# Matched Filtering with Rate Back-off for Low Complexity Communications in Very Large Delay Spread Channels

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Abstract—We study the possibility to transmit data over channels with large delay spreads under the constraint of a very simple receiver which has only one tap. Such a scheme is of interest when a cost-efficient way to transmit potentially high data rates are sought after. We investigate the performance of the optimal pre-filter for this scheme, and compare it to a simplified, so-called time-reversal (TR) pre-filter which has very low complexity. At low SNRs, the TR pre-filter and the optimal pre-filter are equivalent. At high SNRs, the TR prefilter achieves a performance that is independent from the delay spread of the channel and hence its performance is the same for any bandwidth. In applications where bandwidth is abundant, such as ultra-wide band (UWB) communications, any required performance can be obtained by TR pre-filters with a rate back-off transmission (i.e. transmission rate lower than the allowable bandwidth). Similar performance can also be obtained with full-rate transmission using several transmit antennas. This performance is guaranteed, since the high diversity of the large delay spread channel effectively eliminates any fading.

## I. Introduction

In this paper, we examine transmission schemes for channels with large delay spread using a transmitter with moderate complexity, and an absolutely low-cost receiver. Our work is motivated by the fact that communication systems can only compete if they can offer high data rate at a low cost. Whereas the transmitter is regarded as a centralized server and is allowed to have several antennas and to pre-equalize the channel, the receiver has only one antenna and a single tap. In order to support high data rates with this scheme, the transmission bandwidth needs to be large and the channel therefore exhibits a large delay spread.

The drawback of such a cost-efficient scheme with a simple receiver is its limited equalization power which yields, without any further measures, a low signal-to-noise ratio (SNR) at the output of the receiver. We will show, however, that with a simple transmit pre-filter and increasing bandwidth, this SNR converges toward a limit which is independent from the delay spread of the channel, hence it is valid for any transmission rate of the system. Since this SNR limit is not subject to any fading, it guarantees a minimum performance of the system which can be achieved independently of the harshness of the delay spread channel. This limit can be increased to a reasonable output SNR if multiple antennas are employed at the transmitter. Alternatively, it can be increased if rate back-off transmission is employed. We furthermore note that this

single tap receiver does not need to estimate the channel which again helps to reduce its complexity.

We use two different pre-filters for our study. We first derive an optimal pre-filter for a channel with delay spread using a single-tap receiver. This optimal pre-filter has the well-known MMSE-like structure. At low SNRs, it converges to a transmit matched filter [1] or pre-rake [2]; and for long channels, such a filter is also known as time-reversal (TR) filter, where it has the origin in underwater acoustics and ultrasound [3]. This TR filter is the second pre-filter under study. In heavy scattering environments, the large number of multi-paths make a simple TR pre-filter very effective. It produces a strong focus of energy into a single tap which is resistant to fading, and low side lobes which are achieved almost surely for any single channel realization. This was recently demonstrated on Ultrawide band (UWB) data [4]. We compare the performance of the MMSE and the TR pre-filters in a static fading channel with 50 taps. At low SNRs, the TR filter converges to the optimal MMSE filter. At high SNRs, the output SNR of the TR filter saturates at a value that is independent from the delay spread of the channel. This value is a function of the number of transmit antennas and the transmission rate of the system. Higher SNRs can be achieved with the TR pre-filters if the system does not operate at the maximum rate. The prerequisite for such a strategy is that bandwidth is not an overly scarce resource, as in the case of Ultra-wide band transmission [5]. For such systems, a rate back-off factor of two or four still allows a very high transmission rate. We finally show that prefiltering in a high delay spread channel is an excellent measure against fading.

In Section II, we set up the channel and signal models and derive the optimal MMSE and the TR pre-filter structures. In Section III, we introduce the effective SNR as a suitable performance measure for both the TR and MMSE pre-filters. We give an analytical formula for the TR filter's performance as a function of the number of transmit antennas and the rateback off factor. In Section IV, we compare the performance of the TR pre-filters to that of the MMSE pre-filter numerically. We conclude in Section V.

**Notations.** E[.] denotes expectation, \* is conjugation, and \* is convolution.  $\delta_{lm}$  stands for the Kronecker Delta function, and  $\delta(.)$  and  $\delta[.]$  stand for the Dirac Delta function in the continuous and discrete domains respectively.  $\mathbf{A}^H$  denotes the conjugate transpose of the matrix  $\mathbf{A}$ , and  $\parallel \mathbf{a} \parallel$  denotes the  $l^2$ -norm of the vector  $\mathbf{a}$ .  $\hat{\mathbf{e}}_{\Delta}$  is a unit vector with a 1 in position

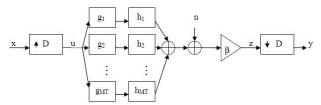


Fig. 1. A block diagram of a MISO system with a pre-equalizer at the transmitter and a single tap receiver.

 $\Delta$  and 0 everywhere else.

## II. PRE-FILTERING FOR LONG DELAY SPREAD CHANNELS

## A. Channel and Signal Model

We consider a frequency selective MISO (multiple-input single-output) channel with  $M_T$  transmit antennas. The channel impulse responses (CIRs) in the continuous domain can be modeled as

$$h_i(\tau) = \sum_{r=1}^{\Gamma} \alpha_{r,i} \delta(\tau - \tau_{r,i}), \tag{1}$$

where  $\alpha_{r,i}$  is the amplitude and  $\tau_{r,i}$  is the delay of a path, r denotes the path index amongst the  $\Gamma$  received paths, and  $i \in \{1,\ldots,M_T\}$  denotes the transmit antennas. We assume that the path delays  $\tau_{r,i}$  are uniformly distributed within a given interval  $[0,T_{max}]$ ; the path amplitudes  $\alpha_{r,i}$  are i.i.d. complex Gaussian random variables with zero mean and a variance  $\sigma_{r,i}^2$  given by the power delay profile (PDP) of the channel. For the exponential PDP we have

$$E[|\alpha_{r,i}|^2] = \sigma_{r,i}^2 = \exp\left(-\frac{\tau_{r,i}}{\sigma_\tau}\right),\tag{2}$$

where  $\sigma_{\tau}$  is the delay spread of the channel. The sampled channel impulse response (CIR),  $h_i[l]$ , is obtained by convolving  $h_i(\tau)$  with the root-raised cosine pulse matched filters  $\phi_{Tx}$  and  $\phi_{Rx}$  with excess bandwidth  $\beta=0.35$  at the transmitter and receiver, and sampling at a sample period  $T_S$ ,

$$h_i[l] = h(lT_S) = (\phi_{Rx} * h * \phi_{Tx})(lT_S).$$
 (3)

We assume that the paths are uncorrelated. Throughout this paper, the discrete channel has L taps such that  $LT_S=T_{max}, \ {\rm i.e.},$  the number of channel taps is chosen to capture contributions from all scatterers. The sampled CIR can then be written as

$$h_i[k] = \sum_{l=0}^{L-1} h_i[l]\delta[k-l]. \tag{4}$$

The block-diagram of the system to be optimized is shown in Fig. 1. One way to combat ISI in channels with long delay spread is to reduce the transmission rate by a factor of D as compared to the system bandwidth. We call this a rate-back off strategy. An input bit stream x is up-sampled by a factor D, put on the  $M_T$  branches of the transmitter, pre-equalized with filters  $g_i$ ,  $i=1,\ldots,M_T$ , and sent over the channel. It is received at the receiver's single antenna and noise is added. At the receiver, the signal is down-sampled by a factor of D. The receiver has a single tap with tap gain denoted by

 $\beta$ . The tap gain  $\beta$  can not change the received SNR, since it scales both the input signal and the noise. However, it will play an important role in the optimization problem where the transmitter balances the channel gain against noise.

Let each FIR filter  $g_i$  have length N  $(N \ge L)$  for each transmit antenna. We denote  $g_i[k]$  as

$$g_i[k] = \sum_{l=0}^{N-1} g_i[l]\delta[k-l].$$
 (5)

Notice that for simplified expressions, we have written  $g_i$  in a way such that the equalized signal has the peak at the same time as the input signal. The CIR of the pre-equalized channel is the convolution of the channel and the pre-filter,

$$(g * h)_{i}[k] = \sum_{l=0}^{L-1} h_{i}[l]g_{i}[-l]\delta[k] + \sum_{m \neq -l} h_{i}[l]g_{i}[m]\delta[k-m-l].$$
 (6)

From (6) we derive the input-output relation of the preequalized channel without up-sampling (D=1) as

$$y[k] = \beta \sum_{i=1}^{M_T} h_i[k] * g_i[k] * x[k] + \beta \tilde{n}[k]$$
 (7)

where  $\tilde{n}[k]$  is white noise.

For up-sampling factor of D, the up-sampled sequence u[k] is related to the input sequence via:

$$u[k] = \begin{cases} x[k/D] & \text{if } k/D \in \mathbb{Z} \\ 0 & \text{otherwise.} \end{cases}$$
 (8)

The received sequence z[k] can hence be written as

$$z[k] = \beta \sum_{i} \sum_{m} u[m](h_i * g_i)[k - m] + \beta \tilde{n}[k].$$
 (9)

The received sequence with rate back-off is then down-sampled by  ${\cal D}$  to give

$$y^{[D]}[k] = z[Dk] = \beta \sum_{i} \sum_{m} u[m](h_i * g_i)[Dk - m] + \beta n[k]$$
(10)

where we use the notation  $^{[D]}$  to denote down-sampling by D and with  $n[k] = \tilde{n}[Dk]$  being the down-sampled noise. After some manipulation we can rewrite the input-output relation of the channel with rate back-off factor  $D, D = 1, \ldots, L$ , as

$$y^{[D]}[k] = \beta \sum_{i=1}^{M_T} \sum_{l} (h_i * g_i)[Dl]x[k-l] + \beta n[k].$$
 (11)

Note that for a fixed bandwidth, rate back-off reduces the ISI power of the received signal, but not that of the main peak. Equation (11) is not used for designing an optimal pre-filter. It is used to demonstrate the amount of ISI reduction when rate back-off is used for a given pre-filter.

## B. Optimal Pre-filtering

To derive the optimal pre-filter, we rewrite (7) in a matrix form

$$y[k] = \beta \mathbf{x}_k \mathbf{H} \mathbf{g} + \beta n[k] \tag{12}$$

with  $\mathbf{x}_k = \begin{bmatrix} x[k] & x[k+1] & \dots & x[k+L+N-2] \end{bmatrix}$ .  $\mathbf{H}$  is a block-Toeplitz matrix of size  $(L+N-1) \times (M_TN)$ ; its block-columns are comprised of the shifted channel vectors  $\begin{bmatrix} h_i[0], \dots, h_i[L-1] \end{bmatrix}^T$ ; and in each block, columns that belong to adjacent antennas are next to each other. The vector  $\mathbf{g}$  is given by

$$\mathbf{g} = \left[\mathbf{g}[0]^T \quad \dots \quad \mathbf{g}[N-1]^T\right]^T \tag{13}$$

with

$$\mathbf{g}[l] = [g_1[l] \dots g_{M_T}[l]]^T; \quad l = 0, \dots, N-1. \quad (14)$$

The product **Hg** is the convolution of the pre-filter and the channel.

We shall find an optimal value for both  ${\bf g}$  and  $\beta$ . In order to simplify receiver processing, the purpose of the set of transmit filters  ${\bf g}$  is to pre-equalize the signal. This means that at the receiver, as much power as possible is focused in the central peak, so that ISI is minimized, subject to a power constraint. This can be mathematically stated as follows

$$\hat{\mathbf{g}}_{\beta,N} = \operatorname{argmin}_{\mathbf{g},\beta} E[\| \mathbf{x}_k \beta \mathbf{H} \mathbf{g} + \beta n[k] - x[k - \Delta] \|^2]$$
s.t.  $E \| \mathbf{g}^H \mathbf{g} \| = 1$ . (15)

Here  $\Delta$  is the delay of the equalizer and is approximately  $\frac{N+L}{2}$ . The difference between this formulation and the one where the equalizer is placed at the receiver is in the transmit power constraint. After taking the expectation, the objective function in (15) becomes

$$P\beta^2 \mathbf{g}^H \mathbf{H}^H \mathbf{H} \mathbf{g} + \beta^2 \sigma^2 + P - \beta P (\mathbf{g}^H \mathbf{H}^H \hat{\mathbf{e}}_{\Delta} + \hat{\mathbf{e}}_{\Delta}^H \mathbf{H} \mathbf{g}).$$
 (16)

The above problem can be solved analytically using the method of Lagrange multipliers to give

$$\hat{\mathbf{g}}_{\beta,N} = \frac{1}{\beta} \left( \mathbf{H}^H \mathbf{H} + \frac{\sigma^2}{P} \mathbf{I} \right)^{-1} \mathbf{H}^H \hat{\mathbf{e}}_{\Delta}$$
 (17)

$$\beta^2 = \hat{\mathbf{e}}_{\Delta}^T \mathbf{H} \left( \mathbf{H}^H \mathbf{H} + \frac{\sigma^2}{P} \mathbf{I} \right)^{-2} \mathbf{H}^H \hat{\mathbf{e}}_{\Delta}$$
 (18)

Equation (17) defines a filter with an MMSE-type solution with a finite number of taps. We notice that the optimized value for  $\beta$  ensures that the power constraint of the filter is valid. In the transmit-receiver chain of Fig. 1,  $\beta$  has no effect on the SNR at the receiver; it ensures, however, that the transmitter can take into account the SNR while optimizing the pre-equalizer. In other words,  $\beta$  compensates for the signal attenuation by the channel, so that a simple symbol-by-symbol detection can be carried out at the receiver.

# C. TR-Prefilter

The TR-pre-filter is a matched pre-filter which follows from the optimal filter (17) for low SNR,

$$\mathbf{g}_{TR} = \lim_{SNR \to 0} \mathbf{g} = \mathbf{h}^*$$

$$s.t. \parallel \mathbf{h} \parallel = 1, \tag{19}$$

or

$$g_{TR,i}[k] = \sqrt{\gamma} h_i^*[-k], \qquad (20)$$

with  $\gamma = \left(\sum_{i=1}^{M_T}\sum_{l=0}^{L-1}|h_i[l]|^2\right)^{-1}$ , i.e., the filter equals the time-reversed and conjugated CIR of the channel, and the

receiver sees the autocorrelation function of the channel. The power in its central peak is maximized, but the ISI suppression is worse than that of the optimal filter.

#### III. PERFORMANCE EVALUATION OF THE TR FILTER

## A. Performance Criteria

The main performance criterion for the comparison of the two filters will be the effective SNR  $\rho_{\rm eff}$ , and the variance of the received SNR due to fading in the channel.

We define  $\rho_{\text{eff}}$  as the ratio of the average useful power at the receiver against the sum of the noise power and the average ISI. We split (11) into the part containing the intended symbol and the other one for the ISI,

$$y^{[D]}[k] = \beta \sum_{i=1}^{M_T} \sum_{l=0}^{L-1} h_i[l] g_i[-l] x[k]$$

$$+ \beta \sum_{i=1}^{M_T} \sum_{k \neq 0} \sum_{m \neq -l} h_i[l] g_i[m] \delta[Dk - l - m] x[n - k]$$

$$+ \beta n[k].$$
(21)

Since  $\beta$  does not affect the received SNR, we omit it in the subsequent derivations. The power of the received, possibly down-sampled, signal can the be expressed as

$$E[|y^{[D]}[k]|^2] = P_x E[P_0] + P_x E[Q^{[D]}] + \sigma^2$$
 (22)

where we used  $E[x[l]x^*[m]] = P_x\delta_{lm}$ , with  $P_x$  denoting the transmitter power, and  $E[n[l]n^*[m]] = \sigma^2\delta_{lm}$  with  $\sigma^2$  as the noise power. The constants  $P_0$  and  $Q^{[D]}$  are the instant useful power and the ISI power, respectively,

$$P_0 = \left| \sum_{i=1}^{M_T} \sum_{l=0}^{L-1} h_i[l] g_i[-l] \right|^2 \tag{23}$$

$$Q^{[D]} = \left| \sum_{i=1}^{M_T} \sum_{l \neq 0} \sum_{k \neq 0} h_i[l] g_i[m] \delta[Dk - m - l] \right|^2$$
 (24)

We can now define the effective SNR with a rate-back off factor D as

$$\rho_{\text{eff,D}} = \frac{P_x E[P_0]}{P_x E[Q^{[D]}] + \sigma^2} = \frac{E[P_0]}{E[Q^{[D]}] + \frac{1}{\sigma^2}}$$
(25)

where  $\rho_0 = \frac{P_x}{\sigma^2}$ .

We furthermore define the SNR  $\rho_{MFB}$  that would be obtained if the receiver had a matched filter and full channel knowledge (i.e. no restriction on the receiver complexity) as

$$\rho_{\text{MFB}} = \frac{P_x E\left[\sum_{i=1}^{M_T} \sum_{l} |h_i[l]|^2\right]}{\sigma^2}.$$
 (26)

As a measure of the fading of the channel, we also introduce the variance of the received SNR as

$$\sigma_{\rm SNR}^2 = {\rm Var} \frac{P_x P_0}{P_x Q^{[D]} + \sigma^2}. \eqno(27)$$

# B. Analytical Bounds on the Performance of the TR Filter

Due to the TR filter's simple structure, we can derive the effective SNR analytically.

The TR filter structure implies that N=L and the prefilter taps have the form as in (20), i.e., the pre-filter taps are simply the scaled time-reverse and conjugation of the channel impulse responses. The peak and ISI power in (23) and (24) become

$$P_{TR} = \gamma \left( \sum_{i=1}^{M_T} \sum_{l=0}^{L-1} |h[l]|^2 \right)^2$$
 (28)

$$Q_{TR}^{[D]} \approx \gamma \sum_{i=1}^{M_T} \sum_{k=\frac{-L+1}{D}}^{\frac{L-1}{D}} \sum_{l=0}^{L-1} \left| h_i[l] h_i[Dk+l] \right|^2$$
 (29)

Equation (28) is the entire channel power gain and equals  $\frac{1}{\rho_0} \rho_{\text{MFB}}$ . For the derivation of (29), we assume that in channels with long delay spread and widely distributed scatterers, the phases of the incoming waves are uniformly distributed. With sufficiently large number of taps, the summation over all the cross-product terms in  $Q_{TR}$  will the approach its mean zero.

In order to compute the effective SNR, we calculate  $E[P_{TR}]$ and  $E[Q_{TR}]$ . We assume that the channel has many scatterers, hence the  $h_i[l]$  are approximately complex Gaussian distributed channel taps with zero mean, and with second and fourth moments given by

$$E[|h_i[l]|^2] \approx \Gamma \exp\left(-\frac{lT_S}{\sigma_\tau}\right),$$
 (30)

$$E[|h_i[l]|^4] = 2E[|h_i[l]|^2]^2.$$
 (31)

From this we obtain

$$\sum_{i=1}^{M_T} \sum_{l=0}^{L-1} E\left[|h_i[l]|^2\right] = M_T \sum_{l=0}^{L-1} \exp\left(-\frac{lT_S}{\sigma_\tau}\right)$$

$$= M_T \frac{1 - \exp\left(-\frac{LT_S}{\sigma_\tau}\right)}{1 - \exp\left(-\frac{1}{\sigma_\tau}\right)}, \quad (32)$$

$$\sum_{i=1}^{M_T} \sum_{l=0}^{L-1} E\left[|h_i[l]|^4\right] = M_T \frac{1 - \exp\left(-\frac{2LT_S}{\sigma_\tau}\right)}{1 - \exp\left(-\frac{2}{\sigma_\tau}\right)}$$
(33)

Thus one can compute the ratio between the useful power against the ISI in the limit of large  $L/\sigma_{\tau}$  as

$$\frac{E[P_{TR}]}{E[Q_{TR}^{[D]}]} = \left(1 - e^{-\frac{D}{\sigma_{\tau}}}\right) \times \frac{\left[M_T^2\left(1 + e^{-\frac{1}{\sigma_{\tau}}}\right) + M_T\left(1 - e^{-\frac{1}{\sigma_{\tau}}}\right)\right]}{2M_T e^{-\frac{D}{\sigma_{\tau}}}\left(1 - e^{-\frac{1}{\sigma_{\tau}}}\right)} (34)$$

Using (34) together with (25) and (26) we arrive at an analytical expression for the effective SNR of a TR communication system with  $M_T$  transmit antennas and a rate back-off factor of D,

$$\rho_{\text{eff,D}} = \rho_{\text{MFB}} \left( 1 + \rho_{\text{MFB}} \frac{E[Q_{TR}^{[D]}]}{E[P_{TR}]} \right)^{-1}.$$
 (35)

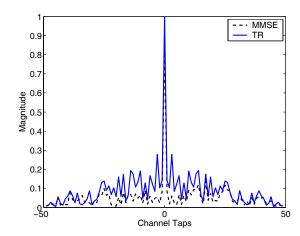


Fig. 2. The effective channel as generated by TR (dashed line) and the MMSE filter (cont. line)

We can derive from this the following limiting cases:

$$\rho_{\text{eff,D}} = \rho_{\text{MFB}} \qquad \text{for } \sigma_{\tau} \longrightarrow 0 \qquad (36)$$

$$\rho_{\text{eff,D}} = \rho_{\text{MFB}} \qquad \text{for } \sigma_{\tau} \longrightarrow 0 \qquad (36)$$

$$\rho_{\text{eff,D}} = \frac{DM_{T}\rho_{\text{MFB}}}{\rho_{\text{MFB}} + DM_{T}} \qquad \text{for } \sigma_{\tau} \longrightarrow \infty. \qquad (37)$$

Equation (36) shows that a TR system is lossless if the delay spread of the channel is very small. Equation (37) can be split into low-SNR and high-SNR regimes as

$$\rho_{\rm eff,D} = \rho_{\rm MFB}$$
 for  $\rho_{\rm MFB}$  small,  $\sigma_{\tau}$  large (38)

$$\rho_{\text{eff,D}} = DM_T \text{ for } \rho_{\text{MFB}}, \sigma_{\tau} \text{ both large.}$$
 (39)

Equation (38) shows that the TR pre-filter is optimal at low SNR; this behavior is expected for a transmit matched filter. Equation (39) demonstrates that in an environment with a large delay spread, the performance of a TR system at high SNR saturates at a value independent from the delay spread of the channel. The TR performance increases by 3 dB per each additional transmit antenna, and by another 3 dB for each rate back-off factor of 2.

### IV. NUMERICAL COMPARISON

Unless otherwise stated, simulations were run with L =50 taps in a channel with  $\Gamma=200$  paths using the model described in Subsection II-A. The delay spread of the channel is  $\sigma_{\tau}=25T_{S}$ ; on average, the PDP of this channel has 10 dB less power at the last tap compared to the first one.

The CIR of a pre-equalized channel is shown in Fig. 2; for the computation of the MMSE filter,  $\rho_0 = 5$  dB was assumed. The dashed line is obtained using the MMSE filter, the continuous one is from the TR pre-filter. Both CIRs are well compressed around their central peak. The ISI of the impulse response obtained with the TR filter is higher than that of the MMSE filter, but its peak is also slightly higher.

In Fig. 3 we plot the effective SNR of the two filters as a function of  $\rho_{\rm eff}$  ranging from -10 to 30 dB. The dashed curves are performances of the MMSE filters, and the solid curves are of the TR filters. The three different curves for each filter are obtained for  $M_T = 1, 2$ , and 4. The results for the TR filter match the analytical results in (34) well. For large  $\rho_{\text{MFB}}$ ,

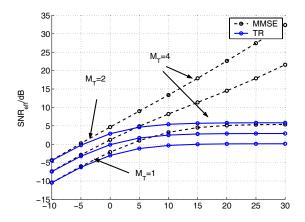


Fig. 3. The effective SNR of the two filters for 1, 2, and 4 transmit antennas.

the effective SNR becomes a constant; it is 0 dB for  $M_T=1$ , and increases by 3 dB for each additional transmit antenna. At low SNRs, the curves increase linearly with  $\rho_{\rm MFB}$ . In this range, the TR filter performs equally well compared to the optimal MMSE filter. The effective SNR for the SISO system is not very good; better performance requires the application of multiple transmit antennas. However, these results are achieved with an extremely simple, one-tap receiver, and in a channel with very long delay spread. At a large  $\rho_{\rm MFB}$ , the TR system performance is independent from the delay spread.

To increase the performance at the cost of a reduced transmission rate, the system can operate with rate back-off. The effective SNR for a single-antenna system with a rate back-off factor of D=1,2,5, and 25 is displayed in Fig. 4. With decreased rate, the saturation of the effective SNR sets in much later; the linear increase of  $\rho_{\rm eff}$  holds up to  $\rho_{\rm MFB}=10$  dB for the system with D=5, and up to 15 dB for the system with D=25. In this linear range, the performance of the TR pre-filter, in terms of the effective SNR, is slightly better than that of the MMSE pre-filter, since the latter is optimized for ISI suppression and has less power in its central peak.

In order to characterize the fading of the channel, we compute the variance of the output SNR according to (27) for both MMSE and TR filters for  $\rho=5$  dB and  $M_T=2$  as a function of the channel length L. The result is shown in Fig. 5 for L in the range between 5 and 50. Both filters have a similar SNR variance, which decreases with increasing number of channel taps toward a very low value. Both pre-filters are able to capture the multi-path diversity of the channel without using any coding.

## V. CONCLUSION

We investigated the possibility of transmitting data over channels with very high delay spread, using a simple one-tap receiver. The optimal pre-filter for this scheme is of an MMSE-type, which converges at low SNR to a time-reversal filter. Due to the simplicity of the receiver and the high delay spread, the effective SNR is very low if transmission at maximum rate is intended and only a single transmit antenna is available.

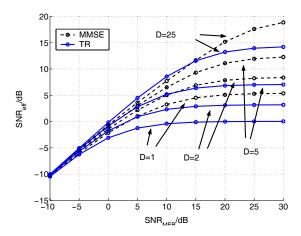


Fig. 4. The impact of rate-back off of a factor of 1,2,5 and 25 for one transmit antenna.

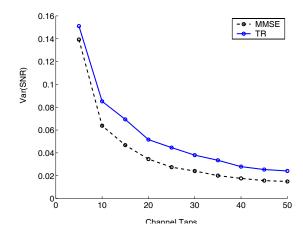


Fig. 5. The variance of the effective SNR for different channel lengths.

However, with several transmit antennas and sufficient rate back-off, reasonable SNRs can be achieved. The effective SNR levels using TR pre-filters are non-fading and do not decrease with increasing delay spread. This makes the scheme not only extremely simple but also very robust. Time Reversal pre-filtering thus offers the possibility to transmit high data rates with low complexity at a very low cost.

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