Interference and outage analysis in a cognitive radio network with beacon

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***Abstract* –This paper presents a study on the interference caused by Secondary Users (SUs) due to miss-detection and its effects on the capacity-outage performance of the Primary User (PU) in a cognitive network with beacon. Investigation by simulation indicates that a Gamma distribution can be used to characterize the total interference power from the SUs, and tight upper-bounds on its mean and variance are derived. Based on these results, a closed-form expression of the capacity-outage probability of the PU is developed to examine the effects of various system parameters on the PU performance in the presence of interference from SUs. Simulation results confirm the validity of the developed analytical models.**

I. INTRODUCTION

In cognitive communications, beacon can be used by the Primary User (PU) prior to its own transmission to facilitate Secondary Users (SUs) in detection of spectrum holes. Upon detecting the beacon, SUs will keep silent to avoid interference to PU. Although beacon is designed to improve its detection performance, there is a non-zero probability of beacon miss-detection due to noise and channel fading, and in such a case, SU transmission will cause interference to PU. How this interference caused by the SUs affects the performance of PU and how it relates to design parameters are of interest [1-3].

In [1], the interference power is analyzed in a network consisting of a single PU at the center and multiple SUs uniformly distributed in a circle around the PU. By assuming random interference power to be Gaussian, a closed-form upper bound for the capacity-outage probability of the PU has been derived.

In this paper, we improve upon the analysis results in [1] to provide tighter bounds on the interference mean and variance with verification by simulation. The probabilistic properties of interference power caused by SU's are investigated by simulation, and it is shown that Gamma distribution is a better fit to characterize the interference power. Furthermore, tighter closed-form upper bounds for the mean and variance of interference power, which are in good agreement with simulation results, are derived and used with the Gamma distribution of the interference power to develop the closed-form capacity-outage probability of the PU.

The remainder of this paper is organized as follows. After a brief overview of the system configuration and model, Section II provides the probability density function (pdf) of the interference power. Section III presents tight closed-form upper bounds on the mean and variance of interference power and compares them with numerical simulations obtained. The closed-form equation for the capacity-outage probability is derived to investigate effects of various system parameters on the outage performance in Section IV. Finally, Section V provides the conclusions.

II. SYSTEM AND INTERFERENCE MODELS

A. System Configuration and Model

We consider a circular area of radius R with one PU and n SUs surrounding the PU. The PU receiver R_x^0 is located at the center of the area and the PU transmitter T_x^0 is at distance R_0 away from R_x^0. Each SU has a transmitter T_x^i and a receiver R_x^i. SUs are distributed uniformly in this circular area with a density of λ SUs per unit area. To limit SU-to-PU interference, a protection radius $\varepsilon > 0$ is assumed around R_x^0 so that the distance between any T_x^i and R_x^0, r_i, is at least ε. Under these assumptions, r_i is a random variable with pdf $f_{r_i}(r_i) = 2r_i(R^2 - \varepsilon^2)^{-1}$, where $\varepsilon \le r_i \le R$, and the angle θ_i which T_x^i makes to the line connecting T_x^0 and R_x^0, is uniformly distributed between 0 and 2π (see Figure 1).

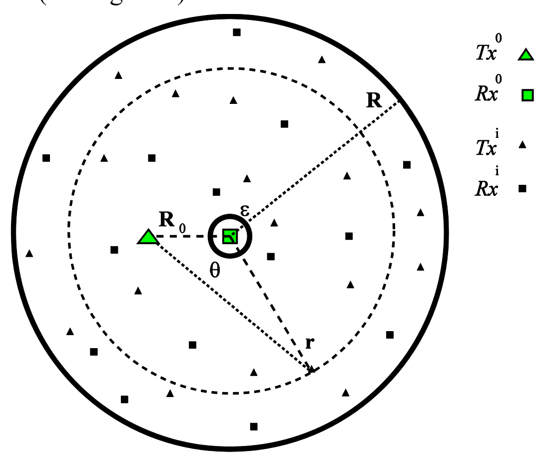

Fig. 1. Network model.

The wireless channel model including path loss and small-scale fading $h = \tilde{h}/d^{\alpha/2}$ is considered where α is the power path-loss exponent, $\tilde{h}$ is the Rayleigh fading gain and d is the distance between the transmitter and the receiver. The PU channel between T_x^0 and R_x^0 and the SU channel between T_x^i and R_x^0 are denoted as h_0 and h_i, respectively.

The PU transmitter T_x^0 is assumed to send a beacon prior to its transmission. If the SU correctly detects the beacon, it will be silent for the whole transmission period of the PU. In the case it miss-detects the beacon, the SU transmits concurrently with the PU, and, as a result, it may introduce interference to the PU. Assume an energy detection scheme in which the PU declares the beacon presence if its received power from the beacon is larger than a threshold. The beacon miss-detection probability of the i^{th} SU is computed in [1] as $P_i = 1 - e^{-\gamma d^{\alpha}(r_i,\theta_i)}$ where γ is the beacon threshold level and $d(r_i,\theta_i) = \sqrt{r_i^2 + R_0^2 - 2r_iR_0 cos\theta_i}$ is the distance between T_x^i and

T_x^0. Let x_0 and x_i, be the transmitted signals of T_x^0 and T_x^i, respectively. The received signal at R_x^0 can be written as

$$y_0 = h_0 x_0 + \sum_{i=1}^{n} F_i h_i x_i + z_0 \tag{1}$$

where $z_0 \sim N(0, \sigma^2)$ is AWGN and F_i which indicates the coincident transmission of the i^{th} cognitive user with the PU transmission, is a Bernoulli random variable as

$$F_i = \begin{cases} 1 & with\ probability \quad P_i \\ 0 & with\ probability \quad 1 - P_i \end{cases} \tag{2}$$

B. Interference from SUs

Since x_i in (1) are independent and zero-mean signals with power P, according to (1) and (2), the total interference power caused by the SUs becomes

$$I_0 = \sum_{i=1}^{n} I_i\,, \quad I_i = P P_i |\tilde{h}_i|^2 r_i^{-\alpha} \tag{3}$$

Based on the probabilistic properties of $P_i, |\tilde{h}_i|^2, r_i^{-\alpha}$, sample values of I_0 can be generated by simulation to obtain the histogram of its distribution as shown in Figure 2. By using mean squared-error curve-fitting for different number of SUs , Gamma distribution is found to have good agreement with the simulation results. Hence, the pdf of the total interference power I_0 can be approximated as $f_{I_0}(i; k, \theta) = i^{k-1} e^{-i/\theta} / \theta^k \Gamma(k), \quad i > 0$ where $\theta = var[I_0]/E[I_0]$ and $k = (E[I_0])^2 / var[I_0]$.

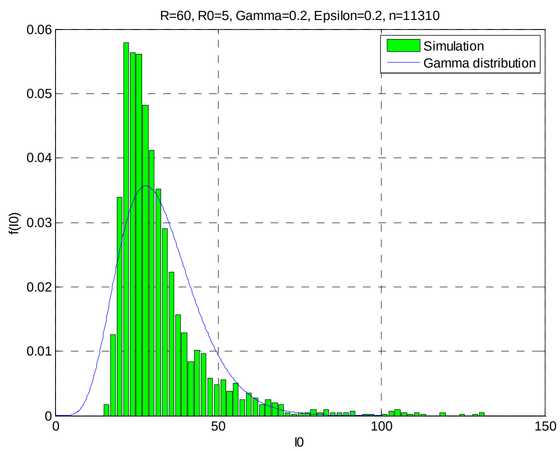

Fig. 2. Histogram and pdf of interference power.

III. MEAN AND VARIANCE OF THE INTERFERENCE POWER

$I_i = P P_i |\tilde{h}_i|^2 r_i^{-\alpha}$ is the interference power caused by the i^{th} SU. Since the channel fading and the location of each cognitive radio is independent, I_i are i.i.d. random variables. Hence,

$$E[I_0] = nE[I_i] \tag{4}$$

$$var[I_0] = E[I_0^2] - E^2[I_0] = n(E[I_i^2] - E^2[I_i]) \tag{5}$$

A. Upper-bound for the mean

Since the fading gains of different channels $\tilde{h}_i$ are independent and $E\left[|\tilde{h}_i|^2\right] = 1$, the mean of interference power caused by n SU's can be simplified to

$$E[I_0] = nP \int_0^{2\pi} \int_\varepsilon^R P_i r_i^{-\alpha} f_{r_i}(r_i) f_{\theta_i}(\theta_i) dr_i\, d\theta_i. \tag{6}$$

In [1], a closed-form upper bound for $E[I_0]$ was derived. In order to attain a tighter upper bound, the integral over the range $[\varepsilon, R]$ in (6) can be split into two parts. The first one contains a finite integral from ε to R_0+ε and represents the major part of interference caused by the SU close to the PU receiver (i.e., at about the same distance of the PU transmitter). Because of the finite integral range, it can be computed numerically with low complexity. The second part involves an integral from R_0+ε to R and can be substituted by the closed-form upper bound in [1], which is valid even as R→ ∞. As a result,

$$E[I_0] \le nP \int_0^{2\pi} \int_\varepsilon^{R_0+\varepsilon} P_i r_i^{-\alpha} f_{r_i}(r_i) f_{\theta_i}(\theta_i) dr_i d\theta_i + 2\pi\lambda P(\alpha-2)^{-1}\varphi \tag{7}$$

where

$$\varphi = (\varepsilon + R_0)^{2-\alpha} - R^{2-\alpha} - e^{-\gamma(\varepsilon+2R_0)^\alpha}(\varepsilon + 2R_0)^{2-\alpha} + e^{-\gamma(R+R_0)^\alpha}(R + R_0)^{2-\alpha} + \gamma^{\frac{\alpha-2}{\alpha}}\omega,$$

$$\omega = \left(\Gamma(2/\alpha\,, \gamma(\varepsilon + 2R_0)^\alpha) - \Gamma(2/\alpha\,, \gamma(R + R_0)^\alpha)\right).$$

Figures 3, 4 and 5 illustrate the simulation results, the upper bound in [1] and the new upper bound in (7) for the mean of the interference power $E[I_0]$ versus, respectively, γ, R_0 and ε. They show that the new upper bound in (7) is much tighter and in good agreement with simulation results.

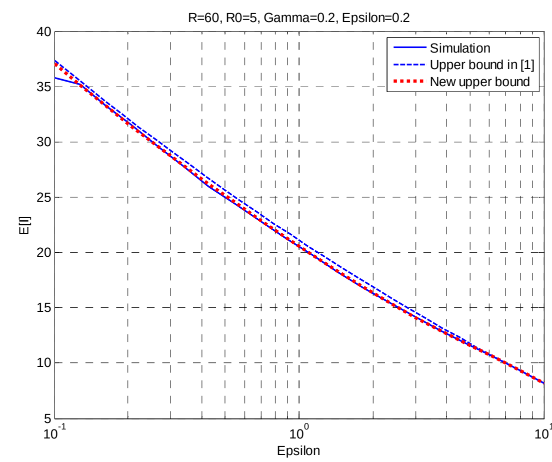

Fig. 3. Interference power mean vs. the receiver guard radius ε.

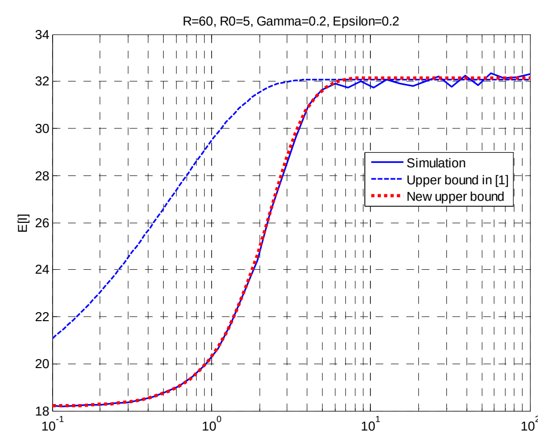

Fig. 4. Interference power mean vs. R_x^0-T_x^0 distance R_0.

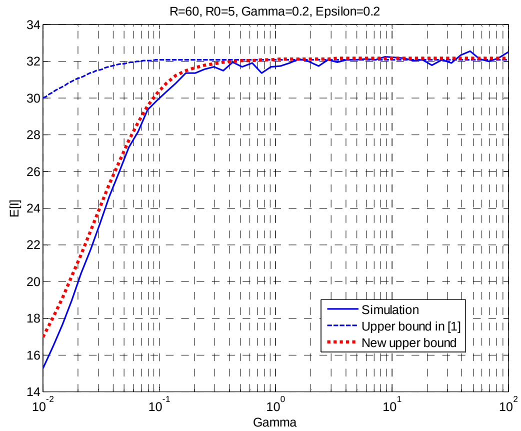

Fig. 5. Interference power mean vs. the beacon detection threshold γ.

B. *Upper-bound for the variance*

According to the results in Figures 3, 4 and 5, the upper bound for $E[I_0]$ is tight enough so that (7) can be used as an approximation for $E[I_0]$. Subsequently, from (5), an upper-bound for $var[I_0]$ can be obtained by using an upper-bound for $E[I_0^2]$ and this approximation for $E[I_0]$.

Since the channel fading gain $\tilde{h}_i$ is independent, and $E\left[\left|\tilde{h}_i\right|^4\right] = 2$, $E[I_0^2] = 2nP^2E[P_i^2 r_i^{-2\alpha}]$, i.e.,

$$E[I_0^2] = 2nP^2\int_0^{2\pi}\int_\varepsilon^R P_i^2 r_i^{-2\alpha} f_{r_i}(r_i) f_{\theta_i}(\theta_i) dr_i d\theta_i \qquad (8)$$

Similar to the case for $E[I_0]$, to obtain a tight upper bound for $E[I_0^2]$, the integral over the range $[\varepsilon, R]$ in (8) can be split into two parts. The first one contains a finite integral from ε to R_0+ε to include the major part of interference caused by SUs closer to the PU, and is computed numerically. For the second part, the closed-form upper bound for $E[I_0^2]$ in [1] is substituted as follows

$$E[I_0^2] \le 2nP^2 \int_0^{2\pi}\int_\varepsilon^{R_0+\varepsilon} P_i^2 r_i^{-2\alpha} f_{r_i}(r_i) f_{\theta_i}(\theta_i) dr_i d\theta_i + \frac{6\beta}{R^2-(\varepsilon+R_0)^2}$$

where (9)

$$\beta = \frac{(\varepsilon+R_0)^{-2(\alpha-1)} - R^{-2(\alpha-1)}}{2(\alpha-1)} - 2F(\alpha, 1-2\alpha, \gamma, \varepsilon+2R_0, R+R_0) + e^{-2\gamma R_0^\alpha} F(\alpha, 1-2\alpha, 2\gamma, \varepsilon+R_0, R)$$

and the function F is defined in [1].

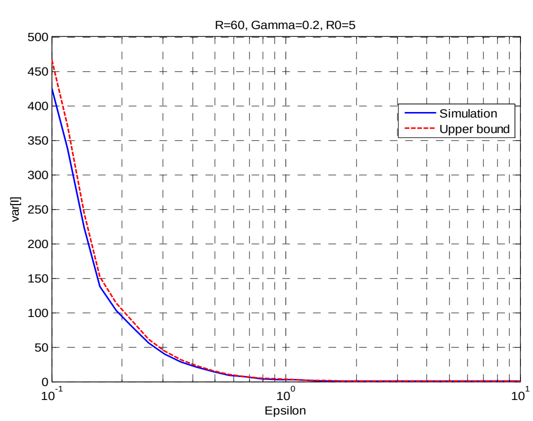

Fig. 6. Interference power variance vs. the receiver guard radius ε.

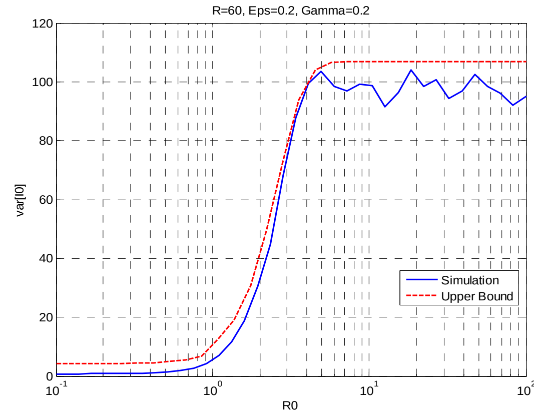

Fig. 7. Interference power variance vs. the R_x^0-T_x^0 distance R_0.

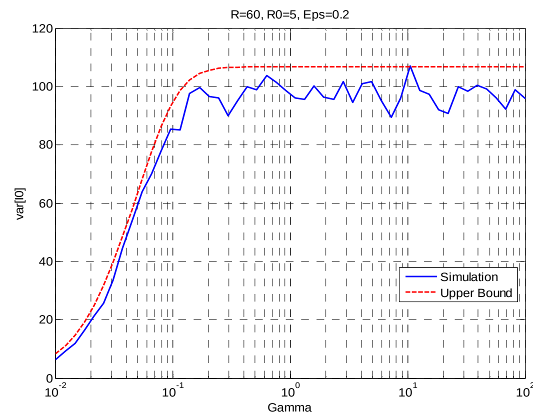

Fig. 8. Interference power variance vs. the beacon detection threshold γ.

Figures 6, 7 and 8 show the simulation results and approximate upper bounds for the variance of interference power, $var[I_0]$, based on (5), (7) and (9), versus γ, R_0 and ε, respectively. The provided upper bound is tight enough to the simulation results for $var[I_0]$.

IV. OUTAGE PROBABILITY

In this section, a closed-form expression for the outage probability of the PU relating to the mean and variance of the interference is derived to quantify the effect of the SUs. Referring to (1), the capacity of the PU with transmit power P_0 is $C_0 = log(1+|h_0|^2P_0/(I_0+\sigma^2))$. Given a PU rate threshold C_{th}, the capacity-outage probability can be calculated as

$$P_{out} = E[Pr(C_0 \le C_{th}|I_0)] \qquad (10)$$

where $Pr(C_0 \le C_{th}|I_0) = \Pr\left[\left|\tilde{h}_0\right|^2 \le P_r^{-1}(I_0+\sigma^2)\right]$ and $P_r = P_0(2^{C_{th}}-1)^{-1}R_0^{-\alpha}$. $\left|\tilde{h}_0\right|^2$ has the exponential distribution with parameter 1 and I_0 has the Gamma distribution represented by its pdf $f_{I_0}(i;k,\theta)$, as previously discussed in Section II.B. As a result,

$$Pr(C_0 \le C_{th}|I_0) = \int_0^{\frac{(i+\sigma^2)}{P_r}} f_{|\tilde{h}_0|^2}(h,1)dh = 1 - e^{-\frac{(i+\sigma^2)}{P_r}} \qquad (11)$$

$$\Rightarrow P_{out} = 1 - e^{-\sigma^2/P_r}E\left[e^{-i/P_r}\right] \qquad (12)$$

Using the moment generating function of the Gamma distribution, we obtain

$$E\left[e^{-\frac{i}{P_r}}\right] = \int_0^{+\infty} e^{-\frac{i}{P_r}} f_{I_0}(i;k,\theta)di = (1+\theta/P_r)^{-k} \tag{13}$$

$$\Rightarrow P_{out} = 1 - e^{-\frac{\sigma^2}{P_r}} \left(1 + \frac{var[I_0]}{P_r E[I_0]}\right)^{-(E[I_0])^2/var[I_0]} \tag{14}$$

Figure 9 shows the plot of the capacity-outage probability versus the PU rate threshold C_{th}. The plot confirms the precision of the analytical derivation in (14) as it closely matches the simulation results. Figure 10-12 illustrate the plots of the capacity-outage probability which are derived analytically from (14) for different system parameters.

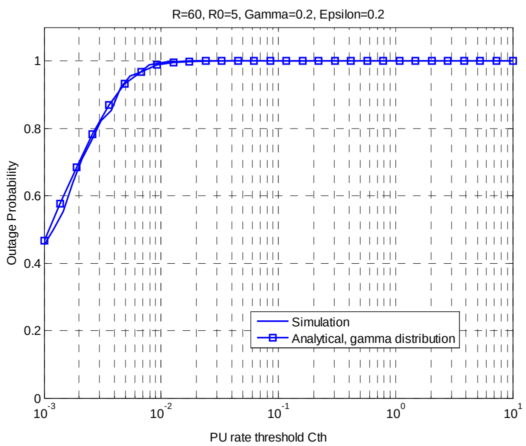

Fig. 9. Capacity-outage probability of the PU.

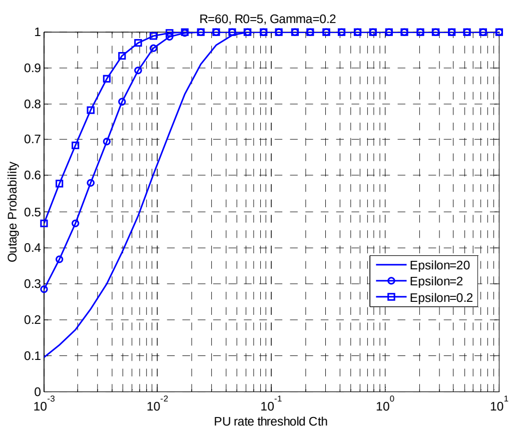

Fig. 10. Capacity-outage probability for different receiver guard radii ε.

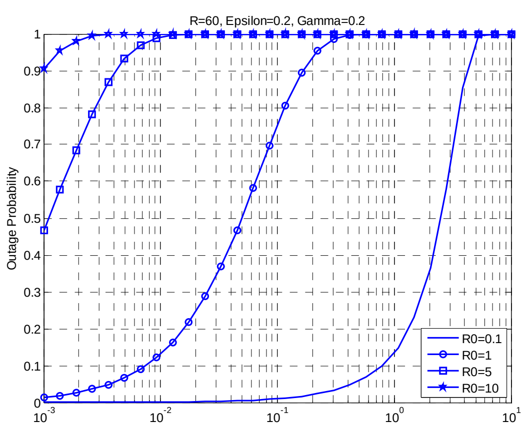

Fig. 11. Outage probability of the PU for different R_x^0-T_x^0 distances R_0.

Figure 10 shows that the performance of the PU improves as ε increases because the number of disrupting SUs around R_x^0 will decrease, as a result, the interference is reduced. Figure 11 illustrates that the capacity-outage probability of the PU is increased (i.e., degraded) with R_0. This is because most interference is caused by the SUs close to R_x^0, within the radius R_0. Hence, when R_0 increases, the number of close SUs increases and, as a result, the interference power increases. In addition, the performance of the PU decreases as path loss from T_x^0 to R_x^0 increases with R_0. Figure 12 shows that the capacity-outage probability of the PU degrades to a fixed limit as γ increases. For larger γ, the SUs are more likely to miss the beacon and hence increase the interference to the PU. However, as γ is larger than a fixed level, e.g., 0.2 in Figure 12, the SUs always miss the beacon and always transmit. Hence, the capacity-outage probability does not decrease any more.

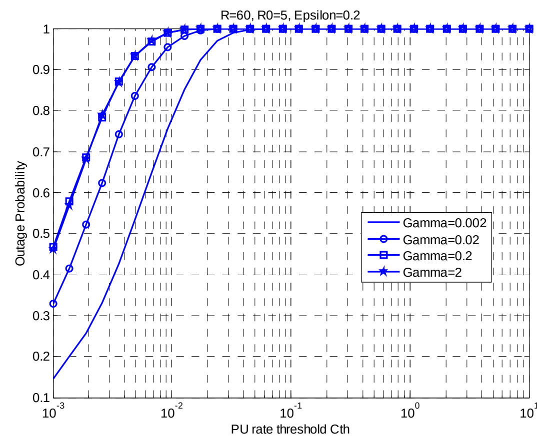

Fig. 12. Capacity-outage probability of the PU for different beacon detection thresholds γ.

V. CONCLUSIONS

In this paper, the interference and its effect on the performance of a network with beacon consisting multiple SUs and single PU is studied mathematically. Since the interference analysis in such networks will be effective in designing the algorithms, tight closed-form upper bounds for the mean and variance of interference are provided in this paper. Then, in order to analyze the performance of the PU in the presence of the SUs interference, a closed-form equation for the outage probability is computed in terms of the mean and variance of the interference power. Furthermore, numerical results are provided to verify the derived closed-form equations.

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