The hat-check problem (a.k.a. coat-check)

- $n$ people leave their hats with an attendant, & get a ticket number for retrieval.

- The attendant loses all ticket info & gives hats back randomly.

How many people do we expect to get their own hats back?
INDICATOR RANDOM VARIABLES

\[ X = \# \text{ people who get their own hat back} = \sum_{k=1}^{n} X_k \]

\[ X_k = \begin{cases} 
1 & \text{if person } k \text{ gets their own hat back} \\
0 & \text{otherwise} 
\end{cases} \]

\[ E(X_k) = \frac{1}{n} \quad \text{(random)} \]

\[ E(X) = E\left( \sum_{k=1}^{n} X_k \right) = \sum_{k=1}^{n} E(X_k) \quad \text{linearity of expectation} \]

\[ = \sum_{k=1}^{n} \frac{1}{n} = 1 \]
The hiring problem: you need one assistant.

- $n$ candidates, interviewed in **random** order.
- No 2 equally skilled.
- Any time you interview someone better than all previous, you hire the new person & fire the current assistant.

How many people do you expect to hire?
**INDICATOR RANDOM VARIABLES**

$X = \# \text{ people do you expect to hire} = \sum_{k=1}^{\hat{n}} X_k$

$X_k = \begin{cases} 
1 & \text{if you hire candidate } k \\
0 & \text{otherwise}
\end{cases}$

$E(X_k) = \frac{1}{k}$ \hspace{1cm} (k is hired iff better than all k-1 previous)

$E(X) = E\left( \sum_{k=1}^{\hat{n}} X_k \right) = \sum_{k=1}^{\hat{n}} E(X_k)$ \hspace{1cm} linearity of expectation

$= \sum_{k=1}^{\hat{n}} \frac{1}{k} = \ln \hat{n} + O(1) < \ln \hat{n} + 1$
The birthday problem (new variant)

How many people do we need in a room so that we expect to have (at least) one birthday match?
**INDICATOR RANDOM VARIABLES**

**X** = \# birthday matches among n people = \[ \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} X_{ij} \]

\[ X_{ij} = \begin{cases} 1 & \text{if persons } i \& j \text{ match} \\ 0 & \text{otherwise} \end{cases} \]

\[ E(X_{ij}) = \frac{1}{365} \]

**E(X) = E\left( \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} X_{ij} \right) = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E(X_{ij}) \quad \text{linearity of expectation} \]

\[ = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{1}{365} = \frac{n \cdot (n-1)}{2} \cdot \frac{1}{365} = 1 \quad \Rightarrow \quad n \approx 28 \]

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