Partial Decode-forward Coding Schemes for the Gaussian Two-Way Relay Channel

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***Abstract*—We design novel partial decode-forward (PDF) schemes for the Gaussian two-way relay channel with direct link. Different from pure decode-forward, each user divides its message into two parts and the relay decodes only one part of each. The relay then generates its codeword as a function of the two decoded parts and forwards to the two users. We propose PDF schemes for both the full- and half-duplex modes. In full duplex, the scheme is based on block Markov encoding and forward joint decoding over 2 consecutive blocks. In half duplex, the transmission is divided into 4 phases, in which one user transmits during the first phase, the other during the second phase, both users transmit during the third phase and the relay transmits during the last phase. The relay decodes a part of the messages from both users at the end of phase 3 and each user decodes only at the end of phase 4. Analysis and simulation show that if for one user, the direct link is stronger than the user-to-relay link, while for the other, the direct link is weaker, then PDF can achieve a rate region strictly larger than the time-shared region of pure decode-forward and direct transmission for both full- and half-duplex modes.**

I. INTRODUCTION

The two-way channel in which two users wish to exchange message was first studied by Shannon [1]. If there is a relay to help the exchange of two users' messages, it is called the two-way relay channel (TWRC). This is practical channel model for wireless communication systems. For example, a dedicated relay station has been proposed in 4G wireless standards to help the mobile and base station exchange messages. In full-duplex transmission, each node can transmit and receive at the same time; whereas for half-duplex transmission, each node can only either transmit or receive at each time. We first study the full-duplex mode to understand optimal coding schemes, then apply them to half-duplex transmission, which is more practical in wireless systems.

A number of coding schemes has been proposed for the full-duplex TWRC. In [2], different relay strategies, including decode-forward, compress-forward and amplify-forward are applied. The author of [3] also proposes a decode-forward scheme for TWRC, but different from [2] in that there is no block Markovity. In both [2] and [3], the relay fully decodes the messages from both users. However, this is suboptimal if for some user, the direct link is stronger than the user-to-relay link. Partial decode-forward is considered in combination with compress-forward in [4] but only for channels without the direct link.

We propose a partial decode-forward scheme for full-duplex channels in [5], in which each user divides its message into two parts and the relay decodes only one part. Numerical results have shown that partial decode-forward outperforms pure decode-forward and direct transmission in general. In the first part of this paper, we present a more detailed analysis on the achievable rate regions of these three schemes. Specifically, we provide the analytical conditions for when partial decode-forward achieves new rates outside the time-shared region of pure decode-forward and direct transmission.

Increasingly more works also focus on the half-duplex TWRC because of practical wireless node constraints. In [6], distributed linear-dispersion space-time coding is considered for the two-way wireless relay networks with several proposed protocols. The authors of [7] propose an opportunistic two-way relaying scheme based on joint network coding and opportunistic relaying, which achieves a better performance than fully distributed space-time two-way relaying. In [8], a new amplify-forward and a novel estimate-forward schemes are proposed, which can outperform decode-forward for some SNR. In [9], a partial decode-forward protocol is proposed for the 2-phase TWRC (without the direct link) that is a superposition of both decode-forward and compress-forward. In [10], based upon both bitwise XOR and symbol-level superposition coding, a new physical layer network coding scheme is proposed, which achieves larger rate region than either individual scheme in asymmetric channels. The authors of [11] propose a novel cooperation strategy for the more general 3-way relay channel in which the relay also has its own message by combining both DF and CF. In [12], three full decode-forward protocols are proposed which has 2, 3 or 4 phases, in which the 4-phase protocol contains the 2- and 3-phase ones as special cases and achieves the largest rate region. In [13], these authors extend the protocols to a mixed relaying strategy which combines CF in one direction and DF in the other.

In all works on the half-duplex mode, partial decode-forward has not been considered for the full TWRC with direct link. As the second part of this paper, we propose a partial decode-forward scheme for the 4-phase transmission protocol, in which each user divides its message into two parts and the relay only decodes one part of each message. The relay then generates its codeword as a function of the decoded parts and forwards to users. This scheme outperforms both the pure DF scheme in [12] and direct transmission.

II. CHANNEL MODEL

A. Full-duplex Gaussian two-way relay channel model

Figure 1 illustrates the full-duplex Gaussian two-way relay channel (GTWRC) model. User 1 and User 2 wish to exchange

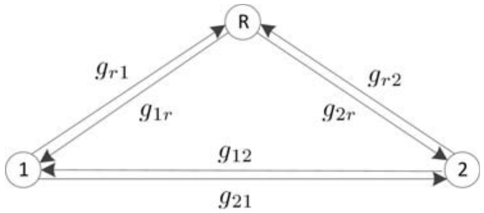

Fig. 1. Full-duplex Gaussian two-way relay channel model.

messages with the help of the relay R. Each node can send and receive at the same time. The channel can be modeled as

$$\begin{aligned} Y_1 &= g_{12}X_2 + g_{1r}X_r + Z_1 \\ Y_2 &= g_{21}X_1 + g_{2r}X_r + Z_2 \\ Y_r &= g_{r1}X_1 + g_{r2}X_2 + Z_r \end{aligned} \tag{1}$$

where X_1, X_2, X_r represent transmitted signals of user 1, user 2 and the relay respectively. The average input power constraints for them are all P. Y_1, Y_2, Y_r represent received signals of user 1, user 2 and the relay respectively. $Z_1 \sim \mathcal{N}(0,1), Z_2 \sim \mathcal{N}(0,1), Z_r \sim \mathcal{N}(0,1)$ are independent noises. $g_{12}, g_{1r}, g_{21}, g_{2r}, g_{r1}, g_{r2}$ are corresponding channel gains.

B. Half-duplex Gaussian two-way relay channel model

For the half-duplex mode, each node can only either send or receive at each time. We consider a 4-phase half-duplex Gaussian two-way relay model as in Figure 2, as motivated by [12] which shows the best performance out of several protocols. During the 1st phase, user 1 transmits. During the 2nd phase, user 2 transmits. During the 3rd phase, both user 1 and user 2 transmit. During the 4th phase, the relay transmits. Assume all nodes listen while not transmitting. The transmitted signals during each phase can be expressed as

$$\begin{aligned} \text{Phase 1:}\quad & Y_{21} = g_{21}X_{11} + Z_{21} \\ & Y_{r1} = g_{r1}X_{11} + Z_{r1} \\ \text{Phase 2:}\quad & Y_{12} = g_{12}X_{22} + Z_{12} \\ & Y_{r2} = g_{r2}X_{22} + Z_{r2} \\ \text{Phase 3:}\quad & Y_{r3} = g_{r1}X_{13} + g_{r2}X_{23} + Z_{r2} \\ \text{Phase 4:}\quad & Y_{14} = g_{1r}X_r + Z_{14} \\ & Y_{24} = g_{2r}X_r + Z_{24}, \end{aligned}$$

where X_{ij} represents the transmitted signal of user i during phase j. Y_{ij} represents received signal of user i during phase j. All the noises Z are independently and identically distributed according to $\mathcal{N}(0,1)$.

III. A FULL-DUPLEX PARTIAL DECODE-FORWARD CODING SCHEME

For the full-duplex GTWRC, two existing coding schemes are direct transmission in which the relay is not used, and full decode-forward scheme in which the relay decodes the whole message of each user then forwards a function of these messages as in [2] [3]. As we will see, similarly to the one-way relay channel, direct transmission achieves strictly larger rate region than decode-forward if for both users, the direct link is stronger than the user-to-relay link. If for both users, the user-to-relay link is sufficiently stronger than the direct link, then decode-forward outperforms direct transmission.

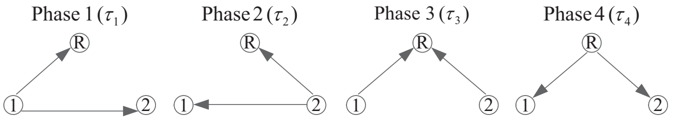

Fig. 2. Half-duplex Gaussian two-way relay channel model.

However, in cases such that for one user, the user-to-relay link is stronger than the direct link, while for the other user, it's the opposite, then neither existing scheme outperforms the other. This motivates us to put forward the partial decode-forward scheme where the relay only decodes a part of the messages and forwards them. Specifically, each message is divided into two parts. At the end of each block, the relay decodes one part of each message and forwards a function of these parts. Each user then decodes the message of the other user transmitted in the previous block based on the signals received in both the current and previous blocks. This PDF scheme supersedes both pure decode-forward and direct transmission for all channel configurations.

A. Partial decode-forward scheme and achievable rate region

For the full-duplex mode, each node can transmit and receive at the same time. We use Gaussian codes and a block coding scheme in which each user sends $B-1$ messages over B blocks of n symbols each. Figure 3 illustrates the transmitted signals in two neighboring blocks.

1) User encoding: In each block, each user splits its message into two parts: $m_1 = (m_{10}, m_{11})$ with rate (R_{10}, R_{11}), and $m_2 = (m_{20}, m_{22})$ with rate (R_{20}, R_{22}). User 1 uses signal U_1 and V_1 to encode message m_{10} and m_{11} respectively. In each block $b \in [1:B]$, the transmitted signal of user 1 is

$$X_1(m_{10}(b), m_{11}(b)) = U_1(m_{10}(b)) + V_1(m_{11}(b)),$$

where $U_1 \sim \mathcal{N}(0, \alpha P)$ and $V_1 \sim \mathcal{N}(0, \bar{\alpha}P)$ are independent. Similarly, the transmitted signal of user 2 in block b is

$$X_2(m_{20}(b), m_{22}(b)) = U_2(m_{20}(b)) + V_2(m_{22}(b)),$$

where $U_2 \sim \mathcal{N}(0, \beta P)$ and $V_2 \sim \mathcal{N}(0, \bar{\beta}P)$ are independent.

2) Relay operation: Decoding: At the end of block b, the relay decodes the message parts $(m_{10}(b), m_{20}(b))$. As in the multiple access channel, it succeeds with high probability if the following conditions are satisfied.

$$\begin{aligned} R_{10} &\le C\left(\frac{g_{r1}^2\alpha P}{g_{r1}^2\bar{\alpha}P + g_{r2}^2\bar{\beta}P + 1}\right) \\ R_{20} &\le C\left(\frac{g_{r2}^2\beta P}{g_{r1}^2\bar{\alpha}P + g_{r2}^2\bar{\beta}P + 1}\right) \\ R_{10} + R_{20} &\le C\left(\frac{g_{r1}^2\alpha P + g_{r2}^2\beta P}{g_{r1}^2\bar{\alpha}P + g_{r2}^2\bar{\beta}P + 1}\right) \end{aligned} \tag{3}$$

Encoding: The relay then generates its codeword based on the decoded $(m_{10}(b), m_{20}(b))$. The codeword can be generated as a function of codewords for $m_{10}(b)$ and $m_{20}(b)$, such as XOR as in [12], but other functions also suffice (for example binning as in [3]). It transmits the encoded signal $X_r(m_{10}(b), m_{20}(b))$ in block $b+1$.

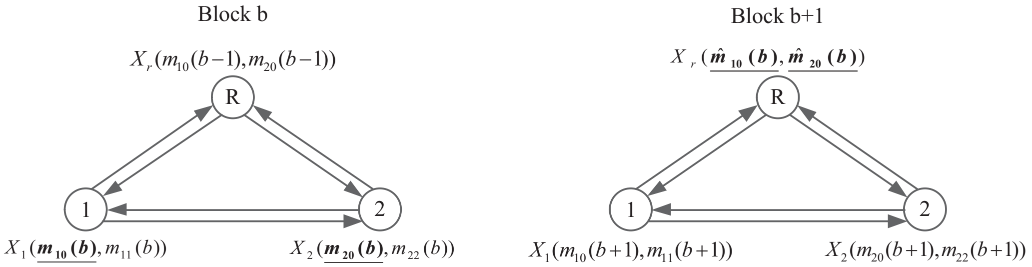

Fig. 3. Full-duplex partial decode-forward transmission diagram.

$$\begin{aligned} R_1 &\leq \min\left\{C\left(\frac{g_{r1}^2\alpha P}{g_{r1}^2\bar{\alpha}P + g_{r2}^2\bar{\beta}P + 1}\right) + C(g_{21}^2\bar{\alpha}P), C(g_{21}^2P + g_{2r}^2P)\right\} \\ R_2 &\leq \min\left\{C\left(\frac{g_{r2}^2\beta P}{g_{r1}^2\bar{\alpha}P + g_{r2}^2\bar{\beta}P + 1}\right) + C(g_{12}^2\bar{\beta}P), C(g_{12}^2P + g_{1r}^2P)\right\} \\ R_1 + R_2 &\leq C\left(\frac{g_{r1}^2\alpha P + g_{r2}^2\beta P}{g_{r1}^2\bar{\alpha}P + g_{r2}^2\bar{\beta}P + 1}\right) + C(g_{21}^2\bar{\alpha}P) + C(g_{12}^2\bar{\beta}P), \end{aligned} \tag{2}$$

$$\text{where } 0 \leq \alpha, \beta \leq 1 \text{ and } C(x) = \frac{1}{2}\log(1+x).$$

3) User decoding: At the end of block $b+1$, user 2 decodes message $m_1 = (m_{10}(b), m_{11}(b))$ based on received signals in both blocks b and $b+1$ with joint decoding rules as in [5], which succeeds with high probability if the following conditions are satisfied:

$$\begin{aligned} R_{11} &\leq C(g_{21}^2\bar{\alpha}P) \\ R_{10} + R_{11} &\leq C(g_{21}^2P + g_{2r}^2P) \end{aligned} \tag{4}$$

Similar for user 1. By applying Fourier-Motzkin Elimination to above inequalities, we can establish the achievable rate region of the partial decode-forward scheme as follows.

Theorem 1. *The rate region in (2) is achievable for the full-duplex Gaussian two-way relay channel.*

Proof: Refer to [5]. ■

B. Analysis of partial decode-forward rate region

In this section, we analyze and compare the rate region achieved by the proposed partial decode-forward scheme with that achieved by pure decode-forward scheme [3] and direct transmission (without using the relay) for different channel conditions. We first present the achievable rate region of pure decode-forward scheme and direct transmission.

Theorem 2. *The following rate region is achievable for the full-duplex Gaussian two-way relay channel with pure decode-forward scheme [3]:*

$$\begin{aligned} R_1 &\leq \min\left\{C(g_{r1}^2P), C(g_{21}^2P + g_{2r}^2P)\right\} \\ R_2 &\leq \min\left\{C(g_{r2}^2P), C(g_{12}^2P + g_{1r}^2P)\right\} \\ R_1 + R_2 &\leq C(g_{r1}^2P + g_{r2}^2P). \end{aligned} \tag{5}$$

If the two users only use direct links to exchange message instead of using the relay, the following rate region is achievable:

$$\begin{aligned} R_1 &\leq C(g_{21}^2P) \\ R_2 &\leq C(g_{12}^2P). \end{aligned} \tag{6}$$

Remark 1. If $\alpha = 1, \beta = 1$, the rate region in (2) reduces to the decode-forward lower bound in (5). If $\alpha = 0, \beta = 0$, the rate region in (2) reduces to the direct transmission lower bound in (6). Thus partial decode-forward region always include both decode-forward and direct transmission regions as special cases.

The following theorem compares the rate region of partial decode-forwards with that of pure decode-forwards and direct transmission for different channel cases.

Theorem 3. *Comparing PDF with pure decode-forward and direct transmission (without using the relay), we have the following 4 cases:*
1) PDF can achieve rates strictly outside the time-shared region of DF and direct transmission if

$$\begin{aligned} & g_{r1}^2 > g_{21}^2 + \min\{g_{21}^2g_{r2}^2P, g_{2r}^2\}, \quad g_{12}^2 > g_{r2}^2 \\ \textit{or} \quad & g_{r2}^2 > g_{12}^2 + \min\{g_{12}^2g_{r1}^2P, g_{1r}^2\}, \quad g_{21}^2 > g_{r1}^2 \end{aligned} \tag{7}$$

2) PDF achieves the time-shared region of DF and direct transmission if

$$\begin{aligned} g_{21}^2 &< g_{r1}^2 \\ g_{12}^2 &< g_{r2}^2 \\ C(g_{21}^2P) + C(g_{12}^2P) &> C(g_{r1}^2P + g_{r2}^2P) \end{aligned} \tag{8}$$

3) PDF achieves the same rate region as pure DF scheme which is strictly larger than direct transmission if

$$\begin{aligned} g_{21}^2 &\leq g_{r1}^2 \\ g_{12}^2 &\leq g_{r2}^2 \\ C(g_{21}^2P) + C(g_{12}^2P) &\leq C(g_{r1}^2P + g_{r2}^2P) \end{aligned} \tag{9}$$

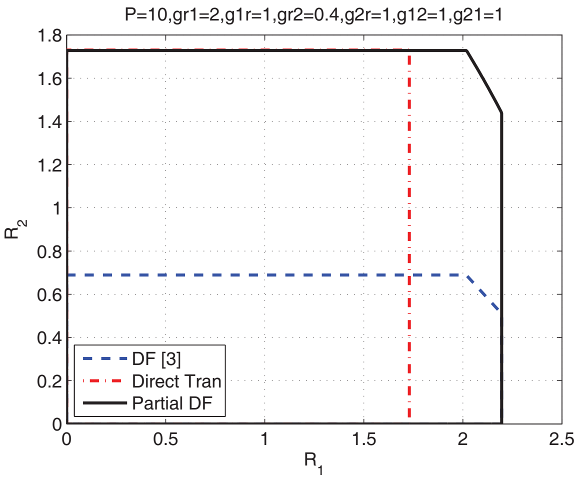

Fig. 4. Partial decode-forward achieves rates outside the time-shared region of decode-forward and direct transmission in the full-duplex TWRC.

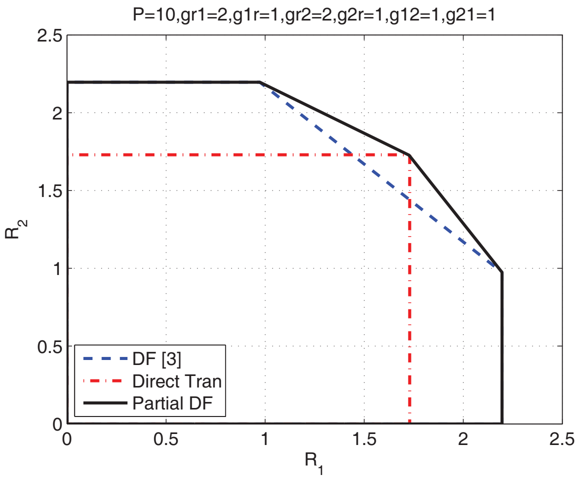

Fig. 5. Partial decode-forward achieves time-shared region of decode-forward and direct transmission in the full-duplex TWRC.

4) PDF achieves the same rate region as direct transmission which is strictly larger than DF if

$$\begin{aligned} g_{21}^2 &\geq g_{r1}^2 \\ g_{12}^2 &\geq g_{r2}^2. \end{aligned} \tag{10}$$

Proof: The proof is omitted because of limited space. ■

C. Discussion and numerical examples

1) Discussion: Some intuition for our proposed partial decode-forward (PDF) scheme can be developed as follows:

- Compared to DF, PDF involves extra superposition encoding which can be easily implemented in practice. It uses joint decoding similar to DF and hence has similar decoding complexity.
- When the user-to-relay link is weaker than the direct link, decoding the whole message at the relay limits the achievable rate. In such a case, partially decoding messages at the relay can relax the constraint and achieve a larger rate region.
- Theorem 3 implies that when both direct links are sufficiently weaker than the user-to-relay links, the relay should fully decode the messages and forward them. When both direct links are stronger than the user-to-relay links, the relay should not be used. If for one user, the direct link is stronger than the user-to-relay link, while for the other one, the direct link is weaker, then the relay should decode only a part of the message from the former.
- Applicability in wireless channels: In the wireless environment, the channel gains fluctuate and can easily cover all cases of Theorem 3. Thus it is useful to know the optimal scheme for each case such that each user can adapt their transmission according to the channel strength.

2) Numerical examples: For cases 1 and 2 in Theorem 3, we provide each an example. Figure 4 shows an example in which partial decode-forward achieves rates outside the time-shared region of decode-forward and direct transmission. Figure 5 shows an example where partial decode-forward achieves the time-shared region of decode-forward and direct transmission. In both cases, the proposed PDF scheme outperforms pure DF.

IV. A 4-PHASE HALF-DUPLEX PARTIAL DECODE-FORWARD CODING SCHEME

In this section, we design a partial decode-forward scheme for the half-duplex case. The half-duplex mode in which each node can either transmit or receive at each time, is more practical in wireless systems. Moreover, transmissions are performed in independent blocks without block Markovity. Each user can decode the message of the other user at the end of each block without any delay.

Three decode-forward protocols for the half-duplex two-way relay channel have been proposed in [12]. The first protocol divides each block into 2 phases, in which both users transmit in the first phase and the relay transmits in the second phase. The second protocol divides each block into 3 phases, in which user 1 transmits in the first phase, user 2 in the second phase and the relay in the third phase. The third protocol divides each block into 4 phases, in which user 1 transmits in the first phase, user 2 in the second phase, both users transmit in the third phase and the relay transmits in the last phase. All nodes listen while not transmitting. It has been shown that the 4-phase achieves the largest rate region among these three protocols.

We will only discuss a 4-phase partial decode-forward scheme as it outperforms the other two. The main difference between our scheme and the scheme in [12] is that the relay only decodes a part of the messages in our scheme, whereas it decodes the full messages in [12]. When a direct link is stronger than the user-to-relay link, the proposed scheme achieves strictly larger rate region.

A. Partial decode-forward scheme and achievable rate region

Consider the transmission at each block, which is divided into four phases as in Figure 6. Each message is divided into two parts for each user. During the 1st phase, user 1 transmits both parts. During the 2nd phase, user 2 transmits both parts. During the 3rd phase, both users transmit only one part of their

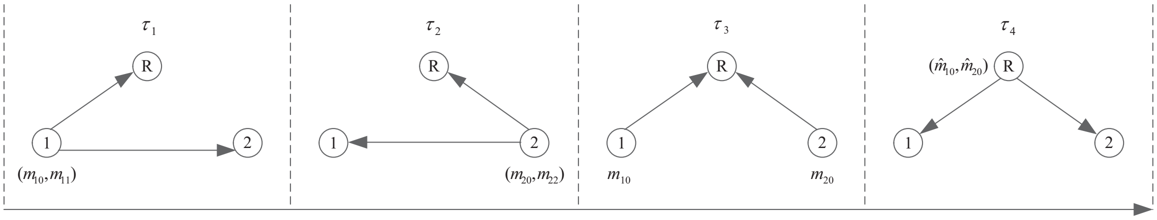

Fig. 6. Half-duplex partial decode-forward transmission diagram.

messages. At the end of the 3rd phase, the relay decodes this part of each message based on the received signals from all first three phases. It then transmits a function of those message parts during the 4th phase. At the end of the 4th phase, user 1 decodes the message of user 2 based on received signals in the 2nd and 4th phases. Similarly for user 2.

1) User encoding: Let the relative time duration of the phases are τ_1, τ_2, τ_3 and τ_4 respectively, where $\tau_1 + \tau_2 + \tau_3 + \tau_4 = 1$. Let m_1 be the message of user 1 to be sent during a specific block. User 1 divides it into two parts (m_{10}, m_{11}) with rate (R_{10}, R_{11}) and encodes m_{10} and m_{11} by U_1 and V_1 respectively. Then the transmitted signals of user 1 during phase 1 and 3 respectively are as follows.

$$\begin{aligned} X_{11} &= \sqrt{\alpha_{11}} U_1(m_{10}) + \sqrt{\beta_{11}} V_1(m_{11}) \\ X_{13} &= \sqrt{\alpha_{13}} U_1(m_{10}) \end{aligned}$$

where $\alpha_{11}, \beta_{11}, \alpha_{13}$ are corresponding power allocations. Similarly, user 2 divides its message m_2 into two parts (m_{20}, m_{22}) with rate (R_{20}, R_{22}) and encodes m_{20} and m_{22} by U_2 and V_2 respectively. Its transmitted signals in the 2nd and 3rd phases respectively are

$$\begin{aligned} X_{22} &= \sqrt{\alpha_{22}} U_2(m_{20}) + \sqrt{\beta_{22}} V_2(m_{22}) \\ X_{23} &= \sqrt{\alpha_{23}} U_2(m_{20}). \end{aligned}$$

2) Relay operation: Decoding: At the end of the 3rd phase, the relay decodes the messages parts (m_{10}, m_{20}) based on received signals from the 1st, 2nd and 3rd phases by joint decoding.

Encoding: The relay then constructs its transmitted signal in the 4th phase as

$$X_r = \sqrt{\gamma} W(m_{10}, m_{20})$$

where $W(m_{10}, m_{20})$ can be generated as a function (for example, XOR or random binning) of the codewords for (m_{10}, m_{20}).

In the above signals, U_1, V_1, U_2, V_2, W are independent and identically distributed according to $\mathcal{N}(0, 1)$. The power constraints for the two users and the relay are as follows.

$$\begin{aligned} \tau_1(\alpha_{11} + \beta_{11}) + \tau_3 \alpha_{13} &= P \\ \tau_2(\alpha_{22} + \beta_{22}) + \tau_3 \alpha_{23} &= P \\ \tau_4 \gamma &= P. \end{aligned} \tag{11}$$

3) User decoding: At the end of phase 4, user 2 uses joint decoding to decode message $m_1 = (m_{10}, m_{11})$ based on received signals from both the 1st and 4th phases. Similarly, user 1 decodes $m_2 = (m_{20}, m_{22})$ based on received signals from both the 2nd and 4th phases.

Theorem 4. *The following rate region is achievable for the half-duplex Gaussian two-way relay channel with partial decode-forward scheme.*

$$R_{10} \le \tau_1 C\left(\frac{g_{r1}^2 \alpha_{11}}{g_{r1}^2 \beta_{11} + 1}\right) + \tau_3 C(g_{r1}^2 \alpha_{13}) = I_1 \tag{12a}$$

$$R_{20} \le \tau_2 C\left(\frac{g_{r2}^2 \alpha_{22}}{g_{r2}^2 \beta_{21} + 1}\right) + \tau_3 C(g_{r2}^2 \alpha_{23}) = I_2 \tag{12b}$$

$$R_{11} \le \tau_1 C(g_{21}^2 \beta_{11}) = I_3 \tag{12c}$$

$$R_{22} \le \tau_2 C(g_{12}^2 \beta_{22}) = I_4 \tag{12d}$$

$$\begin{aligned} R_{10} + R_{20} \le{}& \tau_1 C\left(\frac{g_{r1}^2 \alpha_{11}}{g_{r1}^2 \beta_{11} + 1}\right) + \tau_2 C\left(\frac{g_{r2}^2 \alpha_{22}}{g_{r2}^2 \beta_{21} + 1}\right) \\ &+ \tau_3 C(g_{r1}^2 \alpha_{13} + g_{r2}^2 \alpha_{23}) = I_5 \end{aligned} \tag{12e}$$

$$R_{10} + R_{11} \le \tau_1 C(g_{21}^2(\alpha_{11} + \beta_{11})) + \tau_4 C(g_{2r}^2 \gamma) = I_6 \tag{12f}$$

$$R_{20} + R_{22} \le \tau_2 C(g_{12}^2(\alpha_{22} + \beta_{22})) + \tau_4 C(g_{1r}^2 \gamma) = I_7 \tag{12g}$$

with power constraints in (11), where $\tau_1 + \tau_2 + \tau_3 + \tau_4 = 1$ *and* $C(x) = \frac{1}{2}\log(1 + x)$*. By applying Fourier-Motzkin Elimination, the achievable rates in terms of* $R_1 = R_{10} + R_{11}$ *and* $R_2 = R_{20} + R_{22}$ *can be expressed as*

$$\begin{aligned} R_1 &\le \min\{I_1 + I_3, I_6\} \\ R_2 &\le \min\{I_2 + I_4, I_7\} \\ R_1 + R_2 &\le I_3 + I_4 + I_5. \end{aligned} \tag{13}$$

Proof: At the end of the 3rd phase, the relay decodes (m_{10}, m_{20}) based on received signals from the 1st, 2nd and 3rd phases, which can succeed with high probability if (12a), (12b) and (12e) are satisfied. During the 4th phase, the relay sends $X_r(m_{10}, m_{20})$. Based on the received signals in the 1st phase Y_{21} and the 4th phase Y_{41}, user 2 can decode $m_1 = (m_{10}, m_{11})$ with error probability going to zero if (12c) and (12f) are satisfied. Similarly, user 1 can decode $m_2 = (m_{20}, m_{22})$ with vanishing error if (12d) and (12g) are satisfied. ■

B. Discussion

Several points can be noted for our proposed PDF scheme as follows:

- For the half-duplex mode, there is no block Markovity. Therefore, encoding and decoding are simple and can be done within one block, which is also practical.

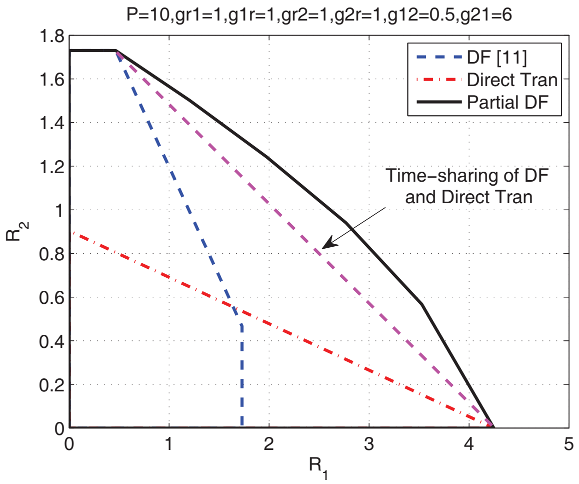

Fig. 7. Rate region comparison between partial decode-forward, pure decode-forward and direct transmission for the half-duplex Gaussian TWRC.

- The signaling for each user again involves only 2-part superposition coding which can be easily implemented in practice.
- It is also interesting to find the optimal power allocations and time slot durations to maximize the achievable rates as these are of directly practical value. These can be topics of future work.
- Similar to the full-duplex case, partial decode-forward helps when the direct link is stronger than the user-to-relay link for one user, while is weaker for the other.
- Our proposed half-duplex PDF scheme again includes both the DF scheme in [12] and direct transmission as special cases.

C. Numerical comparison

We numerically compare the achievable rate regions of partial decode-forward, pure decode-forward [12] and direct transmission. For direct transmission, we divide each block into two phases, where user 1 transmits in the first phase and user 2 in the second phase. For pure decode-forward, we include power scaling to satisfy the power constraint (11), which is different from [12] with fixed power. Hence the DF region here is larger than that in [12]. Figure 7 shows that the proposed partial decode-forward scheme achieves strictly larger rate region than the other two schemes with new rates outside the time-shared region of the other two. This result also agrees with the analysis in Theorem 3 that when the direct link is stronger than the user-to-relay link for one user but is weaker for the other, then PDF strictly outperforms the time-sharing of both DF and direct transmission.

V. CONCLUSION

We have proposed partial decode-forward (PDF) schemes for both full- and half-duplex Gaussian two-way relay channels. Such schemes have not been considered for the full TWRC (with direct link) before. Each user splits its message into 2 parts and the relay decodes only one. Analysis and simulation both show that when the direct link is stronger than the user-to-relay link for one user but is weaker for the other, partial decode-forward in either full- or half-duplex mode can achieve larger rate region than both pure decode-forward and direct transmission. Thus unlike in the one-way Gaussian relay channel, partial decode-forward is beneficial in the two-way relay channel. Furthermore, PDF involves only 2-part superposition coding and hence is simple to implement in practice.

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