{var ids} \to \text{base types} \\
\texttt{class table} & CT & : & \text{class ids} \to \text{meth ids} \to \text{types} \\
\texttt{objects} & O & : & \text{objects} \\
\texttt{object instance} & [\cdot] & : & A \to O \\
 & & & \\
\texttt{method sets} & \mathcal{U} & : & \text{set of user-defined methods} \\
 & \mathcal{L} & : & \text{set of library methods} \\
 & & & \text{where } \mathcal{L} \cap \mathcal{U} = \emptyset
\end{array}
$$

Nil, *Obj*, *Bool*, *True*, *False*, and *Type* are all presumed to be class IDs A. Subtyping is defined as $Nil \leq A$, $A \leq A$, and $A \leq A \sqcup A'$ for all A, A'. $A \sqcup A'$ is the least upper bound of types A and A'.

Figure A.1: λ^C and auxiliary definitions.

A.1 Soundness

We prove soundness of the type system of λ^C by first proving preservation and then progress. The proof of preservation is the more involved step here, and it requires a number of preliminary definitions. First, we define a notion of consistency between the type and dynamic environments:

Definition 1 (Environmental consistency)**.** *Type environment* Γ *is consistent with*

Dynamic semantics $\boxed{\langle E, e, S\rangle \rightsquigarrow \langle E', e', S'\rangle}$

(E-Var) $\langle E, x, S\rangle \rightsquigarrow \langle E, E(x), S\rangle$

(E-Self) $\langle E, \texttt{self}, S\rangle \rightsquigarrow \langle E, E(\texttt{self}), S\rangle$

(E-TSelf) $\langle E, \texttt{tself}, S\rangle \rightsquigarrow \langle E, E(\texttt{tself}), S\rangle$

(E-Seq) $\langle E, v; e, S\rangle \rightsquigarrow \langle E, e, S\rangle$

(E-New) $\langle E, A.\texttt{new}, S\rangle \rightsquigarrow \langle E, [A], S\rangle$

(E-IfTrue) $\langle E, \texttt{if } v \texttt{ then } e_2 \texttt{ else } e_3, S\rangle \rightsquigarrow \langle E, e_2, S\rangle$
if v is not `nil` or `false`

(E-IfFalse) $\langle E, \texttt{if } v \texttt{ then } e_2 \texttt{ else } e_3, S\rangle \rightsquigarrow \langle E, e_3, S\rangle$
if $v = \texttt{nil}$ or $v = \texttt{false}$

(E-EqTrue) $\langle E, v_1 == v_2, S\rangle \rightsquigarrow \langle E, \texttt{true}, S\rangle$
if v_1 and v_2 are equivalent

(E-EqFalse) $\langle E, v_1 == v_2, S\rangle \rightsquigarrow \langle E, \texttt{false}, S\rangle$
if v_1 and v_2 are not equivalent

(E-AppUD) $\langle E, C[v_r.m(v)], S\rangle \rightsquigarrow \langle [\texttt{self} \mapsto v_r, x \mapsto v], e, (E, C) :: S\rangle$
if $\mathit{type\_of}(v_r) = A$ and $A.m \in \mathcal{U}$ and $\mathit{def\_of}(A.m) = x.e$

(E-AppLib) $\langle E, \lceil A \rceil v_r.m(v), S\rangle \rightsquigarrow \langle E, v', S\rangle$
if $\mathit{type\_of}(v_r) = A$ and $A.m \in \mathcal{L}$ and $v' = \mathit{call}(A.m, v_r, v)$ and $\mathit{type\_of}(v') \leq A$

(E-Ret) $\langle E', v, (E, C) :: S\rangle \rightsquigarrow \langle E, C[v], S\rangle$

(E-Context)

$$\frac{\langle E, e, S\rangle \rightsquigarrow \langle E', e', S'\rangle \quad \nexists v.e = v \qquad \nexists v, v_r, A, m.(e = v_r.m(v) \wedge \mathit{type\_of}(v_r) = A \wedge A.m \in \mathcal{U}) \quad \nexists C', e''.e = C'[e'']}{\langle E, C[e], S\rangle \rightsquigarrow \langle E', C[e'], S'\rangle}$$

where $\mathit{def\_of}(A.m) = x.e$ if there exists a definition $\texttt{def } A.m(x) : \sigma \ = \ e$. Additionally, $\mathit{call}(A.m, v, v)$ is our way of dispatching a library method, where $A.m$ identifies the method, the first v is the receiver of the method call, and the second v is the argument of the method call. It returns a value v_r, the value returned by the method call. Finally, we use $\mathit{type\_of}(v)$, where $\mathit{type\_of}(\texttt{nil}) = \mathit{Nil}$, $\mathit{type\_of}(A) = \mathit{Type}$, $\mathit{type\_of}(\texttt{true}) = \mathit{True}$, $\mathit{type\_of}(\texttt{false}) = \mathit{False}$, and $\mathit{type\_of}([A]) = A$.

Figure A.2: Dynamic semantics of λ^C

Type Checking and Check Insertion Rules. $\boxed{\Gamma \vdash_{CT} e \hookrightarrow e : A}$

$$\frac{}{\Gamma \vdash_{CT} \texttt{nil} \hookrightarrow \texttt{nil} : \mathit{Nil}}\ \text{C-Nil} \qquad \frac{}{\Gamma \vdash_{CT} A.\texttt{new} \hookrightarrow A.\texttt{new} : A}\ \text{C-New}$$

$$\frac{}{\Gamma \vdash_{CT} A \hookrightarrow A : \mathit{Type}}\ \text{C-Type} \qquad \frac{}{\Gamma \vdash_{CT} \texttt{true} \hookrightarrow \texttt{true} : \mathit{True}}\ \text{C-True}$$

$$\frac{}{\Gamma \vdash_{CT} \texttt{false} \hookrightarrow \texttt{false} : \mathit{False}}\ \text{C-False} \qquad \frac{}{\Gamma \vdash_{CT} [A] \hookrightarrow [A] : A}\ \text{C-Obj}$$

$$\frac{\Gamma(x) = A}{\Gamma \vdash_{CT} x \hookrightarrow x : A}\ \text{C-Var} \qquad \frac{\begin{array}{c}\Gamma \vdash_{CT} e_1 \hookrightarrow e_1' : A_1 \\ \Gamma_1 \vdash_{CT} e_2 \hookrightarrow e_2' : A_2\end{array}}{\Gamma \vdash_{CT} e_1 == e_2 \hookrightarrow e_1' == e_2' : \mathit{Bool}}\ \text{C-Eq}$$

$$\frac{\begin{array}{c}\Gamma \vdash_{CT} e_1 \hookrightarrow e_1' : A_1 \\ \Gamma_1 \vdash_{CT} e_2 \hookrightarrow e_2' : A_2\end{array}}{\Gamma \vdash_{CT} e_1; e_2 \hookrightarrow e_1'; e_2' : A_2}\ \text{C-Seq} \qquad \frac{\begin{array}{c}CT(A.m) = A_1 \to A_2 \qquad A.m \in \mathcal{U} \\ A_x \leq A_1 \\ \Gamma \vdash_{CT} e \hookrightarrow e' : A \quad \Gamma \vdash_{CT} e_x \hookrightarrow e_x' : A_x\end{array}}{\Gamma \vdash_{CT} e.m(e_x) \hookrightarrow e'.m(e_x') : A_2}\ \text{C-AppUD}$$

$$\frac{\begin{array}{c}\Gamma \vdash_{CT} e_1 \hookrightarrow e_1' : A_1 \\ \Gamma \vdash_{CT} e_2 \hookrightarrow e_2' : A_2 \\ \Gamma \vdash_{CT} e_3 \hookrightarrow e_3' : A_3\end{array}}{\Gamma \vdash_{CT} \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \hookrightarrow \texttt{if } e_1' \texttt{ then } e_2' \texttt{ else } e_3' : (A_2 \sqcup A_3)}\ \text{C-If}$$

$$\frac{\begin{array}{c}CT(A.m) = A_1 \to A_2 \\ A.m \in \mathcal{L} \qquad A_x \leq A_1 \\ \Gamma \vdash_{CT} e \hookrightarrow e' : A \\ \Gamma \vdash_{CT} e_x \hookrightarrow e_x' : A_x\end{array}}{\Gamma \vdash_{CT} e.m(e_x) \hookrightarrow \lceil A_2 \rceil e'.m(e_x') : A_2}\ \text{C-AppLib}$$

$$\frac{\begin{array}{c}CT(A.m) = (a{<:}e_{t1}/A_{t1}) \to e_{t2}/A_{t2} \qquad A.m \in \mathcal{L} \\ \Gamma \vdash_{CT} e \hookrightarrow e' : A \qquad \Gamma \vdash_{CT} e_x \hookrightarrow e_x' : A_x \\ x{:}\mathit{Type}, \texttt{tself}{:}\mathit{Type} \vdash_{\llbracket CT \rrbracket} e_{t1} \hookrightarrow e_{t1}' : \mathit{Type} \\ \langle [x \mapsto A][\texttt{tself} \mapsto A], e_{t1}', \cdot \rangle \rightsquigarrow^* \langle E_1, A_1, \cdot \rangle \qquad A_x \leq A_1 \\ x{:}\mathit{Type}, \texttt{tself}{:}\mathit{Type} \vdash_{\llbracket CT \rrbracket} e_{t2} \hookrightarrow e_{t2}' : \mathit{Type} \\ \langle [x \mapsto A][\texttt{tself} \mapsto A], e_{t2}', \cdot \rangle \rightsquigarrow^* \langle E_2, A_2, \cdot \rangle\end{array}}{\Gamma \vdash_{CT} e.m(e_x) \hookrightarrow \lceil A_2 \rceil e'.m(e_x') : A_2}\ \text{C-App-Comp}$$

Figure A.3: Type checking and check insertion rules for λ^C.

Type checking rules $\boxed{\Gamma \vdash_{CT} e : A}$

$$\frac{}{\Gamma \vdash_{CT} \texttt{nil} : \mathit{Nil}}\ \text{T-Nil} \qquad \frac{}{\Gamma \vdash_{CT} [A] : A}\ \text{T-Obj} \qquad \frac{\Gamma(\texttt{self}) = A}{\Gamma \vdash_{CT} \texttt{self} : A}\ \text{T-Self}$$

$$\frac{}{\Gamma \vdash_{CT} \texttt{true} : \mathit{True}}\ \text{T-True} \qquad \frac{}{\Gamma \vdash_{CT} \texttt{false} : \mathit{False}}\ \text{T-False}$$

$$\frac{}{\Gamma \vdash_{CT} A : \mathit{Type}}\ \text{T-Type} \qquad \frac{\Gamma(x) = A}{\Gamma \vdash_{CT} x : A}\ \text{T-Var}$$

$$\frac{\Gamma \vdash_{CT} e_1 : A_1 \quad \Gamma_1 \vdash_{CT} e_2 : A_2}{\Gamma \vdash_{CT} e_1 == e_2 : \mathit{Bool}}\ \text{T-Eq} \qquad \frac{\Gamma(\texttt{tself}) = A}{\Gamma \vdash_{CT} \texttt{tself} : A}\ \text{T-TSelf}$$

$$\frac{\Gamma \vdash_{CT} e_1 : A_1 \quad \Gamma_1 \vdash_{CT} e_2 : A_2}{\Gamma \vdash_{CT} e_1; e_2 : A_2}\ \text{T-Seq} \qquad \frac{}{\Gamma \vdash_{CT} A.\texttt{new} : A}\ \text{T-New}$$

$$\frac{\Gamma \vdash_{CT} e_1 : A_1 \quad \Gamma \vdash_{CT} e_2 : A_2 \quad \Gamma \vdash_{CT} e_3 : A_3}{\Gamma \vdash_{CT} \texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3 : (A_2 \sqcup A_3)}\ \text{T-If} \qquad \frac{\Gamma \vdash_{CT} e_0 : A \quad A.m \in \mathcal{U} \quad \Gamma \vdash_{CT} e_1 : A \quad CT(A.m) = A_1 \to A_2 \quad A \leq A_1}{\Gamma \vdash_{CT} e_0.m(e_1) : A_2}\ \text{T-App}$$

$$\frac{\Gamma \vdash_{CT} e_0 : A \quad A.m \in \mathcal{L}}{\Gamma \vdash_{CT} \lceil A \rceil e_0.m(e_1) : A}\ \text{T-App-Lib}$$

Figure A.4: Type checking rules for λ^C.

dynamic environment E*, written* $\Gamma \sim E$*, if for all variables* x*,* $x \in dom(E)$ *if and only if* $x \in dom(\Gamma)$*, and for all* $x \in dom(E)$ *there exists* A *such that* $\Gamma \vdash_{CT} E(x) : A$ *and* $A \leq \Gamma(x)$.

We will use the notation $type\_of(v)$, where $type\_of(\texttt{nil}) = Nil$, $type\_of(\texttt{true}) = True$, $type\_of(\texttt{false}) = False$, $type\_of([A]) = A$, and $type\_of(A) = Type$, for any A.

On blame: Our type system does not prevent invoking a method on a value that is `nil`. Additionally, runtime evaluation can fail if an inserted dynamic check fails. In order to retain soundness of our system, we add dynamic semantics rules which step to *blame* in these cases, where $\Gamma \vdash_{CT} blame : Nil$ for any Γ. Additionally, we add rules which take a step to *blame* whenever a subexpression takes a step to *blame*. We omit the rules here for brevity.

Because we make use of a stack in our dynamic semantics, a standard type preservation theorem which says that we always step to an expression which has the same type (or subtype) will not suffice. Rules (E-AppUD) and (E-Ret) push and pop from the stack. In these cases, an expression e may have an entirely different type than the expression that it steps to, e'. To account for this we incorporate a notion of a *type stack* TS to mirror the runtime stack, which is defined Figure A.1. As an example, suppose we want to apply preservation to $C[v_1.m(v_2)]$. The type checking judgment is $\Gamma \vdash_{CT} C[v_1.m(v_2)] : A'$. Because the dynamic semantics rule (E-AppUD) pushes the current environment and context onto the stack, we will push the current typing judgment on to the type stack. Specifically, we will push

an element of the form $(\Gamma[A], A')$, where Γ is the environment of the current typing judgment; A' is the type of the surrounding context; and A is the type of expression $v_1.m(v_2)$, i.e., the type that the method must return.

With this type stack, we can now define what it means for a type to be a subtype of the type stack, which is the crucial preservation invariant we will prove:

Definition 2 (Stack subtyping)**.** $A_0 \leq (\Gamma[A], A') :: TS$ *if* $A_0 \leq A$.

Definition 3 (Stack consistency)**.** *Type stack element* $(\Gamma[A], A')$ *is consistent with dynamic stack element* (E, C)*, written* $(\Gamma[A], A') \sim (E, C)$*, if* $\Gamma \sim E$ *and* $\Gamma[\Box \mapsto A] \vdash_{CT} C : A'$ *(Here we abuse notation and treat* $\Box$ *as if its a variable.)*

Type stack TS is consistent with dynamic stack S, written $TS \sim S$*, is defined inductively as*

1. $\cdot \sim \cdot$
2. $(\Gamma[A], A') :: TS \sim (E, C) :: S$ *if*
   (a) $(\Gamma[A], A') \sim (E, C)$
   (b) $TS \sim S$
   (c) $A' \leq TS$ *if* $TS \neq \cdot$

We will also make use of a notion of class table validity, which tells us that a class table maps fields and methods to the "correct" types:

Definition 4 (Class table validity)**.** *Let A.m be an arbitrary method. We say valid(CT) if*

1. *if* $A.m \in \mathcal{U}$, *then* $A.m \in dom(CT)$, $CT(A.m) = A_1 \to A_2$ *for some* A_1, A_2, *and there exists a single method definition* $\texttt{def}\ A.m(x) : A_1 \to A_2 = e$ *such that* $[\texttt{self} \mapsto A, x \mapsto A_1] \vdash_{CT} e : A_2'$ *for some* A_2' *where* $A_2' \leq A_2$.

2. *if* $A.m \in \mathcal{L}$, *then* $A.m \in dom(CT)$, $CT(A.m) = \sigma$ *for some* A_m, *and there exists a single method declaration* $\texttt{lib}\ A.m(x) : \sigma$ *such that* $\sigma = A_1 \to A_2$ *or* $\sigma = (a <: e_{t1}/A_1) \to e_{t1}/A_2$ *for some* A_1, A_2, e_{t1}, e_{t2}.

3. *For all* A' *such that* $A' \leq A$, *if* $CT(A'.m) = A_1' \to A_2'$ *and* $CT(A.m) = A_1 \to A_2$ *then* $A_1 \leq A_1'$ *and* $A_2' \leq A_2$.

See section A.2 for the programming type checking rules, and a discussion of how to construct and check the validity of a class table.

Finally, we make use of the following lemmas in our proofs of soundness:

Lemma 1 (Contextual substitution)**.** *If*

$$\frac{\begin{array}{c}\Gamma \vdash_{CT} e : A' \\ \vdots \end{array}}{\Gamma_C \vdash_{CT} C[e] : A_C},$$

then $\Gamma_C[\Box \mapsto A'] \vdash_{CT} C : A_C$.

Lemma 2 (Substitution)**.** *If*

1. $\Delta[\Box \mapsto A_C] \vdash_{CT} C : A_C'$

2. $\Delta \vdash_{CT} e : A$

3. $A \leq A_C$

then $\Delta \vdash_{CT} C[e] : A''_C$ *where* $A''_C \leq A'_C$.

With the above definitions and lemmas, we can finally state our preservation theorem:

Theorem 2 (Preservation)**.** *If*

(1) $\langle E, e, S\rangle \rightsquigarrow \langle E', e', S'\rangle$

(2) $\Gamma \vdash_{CT} e : A$

(3) $A \leq TS$

(4) $\Gamma \sim E$

(5) $TS \sim S$

(6) valid(CT)

then there exists Δ, TS', A' *such that*

(a) $\Delta \vdash_{CT} e' : A'$

(b) $A' \leq TS'$

(c) $\Delta \sim E'$

(d) $TS' \sim S'$

(e) If $S = S'$ *then* $A' \leq A$ *and* $\Delta = \Gamma, \Gamma'$ *for some* Γ'

(1) and (2) are standard: they say that some expression e takes a step, and that e is well typed. Conclusion (a) states that e' is also well typed. (3) says that

the type of e is a subtype of the type stack, and (b) says the same of e'; this is the crux of the type preservation proof, and as explained above, it must be phrased in this way in order to account for the use of a stack, which allows us to step to expressions with completely different types. Notice that (e) also tells us that if the stack goes unchanged, then $A' \leq A$; this is much closer to the standard statement of preservation. (e) also gives us that if the stack goes unchanged, then the new type environment is an extension of the old one.

(4) gives us consistency between type and dynamic environments, and (5) gives us consistency between the type stack and stack, the corresponding conclusions (c) and (d) respectively give us the same for the new environments.

(6) gives us validity of the class table (a corresponding conclusion is unnecessary since the class table goes unchanged).

Finally, we proceed with the proof of preservation.

Proof. By induction on $\langle E, e, S\rangle \rightsquigarrow \langle E', e', S'\rangle$

- Case (E-SELF). By assumption we have

  (1) $\langle E, \texttt{self}, S\rangle \rightsquigarrow \langle E, E(\texttt{self}), S\rangle$ by (E-SELF)

  (2) $\Gamma \vdash_{CT} \texttt{self} : A$

  (3) $\Gamma(\texttt{self}) \leq TS$

  (4) $\Gamma \sim E$

  (5) $TS \sim S$

  (6) $valid(CT)$

Since (2) must have been derived by (T-Self), by inversion of this rule we have that $A = \Gamma(\texttt{self})$. Let $\Delta = \Gamma$ and $TS' = TS$. By equality of Δ and Γ, (2), (4), and the definition of environmental consistency, there exists A' such that $\Delta \vdash_{CT} E(\texttt{self}) : A'$ and $A' \leq \Delta(\texttt{self})$. Then (a) holds since $E(\texttt{self})$ is well typed. (b) holds since $A' \leq \Delta(\texttt{self}) = \Gamma(\texttt{self}) \leq TS$ by (3). Because $A' \leq \Gamma(\texttt{self})$ and $\Delta = \Gamma, \emptyset$, we have (e). Finally, (c) holds by (4), and (d) holds by (5).

- Case (E-Var), (E-TSelf). Similar to (E-Self) case.

- Case (E-New). By assumption we have

  (1) $\langle E, A.\texttt{new}, S\rangle \rightsquigarrow \langle E, [A], S\rangle$ by (E-New)

  (2) $\Gamma \vdash_{CT} A.\texttt{new} : A$ by (T-New)

  (3) $A \leq TS$

  (4) $\Gamma \sim E$

  (5) $TS \sim S$

  (6) $\mathit{valid}(CT)$

  Let $\Delta = \Gamma$, $TS' = TS$, and $A' = A$. Then we get (a) from (T-Obj), (b) from (3), (c) from (4), (d) from (5), and (e) immediately by definition of A' and Δ.

- Case (E-Seq). Trivial.

- Case (E-EqTrue), (E-EqFalse). Trivial.

- Case (E-IfTrue). By assumption we have

(1) $\langle E, \texttt{if } v \texttt{ then } e_1 \texttt{ else } e_2, S\rangle \rightsquigarrow \langle E, e_1, S\rangle$

(2) $\Gamma \vdash_{CT} \texttt{if } v \texttt{ then } e_1 \texttt{ else } e_2 : A$

(3) $A \leq TS$

(4) $\Gamma \sim E$

(5) $TS \sim S$

(6) $\mathit{valid}(CT)$

By (2) and inversion of (T-If), there exits A_v, A_1, A_2 such that:

(7) $\Gamma \vdash_{CT} v : A_v$

(8) $\Gamma \vdash_{CT} e_1 : A_1$

(9) $\Gamma \vdash_{CT} e_2 : A_2$

(10) $A = A_1 \cup A_2$

Let $\Delta = \Gamma$, $TS' = TS$, and $A' = A_1$. Then from (8) we trivially have (a). Additionally, $A' = A_1 \leq A_1 \sqcup A_2$, and so $A' \leq TS'$ giving us (b). Finally, we have (c) by (4), (d) by (5), and (e) by the fact that $A' \leq A$ and $\Delta = \Gamma, \emptyset$.

- Case (E-IfFalse). Similar to (E-IfTrue) case.

- Case (E-Context). By assumption we have:

  (1) $\langle E, C[e], S\rangle \rightsquigarrow \langle E', C[e'], S'\rangle$ where

    (1a) $\langle E, e, S\rangle \rightsquigarrow \langle E', e', S'\rangle$

    (1b) $\neg(e = v_r.m(v) \wedge \mathit{type\_of}(x_r) = A \wedge A.m \in \mathcal{U})$ for some v, v_r, A, m

(1c) $e \neq v$ for some v

(1d) $e \neq C'[e']$ for some C', e' by (E-CONTEXT)

(2) $\Gamma \vdash_{CT} C[e] : A$

(3) $A \leq TS$

(4) $\Gamma \sim E$

(5) $TS \sim S$

(6) $valid(CT)$

First, note that we must have $S' = S$, because the only cases where this would not happen would be if (E-APPUD) or (E-RET) were used to derive (1a), and this cannot be the case given (1b) and because (E-RET) only applies to top-level values and thus can't apply to a context. Now note that since (2) gives us $\Gamma \vdash_{CT} C[e] : A$, by inversion, there should exist A_e so that

(7) $\Gamma \vdash_{CT} e : A_e$

Let TS_e be a type stack such that $TS_e \sim S$, $A_e \leq TS_e$, and all type environments in TS_e are the same as those in TS; it is straightforward to construct such a TS_e from the existing TS. Then, by (1a), (7), $A_e \leq TS_e$, (4), $TS_e \sim S$, and (6), we satisfy the premises of the preservation theorem. Therefore, applying the inductive hypothesis, there exists Δ_e, TS'_e, A'_e such that:

(a_i) $\Delta_e \vdash_{CT} e' : A'_e$

(b_i) $A'_e \leq TS'_e$

(c_i) $\Delta_e \sim E'$

(d_i) $TS'_e \sim S'$

(e_i) If $S = S'$ then $A'_e \leq A_e$ and $\Delta_e = \Gamma, \Gamma'$ for some Γ'

Let $\Delta = \Delta_e$ and $TS' = TS$. Now, because $S' = S$, by (g_i) we have $\Delta = \Gamma, \Gamma'$. By Lemma 1 we have $\Gamma[\Box \mapsto A_e] \vdash_{CT} C : A$, and by the weakening lemma, we also have $\Delta[\Box \mapsto A_e] \vdash_{CT} C : A$. By ($e_i$) we also have $A'_e \leq A_e$. These, along with (a_i) and the substitution lemma 2, give us that $\Delta \vdash_{CT} C[e'] : A'$ for some A' where $A' \leq A$. This immediately gives us (a), and because $A' \leq A$ and $A \leq TS$ by (3) we get (b). Because $\Delta = \Gamma, \Gamma'$ and $A' \leq A$, we get (e). We get (c) from (c_i). (d) comes from (5) and the fact that $S' = S$.

- Case (E-AppLib). By assumption we have

  (1) $\langle E, \lceil A_{res} \rceil v_r.m(v_1), S \rangle \rightsquigarrow \langle E, v, S \rangle$ where

  (1a) $\mathit{type\_of}(v_r) = A_{rec}$

  (1b) $A_{rec}.m \in \mathcal{L}$

  (1c) $v = \mathit{call}(A_{rec}.m, v_r, v_1)$

  (1d) $\mathit{type\_of}(v) \leq A_{res}$

  by (E-AppLib).

  (2) $\Gamma \vdash_{CT} \lceil A_{res} \rceil v_r.m(v_1) : A$

  (3) $A \leq TS$

  (4) $\Gamma \sim E$

(5) $TS \sim S$

(6) $valid(CT)$

Because of (1b), we know that (2) must have been derived by (T-App-Lib). Instantiating this rule with the obvious bindings, we get that $A = A_{res}$. Let $\Delta = \Gamma$, $TS' = TS$, and $A' = type\_of(v)$. If v is `nil`, we get (a) immediately by (1d), definition of *type_ of*, and (T-Nil); a similar argument holds for all other potential values of v. In all cases, we get (b) from (1d) and (3). We get (c) from (4), (d) from (5), and (e) from (1d) and definition of Δ.

- Case (E-AppUD). By assumption we have

  (1) $\langle E, C[v_r.m(v)], S\rangle \rightsquigarrow \langle[\texttt{self} \mapsto v_r, x \mapsto v], e, (E, C) :: S\rangle$ where

    (1a) $type\_of(v_r) = A_{rec}$

    (1b) $A_{rec}.m \in \mathcal{U}$

    (1c) $def\_of(A_{rec}.m) = x.e$

  (2) $\Gamma \vdash_{CT} C[v_r.m(v)] : A_C$

  (3) $A_C \leq TS$

  (4) $\Gamma \sim E$

  (5) $TS \sim S$

  (6) $valid(CT)$

  Noting the type checking rules for each context case, we know (2) must have been derived by some rule of the form:

$$\frac{\begin{array}{c}\Gamma \vdash_{CT} v_r.m(v) : A_m \\ \vdots \end{array}}{\Gamma \vdash_{CT} C[v_r.m(v)] : A_C}$$

From this and the contextual substitution lemma 1, we know $\Gamma[\square \mapsto A_m] \vdash_{CT} C : A_C$. Additionally, by inversion, we have

(7) $\Gamma \vdash_{CT} v_r.m(v) : A_m$

By (1a), definition of *type_ of*, and the value type checking rules, we know that $\Gamma \vdash_{CT} v_r : A_{rec}$. Because by (1b) we know that $A_{rec}.m \in \mathcal{U}$ and $\mathcal{U}$ and $\mathcal{L}$ are by definition disjoint, the type checking judgment (7) must have been derived from rule T-App.

By instantiation of rule T-App with $A_m = A_2$ we get

$$\frac{\begin{array}{c}\Gamma \vdash_{CT} v_r : A_{rec} \qquad A.m \in \mathcal{U} \\ \Gamma \vdash_{CT} v : A_{arg} \\ CT(A.m) = A_1 \to A_2 \qquad A_{arg} \leq A_1 \end{array}}{\Gamma \vdash_{CT} v_r.m(v) : A_2}\ \text{T-App}$$

Thus, by inverting T-App, there must exist A_{arg}, A_1, A_2 such that:

(8) $\Gamma \vdash_{CT} v : A_{arg}$

(9) $CT(A.m) = A_1 \to A_2$

(10) $A_{arg} \leq A_1$

Now, let $\Delta = [x \mapsto A_1, \texttt{self} \mapsto A_{rec}]$. Also, let $TS' = (\Gamma[A_2], A_C) :: TS$. By (1b), (1c), (6), (8), and (9), we know there exists A' such that $[\texttt{self} \mapsto A_{rec}, x \mapsto A_1] \vdash_{CT} e : A'$ where $A' \leq A_2$, which gives us (a); that is to say,

the body of the method type checks as expected. (b) holds by construction of TS'. (c) holds by construction of Δ. (e) holds trivially since $S \neq S'$.

Finally, we show (d). By (4) we have $\Gamma \sim E$, and as noted above we have $\Gamma[\Box \mapsto A_2] \vdash_{CT} C : A_C$. This gives us $(\Gamma[A_2], A_C) \sim (E, C)$. By (3) we have $A_C \leq TS$, and by (5) we have $TS \sim S$. Putting this all together, by the definition of stack consistency, this gives us $(\Gamma_C[A_2], A_C) :: TS \sim (E, C) :: S$, which is (d).

- Case (E-RET). By assumption we have

  (1) $\langle E', v, (E, C) :: S\rangle \rightsquigarrow \langle E, C[v], S\rangle$

  (2) $\Gamma \vdash_{CT} v : A$

  (3) $A \leq (\Gamma_C[A_C], A'_C) :: TS$

  (4) $\Gamma \sim E'$

  (5) $(\Gamma_C[A_C], A'_C) :: TS \sim (E, C) :: S$

  (6) $valid(CT)$

  Let $\Delta = \Gamma_C$and $TS' = TS$. By (5) we have $\Gamma_C[\Box \mapsto A_C] \vdash_{CT} C : A'_C$, and by (3) we have $A \leq A_C$. Then by these, (2), and the substitution lemma 2, we have that $\Gamma_C \vdash_{CT} C[v] : A''_C$ where $A''_C \leq A'_C$; this also means that $\Delta \vdash_{CT} C[v] : A''_C$.

  Let $A' = A''_C$ and we have (a). By (5) and the definition of stack consistency we have $A'_C \leq TS$, and since $A''_C \leq A'_C$, we have (b). Finally, (c) and (d) hold by (5), and (e) holds trivially since $S \neq S'$.

□

Before proving progress, we introduce one assumption and one lemma:

Assumption 1 (Library Method Termination)**.** *For any A, m, v_1, and v_2 where $A.m \in \mathcal{L}$, call$(A.m, v_1, v_2)$ will terminate and return a value.*

This assumption is necessary to prove progress since we do not have the mechanism to refer to the step-by-step evaluation of library methods. A straightforward expansion of λ^C would allow us to do so, but we omit such an expansion here for simplicity.

Lemma 3 (Proper context)**.** *For any expression e, if $e = C[e']$ for some C, e', then there exists a proper context C_P and proper subexpression e_P such that $e = C_P[e_P]$, $e_P \neq v$ for any value v, and $\nexists C', e''$ such that $e_P = C'[e'']$.*

Note that it is still possible in Lemma 3 that $C_P = C$ and $e_P = e$. The high-level idea behind the proof of Lemma 3 is that we construct the proper context by recursively pushing the hole [] deeper into subcontexts while possible. With these, we can proceed with defining and proving progress:

Theorem 3 (Progress)**.** *If*

(1) $\Gamma \vdash_{CT} e : A$

(2) $A \leq TS$

(3) $\Gamma \sim E$

(4) $TS \sim S$

(5) valid(CT)

then one of the following holds

1. *e is a value*
2. *There exists E', e', S' such that $\langle E, e, S\rangle \rightsquigarrow \langle E', e', S'\rangle$*
3. *$\langle E, e, S\rangle \rightsquigarrow$ blame*

Proof. By induction on e.

- Case `nil`, `true`, `false`, $[A]$, or A. e is a value.
- Case `self`. By assumption (1) and (T-Self), we know `self` $\in dom(\Gamma)$. By (3), this means `self` $\in dom(E)$. This means rule (ESelf) can be applied, so we can take a step.
- Cases x, `tself`. Similar to `self`.
- Case $e_1 == e_2$ We split into cases on whether or not e_1, e_2 are values, for any v_1, v_2:
  - $e_1 \equiv v_1$ and $e_2 \equiv v_2$. This case is trivial since one of (E-EqTrue) or (E-EqFalse) will always apply.
  - $e_1 \not\equiv v_1$. Then $e \equiv C[e_1]$ with $C \equiv [] == e_2$. By Lemma 3, there exists a proper context C_P and proper subexpression e_P such that $e \equiv C_P[e_P]$. If $e_P \not\equiv v_r.m(v)$ for any v_r, A, m, v where $type\_of(v_r) = A_r \wedge A_r.m \in \mathcal{U}$, then by the inductive hypothesis there exists E', e'_P, S' such that $\langle E, e_P, S\rangle \rightsquigarrow$

$\langle E', e'_P, S' \rangle$ (or such that we step to blame, in which case e steps to blame and we are done). See the (E-Context) case of the preservation proof for a discussion of how to satisfy the premises of the inductive hypothesis, since the premises here a subset of those of preservation. By construction of the proper subexpression as specified in Lemma 3, $\nexists C', e''$ such that $e_P = C'[e'']$, and $e_P \neq v$ for any value v. This satisfies all the premises of (E-Context), therefore this rule would apply to $C_P[e_P]$ to take a step. Otherwise, $e_P \equiv v_r.m(v)$ with $\mathit{type\_of}(v_r) = A_r \wedge A_r.m \in \mathcal{U}$. We know that (1) must have been derived by rule (T-Eq). By inversion of this rule, this means $\Gamma \vdash_{CT} v_r.m(v) : A_m$ for some A_m. By definition of $\mathit{type\_of}$, it must be that $A_m = A_r$. Because $A_r.m \in \mathcal{U}$ and by definition $\mathcal{U} \cap \mathcal{L} = \emptyset$, we therefore know $\Gamma \vdash_{CT} v_r.m(v) : A_m$ must have been derived from (T-App). By inversion of this rule, we know $CT(A_r.m) = A_1 \to A_2$ for some A_1, A_2. By (5), this means $\mathit{def\_of}(A_r.m) = x.e_m$ for some e_m. If v_r is `nil`, then we return blame. Thus, we have satisfied all the premises of rule (E-AppUD), therefore we can apply this rule and we are done.

  - $e_1 \equiv v_1$ and $e_2 \not\equiv v_2$. Then $e \equiv C[e_2]$ with $C \equiv v_1 == []$. By a similar argument to the previous case, either (E-Context) or (E-AppUD) must apply.

- Case $e_1; e_2$. We split cases on whether or not e_1 is a value:

  - $e_1 \equiv v$. Then, the evaluation rule E-Seq on the expression $v; e_2$ applies to take a step.

- Otherwise, $e \equiv C[e_1]$ with $C \equiv [];e_2$. By a similar argument to the $e_1 == e_2$ case, either (E-Context) or (E-AppUD) must apply.

- Case A.`new`. Trivial.

- Case `if` e_0 `then` e_1 `else` e_2. We split cases on the structure of e_0.

  - $e_0 \equiv$ `nil` or $e_0 \equiv$ `false`. The rule E-IfFalse applies to take a step.

  - $e_0 \equiv v$ where v is not `nil` or `false`. The rule E-IfTrue applies to take a step.

  - Otherwise, $e \equiv C[e_0]$ with $C \equiv$ `if` [] `then` e_1 `else` e_2. By a similar argument to the $e_1 == e_2$ case, either (E-Context) or (E-AppUD) must apply.

- Case $e_1.m(e_2)$. We split this case on the structure of e_1, e_2:

  - $e_1 \not\equiv v$. Then $e \equiv C[e_1]$ with $C \equiv [].m(e_2)$. By a similar argument to the $e_1 == e_2$ case, either (E-Context) or (E-AppUD) must apply.

  - $e_1 \equiv v_1$ and $e_2 \not\equiv v_2$. Then $e \equiv C[e_2]$ with $C \equiv e_1.m([])$. By a similar argument to the $e_1 == e_2$ case, either (E-Context) or (E-AppUD) must apply.

  - $e_1 = v_1$ and $e_2 = v_2$. If v_1 is `nil`, we step to *blame*. Because e is a non-checked method call, we know (1) must have been derived by (T-App). By inversion of this rule, $\Gamma \vdash_{CT} v_1 : A_1$ for some A_1, and that $A_1.m \in \mathcal{U}$ and $CT(A.m) = A_{in} \to A_{out}$. By (7), this means $def\_\ of(A.m) = x.e_m$ for

some e_m. This satisfies all the premises of rule (E-AppUD), therefore we can apply this rule.

- Case $\lceil A_{res} \rceil e_1.m(e_2)$. We split this case on the structure of e_1, e_2:
  - $e_1 \not\equiv v$. Then $e \equiv C[e_1]$ with $C \equiv [].m(e_2)$. By a similar argument to the $e_1 == e_2$ case, either (E-Context) or (E-AppUD) must apply.
  - $e_1 \equiv v_1$ and $e_2 \not\equiv v_2$. Then $e \equiv C[e_2]$ with $C \equiv e_1.m([])$. By a similar argument to the $e_1 == e_2$ case, either (E-Context) or (E-AppUD)
  - $e_1 = v_1$ and $e_2 = v_2$. If v_1 is `nil`, we return blame. Because e is a checked method call, we know (1) must have been derived by (T-App-Lib). By inversion of this rule, we know $\Gamma \vdash_{CT} v_1 : A_1$ for some A_1, where $A_1.m \in \mathcal{L}$. Let $v_{res} = \mathit{call}(A.m, v_1, v_2)$; by Assumption 1 we know that this call will terminate and return a value. Then, if $\mathit{type\_of}(v_{res})$ is not a subtype of A_{res}, we will return blame. Otherwise, we will have satisfied all the preconditions of (E-AppLib), therefore we can apply this rule and take a step.

□

We now introduce our theorem of soundness of the type checking judgment.

Theorem 4 (Soundness of Type Checking)**.** *If valid(CT) and* $\emptyset \vdash_{CT} e \hookrightarrow e' : A_C$ *and* $\emptyset \vdash_{CT} e' : A$*, then either* e' *reduces to a value,* e' *reduces to blame, or* e' *does not terminate.*

Program type checking rules $\boxed{CT \models P}$

$$\frac{CT(A.m) = A_1 \to A_2 \qquad [\texttt{self} \mapsto A, x \mapsto A_1] \vdash_{CT} e : A_2' \qquad A_2' \leq A_2}{CT \models \texttt{def}\ A.m(x) : A_1 \to A_2\ =\ e}\ \text{T-PDef}$$

$$\frac{CT(A.m) = \sigma}{CT \models \texttt{lib}\ A.m(x) : \sigma}\ \text{T-PLib}$$

$$\frac{CT \models P_1 \qquad CT \models P_2}{CT \models P_1, P_2}\ \text{T-PSeq}$$

Figure A.5: Program type checking rules.

Proof. Let $\Gamma = \emptyset$, $E = \emptyset$, $S = (\emptyset, \square) :: \cdot$, and $TS = (\Gamma[A], A) :: \cdot$. By construction we have $A \leq TS$, $\Gamma \sim E$, and $TS \sim S$. Thus, we satisfy the preconditions of progress and preservation, and soundness holds by standard argument. □

With this soundness theorem, it is straightforward to extend soundness to the check insertion rules. We make use of a lemma that states that the type assigned by the check insertion rules will be equivalent to the type assigned by the type checking rules.

Lemma 4. *$\Gamma \vdash_{CT} e \hookrightarrow e' : A_C$ and $\Gamma \vdash_{CT} e' : A$ if and only if $\Gamma \vdash_{CT} e \hookrightarrow e' : A$.*

Proof. Straightforward by induction on check insertion rules $\Gamma \vdash_{CT} e \hookrightarrow e' : A$. □

Theorem 5 (Soundness of Check Insertion)**.** *If valid(CT) and $\emptyset \vdash_{CT} e \hookrightarrow e' : A$ then either e' reduces to a value, e' reduces to blame, or e' does not terminate.*

Proof. By Theorem 4 and Lemma 4. □

A.2 Program Type Checking and Class Table Construction

The rules for type checking a program are given in Figure A.5. They rely on using a class table. We omit a formal definition of the class table construction here as it is straightforward. Informally: to construct a class table, traverse a program, adding type annotations from definitions and declarations as you go. We can then check that $CT \models P$ to ensure that the program P type checks under the constructed class table. If $CT \models P$, and the appropriate subtyping relations hold among methods of subclasses, we can conclude that $valid(CT)$ according to definition 4.

Appendix B: `SimTyper` Evaluation Data

This appendix contains the data corresponding to the graphs in Chapter 5, as well as some additional data.

Program	# Meths	LoC	# Args	# Vars	# Usable	# Overly General	# No Type
code.org	0	0	0	1	0	1	0
Discourse	0	0	0	8	8	0	0
Journey	0	0	0	0	0	0	0
Talks	0	0	0	2	1	1	0
MiniMagick	39	271	28	13	45	10	25
Money	54	350	36	38	83	29	16
Ronin	41	367	8	69	92	17	9
TZInfo	10	60	14	57	48	23	10
Total	**144**	**1048**	**86**	**188**	**277**	**81**	**60**

Table B.1: Data on the types that SimTyper inferred, but we did not have gold standards to compare against. For each program, we list the number of method types SimTyper inferred for which there was no gold standard, the lines of code those methods comprised, the number of argument types those methods include, and the number of instance, class, and global variables SimTyper inferred a type for. Note that we did not include a column for the number of return types in the methods, since this is equivalent to the number of methods. Then, of the total argument, return, and variable types SimTyper inferred, we list the number of these that were usable, overly general, and the number for which SimTyper failed to infer any type.

Program	Match	Match up to Param.
	C /CH/CD/CHD	C /CH/CD/CHD
code.org	59 /106/ 87 / 119	14/ 19 / 14 / 19
Discourse	45 / 55 / 54 / 57	2 / 3 / 2 / 3
Journey	39 / 47 / 45 / 48	1 / 2 / 3 / 3
MiniMagick	25 / 29 / 31 / 35	1 / 1 / 2 / 2
Money	58 / 67 / 82 / 81	6 / 6 / 13 / 11
Ronin	102/134/161/ 167	16/ 15 / 43 / 42
Talks	105/136/138/ 140	13/ 20 / 16 / 22
TZInfo	117/166/261/ 266	7 / 7 / 19 / 18
Total	550/740/859/ 913	60/ 73 /112/ 120

(a) Number of inferred matches and matches up to parameter.

Program	Different (Structural)	No Type
	C / CH / CD / CHD	C /CH/CD/CHD
code.org	67(28) / 25(13) / 56(12) / 20(5)	23 / 13 / 6 / 5
Discourse	17(14) / 11(7) / 14(10) / 11(6)	13 / 8 / 7 / 6
Journey	7(7) / 2(2) / 6(3) / 4(0)	10 / 6 / 3 / 2
MiniMagick	8(5) / 8(5) / 12(2) / 10(2)	23 / 19 / 12 / 10
Money	47(41) / 41(26) / 47(25) / 47(12)	41 / 38 / 10 / 13
Ronin	81(53) / 68(29) / 123(23) /119(14)	144/126/ 16 / 15
Talks	38(26) / 21(7) / 37(13) / 29(7)	40 / 19 / 5 / 5
TZInfo	149(127)/ 119(78) / 129(59) /122(26)	178/159/ 42 / 45
Total	414(301)/295(167)/424(147)/362(72)	472/388/101/ 101

(b) Number of inferred different types, and number of positions for which `SimTyper` did not infer any type.

Table B.2: SimTyper evaluation results corresponding to the plots in Figure 5.8. For each benchmark, we list the number of matching, match up to parameter, and different inferred types measured under all configurations (C/CH/CD/CHD). For the different category, in parentheses we show the number of those types that were structural. Additionally, the *No Type* category indicates the algorithm could not find a more usable solution than giving a type variable for that position.

Cat.	Match	Match up to Param.	Different (Structural)
	C /CH/CD/CHD	C /CH/CD/CHD	C / CH / CD / CHD
Args	69 /208/279/ 322	5 / 17 / 23 / 32	339(296)/222(166)/268(146)/219(71)
Vars	52 / 57 / 61 / 61	9 / 11 / 13 / 14	6(4) / 4(2) / 9(1) / 8(1)
Rets	429/475/519/ 530	46/ 45 / 76 / 74	69(0) / 69(0) / 147(0) / 135(0)

Table B.3: This table corresponds to the graphs in Figure 5.9. Measuring SimTyper's performance for arguments, variables, and returns. For each category, we list the number of match, match up to parameter, and different inferred types measured under all for configurations (C/CH/CD/CHD). For the different category, in parentheses we show the number of those types that were structural.

Configuration	Match	Match up to Param.	Different (Structural)
top-1	819	108	449(172)
top-3	859	112	424(147)
top-5	860	112	423(144)
top-7	860	113	422(143)

Table B.4: This table corresponds to the graphs in Figure 5.10a. Measuring SimTyper's performance under top-1, -3, -5, and -7 configurations. Measurements were taken under the CD configuration. For the different category, in parentheses we show the number of those different types that were structural types.

Embedding Method	Match	Match up to Param.	Different (Structural)
All	859	112	424(147)
Head	792	107	463(176)

Table B.5: This table corresponds to the graphs in Figure 5.10b. Measuring SimTyper's performance under for two different methods of generating embeddings for arguments: Averaging vectors for all uses of an argument (All) or using just the vector for the argument in the method header (Head). Measurements were taken under the CD configuration. For the different category, in parentheses we show the number of those different types that were structural types.

Embedding Method	Match	Match up to Param.	Different (Structural)
N+S	859	112	424(147)
N	866	103	432(147)

Table B.6: This table corresponds to the graphs in Figure 5.10c. Measuring SimTyper's performance under for two different methods of generating embeddings for returns: averaging method names and return sites (N+S), or using just method names (N). Measurements were taken under the CD configuration. For the different category, in parentheses we show the number of those different types that were structural types.

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