INDICATOR RANDOM VARIABLES

The hat-check problem (a.k.a. coat-check)

- n people leave their hats with an attendant, & get a ticket = number for retrieval.
- The attendant loses all ticket info & gives hats back randomly.

How many people do we expect to get their own hats back?

INDICATOR RANDOM VARIABLES

X = # people who get their own hat back $= \sum_{k=1}^{n} X_k$

$$X_k = \begin{cases} 1 & \text{if person } k \text{ gets their own hat back} \\ 0 & \text{otherwise} \end{cases}$$

$E(X_k) = \frac{1}{n}$ (random)

$E(X) = E\left(\sum_{k=1}^{n} X_k\right) = \sum_{k=1}^{n} E(X_k)$ linearity of expectation

$= \sum_{k=1}^{n} \frac{1}{n} = 1$

INDICATOR RANDOM VARIABLES

The hiring problem: you need one assistant.

- n candidates, interviewed in <u>random</u> order.
- No 2 equally skilled.
- any time you interview someone better than all previous, you hire the new person & fire the current assistant.

How many people do you expect to hire?

INDICATOR RANDOM VARIABLES

X = # people do you expect to hire $= \sum_{k=1}^{n} X_k$

$$X_k = \begin{cases} 1 & \text{if you hire candidate } k \\ 0 & \text{otherwise} \end{cases}$$

$E(X_k) = \frac{1}{k}$ (k is hired <u>iff</u> better than all $k-1$ previous)

$$E(X) = E\left(\sum_{k=1}^{n} X_k\right) = \sum_{k=1}^{n} E(X_k)$$ linearity of expectation

$$= \sum_{k=1}^{n} \frac{1}{k} = \ln n + O(1) < \ln n + 1$$

INDICATOR RANDOM VARIABLES

The birthday problem (new variant)

How many people do we need in a room so that we expect to have (at least) one birthday match?

INDICATOR RANDOM VARIABLES

X = # birthday matches among n people $= \underbrace{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} X_{ij}}_{\text{all } \binom{n}{2} \text{ pairs}}$

$$X_{ij} = \begin{cases} 1 & \text{if persons } i \,\&\, j \text{ match} \\ 0 & \text{otherwise} \end{cases}$$

$$E(X_{ij}) = \frac{1}{365}$$

$$E(X) = E\left(\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} X_{ij}\right) = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E(X_{ij})$$ linearity of expectation

we said we want $E[X]=1$

$$= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{1}{365} = \frac{n\cdot(n-1)}{2} \cdot \frac{1}{365} = 1 \Rightarrow n \approx 28$$

NOT 23