A bet : I randomly select half of the class...

A bet : I randomly select half of the class...

If any 2 people in that group have the same birthday

you give me a dollar.

A bet : I randomly select half of the class...

If any 2 people in that group have the same birthday

you give me a dollar.

If no birthday match is found

I give you a dollar

Another bet :

I randomly select 10 people born in the same month

Same deal as before

Another bet :

I randomly select 10 people born in the same month

Same deal as before

My chances of winning : $>80\%$

One last bet ?

I randomly select 7 people

If any 2 people in that group have birthdays within a week of each other...

One last bet ?

I randomly select 7 people

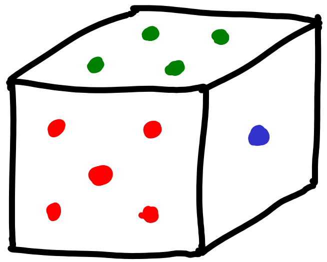
If any 2 people in that group have birthdays within a week of each other...

I win 60% of the time. (52% if within 6 days)

DISCRETE PROBABILITY

DISCRETE PROBABILITY

Roll a die ... Possible outcomes: $\{1,2,3,4,5,6\}$



DISCRETE PROBABILITY

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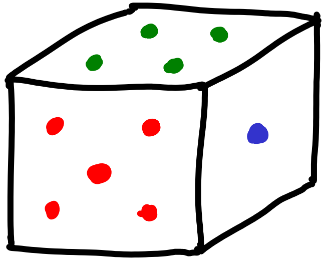
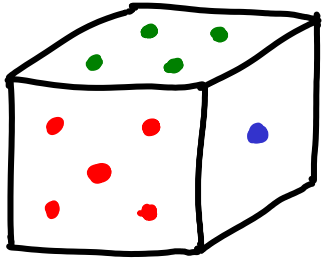


each has a probability: $\{1/6, 1/6, 1/6, 1/6, 1/6, 1/6\}$

DISCRETE PROBABILITY

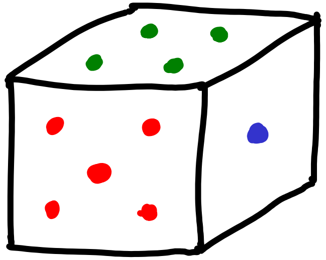
Roll a die ... Possible outcomes: $\overbrace{\{1, 2, 3, 4, 5, 6\}}^{\text{sample space}}$



each has a probability: $\underbrace{\{1/6, 1/6, 1/6, 1/6, 1/6, 1/6\}}_{\text{sum to } 1}$

DISCRETE PROBABILITY

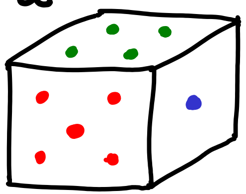
Roll a die ... Possible outcomes: $\overbrace{\{1, 2, 3, 4, 5, 6\}}^{\text{sample space}}$



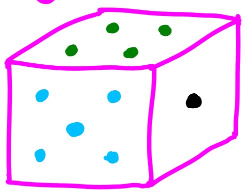
each has a probability: $\underbrace{\{\tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}\}}_{\text{sum to } 1}$

Roll 2 dice ... sample space → ?

a

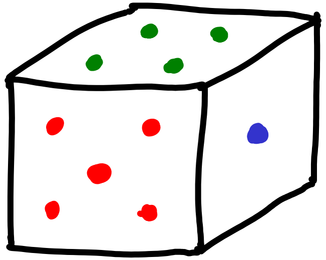


b



DISCRETE PROBABILITY

Roll a die ... Possible outcomes: $\overbrace{\{1,2,3,4,5,6\}}^{\text{sample space}}$



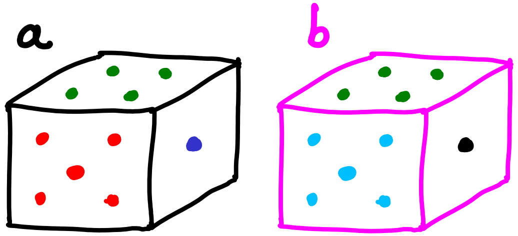
each has a probability: $\underbrace{\{\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6}\}}_{\text{sum to 1}}$

Roll 2 dice ... sample space $\rightarrow$ $\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$

(a,b) $\quad (2,1),(2,2),\ldots$

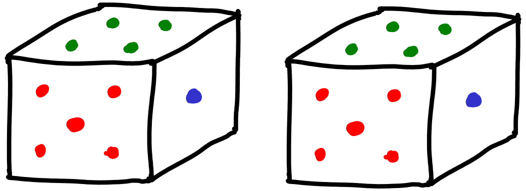
$\vdots$

$(6,1),(6,2),\ldots \quad \ldots(6,6)\}$



prob. $\rightarrow \frac{1}{36}$

Roll 2 indistinguishable dice ...



sample space $\rightarrow$ $\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$

$(2,2), (2,3), (2,4), (2,5), (2,6),$

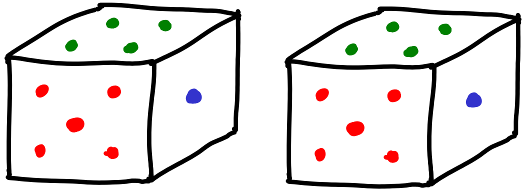
$(3,3), (3,4), (3,5), (3,6),$

$(4,4), (4,5), (4,6),$

$(5,5), (5,6),$

$(6,6)\}$

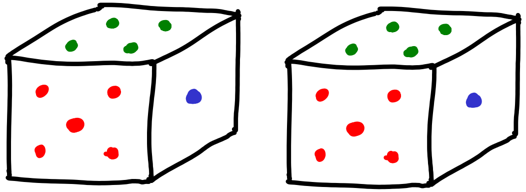
Roll 2 indistinguishable dice ...



sample space $\rightarrow$ $\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,2), (2,3), (2,4), (2,5), (2,6),$
$(3,3), (3,4), (3,5), (3,6),$
$(4,4), (4,5), (4,6),$
$(5,5), (5,6),$
$(6,6)\}$

prob. = ?

Roll 2 indistinguishable dice ...



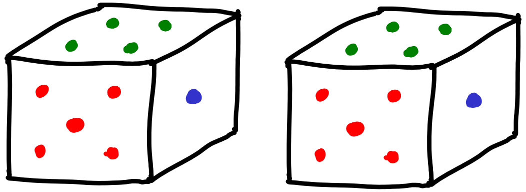
sample space → $\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$
$(2,2),(2,3),(2,4),(2,5),(2,6),$
$(3,3),(3,4),(3,5),(3,6),$
$(4,4),(4,5),(4,6),$
$(5,5),(5,6),$
$(6,6)\}$

prob. $\neq \frac{1}{36}$

if (a,a) then $\frac{1}{36}$

if (a,b) then $\frac{2}{36}$
$a \neq b$

Roll 2 indistinguishable dice ...



sample space $\rightarrow$ $\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,2), (2,3), (2,4), (2,5), (2,6),$
$(3,3), (3,4), (3,5), (3,6),$
$(4,4), (4,5), (4,6),$
$(5,5), (5,6),$
$(6,6)\}$

prob. $\neq \frac{1}{36}$

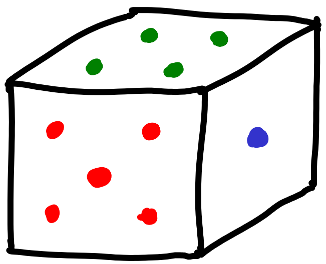
if (a,a) then $\frac{1}{36}$ $\quad \left(6 \cdot \frac{1}{36} = \frac{6}{36}\right)$

$+ \rightarrow 1$

if (a,b) then $\frac{2}{36}$ $\quad \left(15 \cdot \frac{2}{36} = \frac{30}{36}\right)$

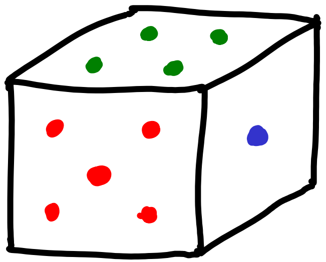
$a \neq b$

Roll a die



$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$

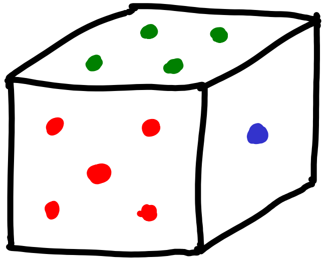
Roll a die



$$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$$

$$P(\underbrace{\text{roll even}}_{\text{event}}) = P(\{2, 4, 6\}) = P(2) + P(4) + P(6) = \frac{1}{2}$$

Roll a die



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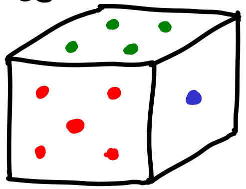
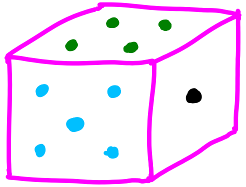
Roll 2 dice... $P(\overbrace{\text{sum} = 7})$ ...

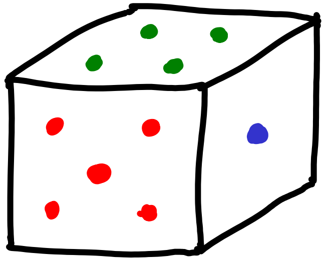
a



b



Roll a die

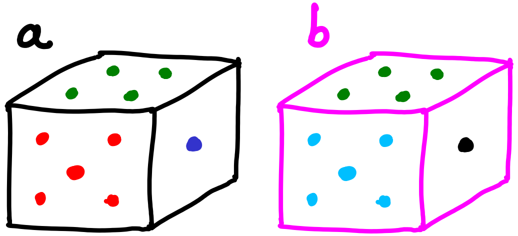


$$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$$

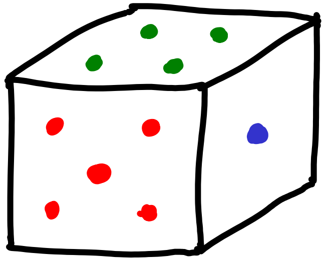
$$P(\underbrace{\text{roll even}}_{\text{event}}) = P(\{2, 4, 6\}) = P(2) + P(4) + P(6) = \frac{1}{2}$$

Roll 2 dice...

$$P(\text{sum} = 7) = P(\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\})$$



Roll a die



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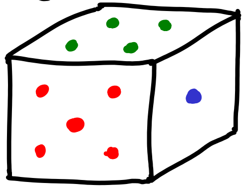
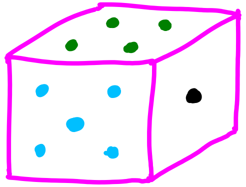
Roll 2 dice...

a



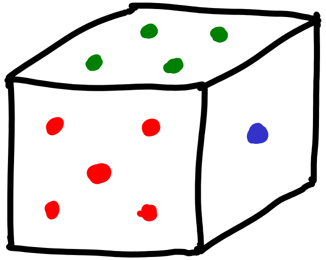
b



$$P(\text{sum} = 7) = P(\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\})$$

$$= P(1,6) + P(2,5) + P(3,4) + P(4,3) + P(5,2) + P(6,1)$$

Roll a die



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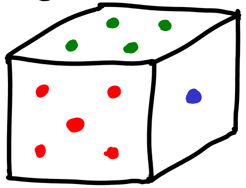
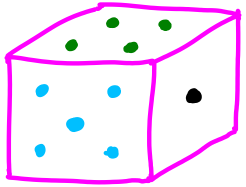
Roll 2 dice...

a



b

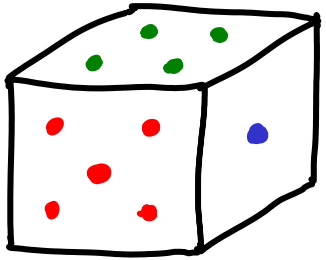


$$P(\text{sum} = 7) = P(\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\})$$

$$= P(1,6) + P(2,5) + P(3,4) + P(4,3) + P(5,2) + P(6,1)$$

$$= 6 \cdot \frac{1}{36} = \frac{1}{6}$$

Roll a die



$$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$$

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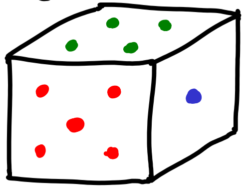
Roll 2 dice...

$$P(\overbrace{\text{sum} = 7}^{\text{event}}) = P(\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\})$$

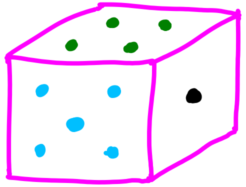
$$= P(1,6) + P(2,5) + P(3,4) + P(4,3) + P(5,2) + P(6,1)$$

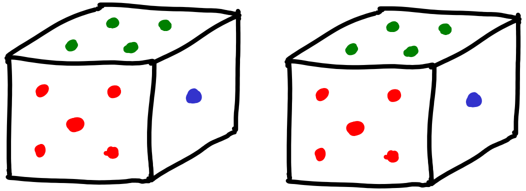
$$= 6 \cdot \frac{1}{36} = \frac{1}{6}$$

a



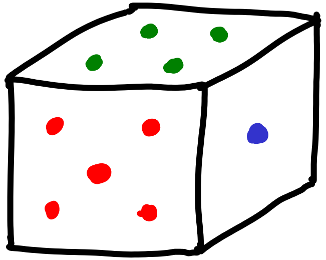
b





$$P(\text{sum} = 7) = ?$$

Roll a die



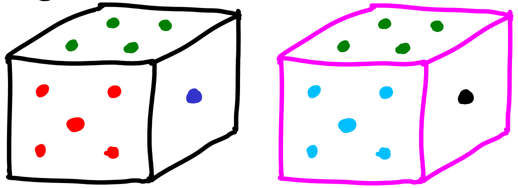
$$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$$

$$P(\underbrace{\text{roll even}}_{\text{event}}) = P(\{2,4,6\}) = P(2) + P(4) + P(6) = \frac{1}{2}$$

Roll 2 dice...

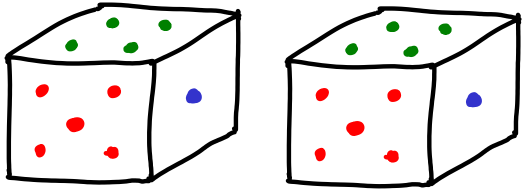
$$P(\overbrace{\text{sum} = 7}^{\text{event}}) = P(\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\})$$

a b



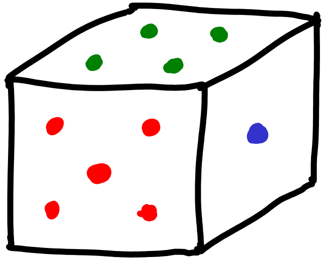
$$= P(1,6) + P(2,5) + P(3,4) + P(4,3) + P(5,2) + P(6,1)$$

$$= 6 \cdot \frac{1}{36} = \frac{1}{6}$$



$$P(\text{sum} = 7) = P(\{(1,6),(2,5),(3,4)\})$$

Roll a die



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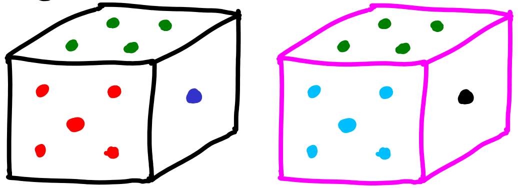
Roll 2 dice...

a b



$$P(\overbrace{\text{sum} = 7}^{\text{event}}) = P(\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\})$$

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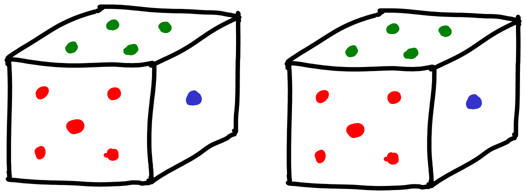
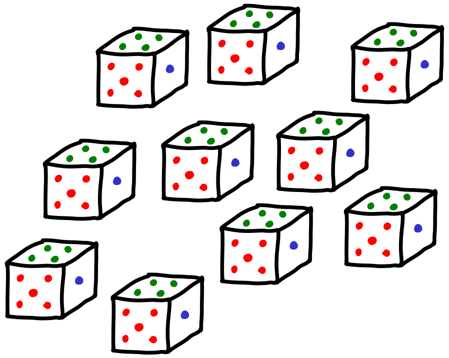
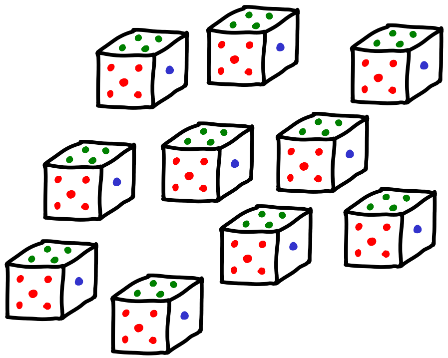


$$P(\text{sum} = 7) = P(\{(1,6),(2,5),(3,4)\}) = 3 \cdot \frac{2}{36} = \frac{1}{6}$$

Roll 10 dice (or 1 die 10 times)

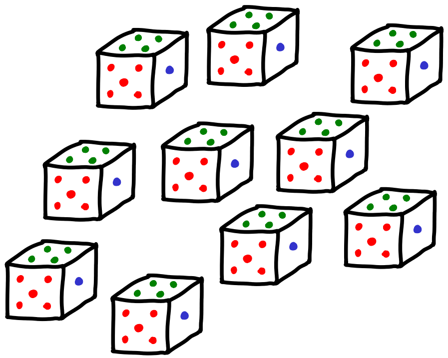


Roll 10 dice (or 1 die 10 times)



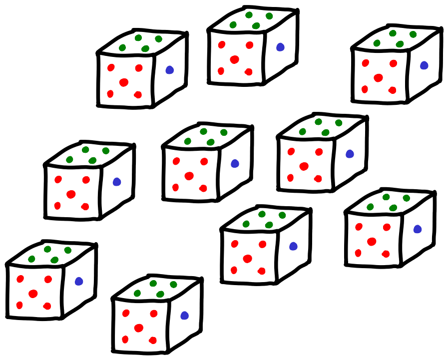
Sample space size : ?

Roll 10 dice (or 1 die 10 times)



Sample space size : 6^{10} >60 million

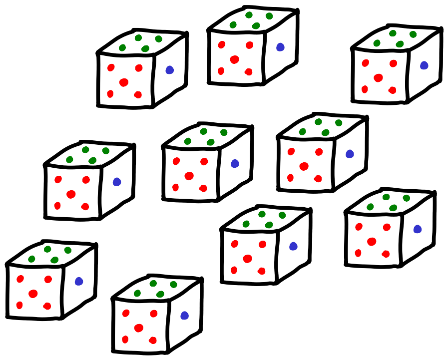
Roll 10 dice (or 1 die 10 times)



Sample space size : 6^{10} >60 million

P(observe no 1's) ?

Roll 10 dice (or 1 die 10 times)

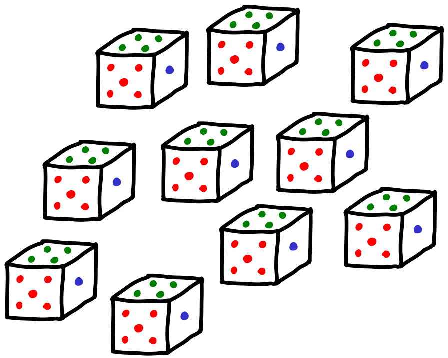


Sample space size : 6^{10} >60 million

P(observe no 1's) ?

How many outcomes have no 1's ?

Roll 10 dice (or 1 die 10 times)

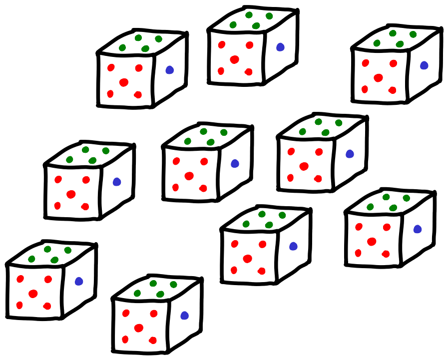


Sample space size : 6^{10} >60 million

P(observe no 1's) ?

How many outcomes have no 1's ? → 5^{10}

Roll 10 dice (or 1 die 10 times)

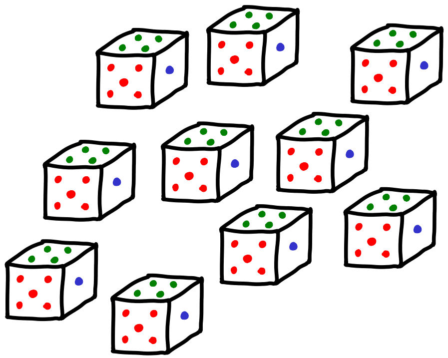


Sample space size : 6^{10} >60 million

P(observe no 1's) ? $\left(\frac{5}{6}\right)^{10}$

How many outcomes have no 1's ? → 5^{10}

Roll 10 dice (or 1 die 10 times)



Sample space size : 6^{10} >60 million

$\downarrow$

P(observe no 1's) ? $\left(\frac{5}{6}\right)^{10}$

How many outcomes have no 1's ? $\rightarrow$ 5^{10}

or, say that each roll/die is independent

so for each roll, $P(\text{no } 1) = \frac{5}{6}$ $\Rightarrow$ $\left(\frac{5}{6}\right)^{10}$ to be discussed further

Poker: 52 cards (4×13 types); select 5.

Poker: 52 cards (4×13 types); select 5.

P(4 of a kind) = P(4 of 5 are of same type)

e.g. 3, 3, 3, 3, 7 or 8, 8, 8, J, 8

= ?

Poker: 52 cards (4×13 types); select 5.

P(4 of a kind) = P(4 of 5 are of same type)

e.g. 3, 3, 3, 3, 7 or 8, 8, 8, J, 8

ans: $\dfrac{\#\text{ 4-of-a-kinds}}{\#\text{ possible outcomes}}$

Poker: 52 cards (4×13 types); select 5.

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e.g. 3, 3, 3, 3, 7 or 8, 8, 8, J, 8

ans: $\dfrac{\text{\# 4-of-a-kinds}}{\text{\# possible outcomes}} \rightarrow ?$ (order doesn't matter)

Poker: 52 cards (4×13 types); select 5.

P(4 of a kind) = P(4 of 5 are of same type)

e.g. 3, 3, 3, 3, 7 or 8, 8, 8, J, 8

ans: $\dfrac{\#\text{ 4-of-a-kinds}}{\#\text{ possible outcomes}} \rightarrow \binom{52}{5}$ } order doesn't matter

what if it did? → hw

Poker: 52 cards $(4 \times 13 \text{ types})$; select 5.

P(4 of a kind) = P(4 of 5 are of same type)

e.g. 3, 3, 3, 3, 7 or 8, 8, 8, J, 8

ans: $\dfrac{\#\text{ 4-of-a-kinds}}{\#\text{ possible outcomes}}$ $\rightarrow$?

$\rightarrow \binom{52}{5}$ } order doesn't matter

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$$\left.\begin{matrix}AAAA\\2222\\\vdots\\KKKK\end{matrix}\right\}$$ 13 types × 48 choices of remaining card

$$\binom{52}{5}$$ } order doesn't matter

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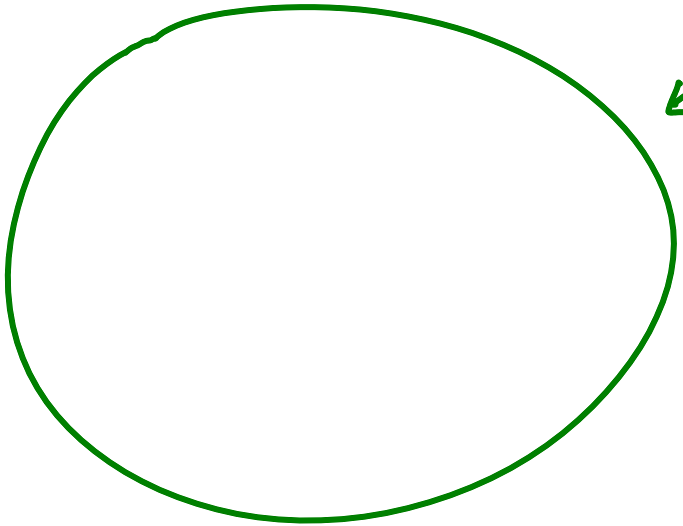
\# 4-of-a-kinds → $\left.\begin{matrix} AAAA \\ 2222 \\ \vdots \\ KKKK \end{matrix}\right\}$ 13 types × 48 choices of remaining card

\# possible outcomes → $\binom{52}{5}$ } order doesn't matter

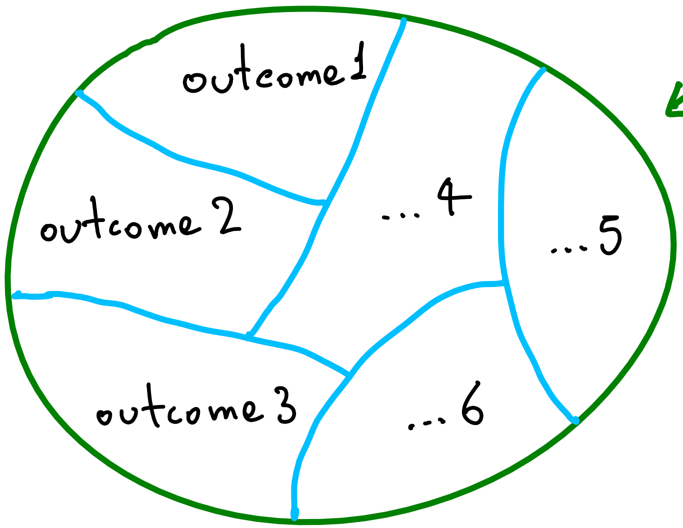
$$\frac{13 \cdot 48}{\binom{52}{5}} = \frac{1}{4165} \sim 0.00024$$

PROBABILITY VISUALIZATION



sample space → area = 1

PROBABILITY VISUALIZATION



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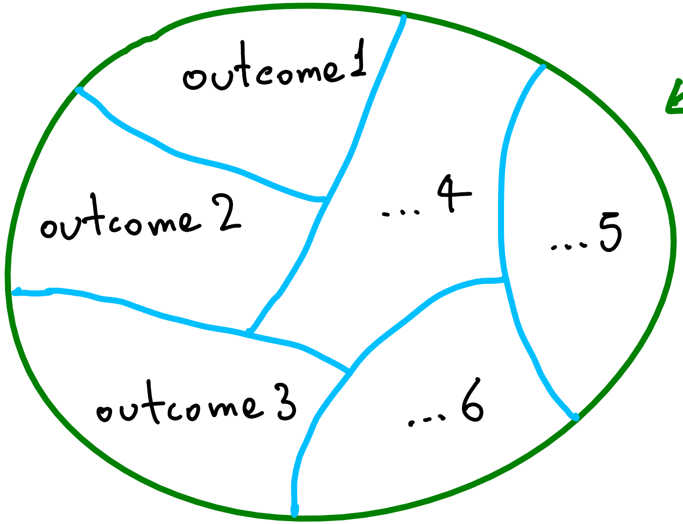
PROBABILITY VISUALIZATION

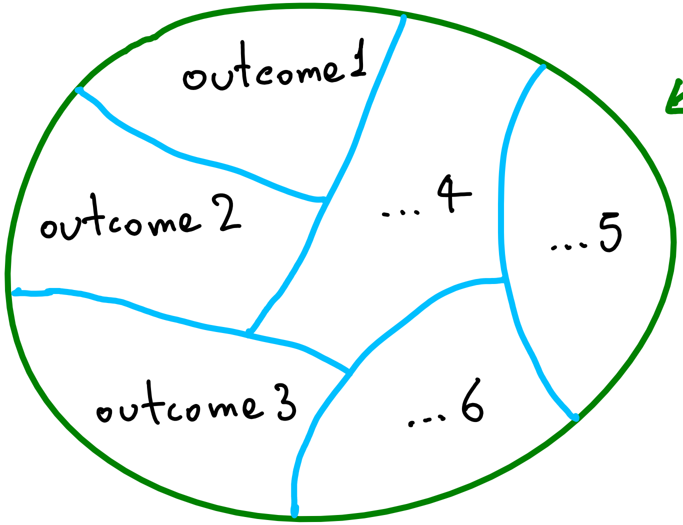


sample space → area = 1

$$P(\text{outcome } i) = \text{area}(i)$$

example: roll one die → all areas equal $= \frac{1}{6}$

PROBABILITY VISUALIZATION



sample space → area = 1

P(outcome i) = area(i)

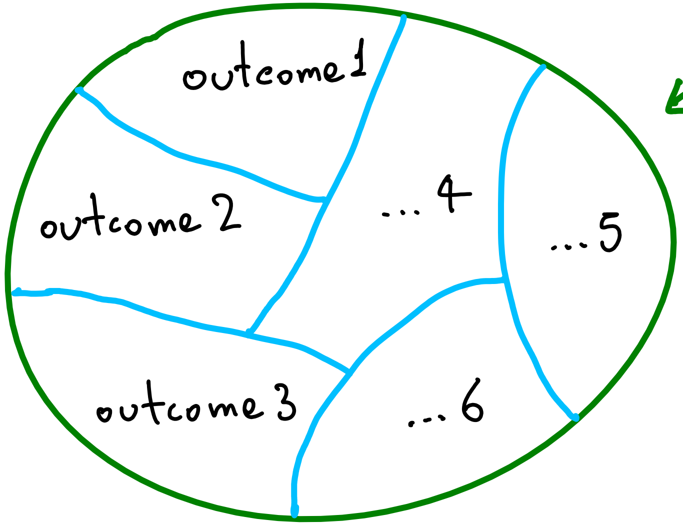
example: roll one die → all areas equal = $\frac{1}{6}$

P(event) = sum of appropriate areas

e.g. P(roll prime # OR even #)

which outcomes?

PROBABILITY VISUALIZATION



sample space $\rightarrow$ area $= 1$

$$P(\text{outcome } i) = \text{area}(i)$$

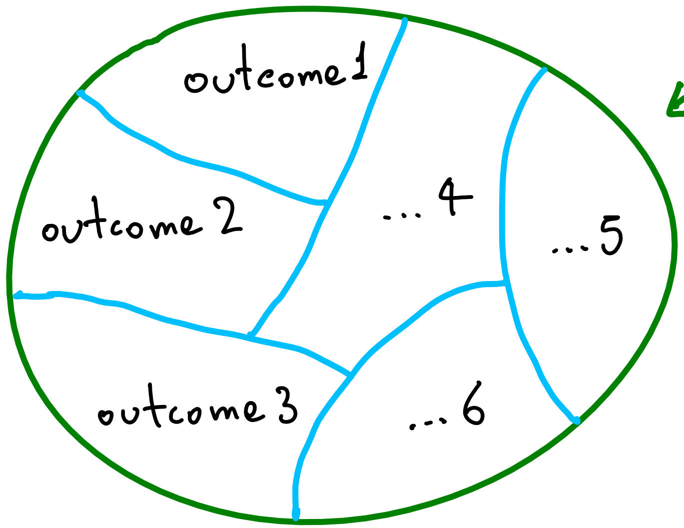
example: roll one die $\rightarrow$ all areas equal $= \frac{1}{6}$

$$P(\text{event}) = \text{sum of appropriate areas}$$

e.g. $P(\underbrace{\text{roll prime \#}}_{2,\,3,\,5} \text{ OR } \underbrace{\text{even \#}}_{2,\,4,\,6})$

$P(\text{prime}) = ?$ $\quad$ $P(\text{even}) = ?$

PROBABILITY VISUALIZATION



sample space → area = 1

$P(\text{outcome } i) = \text{area}(i)$

example: roll one die → all areas equal $= \frac{1}{6}$

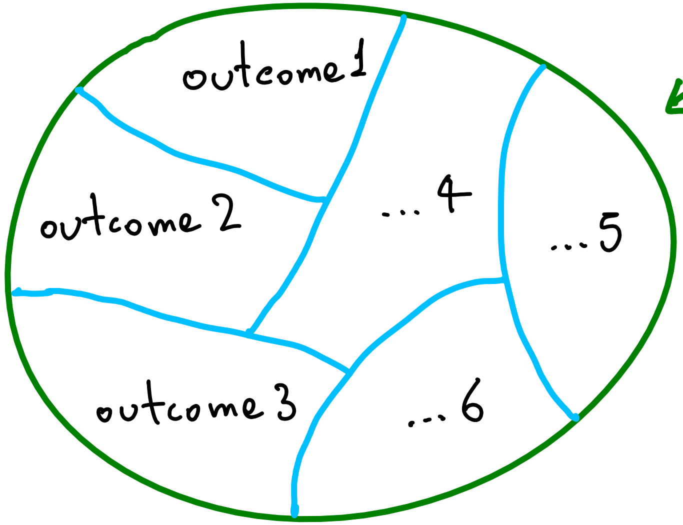
$P(\text{event}) = $ sum of appropriate areas

e.g. $P(\underbrace{\text{roll prime \#}}_{2,\,3,\,5} \text{ OR } \underbrace{\text{even \#}}_{2,\,4,\,6})$

$P(\text{prime}) = \frac{3}{6}$ $\qquad$ $P(\text{even}) = \frac{3}{6}$

?

PROBABILITY VISUALIZATION



sample space → area = 1

P(outcome i) = area(i)

example: roll one die → all areas equal = $\frac{1}{6}$

P(event) = sum of appropriate areas

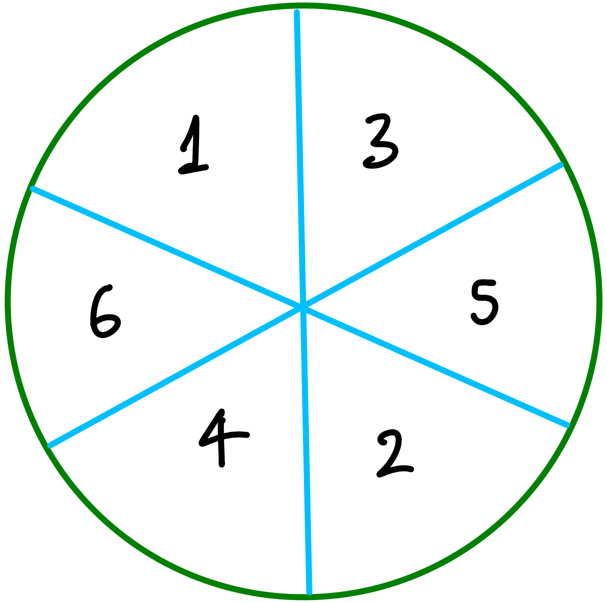
e.g. $P(\underbrace{\text{roll prime \#}}_{2,3,5} \text{ OR } \underbrace{\text{even \#}}_{2,4,6})$ = $\frac{5}{6}$

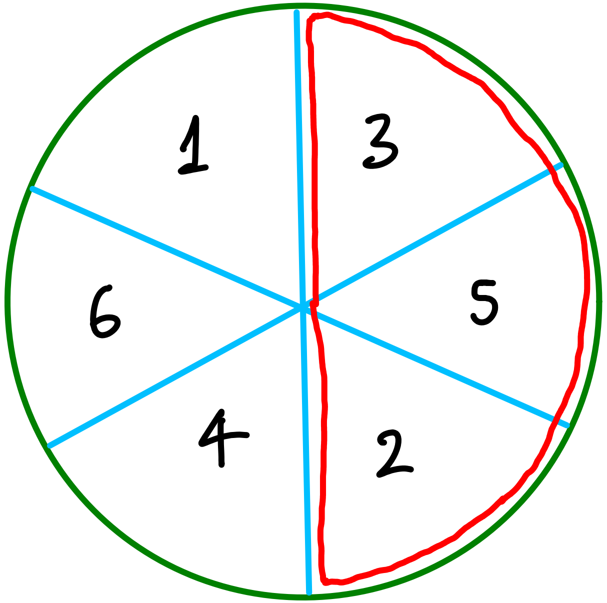
$\underbrace{2,3,5}_{}$ P(prime) = $\frac{3}{6}$

$\underbrace{2,4,6}_{}$ P(even) = $\frac{3}{6}$

avoid doublecounting

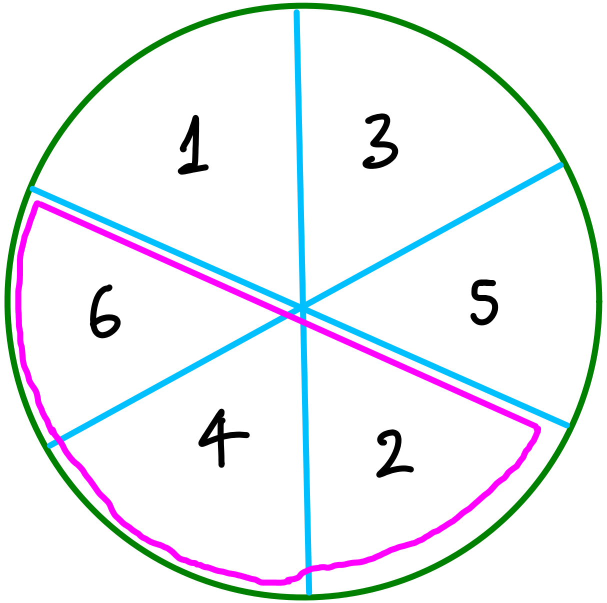
NOT $\frac{3}{6}+\frac{3}{6}$


1
3
6
5
4
2



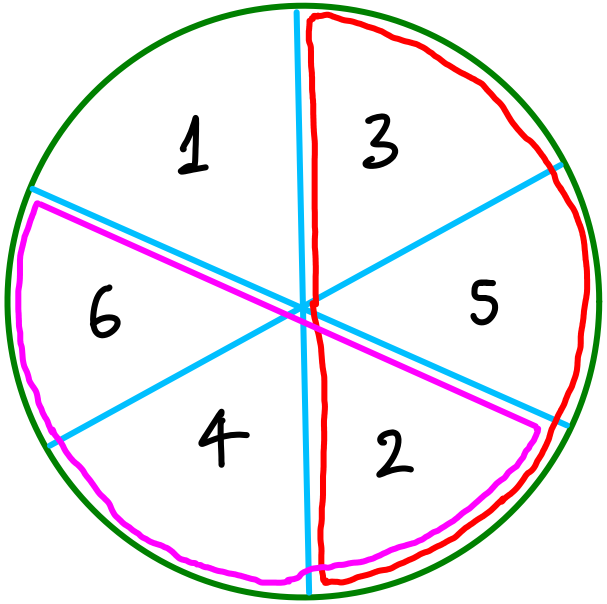
roll prime #

$\underbrace{\quad\quad\quad\quad}_{2,3,5}$

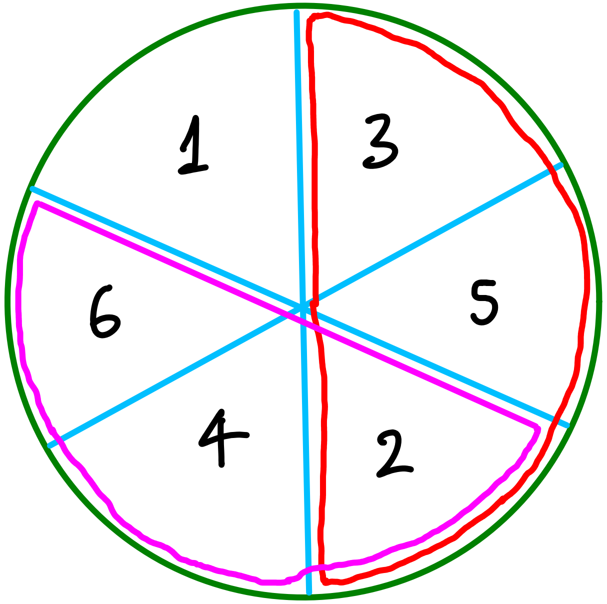


roll even #

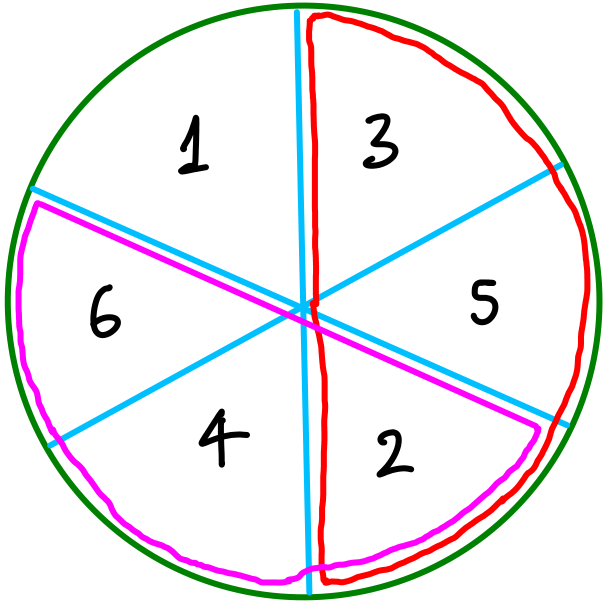
$\underbrace{\text{even \#}}_{2,4,6}$



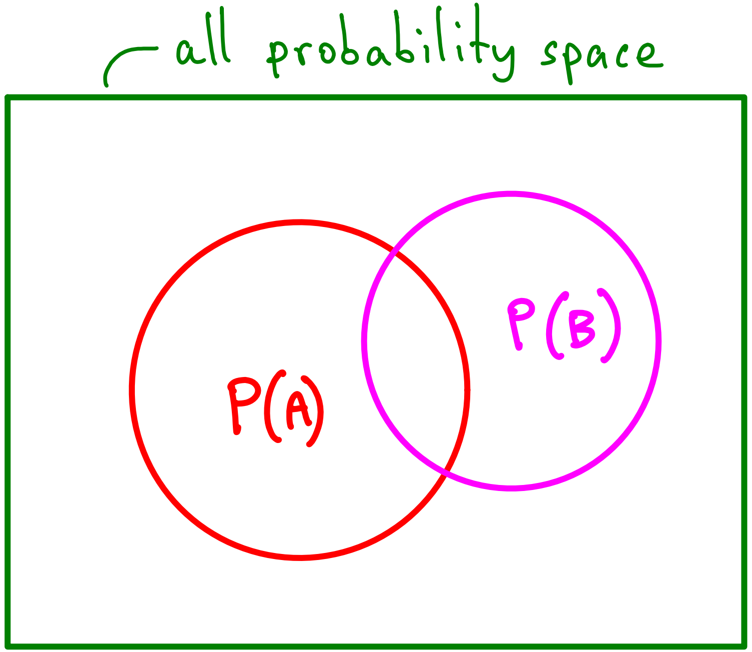
$\underbrace{\text{roll prime \#}}_{2,\,3,\,5}$ $\underbrace{\text{even \#}}_{2,\,4,\,6}$

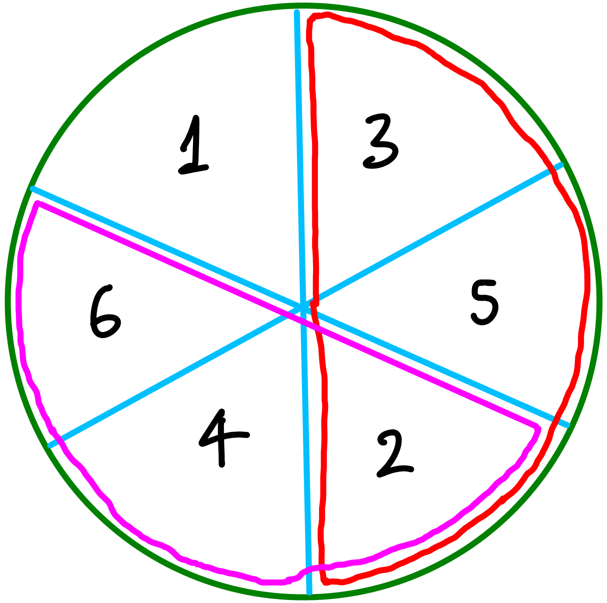


$$P(\underbrace{\text{roll prime \#}}_{2,\,3,\,5} \text{ OR } \underbrace{\text{even \#}}_{2,\,4,\,6}) \rightarrow P(A \cup B)$$



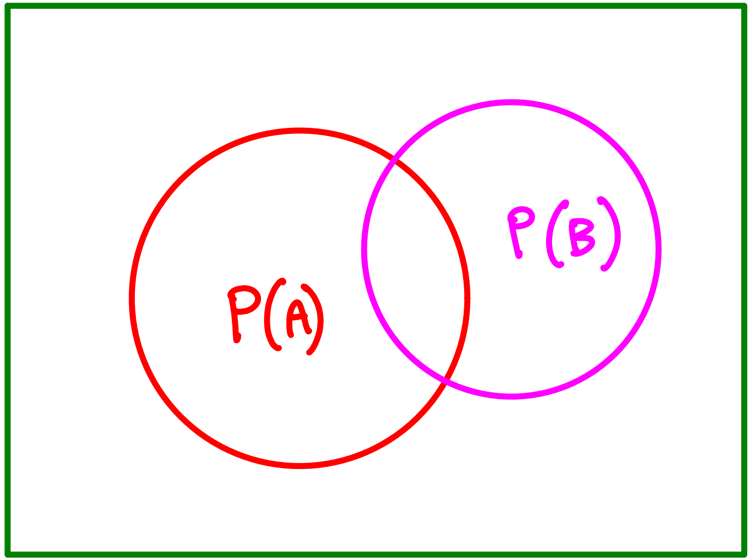
$$P(\underbrace{\text{roll prime \#}}_{2,\,3,\,5} \text{ OR } \underbrace{\text{even \#}}_{2,\,4,\,6}) \rightarrow P(A \cup B)$$



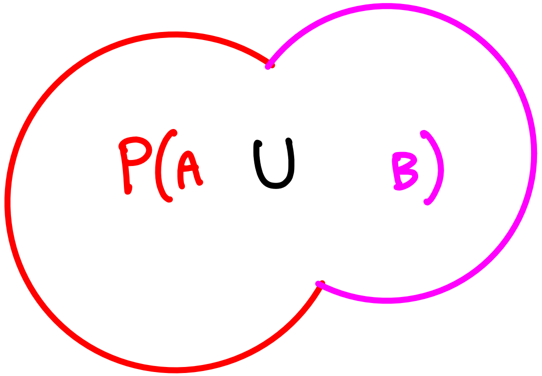


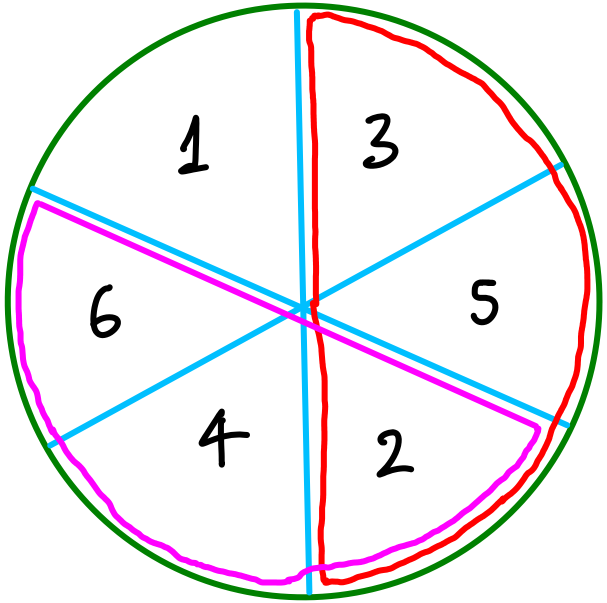
$$P(\underbrace{\text{roll prime \#}}_{2,\,3,\,5} \text{ OR } \underbrace{\text{even \#}}_{2,\,4,\,6}) \rightarrow P(A \cup B)$$

all probability space

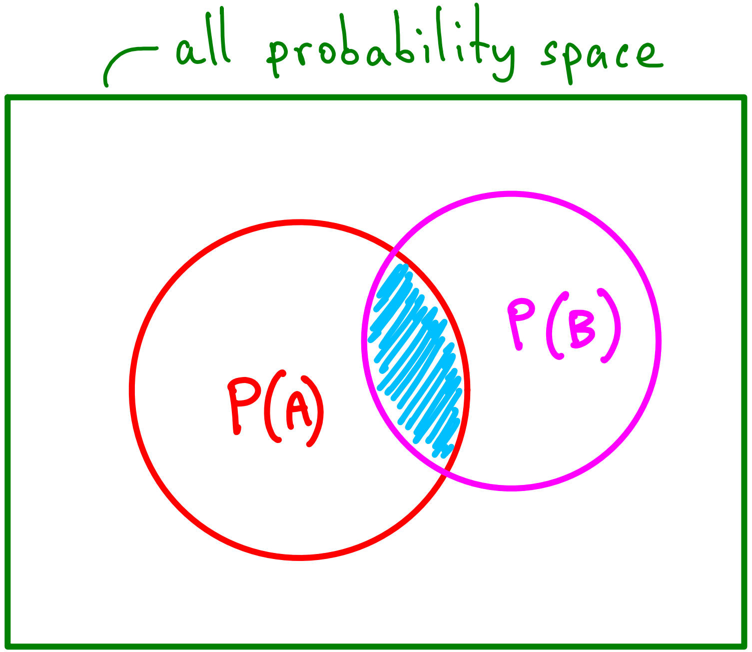


"OR"

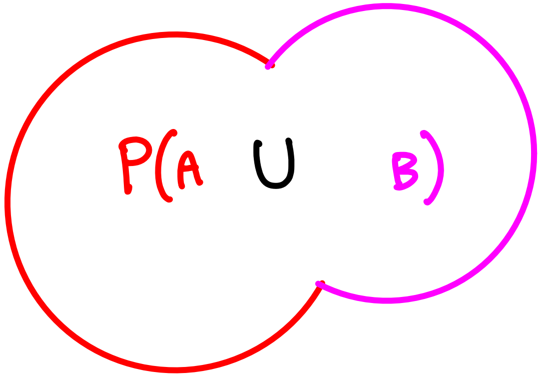




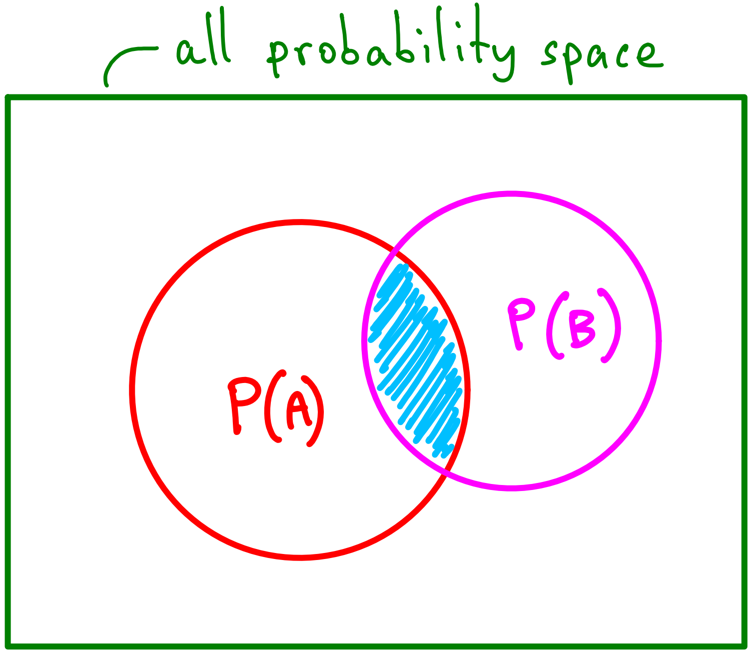
$$P(\underbrace{\text{roll prime \#}}_{2,3,5} \text{ OR } \underbrace{\text{even \#}}_{2,4,6}) \rightarrow P(A \cup B)$$



"OR"

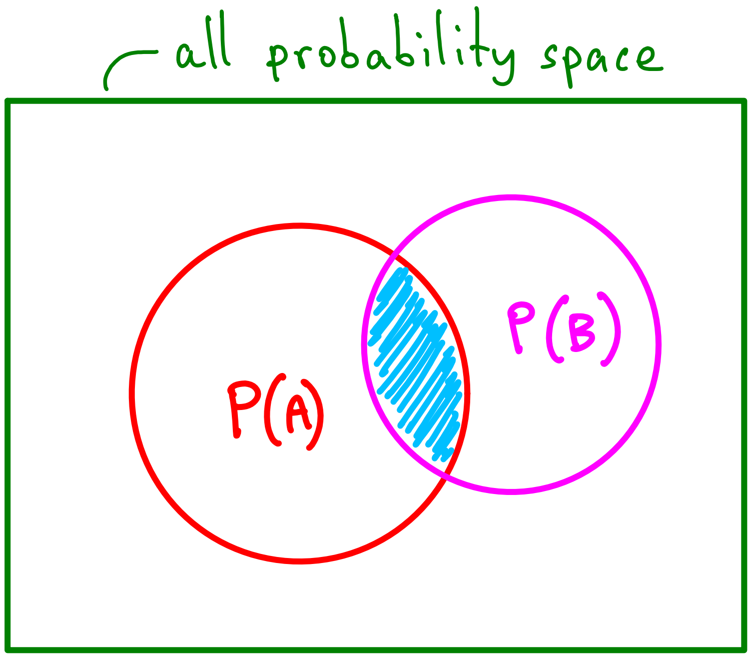


"AND"

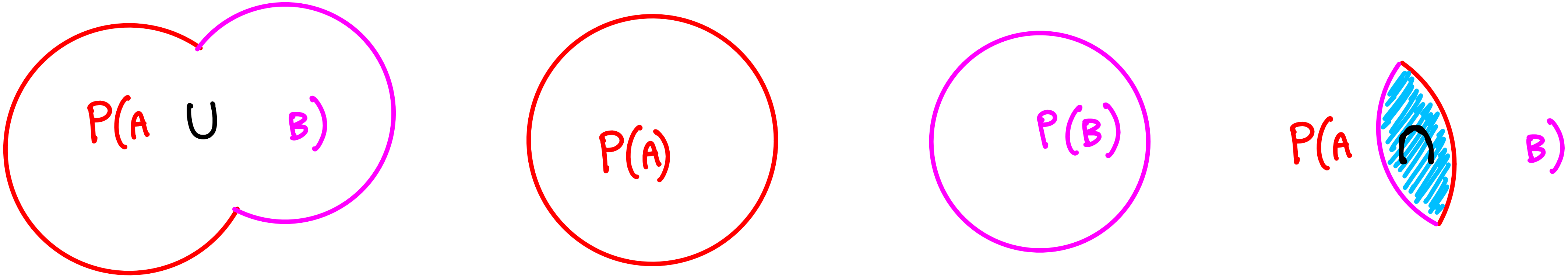


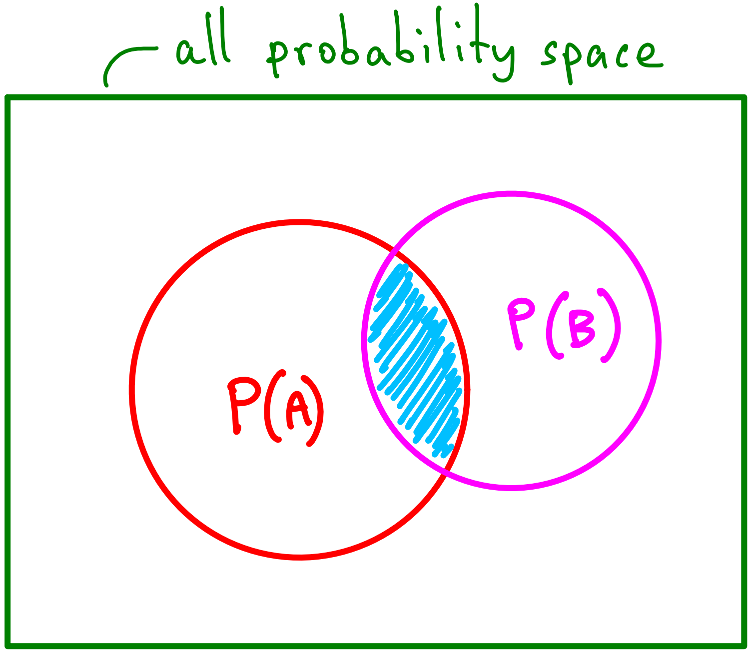
$P(A)$ $P(B)$ vs $P(A \cup B)$ $P(A \cap B)$

?

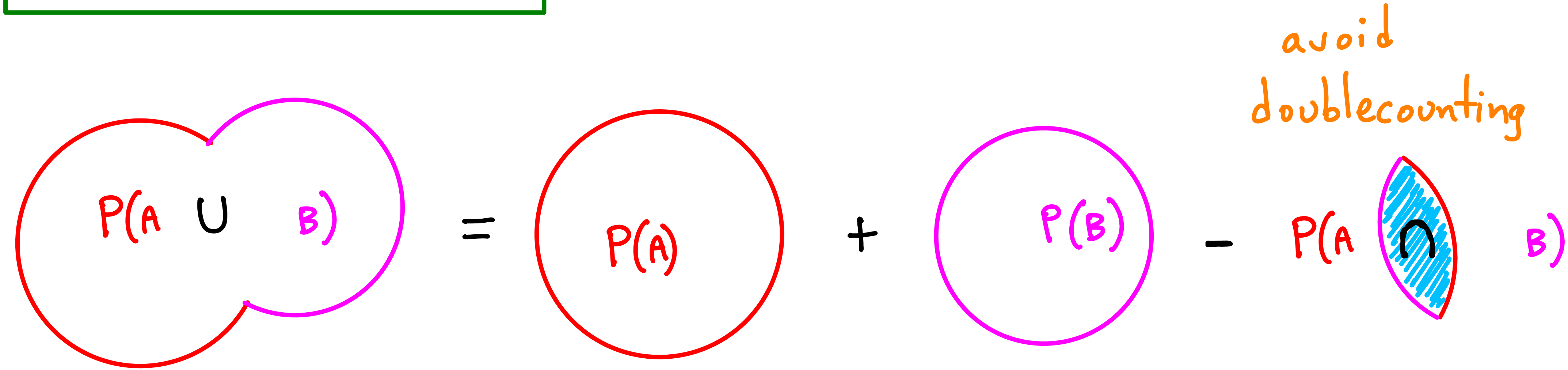


$P(A)$ $P(B)$ $P(A \cup B)$ $P(A \cap B)$





$$P(A) + P(B) = P(A \cup B) + P(A \cap B)$$



P(someone in class was born on Feb. 29)

$$P(\text{someone in class was born on Feb. 29})$$

$$= P\big[(\text{student 1 born on Feb. 29}) \cup (\text{student 2 born on Feb. 29}) \ldots$$

$$\ldots \cup (\text{student 3 born on Feb. 29}) \ldots \cup \ldots (\text{student } k \text{ born on Feb. 29})\big]$$

$P(\text{someone in class was born on Feb. 29})$

$= P\Big[(\text{student 1 born on Feb. 29}) \cup (\text{student 2 born on Feb. 29}) \dots$

$\dots \cup (\text{student 3 born on Feb. 29}) \dots \cup \dots \underbrace{(\text{student } k \text{ born on Feb. 29})}_{\sim \frac{1}{365 \cdot 4 + 1} \sim 0.07\%}\Big]$

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awful

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$$= P\Big[(\text{student 1 born on Feb. 29}) \cup (\text{student 2 born on Feb. 29}) \ldots$$

$$\ldots \cup (\text{student 3 born on Feb. 29}) \ldots \cup \ldots \underbrace{(\text{student } k \text{ born on Feb. 29})}_{\sim \frac{1}{365 \cdot 4 + 1} \sim 0.07\%}\Big]$$

awful but we could say it is $< \sum P(i)$

P(someone in class was born on Feb. 29)

$= P\Big[$(student 1 born on Feb. 29) $\cup$ (student 2 born on Feb. 29) ...

... $\cup$ (student 3 born on Feb. 29) ... $\cup$ ... $\underbrace{\text{(student } \underline{k} \text{ born on Feb. 29)}}_{\sim \frac{1}{365 \cdot 4 + 1} \sim 0.07\%}\Big]$

$\hookrightarrow$ awful but we could say it is $< \sum P(i) \sim 80 \cdot 0.07\%$

$\sim 5.6\%$

$P(\text{someone in class was born on Feb. 29})$

$= P[(\text{student 1 born on Feb. 29}) \cup (\text{student 2 born on Feb. 29}) \ldots$

$\ldots \cup (\text{student 3 born on Feb. 29}) \ldots \cup \ldots \underbrace{(\text{student } k \text{ born on Feb. 29})}_{\sim \frac{1}{365 \cdot 4 + 1} \sim 0.07\%}]$

awful but we could say it is $< \sum P(i) \sim 80 \cdot 0.07\%$
$\sim 5.6\%$

$= 1 - P(\text{nobody in class was born on Feb. 29})$

$P(\text{someone in class was born on Feb. 29})$

$= P\Big[(\text{student 1 born on Feb. 29}) \cup (\text{student 2 born on Feb. 29}) \ldots$

$\ldots \cup (\text{student 3 born on Feb. 29}) \ldots \cup \ldots \underbrace{(\text{student } k \text{ born on Feb. 29})}_{\sim \frac{1}{365\cdot 4+1} \sim 0.07\%}\Big]$

awful but we could say it is $< \sum P(i) \sim 80 \cdot 0.07\%$
$\sim 5.6\%$

$= 1 - P(\text{nobody in class was born on Feb. 29})$

$= 1 - P\Big[(\text{student 1 NOT born on Feb. 29}) \cap (\text{student 2 NOT born on Feb. 29})$

$\ldots \cap (\text{student } k \text{ NOT born on Feb. 29})\Big]$

P(someone in class was born on Feb. 29)

$= P[(\text{student 1 born on Feb. 29}) \cup (\text{student 2 born on Feb. 29}) \ldots$

$\ldots \cup (\text{student 3 born on Feb. 29}) \ldots \cup \ldots \underbrace{(\text{student } k \text{ born on Feb. 29})}_{\sim \underbrace{\frac{1}{365 \cdot 4 + 1}}_{1-\alpha} \sim 0.07\%}]$

$\hookrightarrow$ awful but we could say it is $< \sum P(i) \sim 80 \cdot 0.07\%$
$\sim 5.6\%$

$= 1 - P(\text{nobody in class was born on Feb. 29})$

$= 1 - P[(\text{student 1 NOT born on Feb. 29}) \cap (\text{student 2 NOT born on Feb. 29})$
$\ldots \cap (\text{student } k \text{ NOT born on Feb. 29})]$

$= 1 - \alpha^k$ $\qquad \alpha = P(\text{student } i \text{ NOT born on Feb. 29})$

P(someone in class was born on Feb. 29) (suppose $k=80$ students)

$= P[$(student 1 born on Feb. 29) $\cup$ (student 2 born on Feb. 29) ...

... $\cup$ (student 3 born on Feb. 29) ... $\cup$ ... $\underbrace{\text{(student } k \text{ born on Feb. 29)}}_{\sim \frac{1}{365\cdot 4+1}}]$ $\sim 0.07\%$

awful but we could say it is $< \sum P(i) \sim 80 \cdot 0.07\%$
(approximation) $\sim 5.6\%$

$\underbrace{365\cdot 4+1}$ → assuming all days equally likely & 1 leap year every 4.

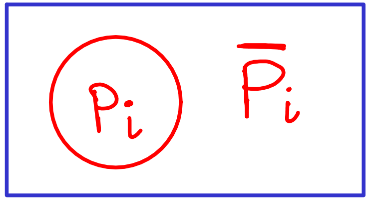
= 1 − P(nobody in class was born on Feb. 29)

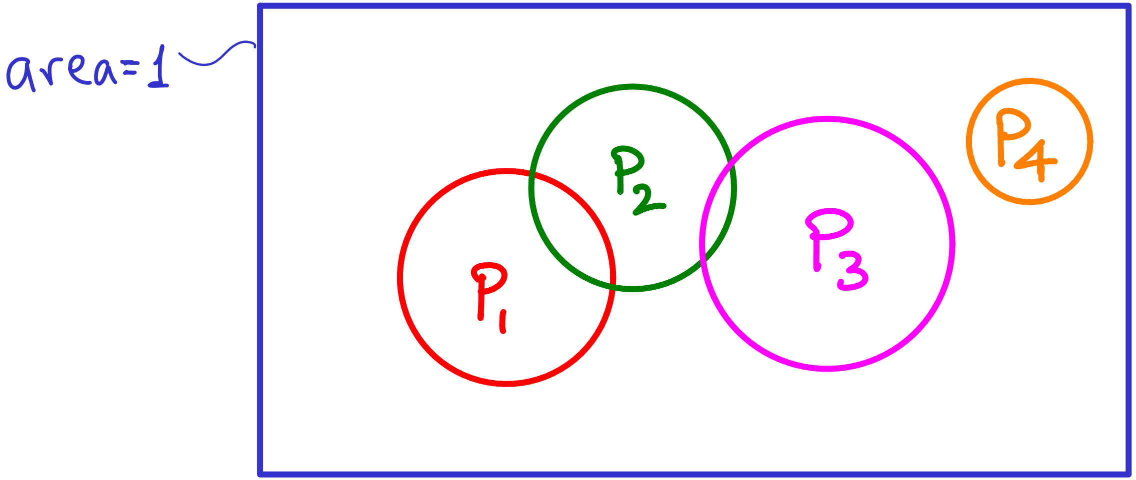
= 1 − P[(student 1 NOT born on Feb. 29) $\cap$ (student 2 NOT born on Feb. 29)
... $\cap$ (student k NOT born on Feb. 29)]

$= 1-\alpha^k \quad = 1-\left(\frac{365\cdot 4}{365\cdot 4+1}\right)^k$ exactly

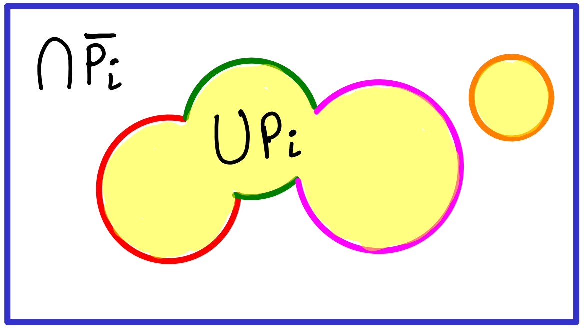
80 students ~ 5%

$$P_i + \bar{P}_i = 1 \quad \left\} \begin{array}{l} \text{area in circle } i \\ + \text{ area outside circle } i \end{array} \right. = 1$$





$$\underbrace{\bigcup P_i}_{\text{inside any circle}} = 1 - \underbrace{\bigcap \bar{P}_i}_{\text{outside every circle}}$$



$P(\geqslant 2$ people in a group of k have same birthday$)$ no Feb. 29 allowed

$P(\geqslant 2$ people in a group of k have same birthday$)$ no Feb. 29 allowed

if $k > 365$ then ?

$P(\geq 2$ people in a group of k have same birthday$)$ no Feb. 29 allowed

$\boxed{\text{if } k > 365 \text{ use pigeonhole}}$

$P(\geq 2$ people in a group of k have same birthday$)$ no Feb. 29 allowed

$\hookrightarrow P[(1,2) \cup (1,3) \cup (1,4) \ldots \cup \underbrace{(1,k)}_{1/365} \cup (2,3) \cup (2,4) \ldots \cup (2,k) \ldots \ldots \cup (k-1,k)]$

$\boxed{\text{if } k > 365 \text{ use pigeonhole}}$

$P(\geq 2 \text{ people in a group of } k \text{ have same birthday})$ no Feb. 29 allowed

$\hookrightarrow P[(1,2) \cup (1,3) \cup (1,4) \ldots \cup \underbrace{(1,k)}_{1/365} \cup (2,3) \cup (2,4) \ldots \cup (2,k) \ldots \ldots \cup (k-1,k)]$

awful

$\boxed{\text{if } k > 365 \text{ use pigeonhole}}$

$= 1 - P(\text{all } k \text{ have distinct birthdays})$

$P(\geq 2$ people in a group of k have same birthday$)$ no Feb. 29 allowed

$\hookrightarrow P[(1,2) \cup (1,3) \cup (1,4) \ldots \cup \underbrace{(1,k)}_{1/365} \cup (2,3) \cup (2,4) \ldots \cup (2,k) \ldots \ldots \cup (k-1,k)]$

awful

$\boxed{\text{if } k > 365 \text{ use pigeonhole}}$

$= 1 - P(\text{all } k \text{ have distinct birthdays})$

$\hookrightarrow P(\text{2nd person has different bday than 1st}) = \dfrac{364}{365} = P(A)$

$P(\geq 2$ people in a group of k have same birthday$)$ no Feb. 29 allowed

$\hookrightarrow P[(1,2) \cup (1,3) \cup (1,4) \ldots \cup \underbrace{(1,k)}_{1/365} \cup (2,3) \cup (2,4) \ldots \cup (2,k) \ldots \ldots \cup (k-1,k)]$

awful

$\boxed{\text{if } k > 365 \text{ use pigeonhole}}$

$= 1 - P(\text{all } k \text{ have distinct birthdays})$

$\hookrightarrow P(\text{2nd person has different bday than 1st}) = \frac{364}{365} = P(A)$

$P(\text{3rd} \ldots \ldots \ldots \text{1st \& 2nd}) = \frac{363}{365} = P(B)$

$P(\geq 2$ people in a group of k have same birthday$)$ no Feb. 29 allowed

$\hookrightarrow P[(1,2) \cup (1,3) \cup (1,4) \ldots \cup \underbrace{(1,k)}_{1/365} \cup (2,3) \cup (2,4) \ldots \cup (2,k) \ldots \ldots \cup (k-1,k)]$

awful

$\boxed{\text{if } k > 365 \text{ use pigeonhole}}$

$= 1 - P(\text{all } k \text{ have distinct birthdays})$

$\hookrightarrow P(\text{2nd person has different bday than 1st}) = \frac{364}{365} = P(A)$

$P(\text{3rd} \ldots \ldots \underbrace{\text{1st \& 2nd}) = \frac{363}{365}}_{\text{assuming?}} = P(B)$

$P(\geq 2$ people in a group of k have same birthday$)$ no Feb. 29 allowed

$\hookrightarrow P[(1,2) \cup (1,3) \cup (1,4) \ldots \cup \underbrace{(1,k)}_{1/365} \cup (2,3) \cup (2,4) \ldots \cup (2,k) \ldots \ldots \cup (k-1,k)]$

awful

$\boxed{\text{if } k > 365 \text{ use pigeonhole}}$

$= 1 - P(\text{all } k \text{ have distinct birthdays})$

$\hookrightarrow P(\text{2nd person has different bday than 1st}) = \dfrac{364}{365} = P(A)$

$P(\text{3rd} \ldots \ldots \ldots \underbrace{\text{1st \& 2nd}}_{\text{assuming 1st \& 2nd differ}}) = \dfrac{363}{365} = P(B)$

this is actually "conditional probability" which will be covered next time.

$P(\geq 2$ people in a group of k have same birthday$)$ no Feb. 29 allowed

$P[(1,2) \cup (1,3) \cup (1,4) \ldots \cup \underbrace{(1,k)}_{1/365} \cup (2,3) \cup (2,4) \ldots \cup (2,k) \ldots \ldots \cup (k-1,k)]$

awful

$\boxed{\text{if } k > 365 \text{ use pigeonhole}}$

$= 1 - P(\text{all } k \text{ have distinct birthdays})$

$P(\text{2nd person has different bday than 1st}) = \frac{364}{365} = P(A)$

$P(\text{3rd} \ldots \ldots \ldots \text{1st \& 2nd}) = \frac{363}{365} = P(B)$

$P(\text{4th} \ldots \ldots \ldots (1-3)) = \frac{362}{365} = P(C)$

etc

$P(\geq 2$ people in a group of k have same birthday$)$ no Feb. 29 allowed

$\hookrightarrow P[(1,2) \cup (1,3) \cup (1,4) \ldots \cup \underbrace{(1,k)}_{1/365} \cup (2,3) \cup (2,4) \ldots \cup (2,k) \ldots \ldots \cup (k-1,k)]$

awful

if $k > 365$ use pigeonhole

$= 1 - P($all k have distinct birthdays$)$

$\hookrightarrow P($2nd person has different bday than 1st$) = \frac{364}{365} = P(A)$

$P($3rd . . . . 1st & 2nd$) = \frac{363}{365} = P(B)$

$P($4th . . . . $(1-3)) = \frac{362}{365} = P(C)$

etc

$= 1 - [P(A) \cap P(B) \cap P(C) \ldots]$

$P(\geq 2$ people in a group of k have same birthday$)$ no Feb. 29 allowed

$\hookrightarrow P[(1,2) \cup (1,3) \cup (1,4) \ldots \cup \underbrace{(1,k)}_{1/365} \cup (2,3) \cup (2,4) \ldots \cup (2,k) \ldots \ldots \cup (k-1,k)]$

awful

$\boxed{\text{if } k > 365 \text{ use pigeonhole}}$

$= 1 - P(\text{all } k \text{ have distinct birthdays})$

$\hookrightarrow P(\text{2nd person has different bday than 1st}) = \frac{364}{365} = P(A)$

$P(\text{3rd} \ldots \ldots \ldots \text{1st \& 2nd}) = \frac{363}{365} = P(B)$

$P(\text{4th} \ldots \ldots \ldots (1-3)) = \frac{362}{365} = P(C)$

etc

$= 1 - [P(A) \cap P(B) \cap P(C) \ldots] = 1 - \dfrac{\frac{365!}{(365-k)!}}{365^k} = 1 - \dfrac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k}$

P(≥ 2 people in a group of k have same birthday)

$$= 1 - \frac{\frac{365!}{(365-k)!}}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k}$$

$P(\geqslant 2 \text{ people in a group of } k \text{ have same birthday})$

$$= 1 - \frac{\frac{365!}{(365-k)!}}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}$$

$P(\geq 2 \text{ people in a group of } k \text{ have same birthday})$

$$= 1 - \frac{\frac{365!}{(365-k)!}}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}$$

$k = 2 \rightarrow P \sim 0{,}27\% \ \left(\frac{1}{365}\right)$

$P(\geq 2 \text{ people in a group of } k \text{ have same birthday})$

$$= 1 - \frac{\frac{365!}{(365-k)!}}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}$$

$k=2 \rightarrow P \sim 0{,}27\% \quad \left(\frac{1}{365}\right)$

$k=4 \rightarrow P \sim 1.64\%$

$P(\geqslant 2 \text{ people in a group of } k \text{ have same birthday})$

$$= 1 - \frac{\frac{365!}{(365-k)!}}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}$$

$k = 2 \rightarrow P \sim 0{,}27\% \quad \left(\frac{1}{365}\right)$

$k = 4 \rightarrow P \sim 1.64\%$

$k = 23 \rightarrow P \sim 50.73\%$

$P(\geq 2$ people in a group of k have same birthday$)$

$$= 1 - \frac{\frac{365!}{(365-k)!}}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}$$

$k=2 \rightarrow P \sim 0.27\%$ $\left(\frac{1}{365}\right)$

$k=4 \rightarrow P \sim 1.64\%$

$k=23 \rightarrow P \sim 50.73\%$

$k=30 \rightarrow P \sim 70.6\%$

$P(\geq 2$ people in a group of k have same birthday$)$

$$= 1 - \frac{\frac{365!}{(365-k)!}}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}$$

$k = 2 \rightarrow P \sim 0{,}27\ \%$ $\left(\frac{1}{365}\right)$

$k = 4 \rightarrow P \sim 1{,}64\ \%$

$k = 23 \rightarrow P \sim 50{.}73\ \%$

$k = 30 \rightarrow P \sim 70{,}6\ \%$

$k = 70 \rightarrow P \sim 99{.}9\ \%$

$P(\geq 2$ people in a group of k have same birthday$)$

$$= 1 - \frac{\frac{365!}{(365-k)!}}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}$$

$k = 2 \rightarrow P \sim 0{,}27\%$ $\left(\frac{1}{365}\right)$

$k = 4 \rightarrow P \sim 1.64\%$

$k = 23 \rightarrow P \sim 50.73\%$

$k = 30 \rightarrow P \sim 70.6\%$

$k = 70 \rightarrow P \sim 99.9\%$

$k \sim 116 \rightarrow P \sim 1 - \frac{1}{10^9}$

$P(\geqslant 2$ people in a group of k have same birthday$)$

$$= 1 - \frac{\frac{365!}{(365-k)!}}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}$$

$k = 2 \rightarrow P \sim 0{,}27\,\%$ $\left(\frac{1}{365}\right)$

$k = 4 \rightarrow P \sim 1.64\,\%$

$k = 23 \rightarrow P \sim 50.73\,\%$

$k = 30 \rightarrow P \sim 70.6\,\%$

$k = 70 \rightarrow P \sim 99.9\,\%$

$k \sim 116 \rightarrow P \sim 1 - \frac{1}{10^9}$

$k = 300 \rightarrow P \sim 1 - \frac{1}{10^{80}}$

$P(\geq 2 \text{ people in a group of } k \text{ have same birthday})$

$$= 1 - \frac{\frac{365!}{(365-k)!}}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}$$

$k = 2 \rightarrow P \sim 0.27\%$ $\left(\frac{1}{365}\right)$

$k = 4 \rightarrow P \sim 1.64\%$

$k = 23 \rightarrow P \sim 50.73\%$

$k = 30 \rightarrow P \sim 70.6\%$

$k = 70 \rightarrow P \sim 99.9\%$

$k \sim 116 \rightarrow P \sim 1 - \frac{1}{10^9}$

$k = 300 \rightarrow P \sim 1 - \frac{1}{10^{80}}$

($10^{80} \sim$ # atoms in universe)

$(k > 365 \rightarrow P = 1)$

$P(\geq 2 \text{ people in a group of } k \text{ have same birthday})$

Didn't cover this slide in class.
It explains bet #2.

$$1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k}$$

For the bet involving k people born in a month w/ 30 days

substitute $365 \rightarrow 30$

$$(k=10) \qquad 1 - \frac{30 \cdot 29 \cdot 28 \cdot \ \cdots \ \cdot 23 \cdot 22 \cdot 21}{30^{10}} \sim 0.815$$