Automatic Voltage Regulation

Chenguang Xi; Dini Hu
Advisor: Prof Usman Khan

Outline

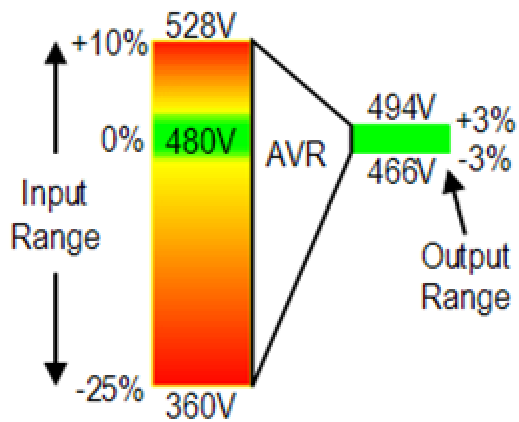
- Regulator
  - Voltage Regulator
  - Need for AVR
  - Category of Voltage Regulator
- Regulation on Linear Control System
  - Problem statement
  - Feedback Control
  - More Complicated system

Outline

- Regulator
    - Voltage Regulator
    - Need for AVR
    - Category of Voltage Regulator
- Regulation on Linear Control System
    - Problem statement
    - Feedback Control
    - More Complicated system

Voltage Regulation

- An automatic voltage regulator, AVR for short, is a device that is designed to automatically control, adjust or maintain a constant voltage level.



- tremendous diversity in the size and type of device that could qualify to be called an AVR.

Need for Voltage Regulator

- the ultimate reason for using voltage regulation is financial – to avoid the costs associated with equipment damage and downtime caused by poor voltage levels.

- Utility Voltage Levels
- Voltage Drop in a Facility
- Sensitivity to Voltage Levels and Voltage Fluctuation

Utility Level

- 120V to a small or large degree
- Factors contributing to voltage level fluctuation such as
  1. location on the local distribution line;
  2. proximity to large electricity consumers;
  3. proximity to utility voltage regulating equipment;
  4. seasonal variations in overall system voltage levels;
  5. load factor on local transmission and distribution system
- Try to Maintain the voltage level within +/-5% (480V+/-5%)
- Sometimes 6% high or 13% low (509V(480+6%) to 420V(480-13%))

Need for Voltage Regulation

- the ultimate reason for using voltage regulation is financial – to avoid the costs associated with equipment damage and downtime caused by poor voltage levels.

- Utility Voltage Levels
- Voltage Drop in a Facility
- Sensitivity to Voltage Levels and Voltage Fluctuation

Voltage Drop in a Facility

- a voltage drop of 3 to 5% to the end user
- due to wiring impedance within a building will always drive voltage levels lower

- Eg, if the incoming utility voltage is 5% low, the voltage at the point of usage might be 8 to 10% (5%+3% to 5%+5%) below nominal due to the voltage drop within a building.

Need for Voltage Regulation

- the ultimate reason for using voltage regulation is financial – to avoid the costs associated with equipment damage and downtime caused by poor voltage levels.

- Utility Voltage Levels
- Voltage Drop in a Facility
- Sensitivity to Voltage Levels and Voltage Fluctuation

Sensitivity to Voltage Levels and Voltage Fluctuation

- Every piece of electrical equipment will operate within a range of voltage levels, however not necessarily with optimal performance. When the voltage level falls outside of its operational range, a device may be unable to start or operate, it may malfunction or the device may be damaged.

- utility voltage levels are very dynamic and will most assuredly change over time and there is no advance warning about when, how much or in which direction they will change, due to the little control over the amount of electricity demanded by any customer at any given time by the utility . Thus, increasing use of relatively sensitive electronics in nearly all facets of business and industry and the growing need for voltage regulation becomes clearer.

Category of Voltage Regulator

- Feed-forward design or negative feedback control loops
- Mechanical voltage regulator or electronic voltage regulator
- used to regulate AC or DC voltages
- Active regulators or shunt regulators
  - Linear regulator
  - Switching regulators

Linear regulator (LR) Switching regulators (SR)

LR: linear regulators are based on devices that operate in their linear region

when low output noise (and low RFI radiated noise) is required;
when a fast response to input and output disturbances is required;
is cheaper and occupies less PCB space at lower levels of power.

SR: a switching regulator is based on a device forced to act as an on/off switch

when power efficiency is critical
when the only power supply is a DC voltage and a higher output voltage is required.
is cheaper (for example, the cost of removing heat generated is less) at high levels of power.

Outline

- Regulator
    - Voltage Regulator
    - Need for AVR
    - Category of Voltage Regulator
- Regulation on Linear Control System
    - Problem statement
    - Feedback Control
    - More Complicated system

Problem Statement

Consider the system

$$\begin{aligned}\dot{x} &= Ax + Bu\\ y &= Cx\end{aligned}$$

where $x \in R^n$, $u \in R^m$, $y \in R^m$. We want to design feedback control to

- stabilize the system, and
- regulate the output y to a set point $r \in R^m$; i.e.,

$$\lim_{t \to \infty} y(t) = r$$

Feedback Control

State feedback:

$$u = Fx + v$$

Closed-loop system:

$$\dot{x} = (A + BF)x + Bv, \quad y = Cx$$

Transfer function from v to y:

$$C[sI - (A + BF)]^{-1}B$$

Take $v = Nr$. Transfer function from r to y:

$$H(s) = C[sI - (A + BF)]^{-1}BN$$

Feedback Control

Final Value Theorem:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\hat{y}(s) = \lim_{s\to 0} sH(s)\frac{r}{s} = H(0)r$$

$$y_{ss} \overset{\text{def}}{=} \lim_{t\to\infty} y(t) = -C(A+BF)^{-1}BNr$$

$(A+BK)$ is nonsingular because all its eigenvalues has negative real parts

Choose N such that

$$-C(A+BF)^{-1}BN = I$$

$$N = -[C(A+BF)^{-1}B]^{-1}$$

Feedback Control (state feedback control)

Design $\boldsymbol{F}$ such that $\boldsymbol{R}_e[\boldsymbol{\lambda}(\boldsymbol{A}+\boldsymbol{B}\boldsymbol{F})] < 0$

$$\text{Take } \boldsymbol{N} = -[\boldsymbol{C}(\boldsymbol{A}+\boldsymbol{B}\boldsymbol{F})^{-1}\boldsymbol{B}]^{-1}$$

$$\boldsymbol{u} = \boldsymbol{F}\boldsymbol{x} + \boldsymbol{N}\boldsymbol{r}$$

Feedback Control (output feedback control)

design K such that

$$R_e[\lambda(A - KC)] < 0$$

$$\begin{aligned} u &= F\hat{x} + Nr \\ \dot{\hat{x}} &= A\hat{x} + Bu + K(y - C\hat{x}) \end{aligned}$$

The closed-loop transfer function from r to y is still

$$H(s) = C[sI - (A + BF)]^{-1}BN$$

Hence, $y_{ss} = r$

Feedback Control Eg

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = x_1 - x_2 + u, \quad y = x_1$$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} -2 & -1 \end{bmatrix} \Rightarrow A + BF = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$

$$\lambda(A + BF) = -1, \; -1$$

$$N = -C(A{+}BF)^{-1}B = -\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1$$

State Feedback:

$$u = -2x_1 - x_2 + r$$

$$H(s) = \frac{1}{(s+1)^2}$$

Feedback Control Eg

Output Feedback:

$$K = \begin{bmatrix} 9 \\ 17 \end{bmatrix} \Rightarrow A - KC = \begin{bmatrix} -9 & 1 \\ -16 & -1 \end{bmatrix}$$

$$\lambda(A - KC) = -5,\ -5$$

$$u = -2\hat{x}_1 - \hat{x}_2 + r$$

$$\dot{\hat{x}} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \hat{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 9 \\ 17 \end{bmatrix} (y - \hat{x}_1)$$

Problem

The problem with feedforward control is that the calculation of the gain N is dependent on the matrices (A, B, C). Any error in the model would result in a matrix N which does not satisfy the condition

$$-C(A + BF)^{-1}BN = I$$

Hence, $y_{ss} \neq r$

More Complicated system

$$\begin{aligned}\dot{x} &= Ax + Bu + \Gamma w \\ y &= Cx + Ew\end{aligned}$$

where $x \in R^n$, $u \in R^m$, $y \in R^m$, and w is a constant disturbance. We want to design feedback control to

- stabilize the system, and
- regulate the output y to a set point $r \in R^m$; i.e.,

$$\lim_{t \to \infty} y(t) = r$$

More Complicated system

Augment the system with an integrator that is driven by the regulation error:

$$\dot{z} = r - y$$

The augmented system is described by

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} \Gamma \\ E \end{bmatrix} w + \begin{bmatrix} 0 \\ I_m \end{bmatrix} r$$

$$\text{Set} \quad \mathcal{X} = \begin{bmatrix} x \\ z \end{bmatrix}, \quad \mathcal{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

$$\dot{\mathcal{X}} = \mathcal{A}\mathcal{X} + \mathcal{B}u + \begin{bmatrix} \Gamma \\ E \end{bmatrix} w + \begin{bmatrix} 0 \\ I_m \end{bmatrix} r$$

More Complicated system

Design state feedback control

$$u = F\mathcal{X} = \begin{bmatrix} F_1 & F_2 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} = F_1 x + F_2 z$$

such that

$$R_e[\lambda(\mathcal{A} + \mathcal{B}F)] < 0$$

More Complicated system

$$\begin{aligned}\dot{x} &= Ax + B(F_1 x + F_2 z) + \Gamma w \\ &= (A + BF_1)x + BF_2 z + \Gamma w \\ \dot{z} &= r - y \;=\; -Cx - Ew + r\end{aligned}$$

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} (A+BF_1) & BF_2 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} \Gamma \\ -E \end{bmatrix} w + \begin{bmatrix} 0 \\ I_m \end{bmatrix} r$$

Equilibrium point:

$$\begin{aligned}0 &= (A + BF_1)\bar{x} + BF_2 \bar{z} + \Gamma w \\ 0 &= r - \bar{y} \;=\; -C\bar{x} - Ew + r\end{aligned}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (A+BF_1) & BF_2 \\ -C & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{z} \end{bmatrix} + \begin{bmatrix} \Gamma \\ -E \end{bmatrix} w + \begin{bmatrix} 0 \\ I_m \end{bmatrix} r$$

More Complicated system

$$x_\delta = x - \bar{x}, \quad z_\delta = z - \bar{z}$$

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} (A + BF_1) & BF_2 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} \Gamma \\ -E \end{bmatrix} w + \begin{bmatrix} 0 \\ I_m \end{bmatrix} r$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (A + BF_1) & BF_2 \\ -C & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{z} \end{bmatrix} + \begin{bmatrix} \Gamma \\ -E \end{bmatrix} w + \begin{bmatrix} 0 \\ I_m \end{bmatrix} r$$

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} (A + BF_1) & BF_2 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x_\delta \\ z_\delta \end{bmatrix}$$

$$\lim_{t \to \infty} x_\delta(t) = 0, \quad \lim_{t \to \infty} z_\delta(t) = 0$$

More Complicated system

Once reach the equilibrium point, The system will stay in that state; Regulation achieved