A bet: I randomly select half of the class...
A bet: I randomly select half of the class...

If any 2 people in that group have the same birthday you give me a dollar.
A bet: I randomly select half of the class...

If any 2 people in that group have the same birthday you give me a dollar.

If no birthday match is found I give you a dollar
Another bet:

I randomly select 10 people born in the same month.

Same deal as before.
Another bet:

I randomly select 10 people born in the same month.

Same deal as before.

My chances of winning: >80%
One last bet?

I randomly select 7 people.

If any 2 people in that group have birthdays within a week of each other...
One last bet?

I randomly select 7 people

If any 2 people in that group have birthdays within a week of each other...

I win 60% of the time. (52% if within 6 days)
DISCRETE PROBABILITY
Roll a die ... Possible outcomes: \{1, 2, 3, 4, 5, 6\}
Discrete Probability

Roll a die ... Possible outcomes: \{1, 2, 3, 4, 5, 6\}

Each has a probability: \{\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\}
DISCRETE PROBABILITY

Roll a die ... Possible outcomes: \{1, 2, 3, 4, 5, 6\}

Each has a probability: \{\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\}

Sample space

Sum to 1
DISCRETE PROBABILITY

Roll a die ... Possible outcomes: \{1, 2, 3, 4, 5, 6\}
Each has a probability: \{\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\}
Sample space
Sum to 1

Roll 2 dice ... Sample space \rightarrow ?

\[\text{a} \quad \text{b}\]
DISCRETE PROBABILITY

Roll a die ... Possible outcomes: \( \{1, 2, 3, 4, 5, 6\} \)

Each has a probability: \( \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6} \) \( \text{sum to 1} \)

Roll 2 dice ... Sample space \( \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \)
\( (2,1), (2,2), \ldots \) \( \vdots \)
\( (6,1), (6,2), \ldots \) \( \ldots (6,6) \) \}
Roll 2 indistinguishable dice...

Sample space → \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,3), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6), (4,4), (4,5), (4,6), (5,5), (5,6), (6,6)\}
Roll 2 indistinguishable dice...

Sample space → \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
                 (2,2), (2,3), (2,4), (2,5), (2,6),
                 (3,3), (3,4), (3,5), (3,6),
                 (4,4), (4,5), (4,6),
                 (5,5), (5,6),
                 (6,6)\}

prob. = ?
Roll 2 indistinguishable dice...

Sample space $\mathcal{S} = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,3), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6), (4,4), (4,5), (4,6), (5,5), (5,6), (6,6)\}$

- If $(a,a)$ then $\frac{1}{36}$
- If $(a,b)$ then $\frac{2}{36}$, $a \neq b$
Roll 2 indistinguishable dice...

Sample space $\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,3), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6), (4,4), (4,5), (4,6), (5,5), (5,6), (6,6)\}$

- If $(a,a)$ then $\frac{1}{36}$

- If $(a,b)$ then $\frac{2}{36}$ if $a \neq b$

$6 \cdot \frac{1}{36} = \frac{6}{36} \rightarrow 1$

$15 \cdot \frac{2}{36} = \frac{30}{36}$
Roll a die

\[ P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6} \]
Roll a die

\[ P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6} \]

\[ P(\text{roll even}) = P(\{2, 4, 6\}) = P(2) + P(4) + P(6) = \frac{1}{2} \]
Roll a die

\[ P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6} \]

\[ P(\text{roll even}) = P(\{2, 4, 6\}) = P(2) + P(4) + P(6) = \frac{1}{2} \]

Roll 2 dice...

\[ P(\text{sum = 7}) \]
Roll a die

\[ P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6} \]

\[ P(\text{roll even}) = P(\{2, 4, 6\}) = P(2) + P(4) + P(6) = \frac{1}{2} \]

Roll 2 dice...

\[ P(\text{sum = 7}) = P(\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}) \]
Roll a die:

\[ P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6} \]

\[ P(\text{roll even}) = P(\{2, 4, 6\}) = P(2) + P(4) + P(6) = \frac{1}{2} \]

Roll 2 dice...

\[ P(\text{sum} = 7) = P(\{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}) \]

\[ = P(1,6) + P(2,5) + P(3,4) + P(4,3) + P(5,2) + P(6,1) \]
Roll a die:

- \( P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6} \)
- \( P(\text{roll even}) = P(\{2, 4, 6\}) = P(2) + P(4) + P(6) = \frac{1}{2} \)

Roll 2 dice:

- \( P(\text{sum} = 7) = P(\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}) \)
- \( = P(1,6) + P(2,5) + P(3,4) + P(4,3) + P(5,2) + P(6,1) \)
- \( = 6 \cdot \frac{1}{36} = \frac{1}{6} \)
Roll a die

\[ P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6} \]

\[ P(\text{roll even}) = P(\{2, 4, 6\}) = P(2) + P(4) + P(6) = \frac{1}{2} \]

Roll 2 dice...

\[ P(\text{sum} = 7) = P(\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}) \]

\[ = P(1,6) + P(2,5) + P(3,4) + P(4,3) + P(5,2) + P(6,1) \]

\[ = 6 \cdot \frac{1}{36} = \frac{1}{6} \]

\[ P(\text{sum} = 7) = \text{?} \]
Roll a die

$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$

$P(\text{roll even}) = P(\{2, 4, 6\}) = P(2) + P(4) + P(6) = \frac{1}{2}$

Roll 2 dice...

$P(\text{sum} = 7) = P(\{1,6\}, \{2,5\}, \{3,4\}, \{4,3\}, \{5,2\}, \{6,1\})$

$= P(1,6) + P(2,5) + P(3,4) + P(4,3) + P(5,2) + P(6,1)$

$= 6 \cdot \frac{1}{36} = \frac{1}{6}$

$P(\text{sum} = 7) = P(\{1,6\}, \{2,5\}, \{3,4\})$
Roll a die

\[ P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6} \]

\[ P(\text{roll even}) = P(\{2, 4, 6\}) = P(2) + P(4) + P(6) = \frac{1}{2} \]

Roll 2 dice...

\[ P(\text{sum} = 7) = P(\{1,6\}, (2,5), (3,4), (4,3), (5,2), (6,1)\}) \]

\[ = P(1,6) + P(2,5) + P(3,4) + P(4,3) + P(5,2) + P(6,1) \]

\[ = 6 \cdot \frac{1}{36} = \frac{1}{6} \]

\[ P(\text{sum} = 7) = P(\{1,6\}, \{2,5\}, \{3,4\}) = 3 \cdot \frac{2}{36} = \frac{1}{6} \]
Roll 10 dice (or 1 die 10 times)
Roll 10 dice (or 1 die 10 times)

Sample space size: ?
Roll 10 dice (or 1 die 10 times)

Sample space size: $6^{10}$ > 60 million
Roll 10 dice (or 1 die 10 times)

Sample space size: $6^{10} > 60$ million

$P(\text{observe no 1's})$?
Roll 10 dice (or 1 die 10 times)

Sample space size: $6^{10}$ > 60 million

$P$(observe no 1's) ?

How many outcomes have no 1's?
Roll 10 dice (or 1 die 10 times)

Sample space size: \(6^{10}\) > 60 million

\[ P(\text{observe no 1's}) \]

How many outcomes have no 1's? \(5^{10}\)
Roll 10 dice (or 1 die 10 times)

Sample space size: $6^{10} > 60$ million

$P(\text{observe no 1's}) = \left(\frac{5}{6}\right)^{10}$

How many outcomes have no 1's? $5^{10}$
Roll 10 dice (or 1 die 10 times)

Sample space size: $6^{10} > 60$ million

$P(\text{observe no 1's})$?

How many outcomes have no 1's? $\rightarrow 5^{10}$

Or, say that each roll/die is independent

so for each roll, $P(\text{no 1}) = \frac{5}{6} \Rightarrow (\frac{5}{6})^{10}$

\text{\{to be discussed further\}}
Poker: 52 cards (4 x 13 types); select 5.
Poker: 52 cards (4x13 types); select 5.

\[ P(4 \text{ of a kind}) = P(4 \text{ of 5 are of same type}) \]

e.g. 3,3,3,3,7 or 8,8,8,J,8

= ?
Poker: 52 cards (4 x 13 types); select 5.

\[ P(4 \text{ of a kind}) = P(4 \text{ of 5 are of same type}) \]

e.g. 3,3,3,3,7 or 8,8,8,J,8

ans: \[
\frac{\# \text{4-of-a-kinds}}{\# \text{possible outcomes}}
\]
Poker: 52 cards (4 \times 13 types); select 5.

\[
P(4 \text{ of a kind}) = P(4 \text{ of 5 are of same type})
\]

e.g. 3,3,3,3,7 or 8,8,8,J,8

ans: \[
\frac{\text{# 4-of-a-kinds}}{\text{#possible outcomes}} \rightarrow ? \quad \text{(order doesn't matter)}
\]
Poker: 52 cards (4x13 types); select 5.

\[ P(4 \text{ of a kind}) = P(4 \text{ of 5 are of same type}) \]

e.g. 3,3,3,3,7 or 8,8,8,J,8

ans: \[
\frac{\text{# 4-of-a-kinds}}{\text{# possible outcomes}} \rightarrow (\binom{52}{5}) \quad \text{order doesn't matter}
\]

what if it did? \rightarrow \text{hw}
Poker: 52 cards (4 x 13 types); select 5.

\[ P(4 \text{ of a kind}) = P(4 \text{ of 5 are of same type}) \]

e.g. 3,3,3,3,7 or 8,8,8,J,8

\[ \frac{\# 4-\text{o}l-a-kinds}{\# \text{possible outcomes}} \rightarrow ? \]

\[ \binom{52}{5} \] order doesn't matter
Poker: 52 cards (4 x 13 types) ; select 5.

\[ P(4 \text{ of a kind}) = P(4 \text{ of 5 are of same type}) \]

\[ \text{e.g. } 3,3,3,3,7 \text{ or } 8,8,8,J,8 \]

\[ \text{ans: } \frac{\# \text{ 4-of-a-kinds}}{\# \text{possible outcomes}} \rightarrow \frac{\text{AAAAA}}{\text{KKKK}} \leq 13 \text{ types} \times 48 \text{ choices of remaining card} \]

\[ (\frac{52}{5}) \geq \text{order doesn’t matter} \]
Poker: 52 cards (4 x 13 types); select 5.

\[ P(4 \text{ of a kind}) = P(4 \text{ of 5 are of same type}) \]

\[ \text{e.g. } 3,3,3,3,7 \text{ or } 8,8,8,J,8 \]

\[ \text{ans:} \quad \frac{\# 4-o-f-a-kinds}{\# \text{possible outcomes}} \quad \rightarrow \quad \begin{cases} \text{13 types} \times 48 \text{ choices of remaining card} \\ \frac{2222}{KKKK} \end{cases} \]

\[ = \frac{13 \cdot 48}{\binom{52}{5}} = \frac{1}{4165} \sim 0.00024 \]
Probability visualization

Sample space $\rightarrow$ area = 1
Probability visualization

Sample space → area = 1
Probability Visualization

Sample space $\rightarrow$ area = 1

$P(\text{outcome } i) = \text{area}(i)$

Example: roll one die $\rightarrow$ all areas equal $= \frac{1}{6}$
**Probability Visualization**

Sample space → area = 1

\[ P(\text{outcome } i) = \text{area}(i) \]

Example: roll one die → all areas equal = \( \frac{1}{6} \)

\[ P(\text{event}) = \text{sum of appropriate areas} \]

E.g. \[ P(\text{roll prime \# OR even \#}) \]

Which outcomes?
**Probability Visualization**

Sample space → area = 1

\[ P(\text{outcome } i) = \text{area}(i) \]

Example: roll one die → all areas equal = \( \frac{1}{6} \)

\[ P(\text{event}) = \text{sum of appropriate areas} \]

E.g. \( P(\text{roll prime \# or even \#}) \)

\[ 2, 3, 5 \]

\[ 2, 4, 6 \]

\[ P(\text{prime}) = ? \]

\[ P(\text{even}) = ? \]
Probability Visualization

Sample space → area = 1

\[ P(\text{outcome } i) = \text{area}(i) \]

Example: Roll one die → all areas equal = \( \frac{1}{6} \)

\[ P(\text{event}) = \text{sum of appropriate areas} \]

E.g. \( P(\text{roll prime# or even#}) \)

\[ \begin{align*} 
\text{Prime:} & \quad 2, 3, 5 \\
\text{Even:} & \quad 2, 4, 6 \\
P(\text{prime}) & = \frac{3}{6} \\
P(\text{even}) & = \frac{3}{6}
\end{align*} \]
**Probability Visualization**

Sample space \( \rightarrow \text{area} = 1 \)

\[
P(\text{outcome } i) = \text{area}(i)
\]

**Example:** Roll one die \( \rightarrow \text{all areas equal} = \frac{1}{6} \)

\[
P(\text{event}) = \text{sum of appropriate areas}
\]

**e.g.** \[P(\text{roll prime\# or even\#}) = \left\{ \begin{array}{l}
2, 3, 5 \quad \text{P(prime)} = \frac{3}{6} \\
2, 4, 6 \quad \text{P(even)} = \frac{3}{6}
\end{array} \right\} = \frac{5}{6}
\]

Avoid double-counting

\( \text{NOT } \frac{3}{6} + \frac{3}{6} \)
Roll prime #
2, 3, 5
roll even #: 2, 4, 6
Roll prime #: 2, 3, 5

Even #: 2, 4, 6
\[ P(\text{roll prime\# or even\#}) \rightarrow P(A \cup B) \]
\[ P(\text{roll prime\# or even\#}) \rightarrow P(A \cup B) \]

All probability space
$P(\text{roll prime # or even #}) \rightarrow P(A \cup B)$

- Prime numbers: 2, 3, 5
- Even numbers: 2, 4, 6

- All probability space

- $P(A)$, $P(B)$
- $P(A \cup B)$

"OR"
\[ P(\text{roll prime } \# \text{ OR even } \#) \rightarrow P(A \cup B) \]

- Events:
  - \( A \): Roll prime numbers 2, 3, 5
  - \( B \): Roll even numbers 2, 4, 6

Venn Diagram:
- \( P(A) \) and \( P(B) \)
- \( P(A \cap B) \)

Probability Space:
- All possible outcomes

Geometric Interpretation:
- \( P(A \cup B) \) includes both \( P(A) \) and \( P(B) \), with overlap for \( P(A \cap B) \)
$P(A)$  $P(B)$  $\text{vs}$  $P(A \cup B)$  $P(A \cap B)$
$P(A)$  $P(B)$  $P(A \cup B)$  $P(A \cap B)$
\[ P(A) + P(B) = P(A \cup B) + P(A \cap B) \]
P(someone in class was born on Feb. 29)
\[ P(\text{someone in class was born on Feb. 29}) = P\left[ (\text{student 1 born on Feb. 29}) \cup (\text{student 2 born on Feb. 29}) \cup \cdots \cup (\text{student k born on Feb. 29}) \right] \]
\[
P(\text{someone in class was born on Feb. 29}) = P\left[\left(\text{student 1 born on Feb. 29}\right) \cup \left(\text{student 2 born on Feb. 29}\right) \ldots \right]
\]
\[
\ldots \cup \left(\text{student 3 born on Feb. 29}\right) \ldots \cup \ldots \left(\text{student } k \text{ born on Feb. 29}\right)
\]
\[
\sim \frac{1}{365 \cdot 4 + 1} \sim 0.07\%
\]
\[ P(\text{someone in class was born on Feb. 29}) = P\left(\text{student 1 born on Feb. 29}\right) \cup \ldots \cup \text{student 3 born on Feb. 29} \ldots \cup \ldots \cup \text{student k born on Feb. 29} \]

\[ \sim \frac{1}{365 \cdot 4 + 1} \sim 0.07\% \]

\( \rightarrow \) awful
\[ P(\text{someone in class was born on Feb. 29}) = P\left(\bigcup \text{student 1 born on Feb. 29} \right) \cup \cdots \cup \text{student 2 born on Feb. 29} \cup \cdots \cup \text{student k born on Feb. 29}\]

\[ \sim \frac{1}{365 \cdot 4 + 1} \sim 0.07\% \]

\[ \text{awful but we could say it is } < \sum P(i) \]
\[ P(\text{someone in class was born on Feb. 29}) \]

\[ = P\left( \text{student 1 born on Feb. 29) U (student 2 born on Feb. 29) ...} \right. \]
\[ \left. \ldots \text{U (student 3 born on Feb. 29) ... U ... (student k born on Feb. 29)} \right] \]

\[ \leq \text{awful but we could say it is } \sum P(i) \approx 80 \cdot 0.07\% \]
\[ \approx 5.6\% \]

\[ \approx \frac{1}{365 \cdot 4 + 1} \approx 0.07\% \]
\[ P(\text{someone in class was born on Feb. 29}) \]
\[ = P\left(\text{(student 1 born on Feb. 29)} \cup \text{(student 2 born on Feb. 29)} \ldots \cup \text{(student 3 born on Feb. 29)} \ldots \cup \ldots \text{(student } k \text{ born on Feb. 29)\right) \]

\[ \leq \text{awful but we could say it is } < \sum P(i) \approx 0.07\% \]
\[ \approx \frac{1}{365 \cdot 4 + 1} \approx 0.07\% \]

\[ = 1 - P(\text{nobody in class was born on Feb. 29}) \]
\[ P(\text{someone in class was born on Feb. 29}) \]

\[ = P\left(\text{student 1 born on Feb. 29}\right) \cup \text{(student 2 born on Feb. 29)} \ ...
\]

\[ \cup \text{(student 3 born on Feb. 29)} \ ...
\]

\[ \cup \text{(student k born on Feb. 29)} \]

\[ \leftarrow \text{awful but we could say it is } \sum P(i) \approx 80 \cdot 0.07\% \approx 5.6\%
\]

\[ \approx \frac{1}{365 \cdot 4 + 1} \approx 0.07\%
\]

\[ = 1 - P(\text{nobody in class was born on Feb. 29})
\]

\[ = 1 - P\left(\text{student 1 NOT born on Feb. 29}\right) \cap \text{(student 2 NOT born on Feb. 29)}
\]

\[ \cap \text{(student k NOT born on Feb. 29)} \]
\[ P(\text{someone in class was born on Feb. 29}) \]
\[ = P[(\text{student 1 born on Feb. 29}) \cup (\text{student 2 born on Feb. 29}) \ldots \cup (\text{student 3 born on Feb. 29}) \ldots \cup \ldots (\text{student } k \text{ born on Feb. 29})] \]

\[ \leq \text{awful} \quad \text{but we could say it is } < \sum P(i) \sim 0.07\% \sim 5.6\% \]

\[ = 1 - P(\text{nobody in class was born on Feb. 29}) \sim \frac{1}{365 \cdot 4 + 1} \sim 0.07\% \]

\[ = 1 - P[(\text{student 1 NOT born on Feb. 29}) \cap (\text{student 2 NOT born on Feb. 29}) \ldots \cap (\text{student } k \text{ NOT born on Feb. 29})] \]

\[ = 1 - \alpha^k \quad \alpha = P(\text{student i NOT born on Feb. 29}) \]
\[ P(\text{someone in class was born on Feb. 29}) \quad \text{(suppose k=80 students)} \]
\[ = P\left[ (\text{student 1 born on Feb. 29}) \cup (\text{student 2 born on Feb. 29}) \cup \ldots \cup (\text{student k born on Feb. 29}) \right] \]

awful but we could say it is \( \approx \sum P(i) \approx 0.07\% \)
(approximation) \( \approx 5.6\% \)

\[ = 1 - P(\text{nobody in class was born on Feb. 29}) \]
\[ = 1 - P\left[ (\text{student 1 not born on Feb. 29}) \cap (\text{student 2 not born on Feb. 29}) \cap \ldots \cap (\text{student k not born on Feb. 29}) \right] \]
\[ = 1 - \alpha^k = 1 - \left( \frac{365 - 4}{365 \cdot 4 + 1} \right)^k \quad \text{exactly} \]

80 students \( \approx 5\% \)
\[ P_i + \overline{P_i} = 1 \] \\
\text{area in circle } i + \text{area outside circle } i = 1

\[ \bigcup P_i = 1 - \bigcap \overline{P_i} \]

inside any circle

outside every circle
\[ P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \] no Feb. 29 allowed
\[ P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \]

no Feb.29 allowed

if \( k > 365 \) then ?
\( P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \) no Feb.29 allowed

if \( k \geq 365 \) use pigeonhole
\[ P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \]

\[ \sum_{i=2}^{k} P[(1,i) \cup (1,i+1) \cup \cdots \cup (1,k) \cup (2,i) \cup (2,i+1) \cup \cdots \cup (2,k) \cup \cdots \cup (k-1,i)] \]

\[ \leq \frac{1}{365} \]

If \( k > 365 \), use pigeonhole principle.
\[ P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \quad \text{no Feb.29 allowed} \]

\[ \approx P\left( (1,2) \cup (1,3) \cup (1,4) \cdots \cup (1,k) \cup (2,3) \cup (2,4) \cdots \cup (2,k) \cdots \cdots \cup (k-1,k) \right) \]

awful

\[ \approx \frac{1}{365} \]

if \( k \geq 365 \) use pigeonhole

\[ = 1 - P(\text{all } k \text{ have distinct birthdays}) \]
\[ P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \quad \text{no Feb.29 allowed} \]

\[ \sum P[(1,2) \cup (1,3) \cup (1,4) \ldots \cup (1,k) \cup (2,3) \cup (2,4) \ldots \cup (2,k) \ldots \ldots \cup (k-1,k)] \]

\[ \frac{1}{365} \]

awful

if \(k > 365\) use pigeonhole

\[ = 1 - P(\text{all } k \text{ have distinct birthdays}) \]

\[ P(\text{2nd person has different bday than 1st}) = \frac{364}{365} = P(A) \]
$P(\geq 2 \text{ people in a group of } k \text{ have same birthday})$  no Feb.29 allowed

$= P[(1,2) U (1,3) U (1,4) \cdots U (1,k) U (2,3) U (2,4) \cdots U (2,k) \cdots \cdots U (k-1,k)] \\
\frac{1}{365}$

awful

if $k>365$ use pigeonhole

$= 1 - P(\text{all } k \text{ have distinct birthdays})$

$= P(2\text{nd person has different bday than 1st}) = \frac{364}{365} = P(A)$

$P(3\text{rd } \cdots \cdots \cdots \cdots \cdots \cdots \text{1st & 2nd}) = \frac{363}{365} = P(B)$
\[
P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \quad \text{no Feb.}29 \text{ allowed}
\]
\[
P[(1,2) \cup (1,3) \cup (1,4) \ldots \cup (1,k) \cup (2,3) \cup (2,4) \ldots \cup (2,k) \ldots \ldots \cup (k-1,k)]
\]
\[
\frac{1}{365}
\]
\[
= 1 - P(\text{all } k \text{ have distinct birthdays})
\]
\[
P(\text{2nd person has different bday than 1st}) = \frac{364}{365} = P(A)
\]
\[
P(\text{3rd \ldots \ldots \ldots \ldots \ldots \ldots \ldots\ 1st \& 2nd}) = \frac{363}{365} = P(B)
\]
if \(k \geq 365\) use pigeonhole

awful
\[ P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \quad \text{no Feb.29 allowed} \]

\[ \leq P[(1,2) \cup (1,3) \cup (1,4) \ldots \cup (1,k) \cup (2,3) \cup (2,4) \ldots \cup (2,k) \ldots \ldots \cup (k-1,k)] \]

\[ \approx \frac{1}{365} \]

if \( k > 365 \) use pigeonhole

\[ = 1 - P(\text{all } k \text{ have distinct birthdays}) \]

\[ \leq P(\text{2nd person has different bday than 1st}) = \frac{364}{365} = P(A) \]

\[ P(\text{3rd \ldots 1st & 2nd}) = \frac{363}{365} = P(B) \]

assuming 1st & 2nd differ

this is actually "conditional probability" which will be covered next time.
\[ P(\text{\# of people in a group of } k \text{ have same birthday}) \quad \text{no Feb.29 allowed} \]

\[ \mathbb{P}\left[(1,2) \cup (1,3) \cup (1,4) \ldots \cup (1,k) \cup (2,3) \cup (2,4) \ldots \cup (2,k) \ldots \ldots \cup (k-1,k)\right] \]

awful

\[ \approx \frac{1}{365} \]

If \( k > 365 \) use pigeonhole

\[ = 1 - P(\text{all } k \text{ have distinct birthdays}) \]

\[ P(\text{2nd person has different bday than 1st}) = \frac{364}{365} = P(A) \]

\[ P(\text{3rd ... } \ldots \ldots \ldots \ldots \ldots \text{ 1st & 2nd}) = \frac{363}{365} = P(B) \]

\[ P(\text{4th ... } \ldots \ldots \ldots \ldots \ldots (1-3)) = \frac{362}{365} = P(C) \]

etc
\[ P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \quad \text{no Feb.29 allowed} \]

\[ \Phi P[(1,2) \cup (1,3) \cup (1,4) \ldots \cup (1,k) \cup (2,3) \cup (2,4) \ldots \cup (2,k) \ldots \ldots \cup (k-1,k)] \]

\[ \approx \frac{1}{365} \]

\[ = 1 - P(\text{all } k \text{ have distinct birthdays}) \]

\[ = 1 - P(\text{2nd person has different bday than 1st}) \]

\[ = 1 - P(\text{3rd person has different bday than 1st & 2nd}) \]

\[ = 1 - P(\text{4th person has different bday than 1st - 3rd}) \]

\[
\leq 1 - \left[ P(A) \cap P(B) \cap P(C) \ldots \right]
\]

\[ \text{if } k > 365 \text{ use pigeonhole} \]
\( P(\text{\geq 2 people in a group of } k \text{ have same birthday}) \) no Feb. 29 allowed

\[
P \left[ (1,2) \cup (1,3) \cup (1,4) \ldots \cup (1,k) \cup (2,3) \cup (2,4) \ldots \cup (2,k) \ldots \ldots \cup (k-1,k) \right] \sim \frac{1}{365}
\]

awful

\[
= 1 - P(\text{all } k \text{ have distinct birthdays})
\]

\[
P(2\text{nd person has different bday than 1st}) = \frac{364}{365} = P(A)
\]

\[
P(3\text{rd } \ldots \ldots \ldots \ 1\text{st } \& \ 2\text{nd}) = \frac{363}{365} = P(B)
\]

\[
P(4\text{th } \ldots \ldots \ldots \ (1-3)) = \frac{362}{365} = P(C)
\]

dec

\[
= 1 - \left[ P(A) \land P(B) \land P(C) \ldots \right] = 1 - \frac{365!}{(365-k)!} \frac{1}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \ldots (363-k+1)}{365^k}
\]

if \( k > 365 \) use pigeonhole
\[ P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \]

\[ = 1 - \frac{365!}{(365-k)!} \cdot \frac{1}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k} \]
P(\geq 2 \text{ people in a group of } k \text{ have same birthday})

= 1 - \frac{365!}{(365-k)!365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}
\[
P(\geq 2 \text{ people in a group of } k \text{ have same birthday})
\]
\[
= 1 - \frac{365!}{(365-k)!} \cdot \frac{1}{365^k}
= 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k}
= 1 - \frac{(365)_k}{365^k}
\]

\[k = 2 \quad \rightarrow \quad P \sim 0.27\% \quad \left(\frac{1}{365}\right)\]
\[
P(\geq 2 \text{ people in a group of } k \text{ have same birthday})
= 1 - \frac{365!}{(365-k)!} \cdot \frac{1}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}
\]

\[k = 2 \quad \Rightarrow \quad P \approx 0.27\% \quad \left(\frac{1}{365}\right)\]

\[k = 4 \quad \Rightarrow \quad P \approx 1.64\%\]
The probability that \( k \) people in a group of \( k \) have the same birthday is:

\[
P(\text{\textgreater} 2 \text{ people in a group of } k \text{ have same birthday}) = 1 - \frac{365!}{(365-k)! \cdot 365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k}
\]

\[
= 1 - \frac{(365)_k}{365^k}
\]

For \( k = 2 \), \( P \approx 0.27\% \) \( \left( \frac{1}{365} \right) \)

For \( k = 4 \), \( P \approx 1.64\% \)

For \( k = 23 \), \( P \approx 50.73\% \)
$P(\geq 2 \text{ people in a group of } k \text{ have same birthday})$

$$= 1 - \frac{365!}{(365-k)!} \frac{1}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}$$

$k = 2 \implies P \approx 0.27\% \left(\frac{1}{365}\right)$

$k = 4 \implies P \approx 1.64\%$

$k = 23 \implies P \approx 50.73\%$

$k = 30 \implies P \approx 70.6\%$
\[ P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \]

\[ = 1 - \frac{365!}{(365-k)!} \frac{1}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k} \]

\[ k = 2 \quad \rightarrow \quad P \approx 0.27\% \quad \left( \frac{1}{365} \right) \]

\[ k = 4 \quad \rightarrow \quad P \approx 1.64\% \]

\[ k = 23 \quad \rightarrow \quad P \approx 50.73\% \]

\[ k = 30 \quad \rightarrow \quad P \approx 70.6\% \]

\[ k = 70 \quad \rightarrow \quad P \approx 99.9\% \]
$P(\geq 2 \text{ people in a group of } k \text{ have same birthday})$

$$= 1 - \frac{365!}{(365-k)!} \cdot \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}$$

$k = 2 \rightarrow P \approx 0.27\% \left(\frac{1}{365}\right)$

$k = 4 \rightarrow P \approx 1.64\%$

$k = 23 \rightarrow P \approx 50.73\%$

$k = 30 \rightarrow P \approx 70.6\%$

$k = 70 \rightarrow P \approx 99.9\%$

$k \approx 116 \rightarrow P \approx 1 - \frac{1}{109}$
\[ P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \]

\[
= 1 - \frac{365!}{(365-k)!} \cdot \frac{1}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}
\]

\[
k = 2 \quad \rightarrow \quad P \sim 0.27\% \quad \left(\frac{1}{365}\right)
\]

\[
k = 4 \quad \rightarrow \quad P \sim 1.64\%
\]

\[
k = 23 \quad \rightarrow \quad P \sim 50.73\%
\]

\[
k = 30 \quad \rightarrow \quad P \sim 70.6\%
\]

\[
k = 70 \quad \rightarrow \quad P \sim 99.9\%
\]

\[
k \approx 116 \quad \rightarrow \quad P \sim 1 - \frac{1}{10^9}
\]

\[
k = 300 \quad \rightarrow \quad P \sim 1 - \frac{1}{10^{80}}
\]
\[ P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \]

\[ = 1 - \frac{365!}{(365-k)!} \cdot \frac{1}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k} \]

\[ k = 2 \quad \rightarrow \quad P \sim 0.27\% \quad \left(\frac{1}{365}\right) \]

\[ k = 4 \quad \rightarrow \quad P \sim 1.64\% \]

\[ k = 23 \quad \rightarrow \quad P \sim 50.73\% \]

\[ k = 30 \quad \rightarrow \quad P \sim 70.6\% \]

\[ k = 70 \quad \rightarrow \quad P \sim 99.9\% \]

\[ k \approx 116 \quad \rightarrow \quad P \sim 1 - \frac{1}{10^9} \]

\[ k = 300 \quad \rightarrow \quad P \sim 1 - \frac{1}{10^{80}} \]

\[ \left(10^{80} \sim \# \text{ atoms in universe}\right) \]

\[ (k > 365 \rightarrow P = 1) \]
P(\geq 2 \text{ people in a group of } k \text{ have same birthday})

\[ 1 - \frac{365 \cdot 364 \cdot 363 \cdots (363-k+1)}{365^k} \]

Didn't cover this slide in class. It explains bet #2.

For the bet involving \( k \) people born in a month w/ 30 days substitute 365 \( \rightarrow \) 30

\[(k=10) \quad 1 - \frac{30 \cdot 29 \cdot 28 \cdots \cdot 23 \cdot 22 \cdot 21}{30^{10}} \approx 0.815\]