cs242

# TYPES

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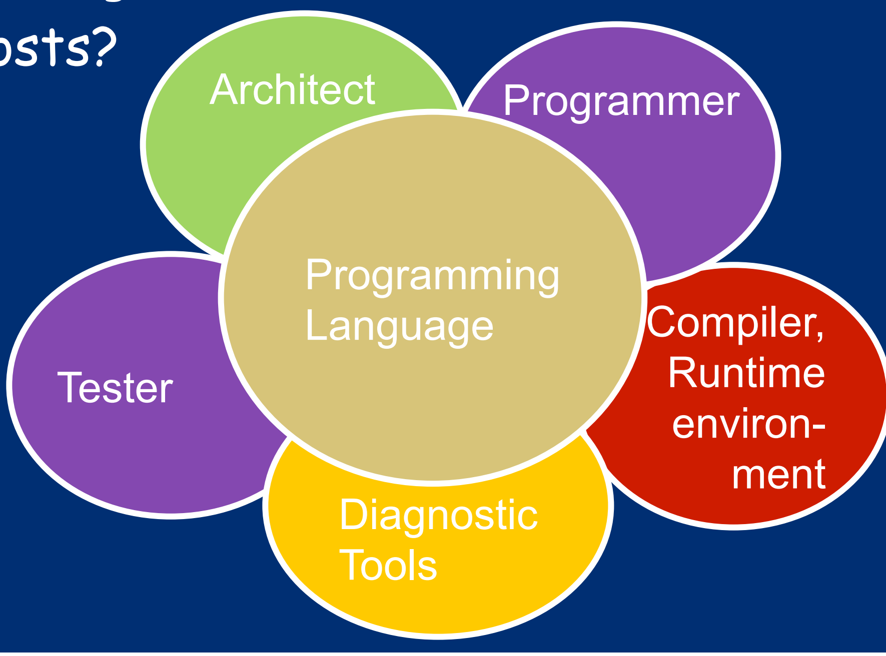
Reading: "Concepts in Programming Languages", Chapter 6

# Outline

- General discussion of types
  - What is a type?
  - Compile-time vs run-time checking
  - Conservative program analysis
- Type inference
  - Will study algorithm and examples
  - Good example of static analysis algorithm
- Polymorphism
  - Uniform vs non-uniform impl of polymorphism
  - Polymorphism vs overloading

# Language Goals and Trade-offs

- Thoughts to keep in mind
  - What features are convenient for programmer?
  - What other features do they prevent?
  - What are design tradeoffs?
    - Easy to write but harder to read?
    - Easy to write but poorer error messages?
  - What are the implementation costs?



# What is a type?

A type is a collection of computable values that share some structural property.

- Examples

```
Integer
String
Int → Bool
(Int → Int) → Bool
```

- Non-examples

```
{3, True, \x->x}
Even integers
{f:Int → Int | x>3 =>
        f(x) > x *(x+1)}
```

Distinction between sets of values that are types and sets that are not types is *language dependent*.

# Uses for Types

- Program organization and documentation
  - Separate types for separate concepts
    - Represent concepts from problem domain
  - Indicate intended use of declared identifiers
    - Types can be checked, unlike program comments
- Identify and prevent errors
  - Compile-time or run-time checking can prevent meaningless computations such as 3 + true – "Bill"
- Support optimization
  - Example: short integers require fewer bits
  - Access record component by known offset

# What is a type error?

- A type error is something the compiler/interpreter reports when I make a mistake in my syntactically correct program?
- Languages represent values as sequences of bits. A type error occurs when a bit sequence written for one type is used as a bit sequence for another type?
- A type error occurs when a value is used in a way inconsistent with its definition.

# Type errors are language dependent

- Array out of bounds access
  - C/C++: runtime errors.
  - Haskell/Java: dynamic type errors.
- Null pointer dereference
  - C/C++: null pointer dereferences are run-time errors.
  - In Haskell/ML, pointers are hidden inside datatypes. Null pointer dereferences correspond to incorrect use of these datatypes. Such errors are static type errors.

# Compile-time vs Run-time Checking

- JavaScript and Lisp use run-time type checking

  f(x) *Make sure f is a function* ***before*** *calling f.*

```
js> var f= 3;
js> f(2);
typein:3: TypeError: f is not a function
js>
```

- ML and Haskell use compile-time type checking

  f(x) *Must have f :: A → B and x :: A*

- Basic tradeoff
  - Both kinds of checking prevent type errors.
  - Run-time checking slows down execution.
  - Compile-time checking restricts program flexibility.

    JavaScript array: elements can have different types

    Haskell list: all elements must have same type
  - Which gives better programmer diagnostics?

# Expressiveness

- In JavaScript, we can write a function like

```
function f(x) { return x < 10 ? x : x(); }
```

Some uses will produce type error, some will not.

- Static typing always conservative

```
if  (big-hairy-boolean-expression)
        then  f(5);
        else  f(15);
```

Cannot decide at compile time if run-time error will occur!

# Relative Type-Safety of Languages

- Not safe: BCPL family, including C and C++
  - Casts, pointer arithmetic
- Almost safe: Algol family, Pascal, Ada.
  - Dangling pointers.
    - Allocate a pointer p to an integer, deallocate the memory referenced by p, then later use the value pointed to by p.
    - No language with explicit deallocation of memory is fully type-safe.
- Safe: Lisp, Smalltalk, ML, Haskell, Java, JavaScript
  - Dynamically typed: Lisp, Smalltalk, JavaScript
  - Statically typed: ML, Haskell, Java

# Type Checking vs. Type Inference

- Standard type checking:

```
int f(int x) { return x+1; };
int g(int y) { return f(y+1)*2; };
```

  - Examine body of each function. Use declared types to *check* agreement.

- Type inference:

```
int f(int x) { return x+1; };
int g(int y) { return f(y+1)*2; };
```

  - Examine code *without* type information. *Infer* the most general types that could have been declared.

ML and Haskell are *designed* to make type inference feasible.

# Why study type inference?

- Types and type checking
  - Improved steadily since Algol 60
    - Eliminated sources of unsoundness.
    - Become substantially more expressive.
  - Important for modularity, reliability and compilation
- Type inference
  - Reduces syntactic overhead of expressive types.
  - Guaranteed to produce *most general type*.
  - Widely regarded as important language innovation.
  - Illustrative example of a flow-insensitive static analysis algorithm.

# History

- Original type inference algorithm was invented by Haskell Curry and Robert Feys for the simply typed lambda calculus in 1958.
- In 1969, Hindley extended the algorithm to a richer language and proved it always produced the *most general type*.
- In 1978, Milner independently developed equivalent algorithm, called algorithm W, during his work designing ML.
- In 1982, Damas proved the algorithm was *complete*.
- Already used in many languages: ML, Ada, Haskell, C# 3.0, F#, Visual Basic .Net 9.0, and soon in: Fortress, Perl 6, C++0x

# uHaskell

- Subset of Haskell to explain type inference.
  - Haskell and ML both have overloading, which slightly complicates type inference. We won't worry about type inference with overloading.

```
<decl> ::= [<name> <pat>  = <exp>]
<pat>  ::=Id  | (<pat>, <pat>) | <pat> : <pat> | []
<exp>  ::= Int | Bool | [] | Id | (<exp>)
         | <exp> <op> <exp>
         | <exp> <exp>  | (<exp>, <exp>)
         | if <exp> then <exp> else <exp>
```

# Type Inference: Basic Idea

- Example

```
f x = 2 + x
> f :: Int -> Int
```

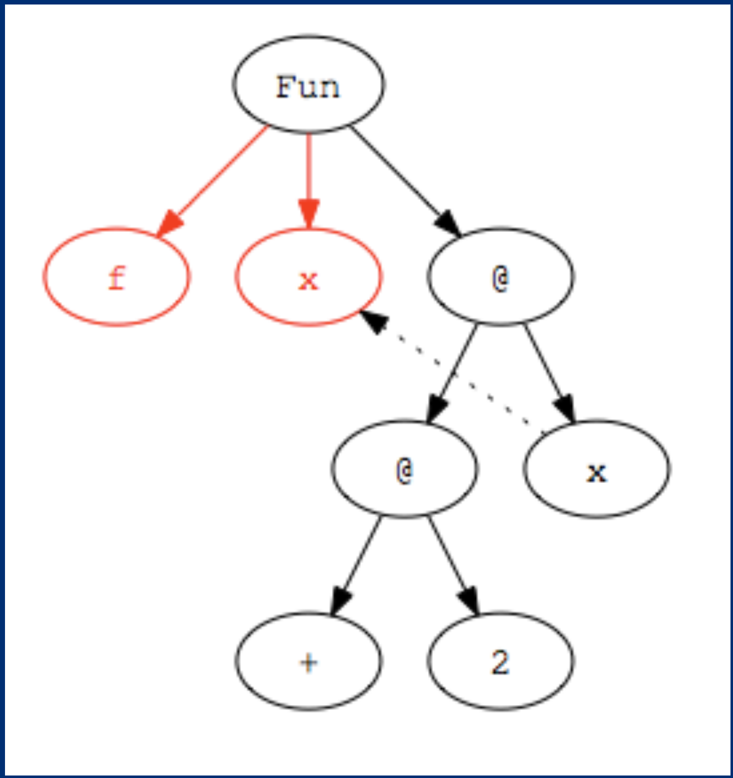
- What is the type of `f`?
  - `+` has type: `Int` → `Int` → `Int`
  - `2` has type: `Int`
  - Since we are applying `+` to `x` we need `x :: Int`
  - Therefore `f x = 2 + x` has type `Int` → `Int`

Overloaded functions introduce more cases to consider.

# Step 1: Parse Program

- Parse program text to construct parse tree.

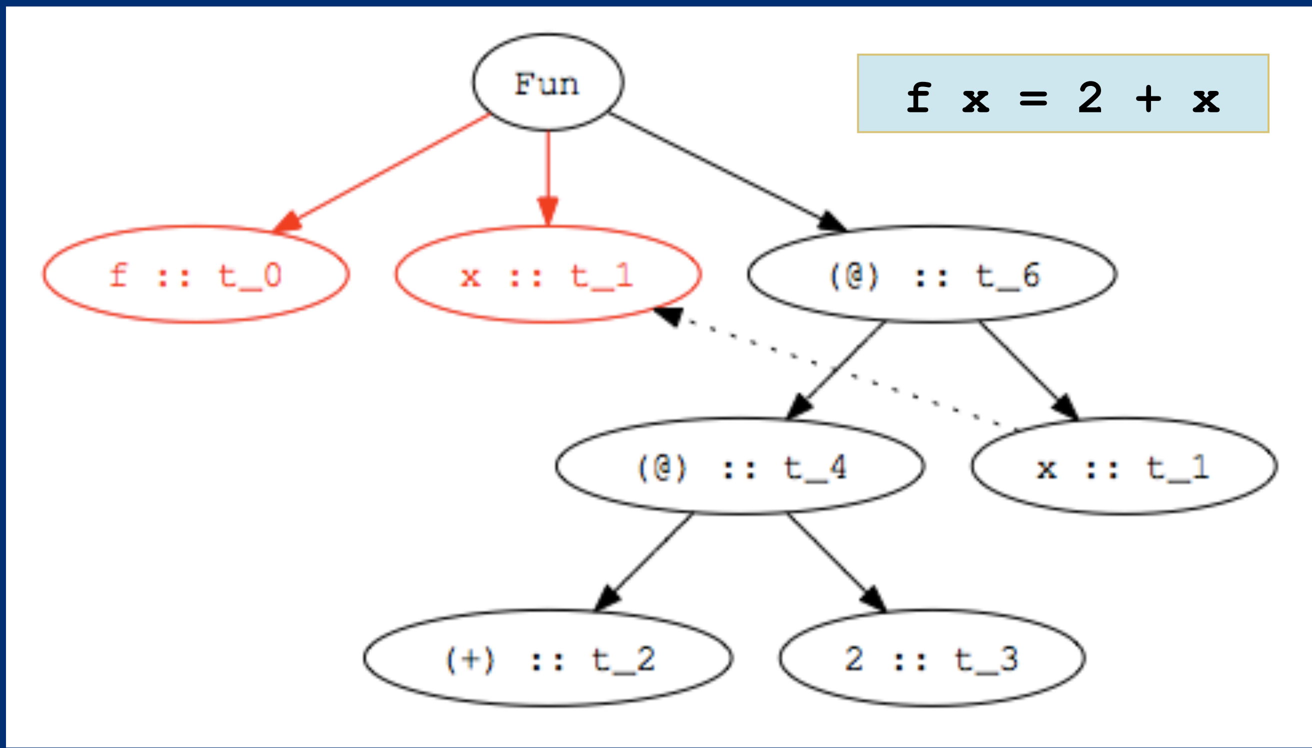
```
f x = 2 + x
```




Infix operators are converted to normal function application during parsing:

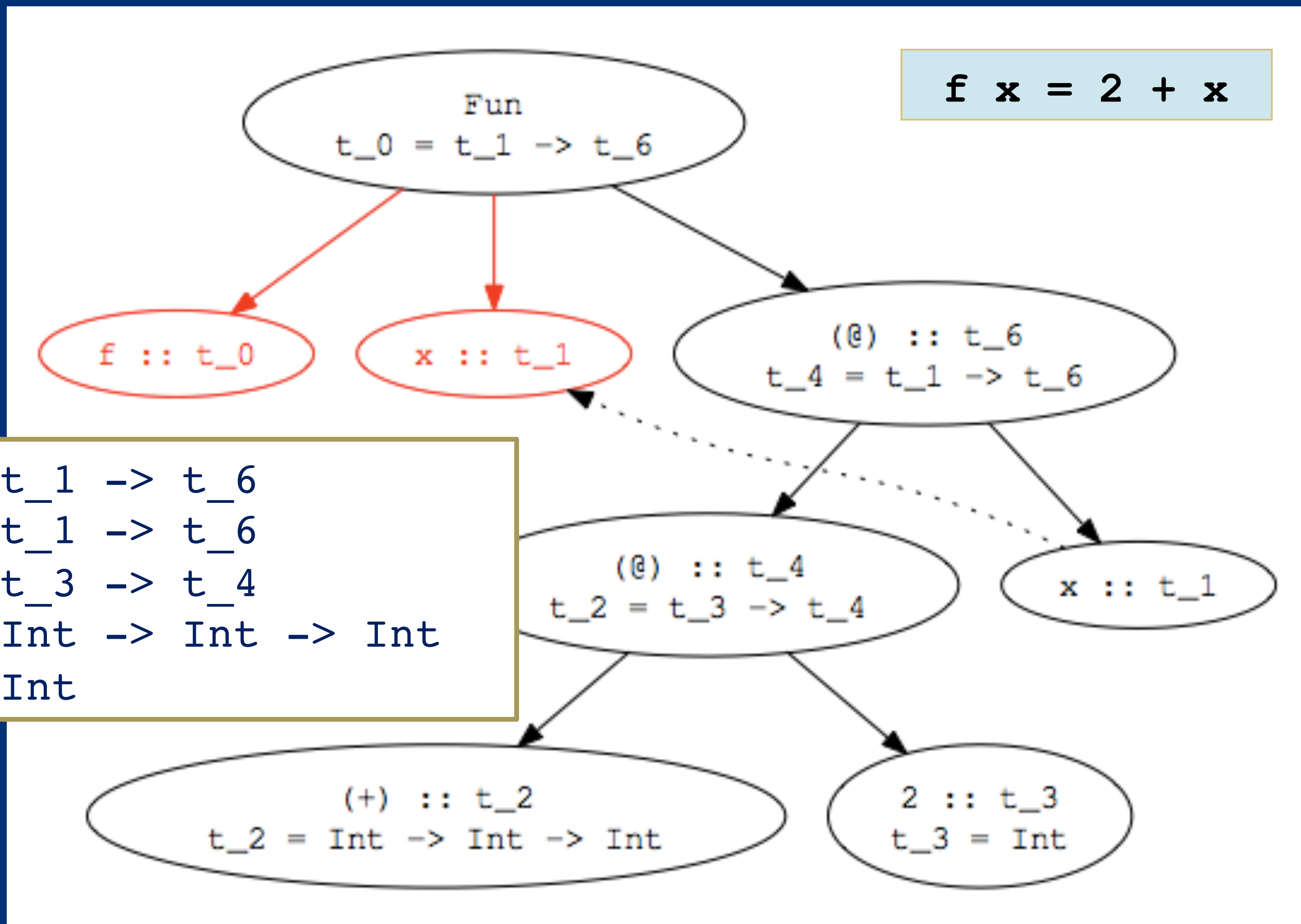
2 + x --> (+) 2 x

# Step 2:
# Assign type variables to nodes




Variables are given same type as binding occurrence.

# Step 3: Add Constraints




```
t_0 = t_1 -> t_6
t_4 = t_1 -> t_6
t_2 = t_3 -> t_4
t_2 = Int -> Int -> Int
t_3 = Int
```

# Step 4: Solve Constraints

```
t_0 = t_1 -> t_6
t_4 = t_1 -> t_6
t_2 = t_3 -> t_4
t_2 = Int -> Int -> Int
t_3 = Int
```

```
t_3 -> t_4 = Int -> (Int -> Int)
```

```
t_3 = Int
t_4 = Int -> Int
```

```
t_0 = t_1 -> t_6
t_4 = t_1 -> t_6
t_4 = Int -> Int
t_2 = Int -> Int -> Int
t_3 = Int
```

```
t_1 -> t_6 = Int -> Int
```

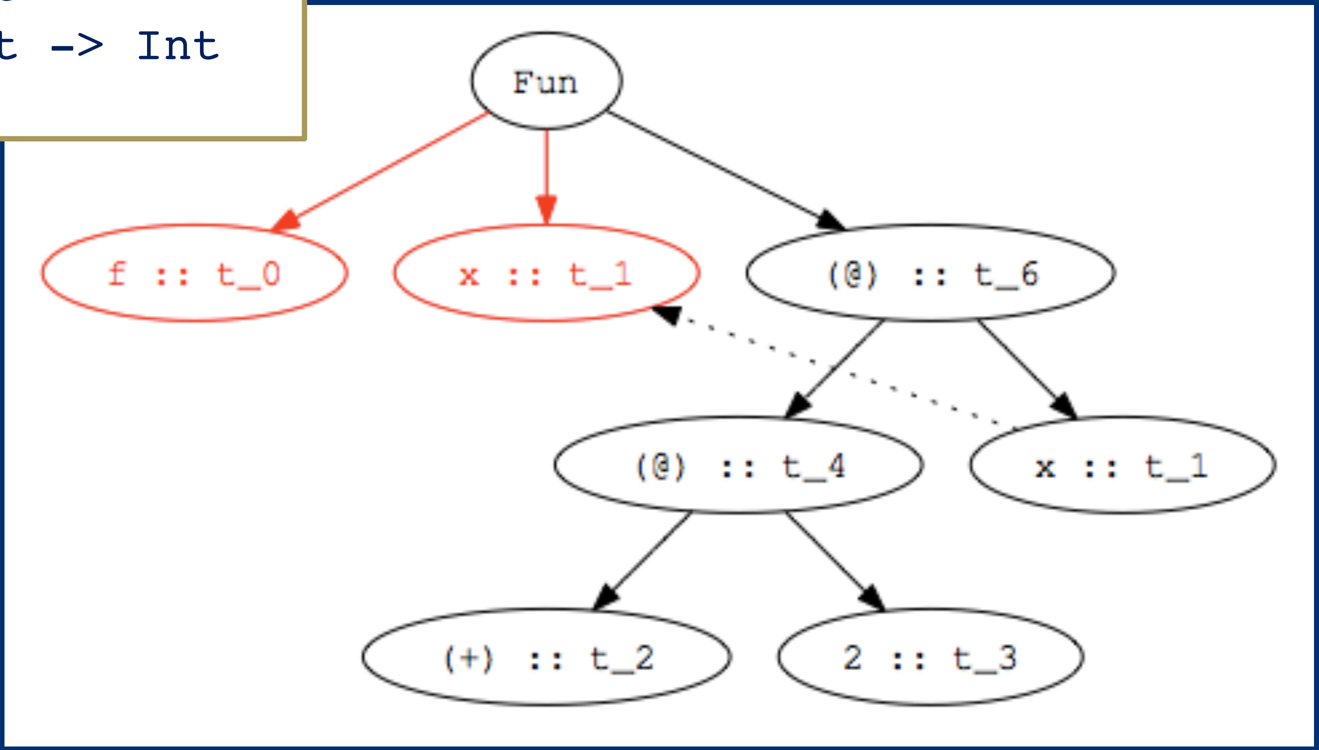
```
t_1 = Int
t_6 = Int
```

```
t_0 = Int -> Int
t_1 = Int
t_6 = Int
t_4 = Int -> Int
t_2 = Int -> Int -> Int
t_3 = Int
```

# Step 5:
# Determine type of declaration

```
t_0 = Int -> Int
t_1 = Int
t_6 = Int -> Int
t_4 = Int -> Int
t_2 = Int -> Int -> Int
t_3 = Int
```

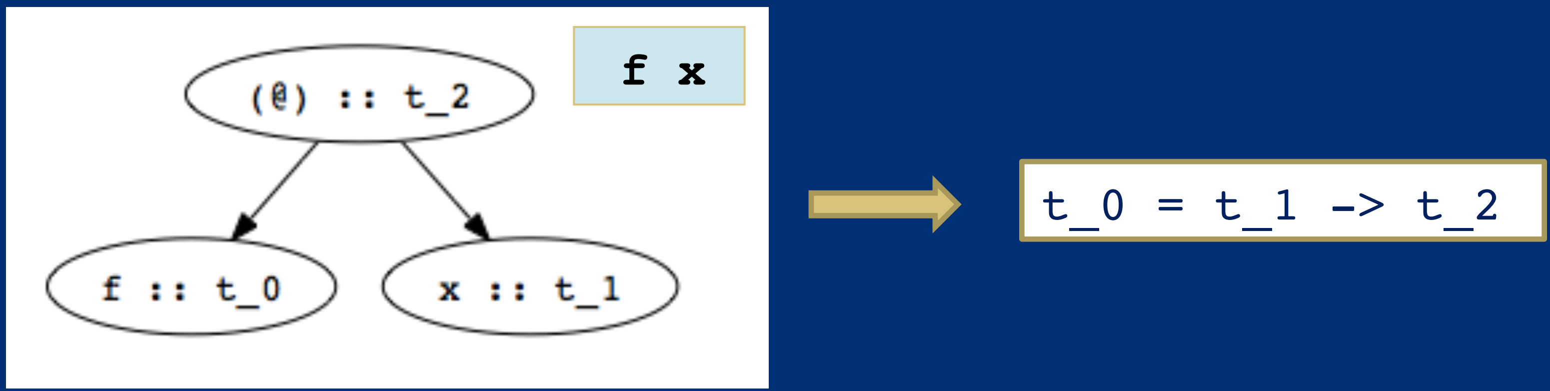
```
f x = 2 + x
> f :: Int -> Int
```



# Type Inference Algorithm

- Parse program to build parse tree
- Assign type variables to nodes in tree
- Generate constraints:
  - From environment: constants (`2`), built-in operators (`+`), known functions (`tail`).
  - From form of parse tree: e.g., application and abstraction nodes.
- Solve constraints using unification.
- Determine types of top-level declarations.

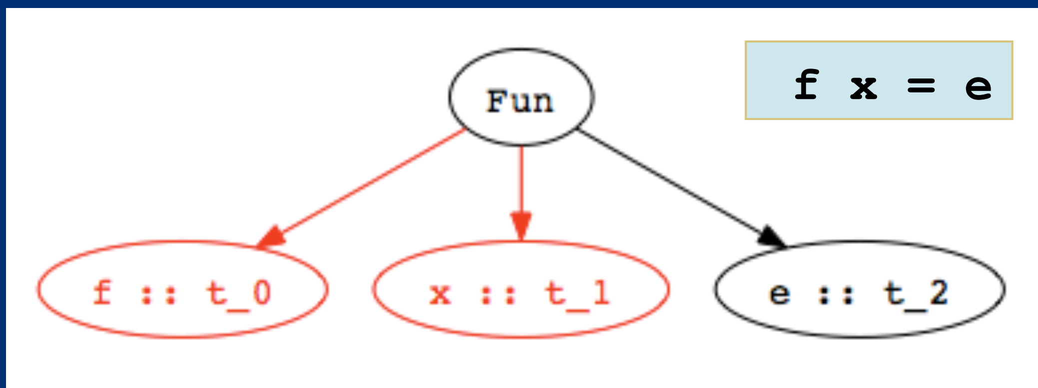
# Constraints from Application Nodes




Apply function **f** to argument **x**:

- Because **f** is being applied, its type (**t_0** in figure) must be a function type: domain → range.
- Domain of **f** must be type of argument **x** (**t_1** in figure).
- Range of **f** must be result type of expression (**t_2** in figure).
- Hence we get the constraint: **t_0 = t_1 -> t_2**

# Constraints from Abstractions




`t_0 = t_1 -> t_2`

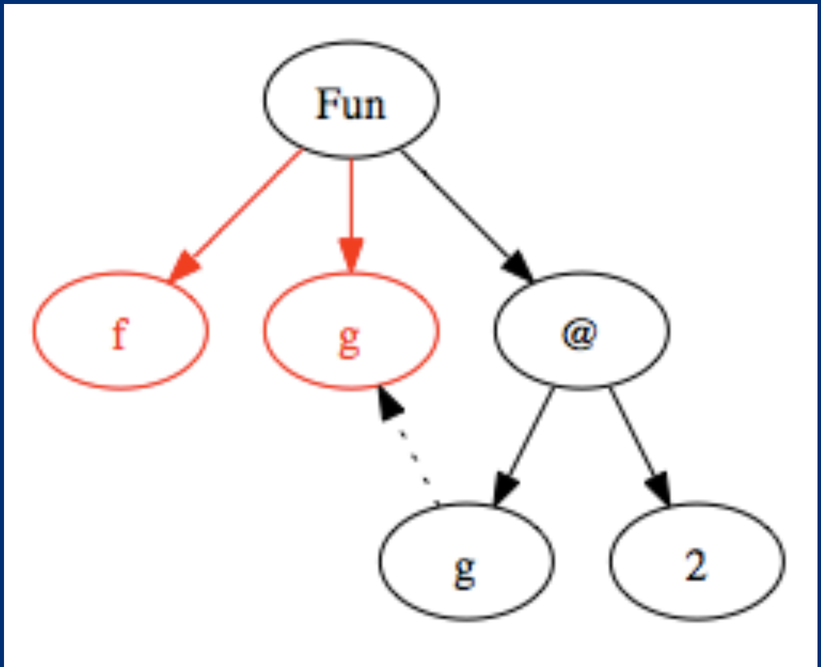
- Function declaration:
  - Type of function `f` (`t_0` in figure) must be a function type: domain → range.
  - Domain is type of abstracted variable `x` (`t_1` in figure).
  - Range is type of function body `e` (`t_2` in figure).
  - Hence we get the constraint: `t_0 = t_1 -> t_2`.

# Inferring Polymorphic Types

- Example:

```
f g = g 2
> f :: (Int -> t_4) -> t_4
```

- Step 1:
  Build Parse Tree

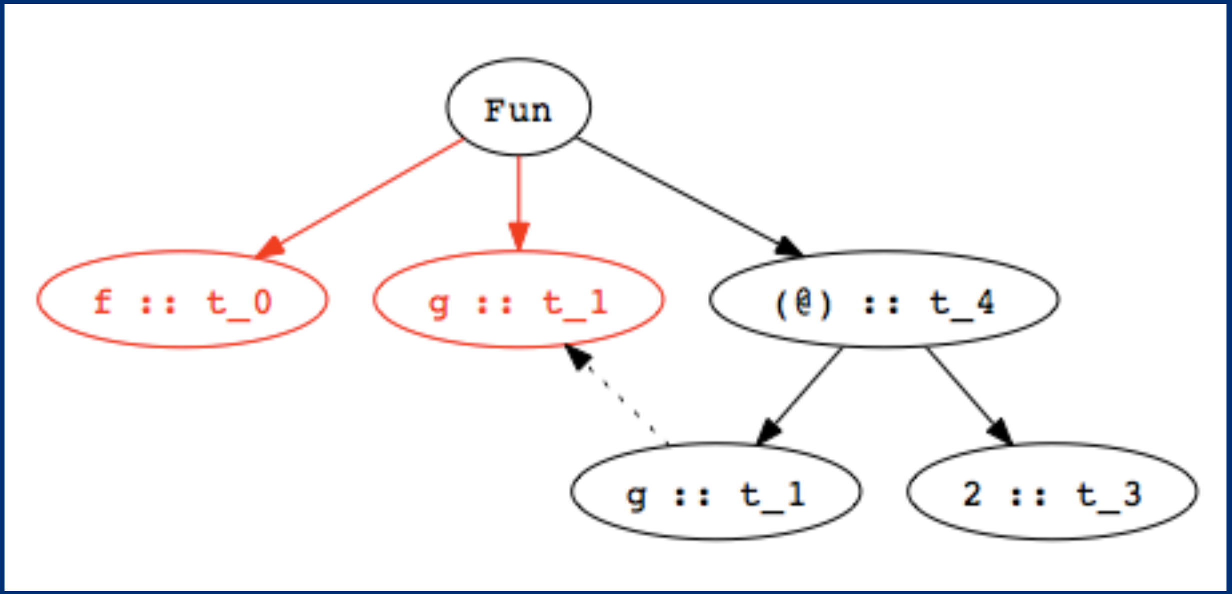


# Inferring Polymorphic Types

- Example:

```
f g = g 2
> f :: (Int -> t_4) -> t_4
```

- Step 2:
  Assign type variables



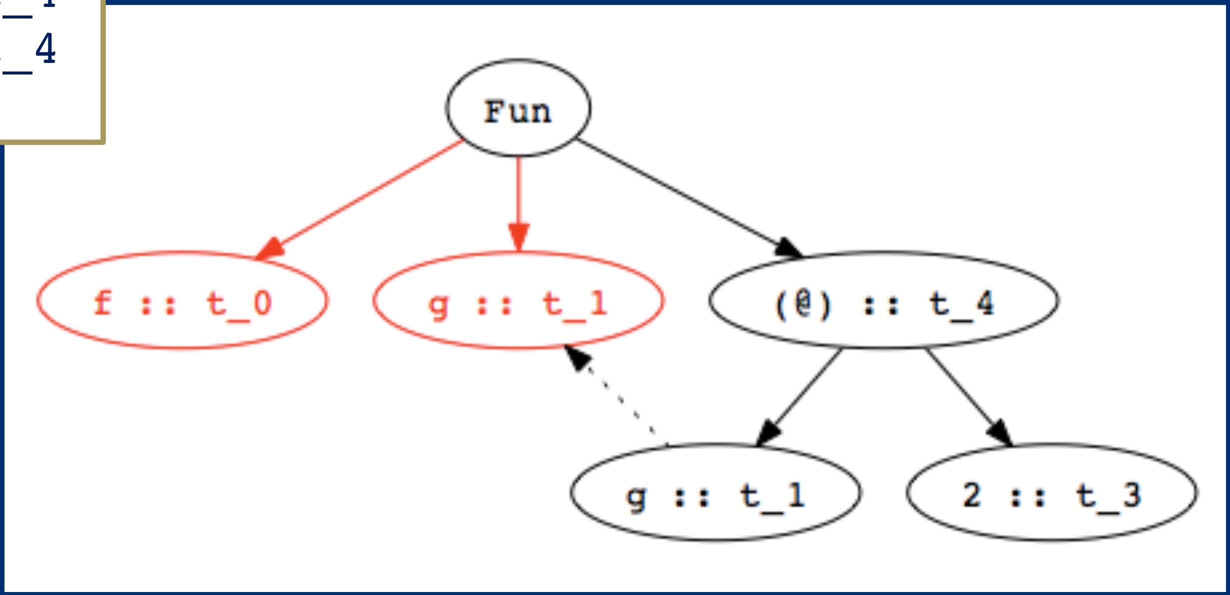
# Inferring Polymorphic Types

- Example:

```
f g = g 2
> f :: (Int -> t_4) -> t_4
```

- Step 3:
  Generate constraints

```
t_0 = t_1 -> t_4
t_1 = t_3 -> t_4
t_3 = Int
```



# Inferring Polymorphic Types

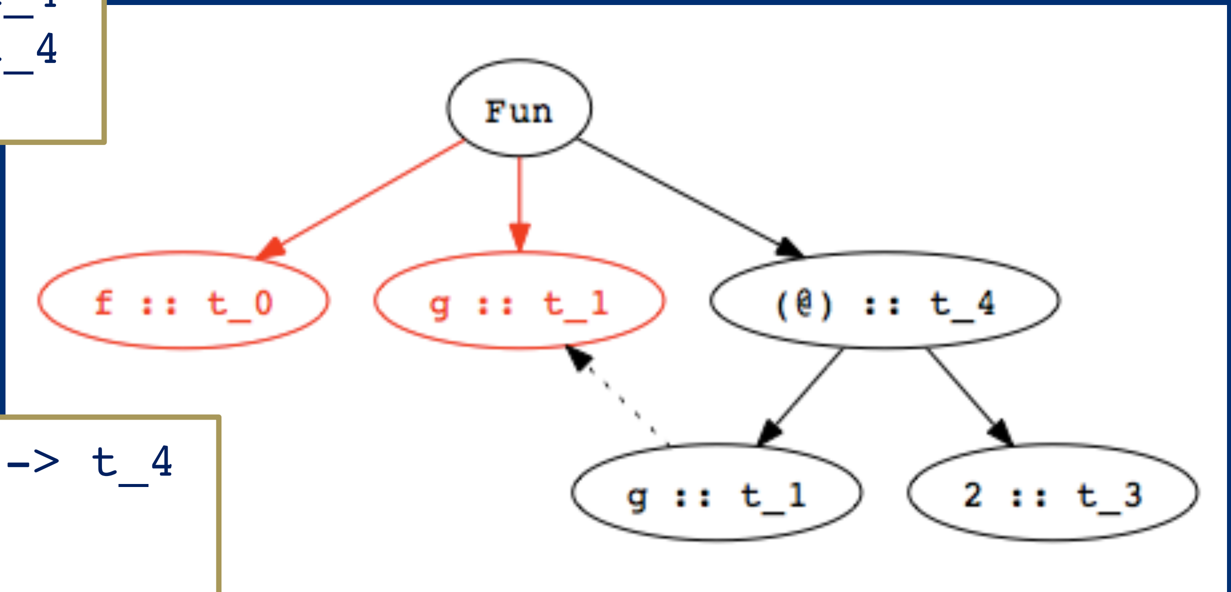
- Example:

```
f g = g 2
> f :: (Int -> t_4) -> t_4
```

- Step 4:
  Solve constraints

```
t_0 = t_1 -> t_4
t_1 = t_3 -> t_4
t_3 = Int
```

```
t_0 = (Int -> t_4) -> t_4
t_1 =  Int -> t_4
t_3 =  Int
```



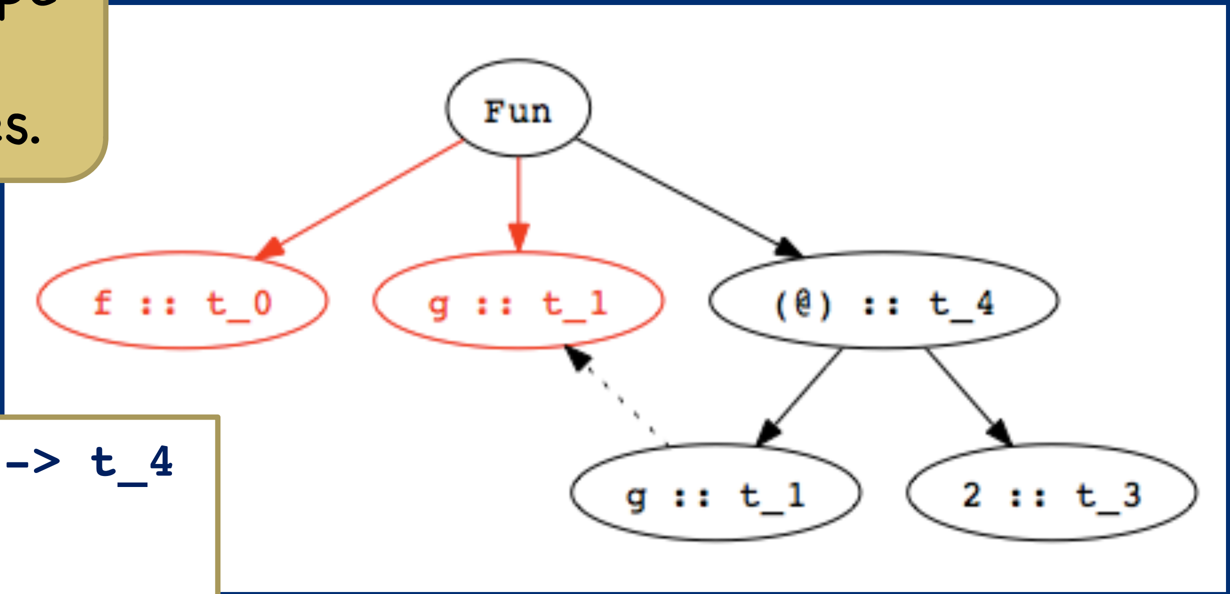
# Inferring Polymorphic Types

- Example:

```
f g = g 2
> f :: (Int -> t_4) -> t_4
```

- Step 5:
  Determine type of top-level declaration

Unconstrained type variables become polymorphic types.




```
t_0 = (Int -> t_4) -> t_4
t_1 =  Int -> t_4
t_3 =  Int
```

# Using Polymorphic Functions

- Function:

```
f g = g 2
> f :: (Int -> t_4) -> t_4
```

- Possible applications:

```
add x = 2 + x
> add :: Int -> Int

f add
> 4 :: Int
```

```
isEven x = mod (x, 2) == 0
> isEven:: Int -> Bool

f isEven
> True :: Int
```

# Recognizing Type Errors

- Function

```
f g = g 2
> f :: (Int -> t) -> t
```

- Incorrect use

```
not x = if x then True else False
> not :: Bool -> Bool

f not
> Error: operator and operand don't agree
  operator domain: Int -> a
  operand:         Bool -> Bool
```

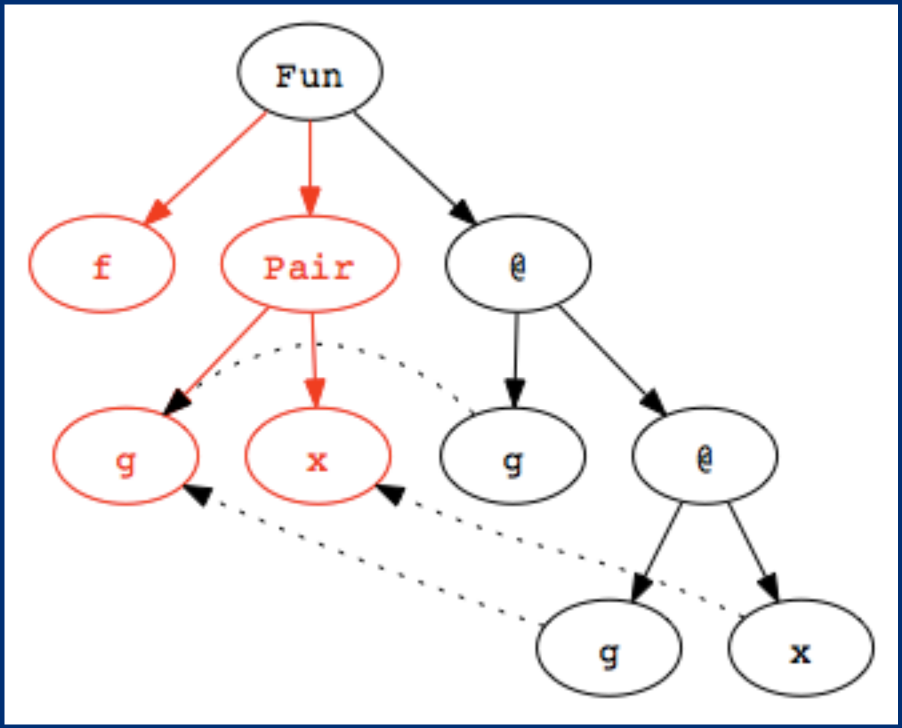
- Type error:
  cannot unify **Bool → Bool** and **Int → t**

# Another Example

- Example:

```
f (g,x) = g (g x)
> f :: (t_8 -> t_8, t_8) -> t_8
```

- Step 1:
  Build Parse Tree

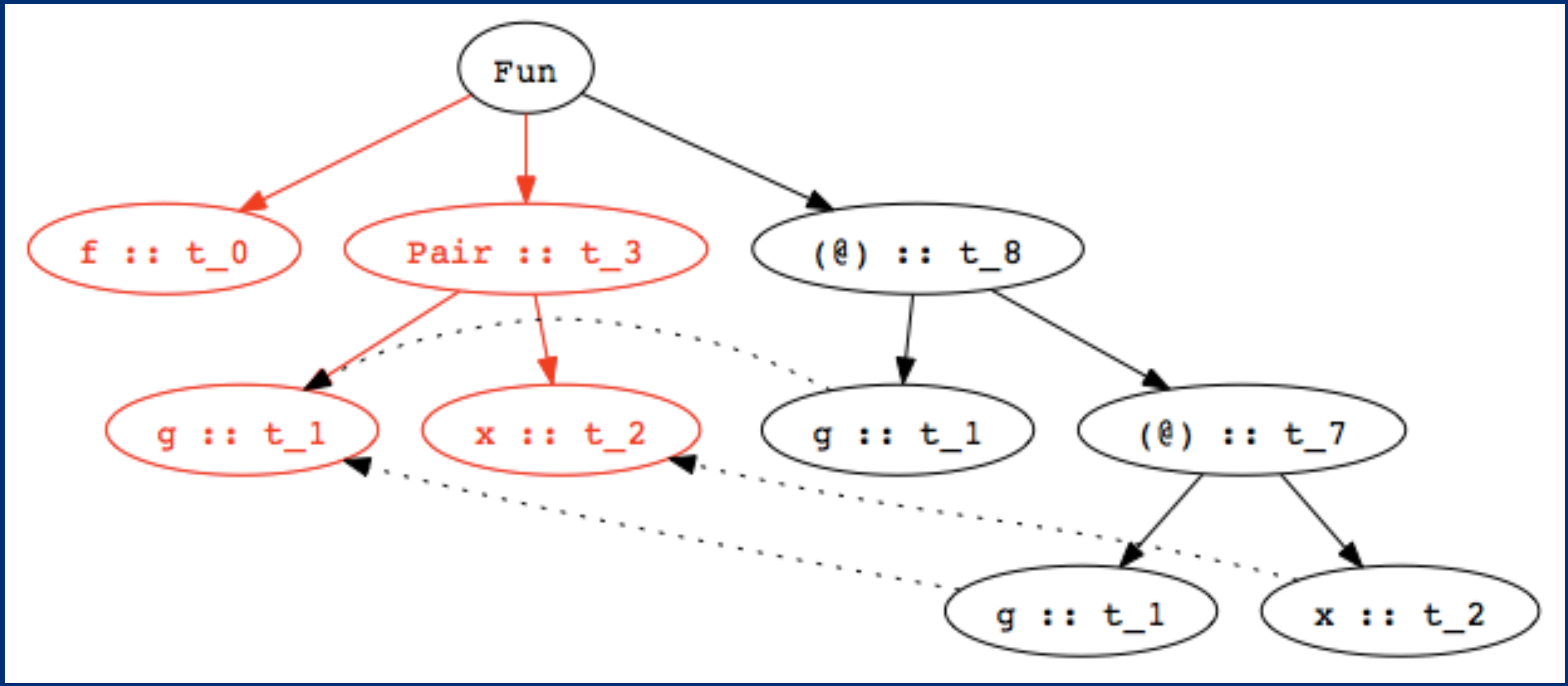


# Another Example

- Example:

```
f (g,x) = g (g x)
> f :: (t_8 -> t_8, t_8) -> t_8
```

- Step 2: *Assign type variables*



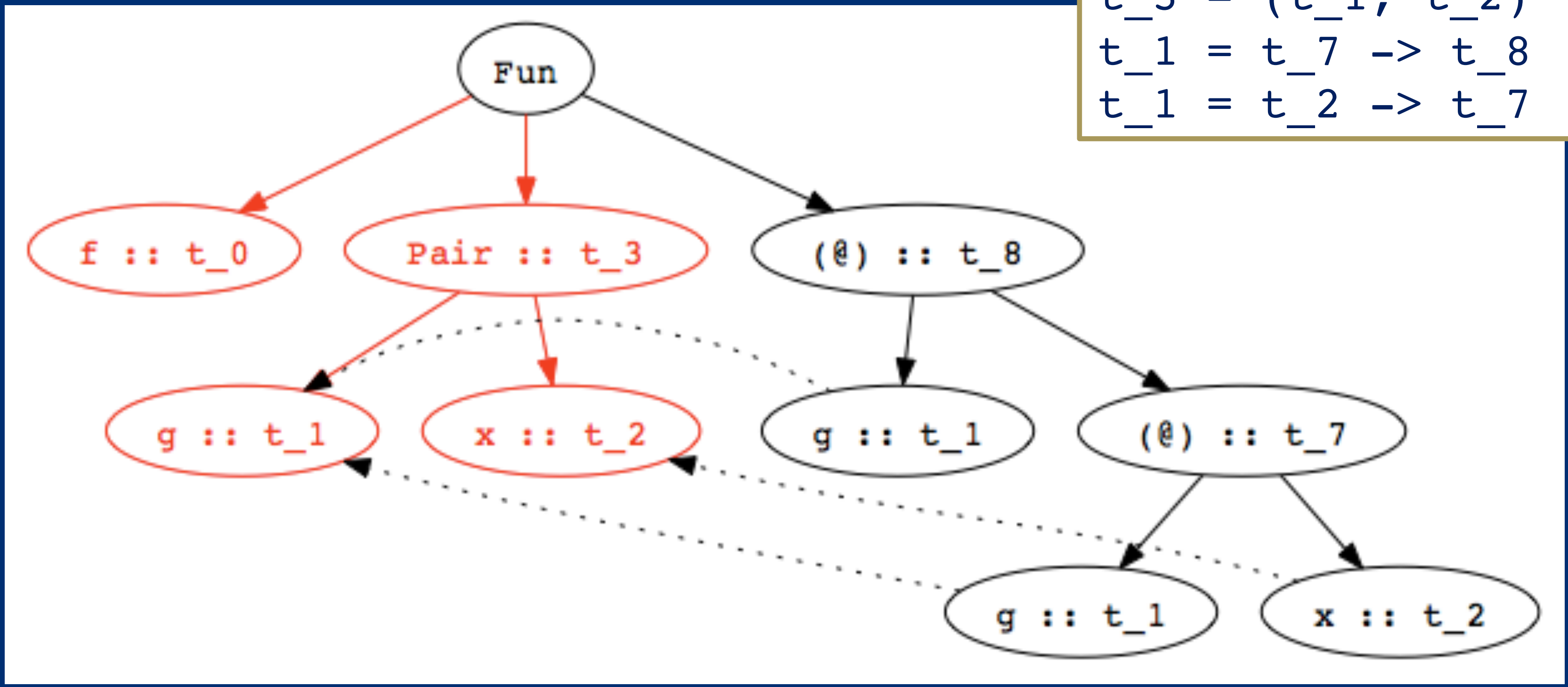
# Another Example

- Example:

```
f (g,x) = g (g x)
> f :: (t_8 -> t_8, t_8) -> t_8
```

- Step 3: Generate constraints

```
t_0 = t_3 -> t_8
t_3 = (t_1, t_2)
t_1 = t_7 -> t_8
t_1 = t_2 -> t_7
```



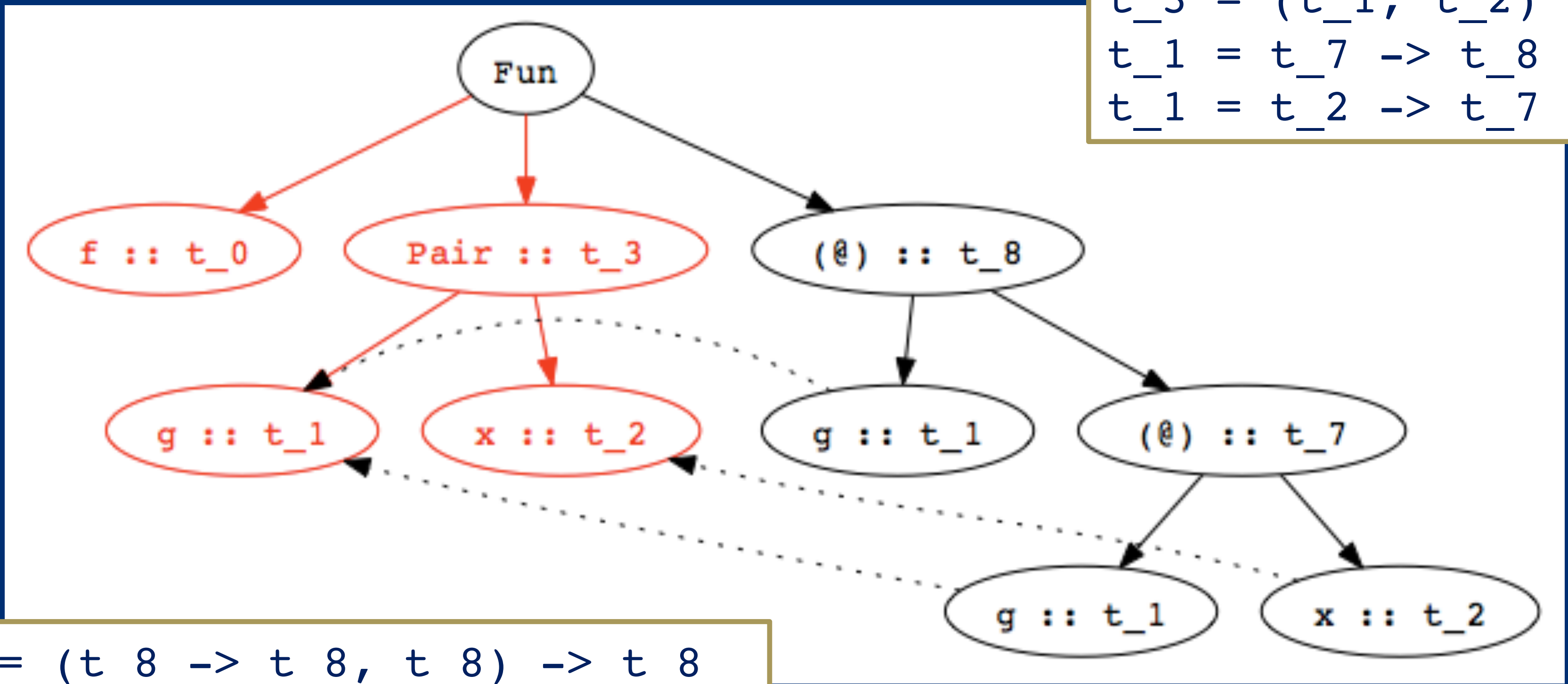
# Another Example

- Example:

```
f (g,x) = g (g x)
> f :: (t_8 -> t_8, t_8) -> t_8
```

- Step 4: Solve constraints

```
t_0 = t_3 -> t_8
t_3 = (t_1, t_2)
t_1 = t_7 -> t_8
t_1 = t_2 -> t_7
```




```
t_0 = (t_8 -> t_8, t_8) -> t_8
```

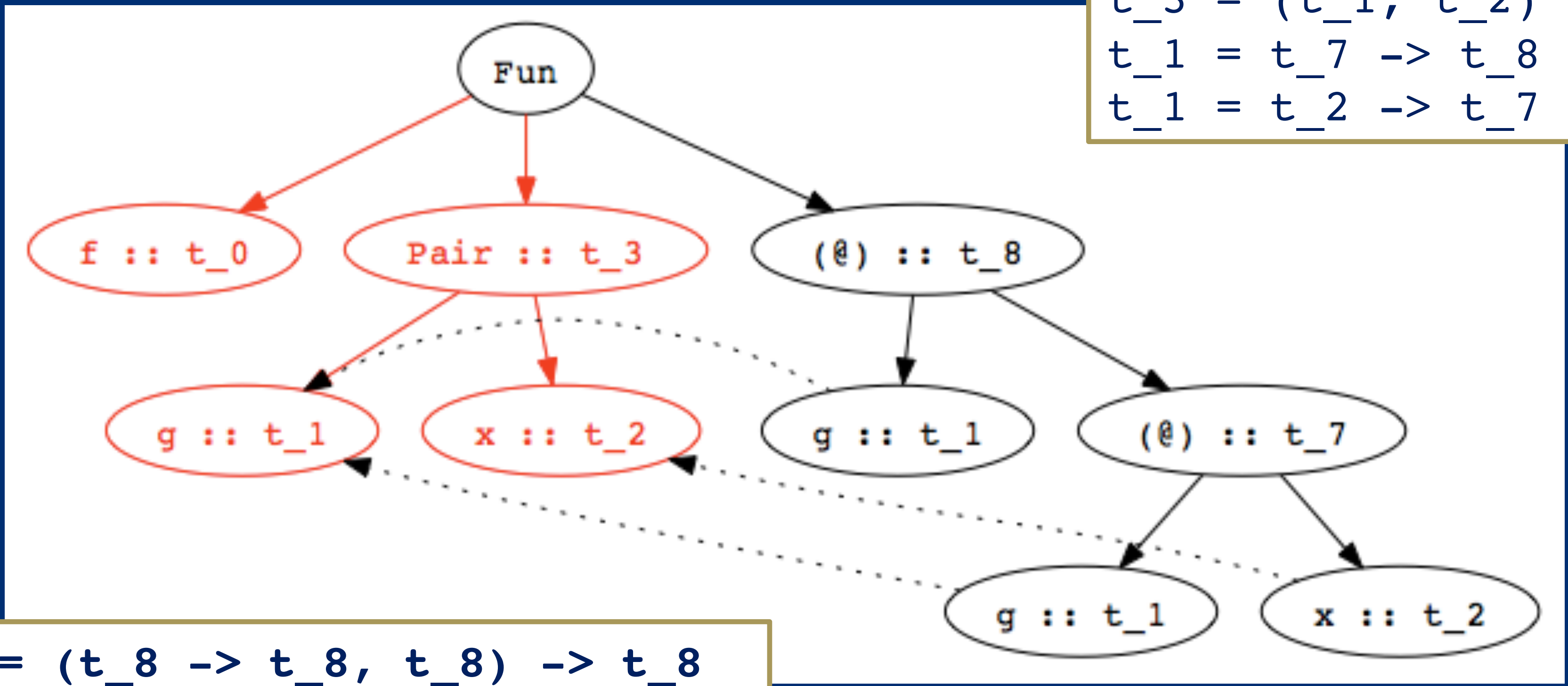
# Another Example

- Example:

```
f (g,x) = g (g x)
> f :: (t_8 -> t_8, t_8) -> t_8
```

- Step 5: Determine type of f

```
t_0 = t_3 -> t_8
t_3 = (t_1, t_2)
t_1 = t_7 -> t_8
t_1 = t_2 -> t_7
```




```
t_0 = (t_8 -> t_8, t_8) -> t_8
```

# Polymorphic Datatypes

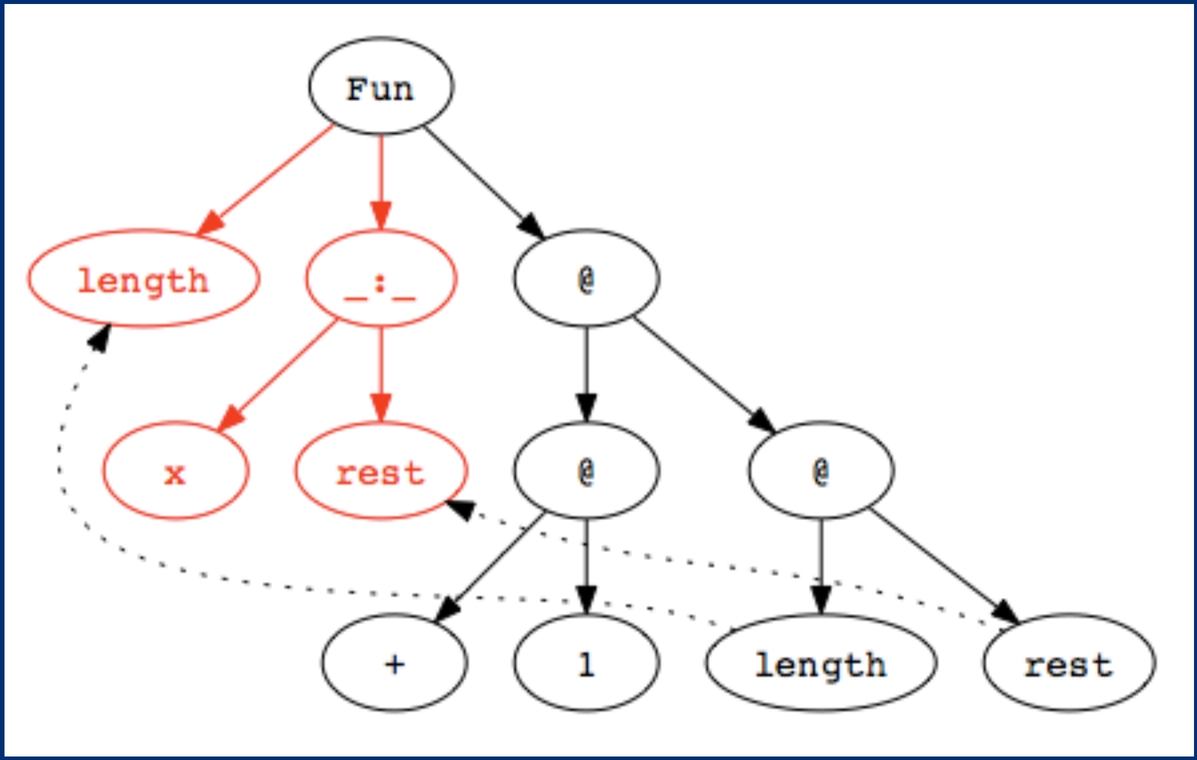
- Often, functions over datatypes are written with multiple clauses:

```
length [] = 0
length (x:rest) = 1 + (length rest)
```

- Type inference
  - Infer separate type for each clause
  - Combine by adding constraint that the types of the branches must be equal.

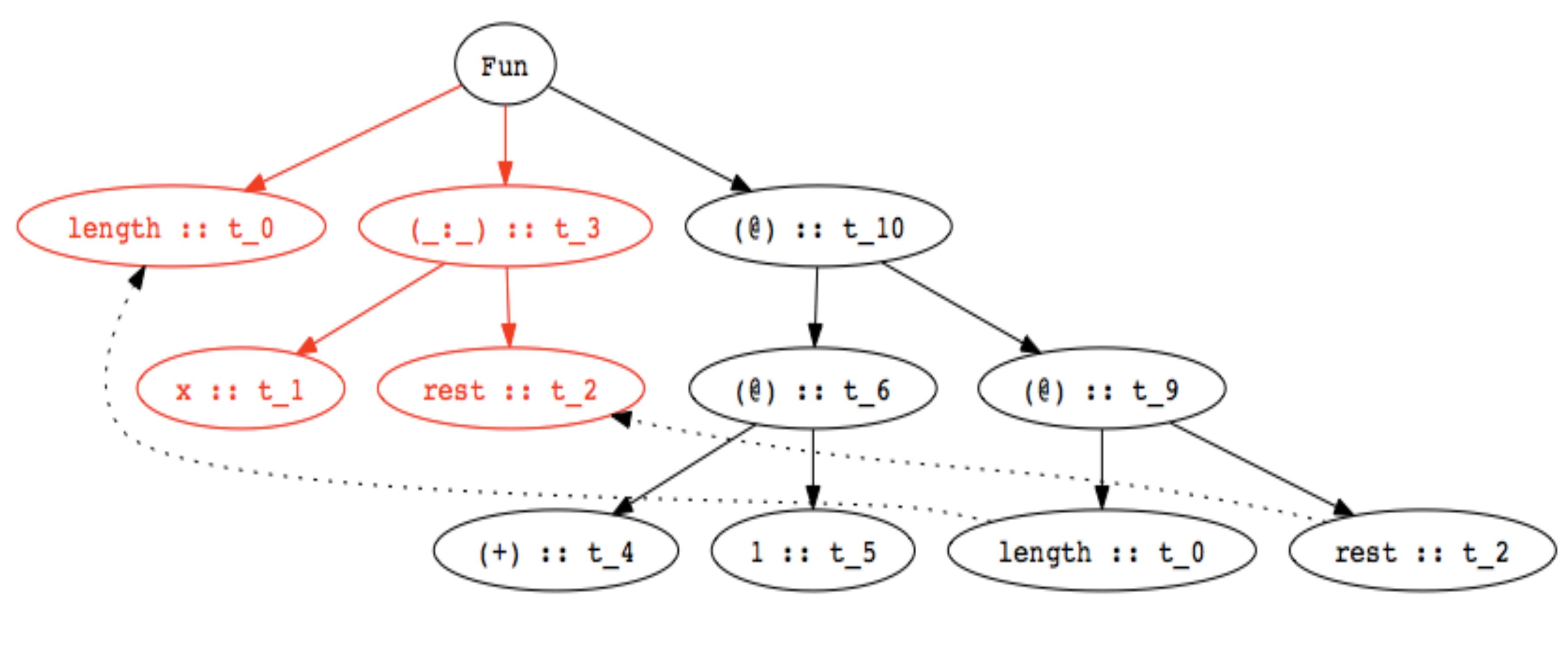
# Type Inference with Datatypes

- Example: `length (x:rest) = 1 + (length rest)`
- Step 1: Build Parse Tree



# Type Inference with Datatypes

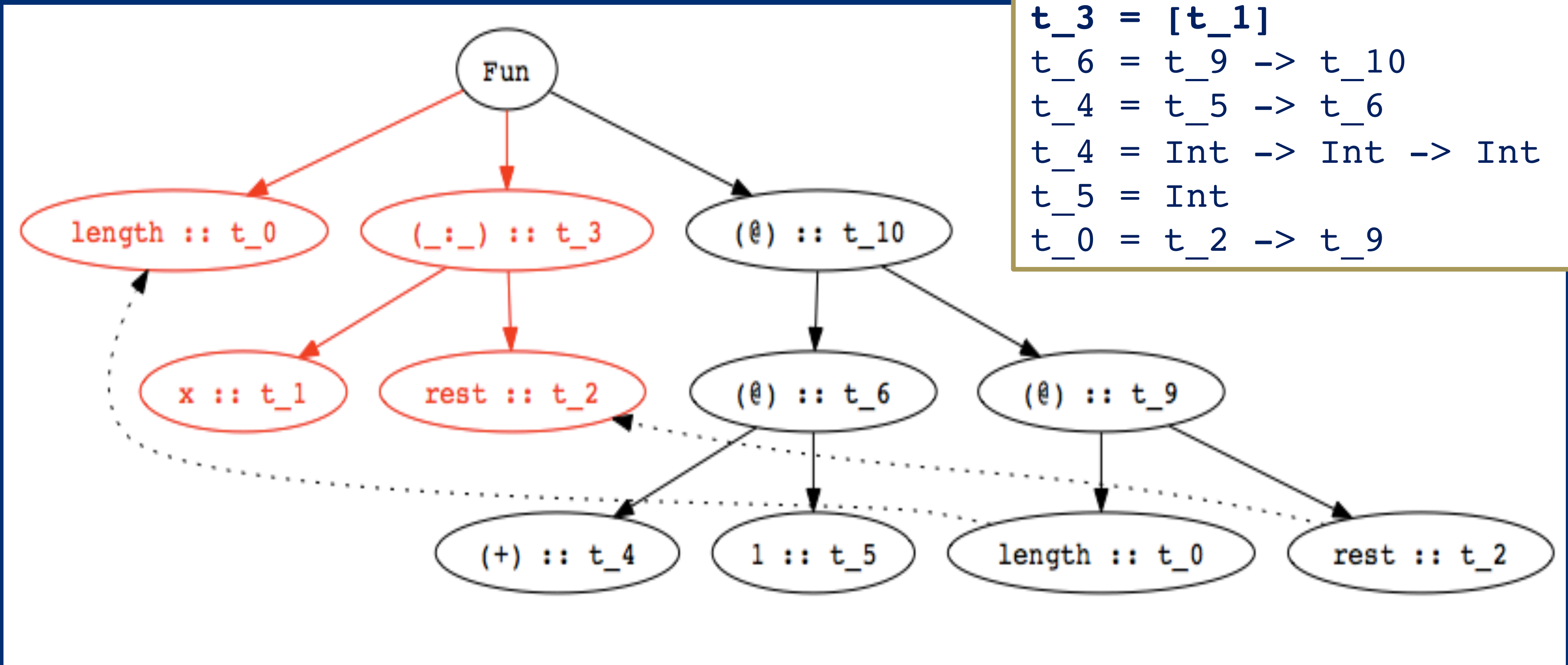
- Example: `length (x:rest) = 1 + (length rest)`
- Step 2: Assign type variables



# Type Inference with Datatypes

- Example: `length (x:rest) = 1 + (length rest)`
- Step 3: Gen. constraints

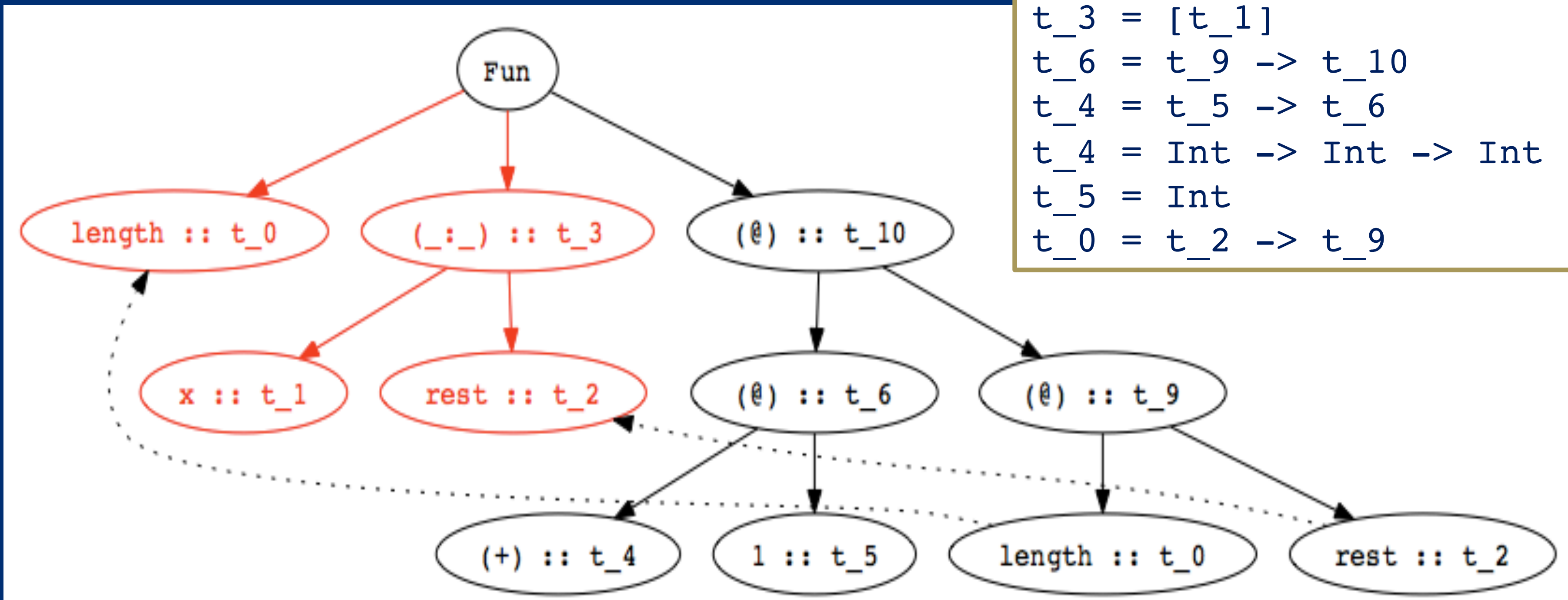
```
t_0 = t_3 -> t_10
t_3 = t_2
t_3 = [t_1]
t_6 = t_9 -> t_10
t_4 = t_5 -> t_6
t_4 = Int -> Int -> Int
t_5 = Int
t_0 = t_2 -> t_9
```



# Type Inference with Datatypes

- Example: `length (x:rest) = 1 + (length rest)`
- Step 3: Solve Constraints

```
t_0 = t_3 -> t_10
t_3 = t_2
t_3 = [t_1]
t_6 = t_9 -> t_10
t_4 = t_5 -> t_6
t_4 = Int -> Int -> Int
t_5 = Int
t_0 = t_2 -> t_9
```




```
t_0 = [t_1] -> Int
```

# Multiple Clauses

- Function with multiple clauses

```
append ([],r) = r
append (x:xs, r) = x : append (xs, r)
```

- Infer type of each branch
  - First branch:

```
> append :: ([t_1], t_2) -> t_2
```

  - Second branch:

```
> append :: ([t_3], t_4) -> [t_3]
```

- Combine by equating types of two branches:

```
> append :: ([t_1], [t_1]) -> [t_1]
```

# Most General Type

- Type inference is *guaranteed* to produce the *most general type*:

```
map (f, []  ) = []
map (f, x:xs) = f x : map (f, xs)
> map :: (t_1 -> t_2, [t_1]) -> [t_2]
```

- Function has many other, less general types:

```
> map :: (t_1  -> Int, [t_1])  -> [Int]
> map :: (Bool -> t_2, [Bool]) -> [t_2]
> map :: (Char -> Int, [Char]) -> [Int]
```

- Less general types are all *instances* of most general type, also called the *principal type*.

# Type Inference Algorithm

- When the Hindley/Milner type inference algorithm was developed, its complexity was unknown.
- In 1989, Mairson proved that the problem was exponential-time complete.
- Tractable in practice though…

# Information from Type Inference

- Consider this function…

```
reverse [] = []
reverse (x:xs) = reverse xs
```

- … and its most general type:

```
> reverse :: [t_1] -> [t_2]
```

- What does this type mean?

Reversing a list does not change its type, so there must be an error in the definition of `reverse`!

See Koenig paper on "Reading" page of CS242 site

# Type Inference: Key Points

- Type inference computes the types of expressions
  - Does not require type declarations for variables
  - Finds the *most general type* by solving constraints
  - Leads to polymorphism
- Sometimes better error detection than type checking
  - Type may indicate a programming error even if no type error.
- Some costs
  - More difficult to identify program line that causes error.
  - Natural implementation requires uniform representation sizes.
  - Complications regarding assignment took years to work out.
- Idea can be applied to other program properties
  - Discover properties of program using same kind of analysis

# Haskell Type Inference

- Haskell uses type classes to support user-defined overloading, so the inference algorithm is more complicated.
- ML restricts the language to ensure that no annotations are required, ever.
- Haskell provides various features like *polymorphic recursion* for which types cannot be inferred and so the user must provide annotations.

# Parametric Polymorphism: Haskell vs C++

- Haskell polymorphic function
  - Declarations (generally) require no type information.
  - Type inference uses type variables to type expressions.
  - Type inference substitutes for variables as needed to instantiate polymorphic code.
- C++ function template
  - Programmer must declare the argument and result types of functions.
  - Programmers must use explicit type parameters to express polymorphism.
  - Function application: type checker does instantiation.

# Example: Swap Two Values

- Haskell

```
swap :: (IORef a, IORef a) -> IO ()
swap (x,y) = do {
  val_x <- readIORef x; val_y <- readIORef y;
  writeIORef y val_x;   writeIORef x val_y;
  return () }
```

- C++

```
template <typename T>
void swap(T& x, T& y){
      T tmp = x;  x=y;  y=tmp;
}
```

Declarations both swap two values polymorphically, but they are compiled very differently.

# Implementation

- Haskell
  - `Swap` is compiled into one function
  - Typechecker determines how function can be used
- C++
  - `Swap` is compiled into linkable format
  - Linker duplicates code for each type of use
- Why the difference?
  - Haskell ref cell is passed by pointer. The local x is a pointer to value on heap, so its size is constant.
  - C++ arguments passed by reference (pointer), but local x is on the stack, so its size depends on the type.

# Another Example

- C++ polymorphic sort function

```
template <typename T>
void sort( int count, T * A[count ] ) {
    for (int i=0; i<count-1; i++)
            for (int j=i+1; j<count-1; j++)
                if (A[j] < A[i]) swap(A[i],A[j]);
}
```

- What parts of code depend on the type?
  - Indexing into array
  - Meaning and implementation of <

# Polymorphism vs Overloading

- Parametric polymorphism
  - Single algorithm may be given many types
  - Type variable may be replaced by *any* type
  - if `f::t→t` then `f::Int→Int`, `f::Bool→Bool`, ...
- Overloading
  - A single symbol may refer to more than one algorithm
  - Each algorithm may have different type
  - Choice of algorithm determined by type context
  - Types of symbol may be arbitrarily different
  - In ML, + has types `int*int→int`, `real*real→real`, *no others*

# Summary

- Types are important in modern languages
  - Program organization and documentation
  - Prevent program errors
  - Provide important information to compiler
- Type inference
  - Determine best type for an expression, based on known information about symbols in the expression
- Polymorphism
  - Single algorithm (function) can have many types
- Overloading
  - One symbol with multiple meanings, resolved at compile time