PLANAR GRAPHS

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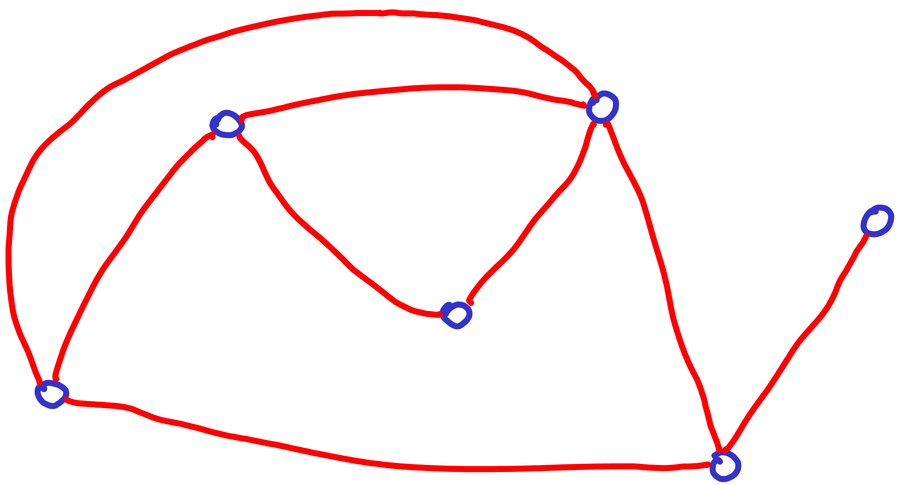
Any graph that can be drawn in the plane without crossings



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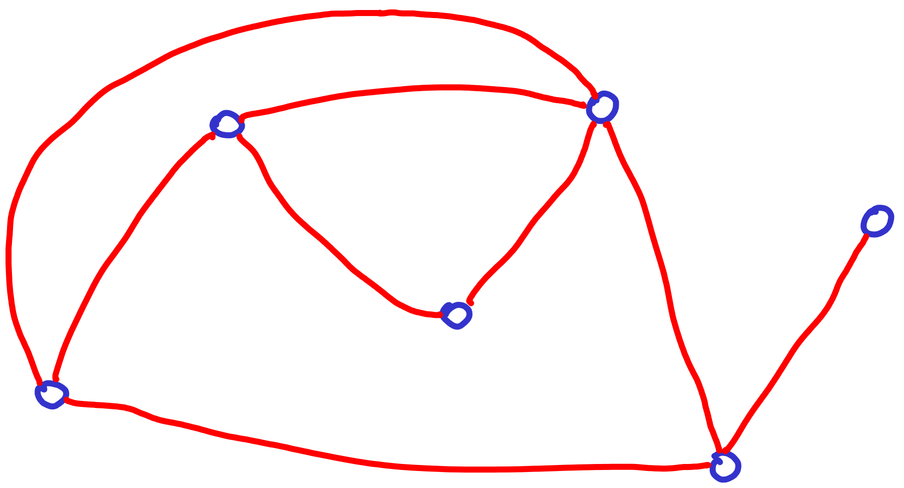

A planar graph that is "embedded" (drawn) without crossings
is a plane graph.

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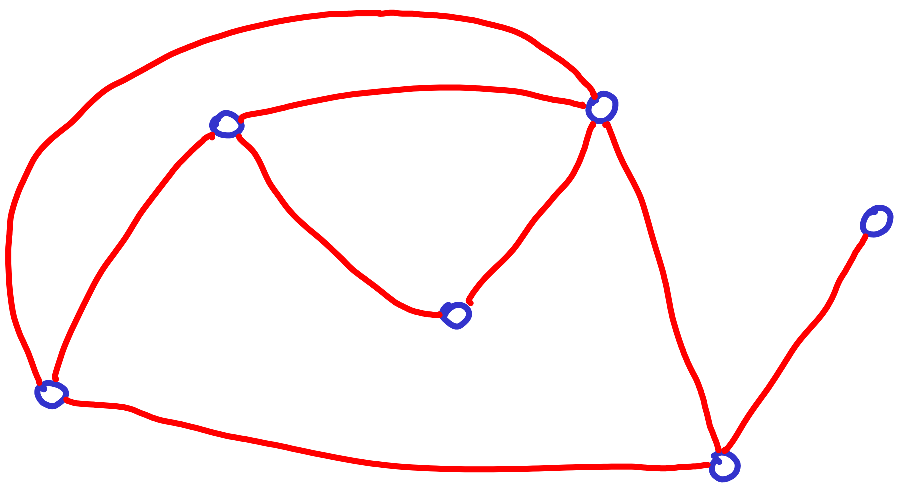
$G=(V,E)$

$V=6$

$E=8$



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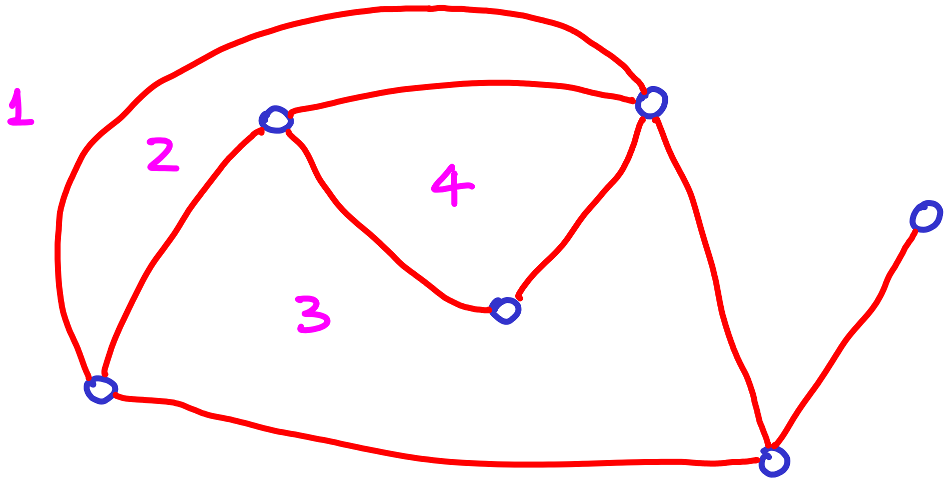

disjoint regions, one of which is unbounded

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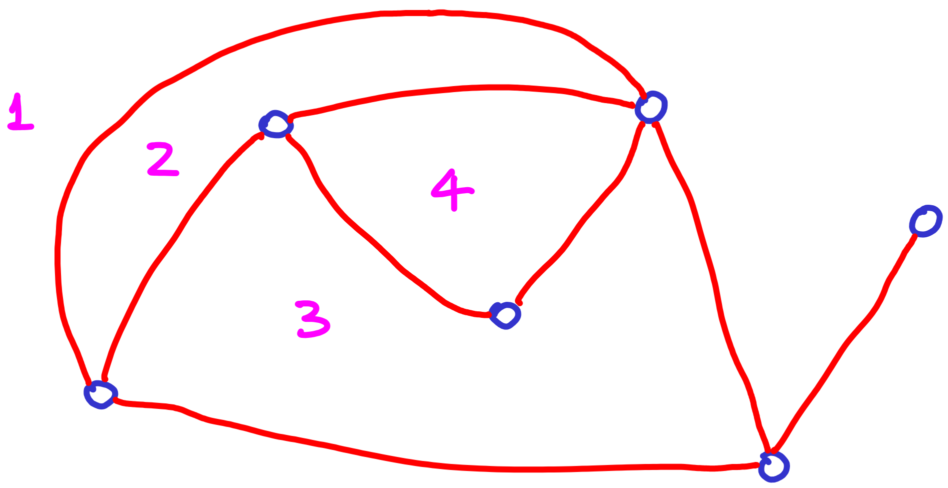

claim:

any planar graph can be drawn w/ straight edges

A planar graph that is "embedded" (drawn) without crossings is a **plane** graph.

EULER FORMULA for planar connected graphs: $\underline{V - E + F = 2}$

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Proof by induction on number of faces:

Base case → $F = 1$ → ?

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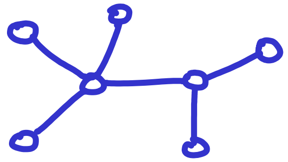

Base case → $F = 1$ → G is a tree → ?

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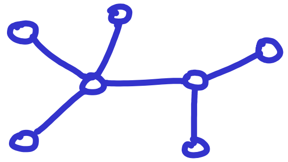

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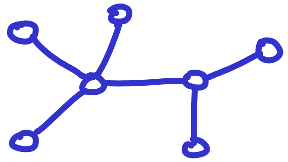

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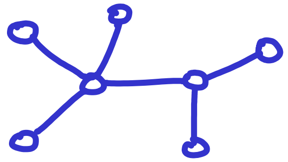

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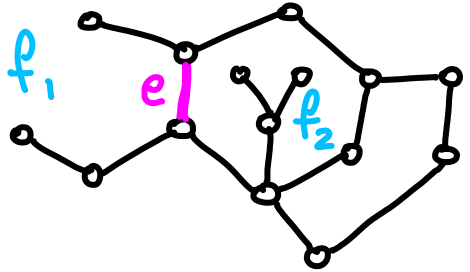

remove an edge e between 2 faces, f_1 & f_2.

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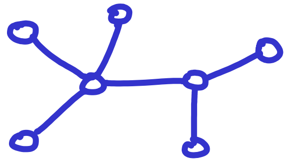

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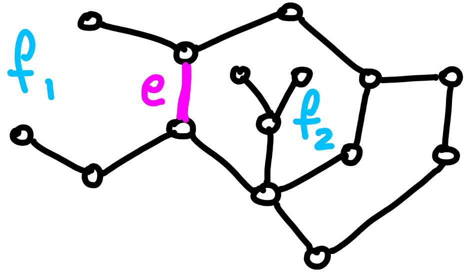

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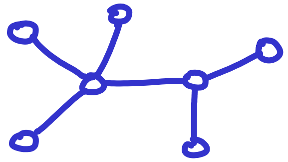

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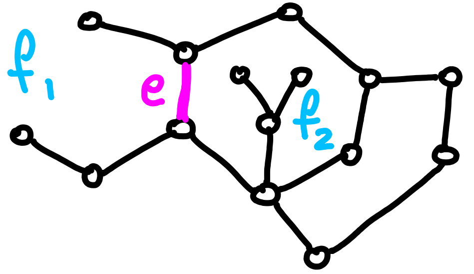

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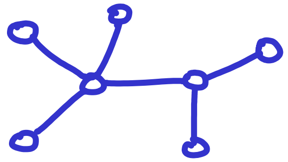

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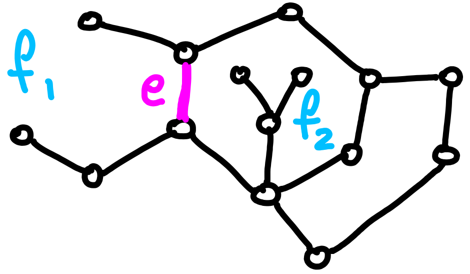

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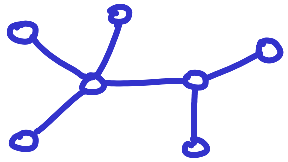

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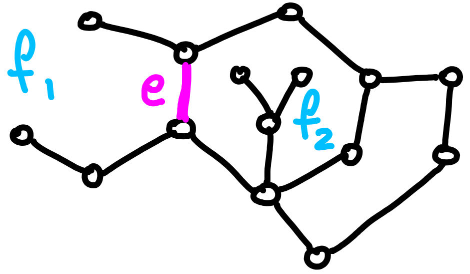

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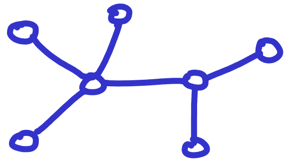

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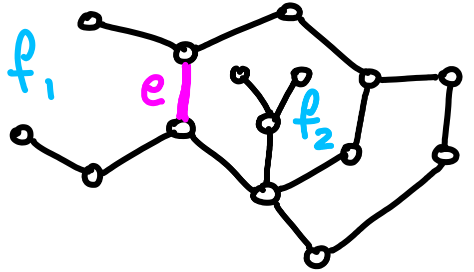

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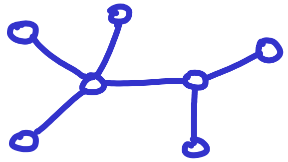

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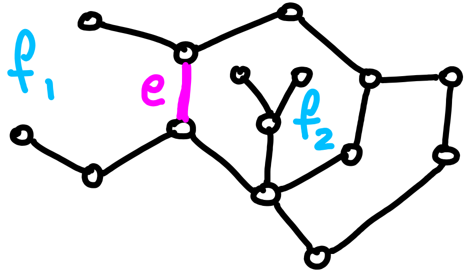

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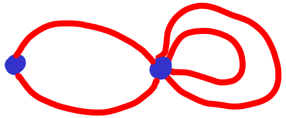
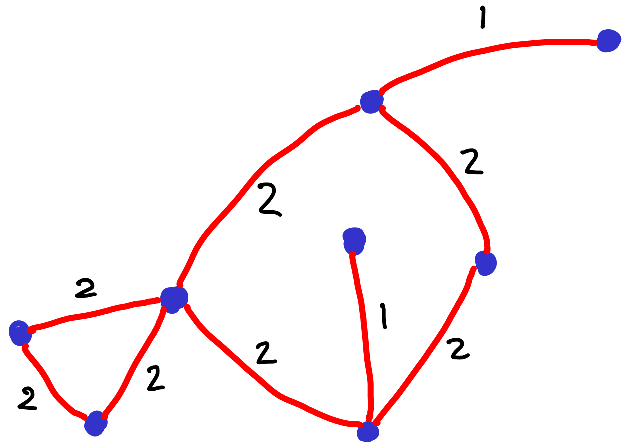
Note that this also holds for multigraphs



For any planar connected graph,

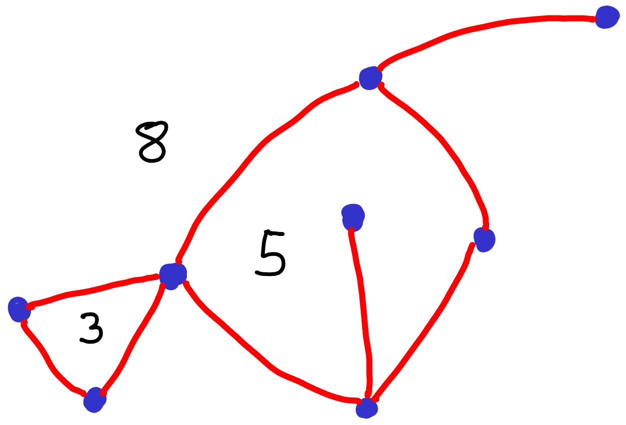
Every edge belongs to 1 or 2 faces



For any planar connected graph,

Every edge belongs to 1 or 2 faces $\quad \sum_{\text{all faces}} e \leq 2E$

$5+8+3 \leq 2\cdot 9$



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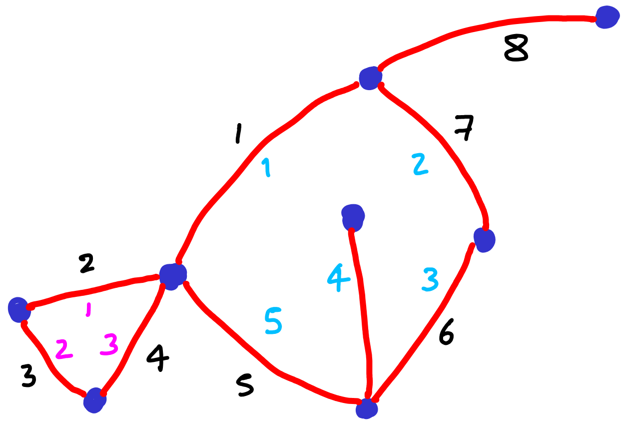

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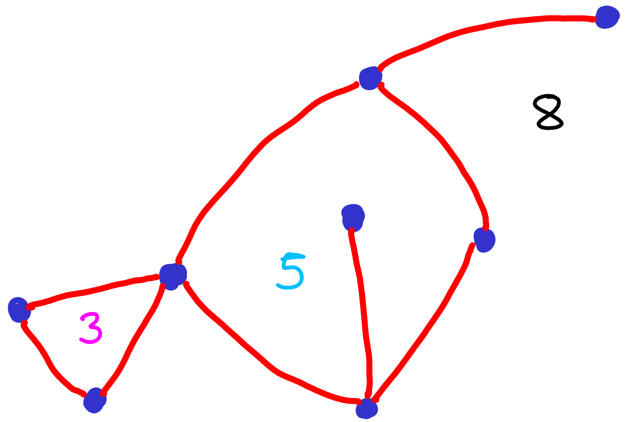
Every face has ≥ 3 edges (for $V > 3$) $\sum_{\text{all faces}} e \geq 3F$

$5+8+3 \geq 3 \cdot 3$



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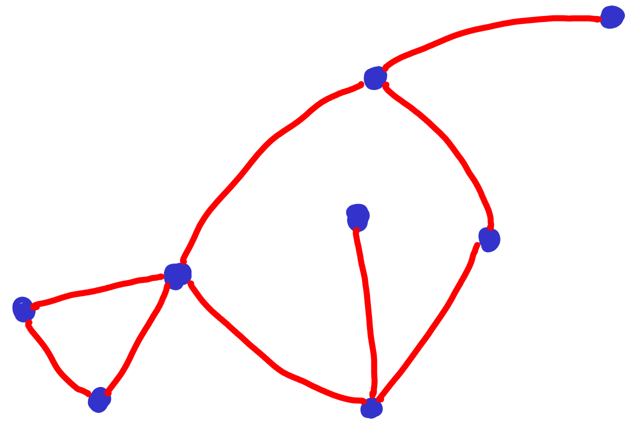

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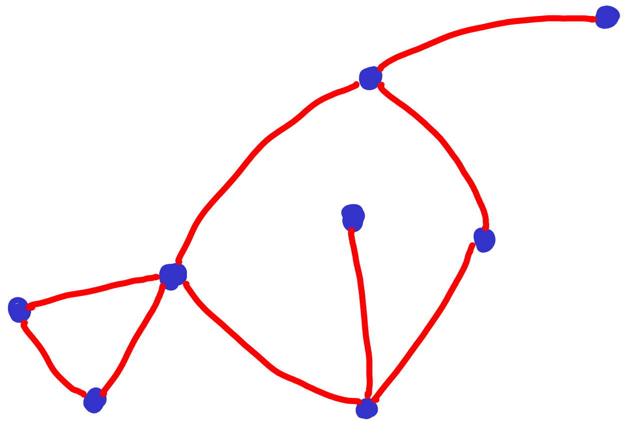
$V - E + F = 2$



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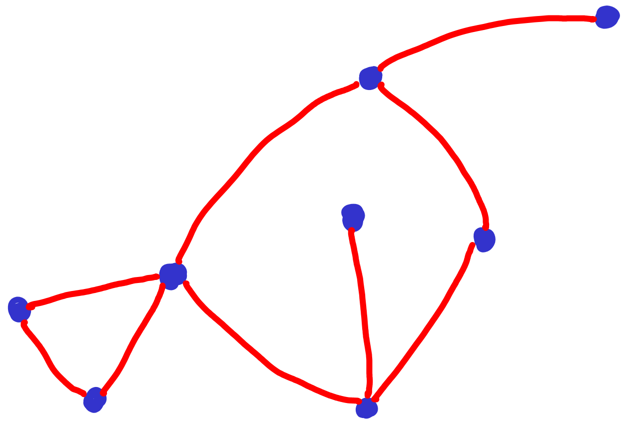
$V - 2 = E - F$



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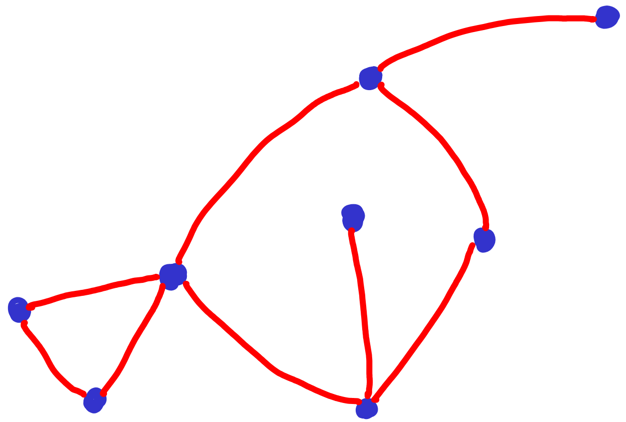
$2E \geq 3F$

$F \leq \frac{2E}{3}$

$V - E + F = 2$

$V - 2 = E - F$

$V - 2 \geq E - \frac{2E}{3}$



For any planar connected graph w/ $V>3$, $E \leq 3V-6$

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$V-E+F=2$

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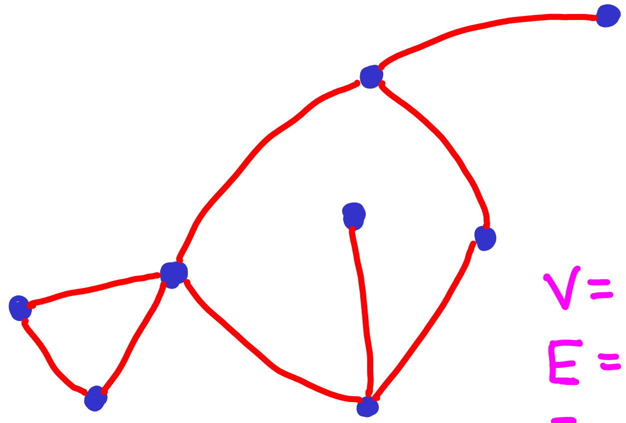
$V-2 \geq E - \frac{2E}{3}$

$3V-6 \geq E$



$V=8$
$E=9$
$F=3$

For any planar connected graph w/ $V>3$, $E \leq 3V-6$

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Every face has $\geqslant 3$ edges (for $V>3$) $\quad \sum_{\text{all faces}} e \geqslant 3F$

$2E \geqslant 3F$

$F \leq \frac{2E}{3}$

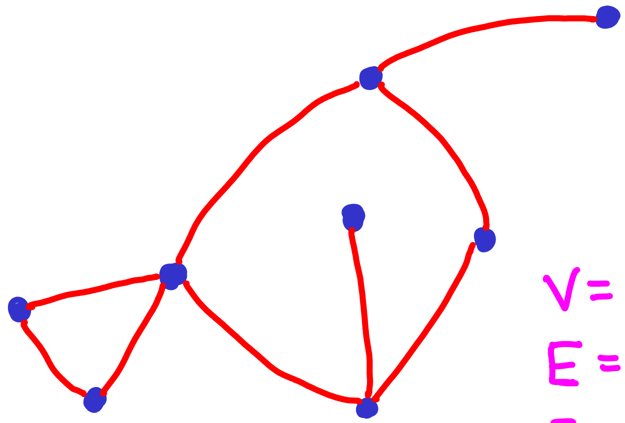
$V - E + F = 2$

$V-2 = E-F$

$V-2 \geqslant E - \frac{2E}{3}$

$3V-6 \geqslant E$

$\boxed{\text{Also } V-2 \geqslant \frac{3F}{2} - F}$



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For any planar connected graph w/ $V>3$, $E \leqslant 3V-6$ $(\& \ F \leqslant 2V-4)$

Every edge belongs to 1 or 2 faces $\displaystyle\sum_{\text{all faces}} e \leqslant 2E$

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$2E \geqslant 3F$

$F \leqslant \dfrac{2E}{3}$

$V - E + F = 2$

$V - 2 = E - F$

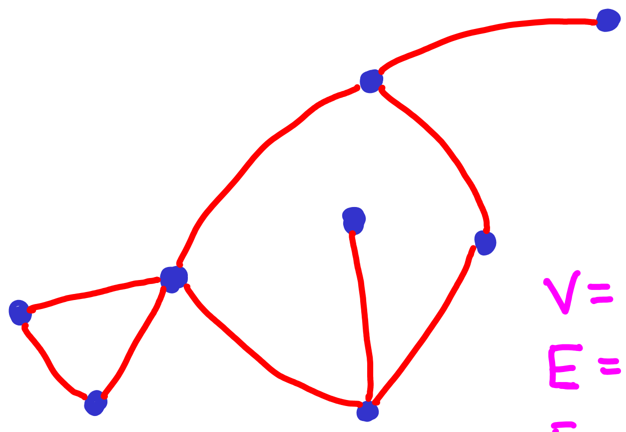
$V - 2 \geqslant E - \dfrac{2E}{3}$

$3V - 6 \geqslant E$

Also

$V - 2 \geqslant \dfrac{3F}{2} - F$

$2V - 4 \geqslant F$



$V = 8$
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For any planar connected graph w/ $V>3$, $E \leq 3V-6$ ($\&\ F \leq 2V-4$)

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Every face has ≥ 3 edges (for $V>3$) $\quad \sum_{\text{all faces}} e \geq 3F$

$2E \geq 3F$

$F \leq \frac{2E}{3}$

what if $V \leq 3$?

↳ then $E \leq V$

$V - E + F = 2$

$V - 2 = E - F$

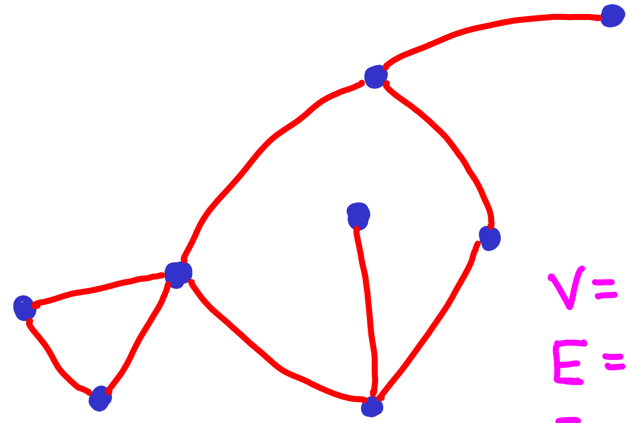
$V - 2 \geq E - \frac{2E}{3}$

$3V - 6 \geq E$

Also

$V - 2 \geq \frac{3F}{2} - F$

$2V - 4 \geq F$



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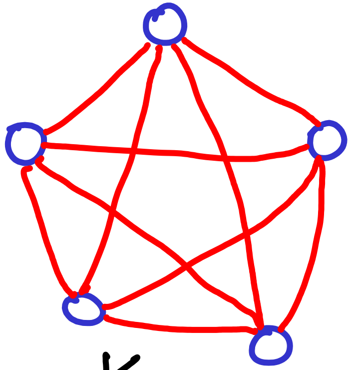
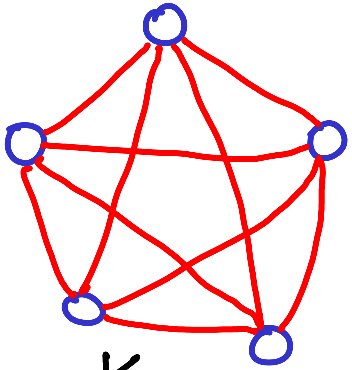


K_5

$E \leq 3V - 6$

$10 \leq 15 - 6$

!!!



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Not planar

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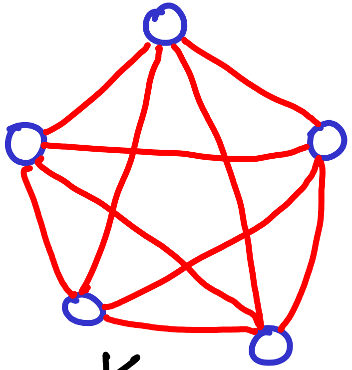

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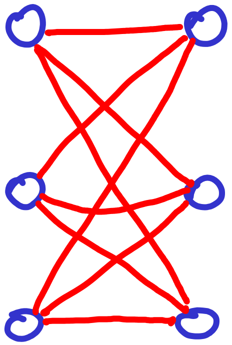
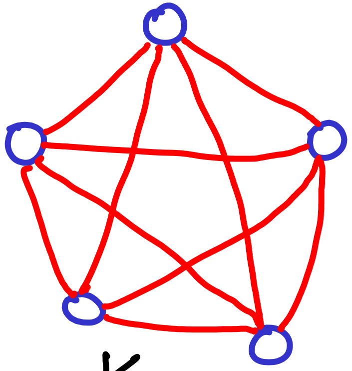


$K_{3,3}$

$E \leq 3V - 6$

$10 \leq 15 - 6$

!!!

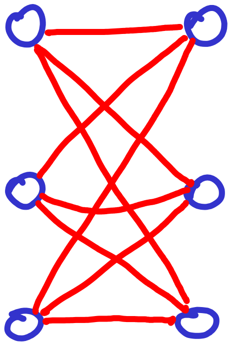


K_5

Not planar

$E \leq 3V - 6$

$9 \leq 18 - 6$ ok!



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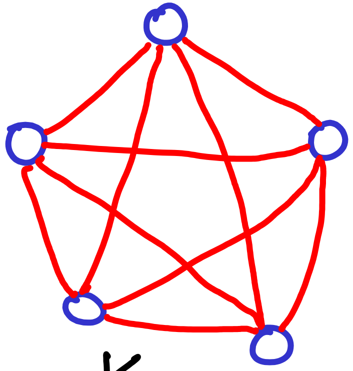

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Not planar

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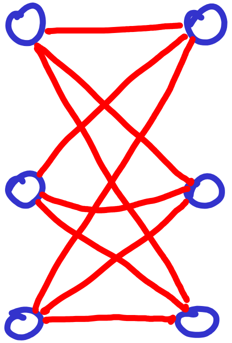
Inconclusive



$K_{3,3}$

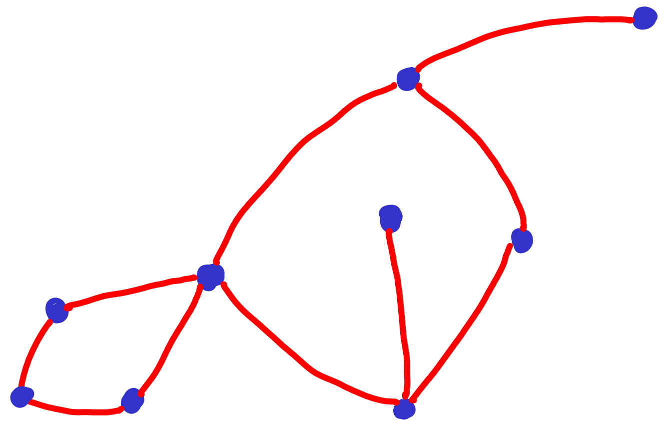
<u>not iff</u>

All planar graphs have $E \leq 3V-6$

Some non-planar graphs can too

$V - E + F = 2$

What if G has no triangles?



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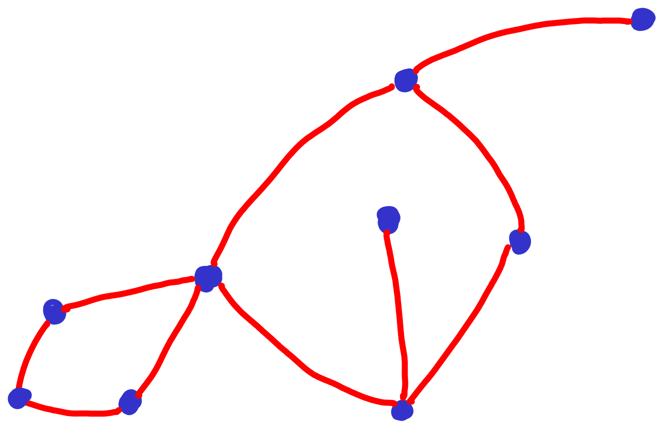
Every edge belongs to 1 or 2 faces $\qquad \sum_{\text{all faces}} e \leq 2E$

Every face has $\cancel{\geq 3}$ ≥ 4 edges (for $V > 4$) $\qquad \sum_{\text{all faces}} e \geq 4F$



$V - E + F = 2$

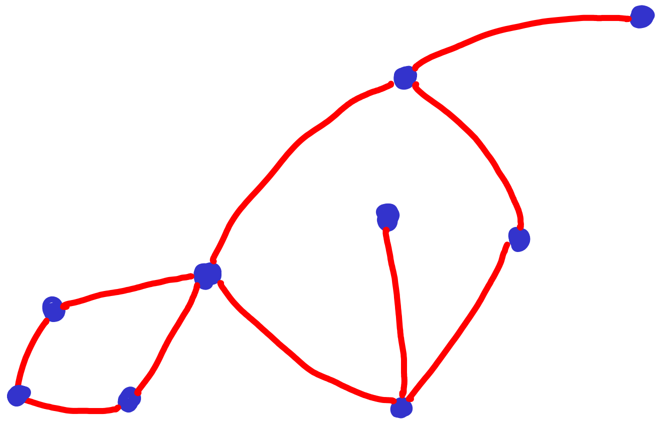
What if G has no triangles?

Every edge belongs to 1 or 2 faces

Every face has $\cancel{\geq 3}$ ≥ 4 edges (for $\underline{V > 4}$)

$$\left.\begin{aligned} \sum_{\text{all faces}} e &\leq 2E \\ \sum_{\text{all faces}} e &\underline{\geq 4F} \end{aligned}\right\}$$

$\underline{E \geq 2F}$



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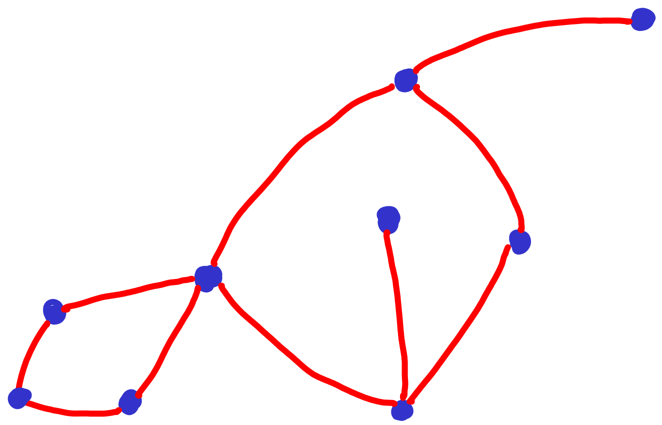
$E \geq 2F$

$E - F = V - 2$

$E - \frac{E}{2} \leq V - 2$

$E \leq 2V - 4$

instead of $\leq 3V - 6$

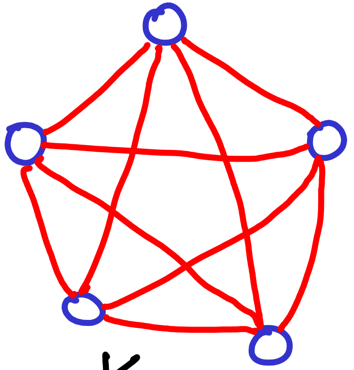


(recap)

$E \leq 3V-6$

$10 \leq 15-6$

!!!



K_5

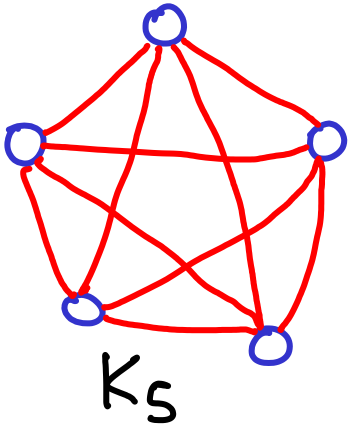
NOT PLANAR

for triangle free: $E \leq 2V-4$

$E \leq 3V-6$

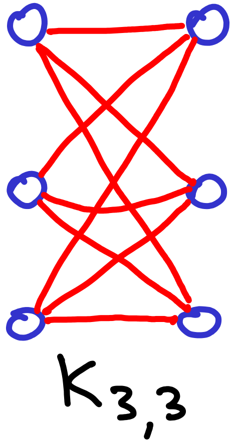
$10 \leq 15-6$

!!!



NOT PLANAR

for triangle free: $E \leq 2V-4$



$E \leq 3V - 6$

$10 \leq 15 - 6$

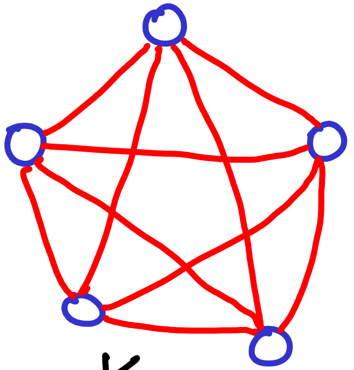
!!!

for triangle free: $E \leq 2V - 4$

$9 \leq 2 \cdot 6 - 4$

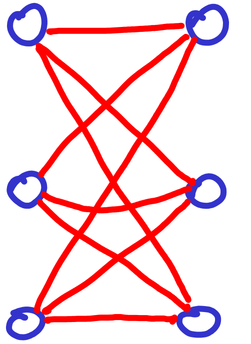
!!!

NOT PLANAR

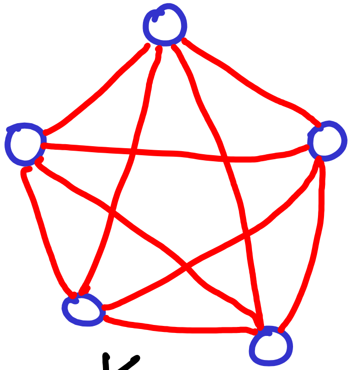


K_5

$V=6, \; E=9$

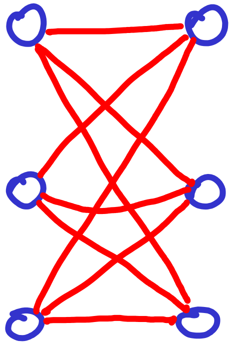


$K_{3,3}$



K_5

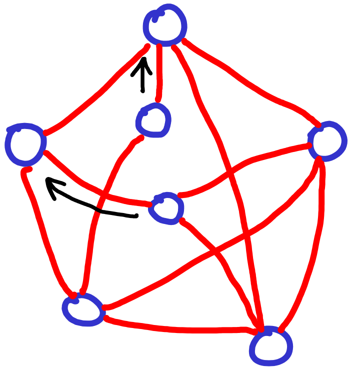
← non-planar →



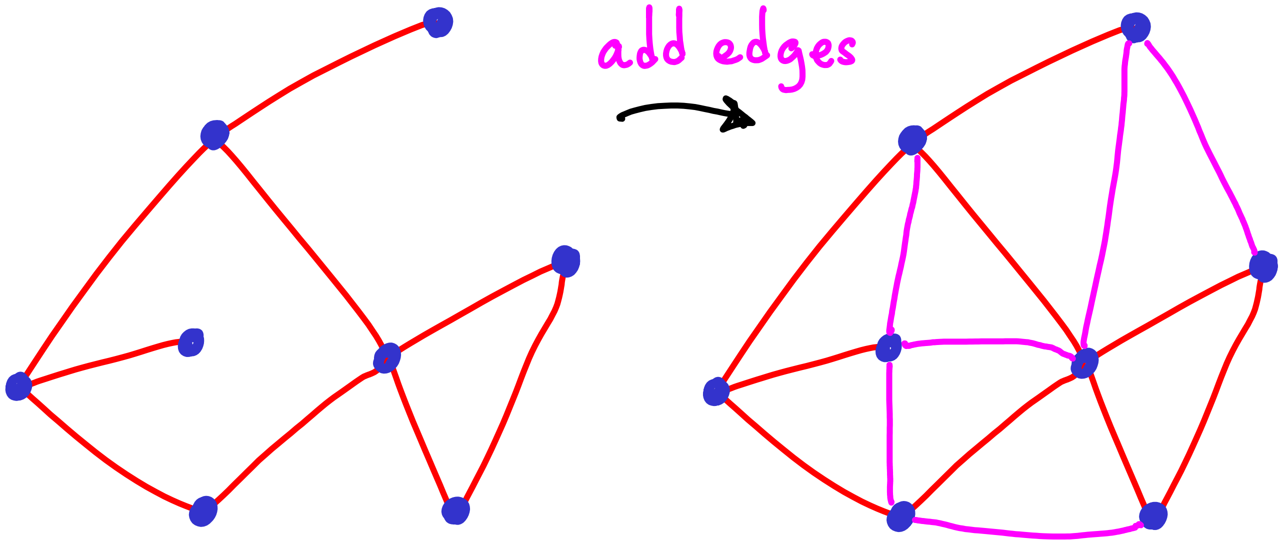
$K_{3,3}$

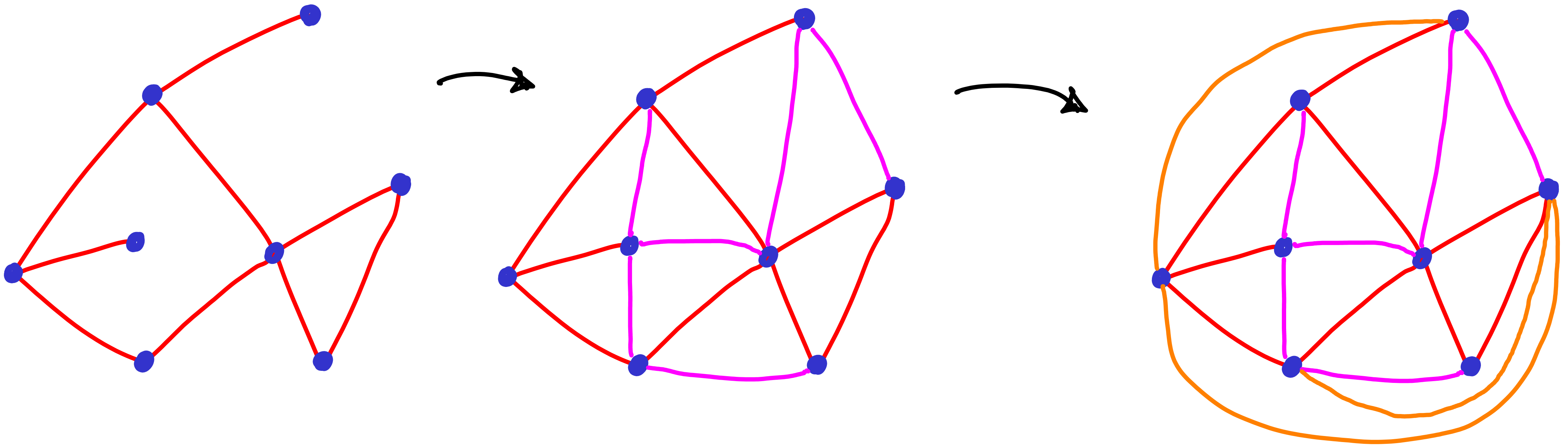
It turns out that every non-planar graph "contains" one of these two shapes.

e.g:



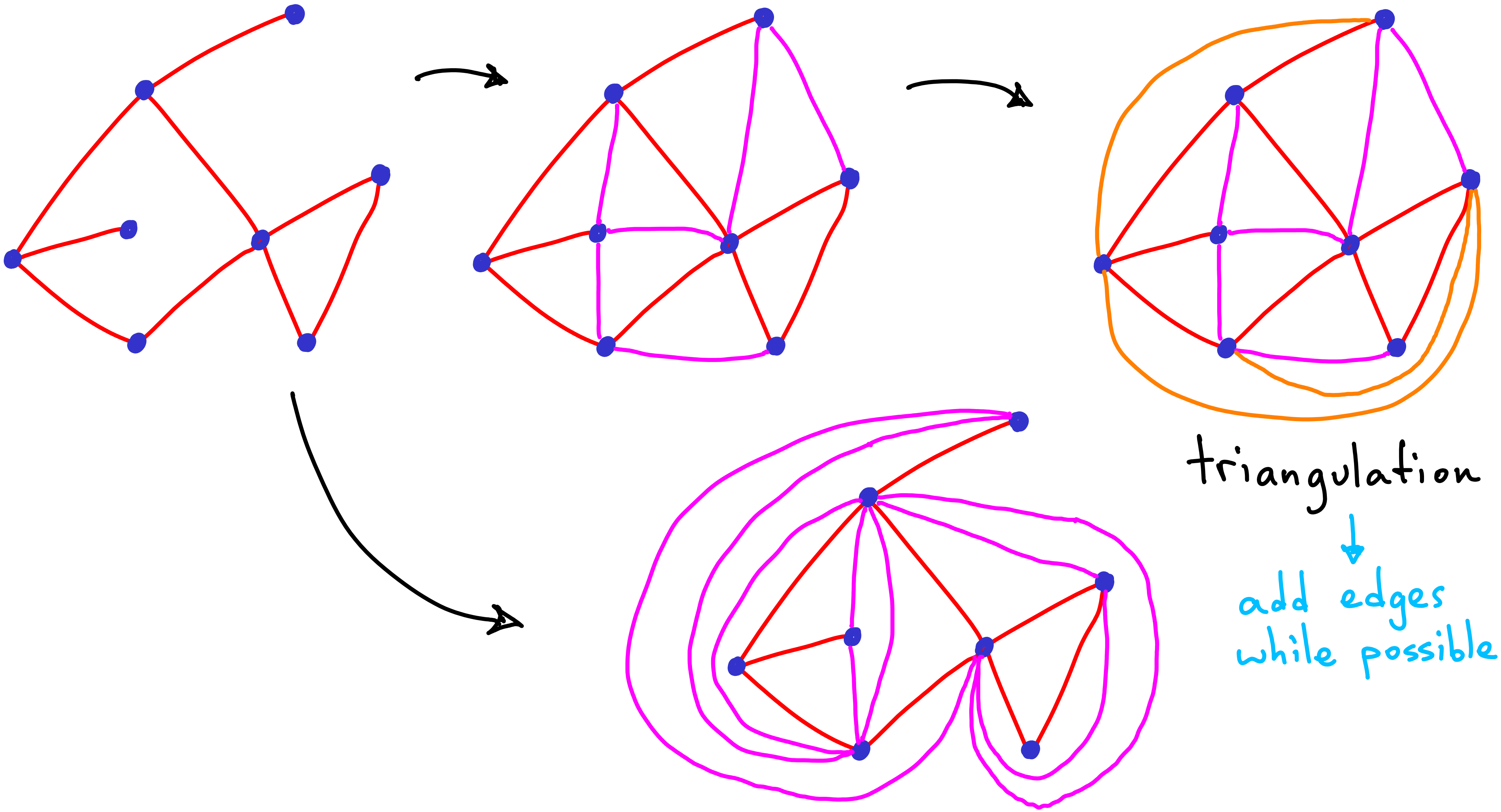
see links


add edges

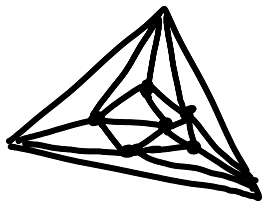


triangulation

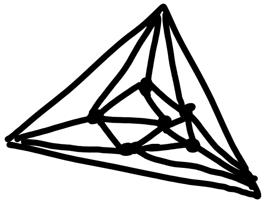
add edges while possible


triangulation
add edges
while possible

$E = 3V-6$



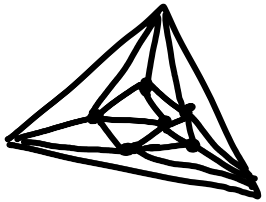
Why?

$E = 3V-6$  ... for triangulations

Why?

$$\left.\begin{array}{ll} \text{Every edge belongs to 1 or 2 faces} & \displaystyle\sum_{\text{all faces}} e \leq 2E \\ \text{Every face has } \geq 3 \text{ edges (for } V>3\text{)} & \displaystyle\sum_{\text{all faces}} e \geq 3F \end{array}\right\} \; 2E \geq 3F$$

$E = 3V-6$



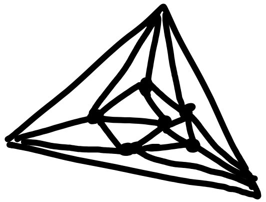
... for triangulations

Why?

Every edge belongs to ~~1 or~~ 2 faces $\quad \sum_{\text{all faces}} e \; \cancel{\leq} \overset{=}{} \; 2E$

Every face has $\cancel{\geq}\overset{=}{}$ 3 edges ~~(for V>3)~~ $\quad \sum_{\text{all faces}} e \; \cancel{\geq} \overset{=}{} \; 3F$

$2E \; \cancel{\geq} \overset{=}{} \; 3F$

$E = 3V - 6$  ... for triangulations

Every edge belongs to ~~1 or~~ 2 faces $\qquad \sum_{\text{all faces}} e \overset{=}{\cancel{\le}} 2E$

Every face has $\overset{=}{\cancel{\ge}}$ 3 edges ~~(for V>3)~~ $\qquad \sum_{\text{all faces}} e \overset{=}{\cancel{\ge}} 3F$

$$2E \overset{=}{\cancel{\ge}} 3F$$

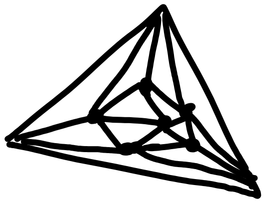
$$V - E + F = 2$$

$$E - F = V - 2$$

$$E - \frac{2E}{3} = V - 2$$

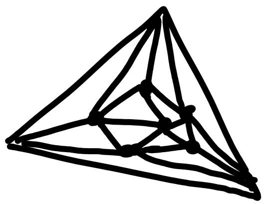
$$E = 3V - 6$$

$E = 3V - 6$



What is the average degree of a triangulation?

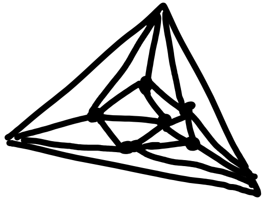
$E = 3V - 6$



What is the average degree of a triangulation?

$$\frac{1}{V} \cdot \sum_{i=1}^{V} d(v_i)$$

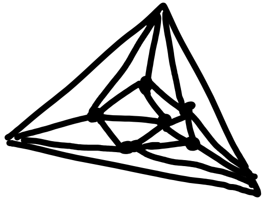
$E = 3V - 6$



What is the average degree of a triangulation?

$$\frac{1}{V} \cdot \sum_{i=1}^{V} d(v_i) = \frac{1}{V} \cdot 2E$$

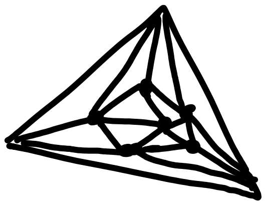
$E = 3V - 6$



What is the average degree of a triangulation?

$$\frac{1}{V} \cdot \sum_{i=1}^{V} d(v_i) = \frac{1}{V} \cdot 2E = \frac{6V-12}{V} \quad \underline{\underline{< 6}}$$

$E = 3V-6$

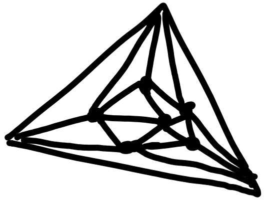


What is the average degree of a triangulation?

$$\frac{1}{V}\cdot\sum_{i=1}^{V} d(v_i) = \frac{1}{V}\cdot 2E = \frac{6V-12}{V} < 6$$

↳ Every triangulation has a vertex with degree ≤ 5

$E = 3V - 6$



What is the average degree of a triangulation?

$$\frac{1}{V} \cdot \sum_{i=1}^{V} d(v_i) = \frac{1}{V} \cdot 2E = \frac{6V-12}{V} < 6$$

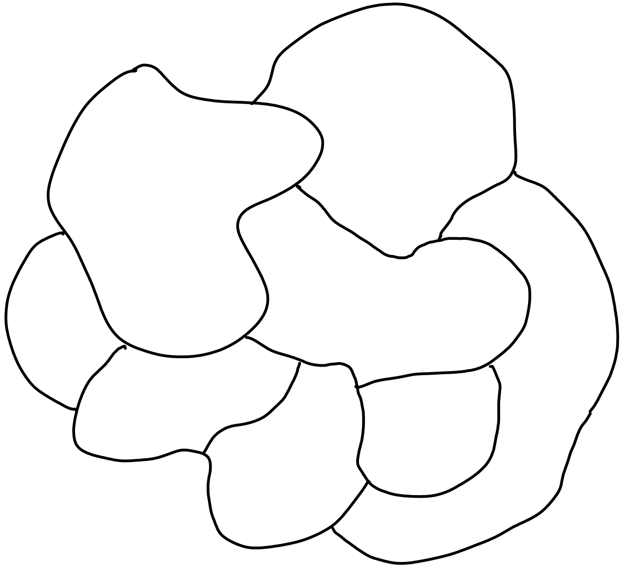
Every triangulation has a vertex with degree ≤ 5

Implies the same for any planar graph

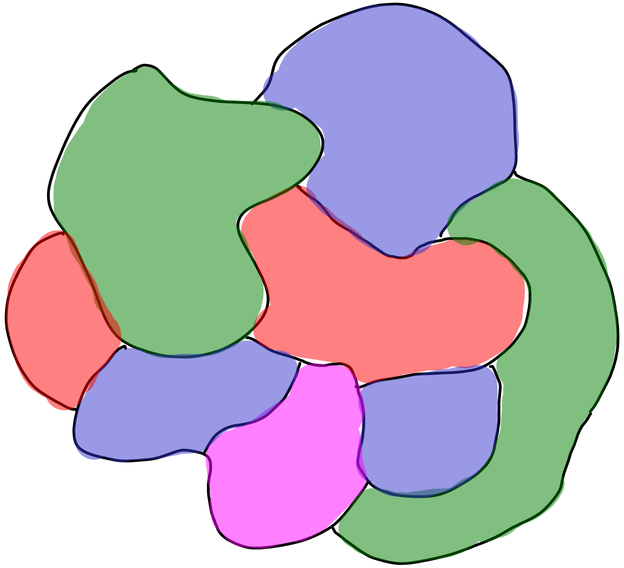
(every graph is a spanning subgraph of a triangulation)

MAP COLORING

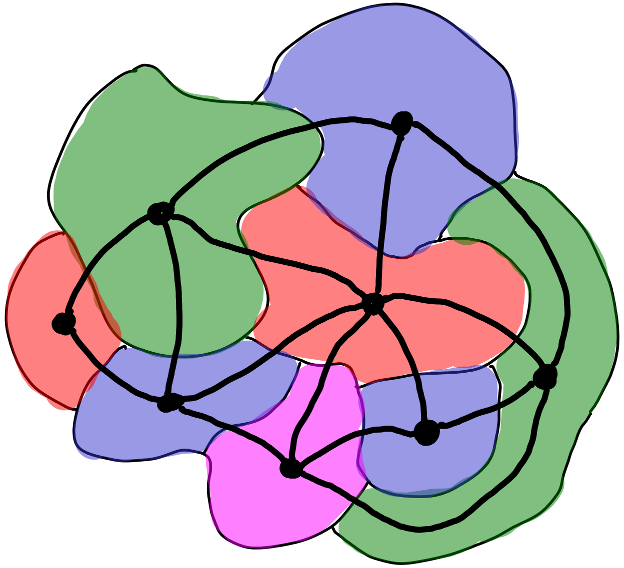
MAP COLORING



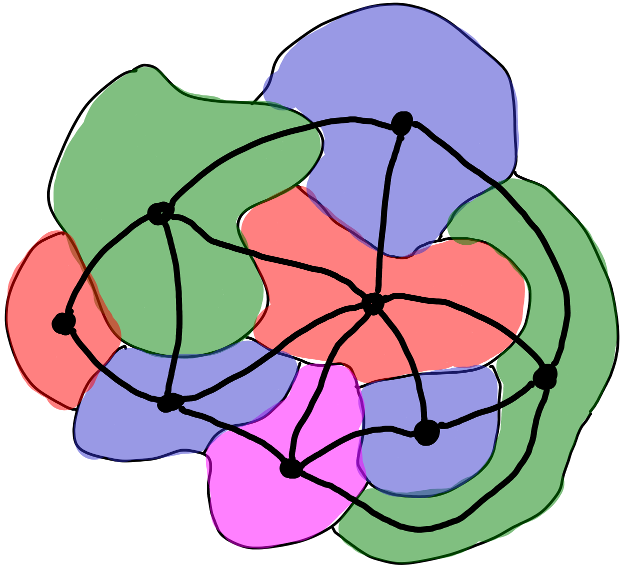
MAP COLORING



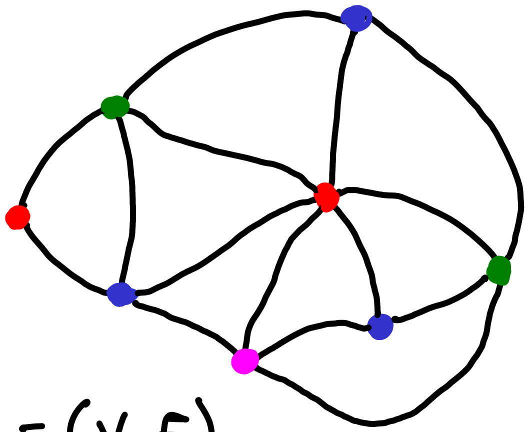
MAP COLORING



MAP COLORING



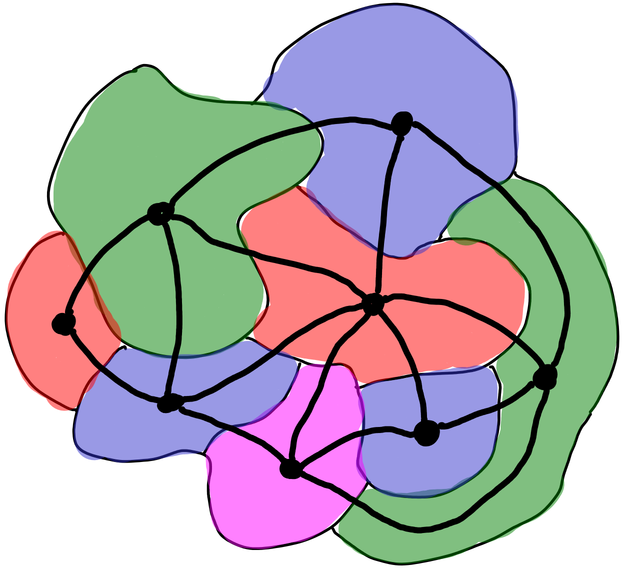
$\Rightarrow$



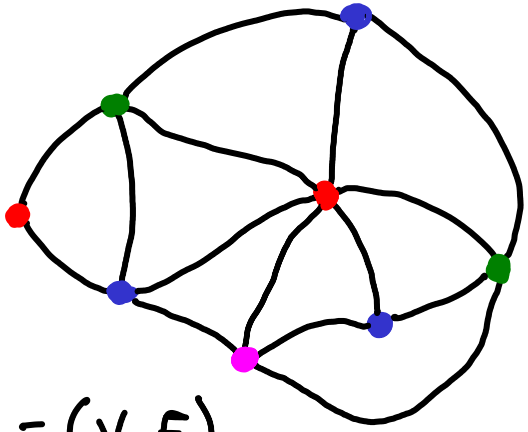
$G = (V, E)$

COLORING G → no adjacent vertices get same color

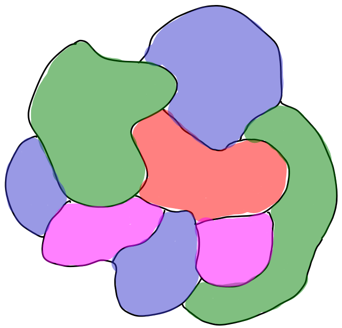
MAP COLORING



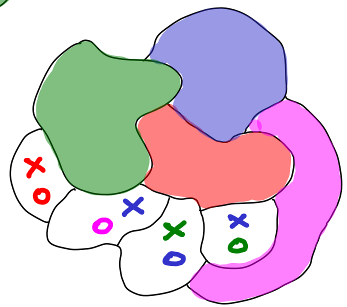
$\Rightarrow$



$G = (V, E)$



etc



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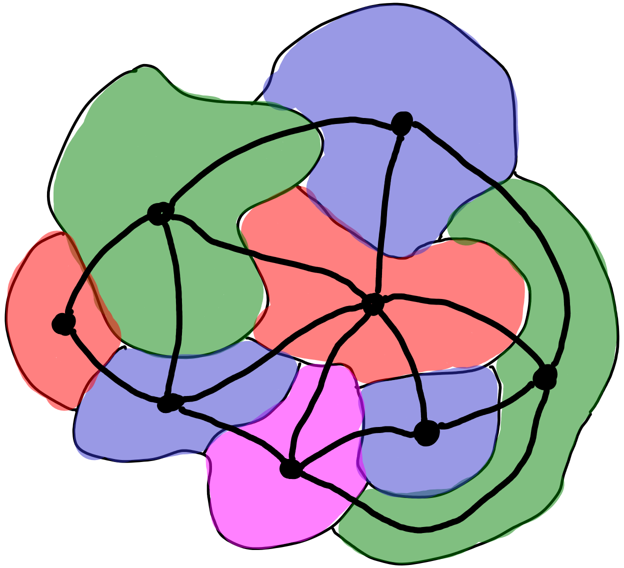
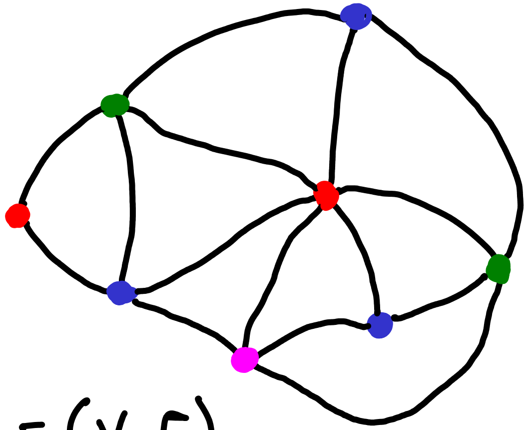
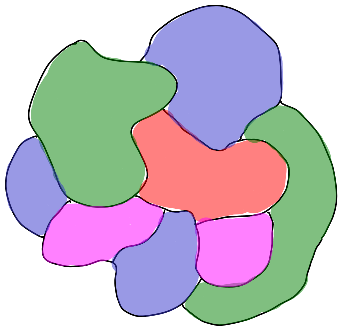
MAP COLORING



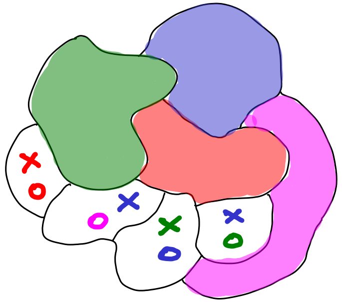
$\Rightarrow$



$G = (V, E)$



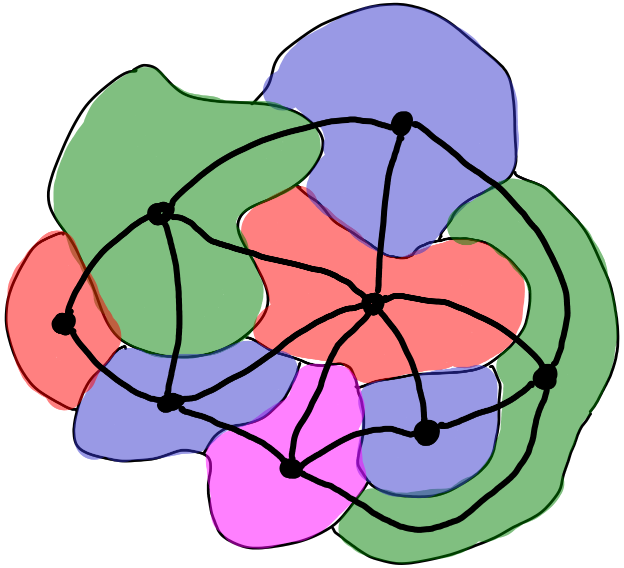
etc



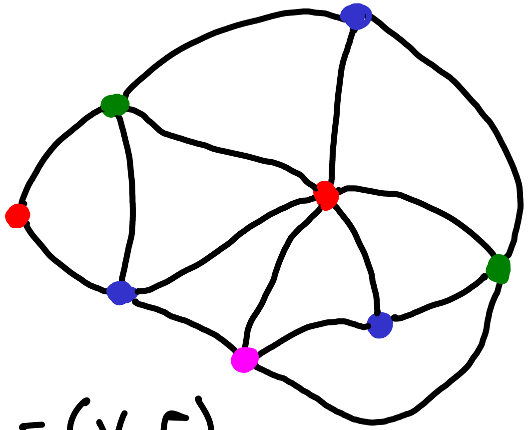
COLORING G $\rightarrow$ no adjacent vertices get same color

G is k-colorable if we can use $\leq k$ colors

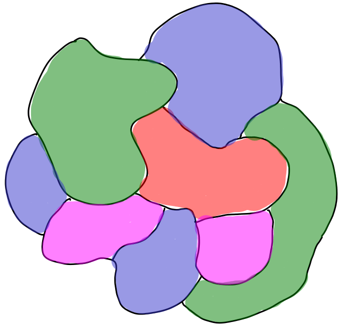
MAP COLORING



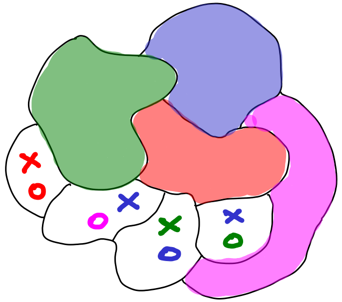
$\Rightarrow$



$G = (V, E)$



etc



COLORING G → no adjacent vertices get same color

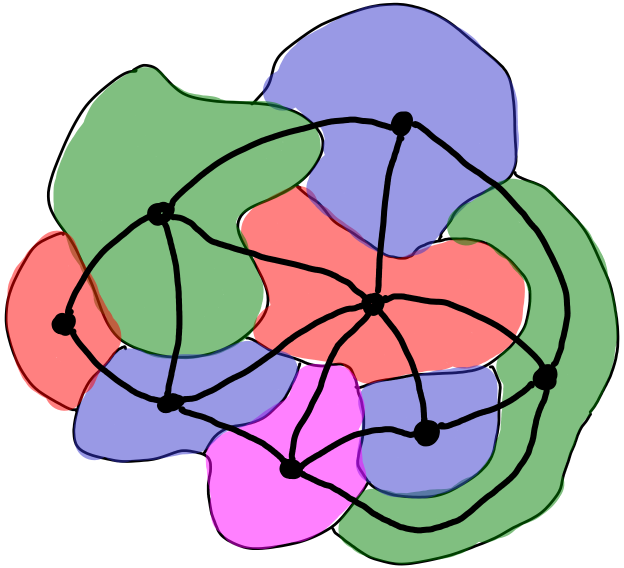
G is k-colorable if we can use $\leq k$ colors

$\boxed{\chi(G)}$: min # colors we can use to color G

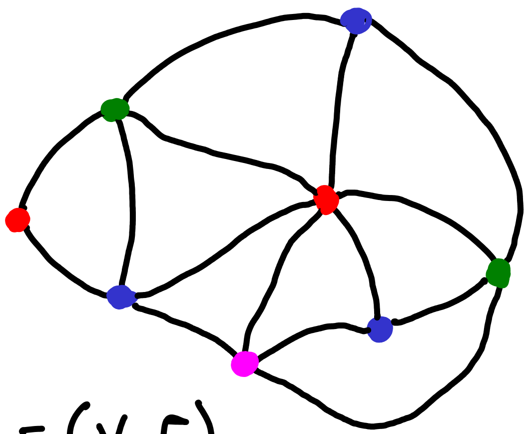
chromatic number

χρὼμα = color

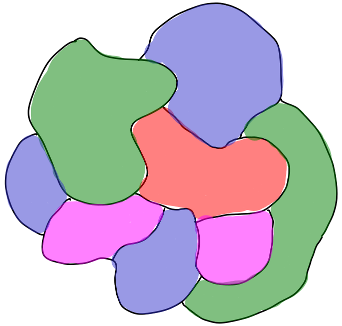
MAP COLORING



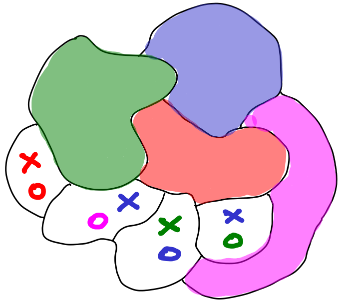
$\Rightarrow$



$G=(V,E)$



etc



COLORING G → no adjacent vertices get same color

G is k-colorable if we can use $\leq k$ colors

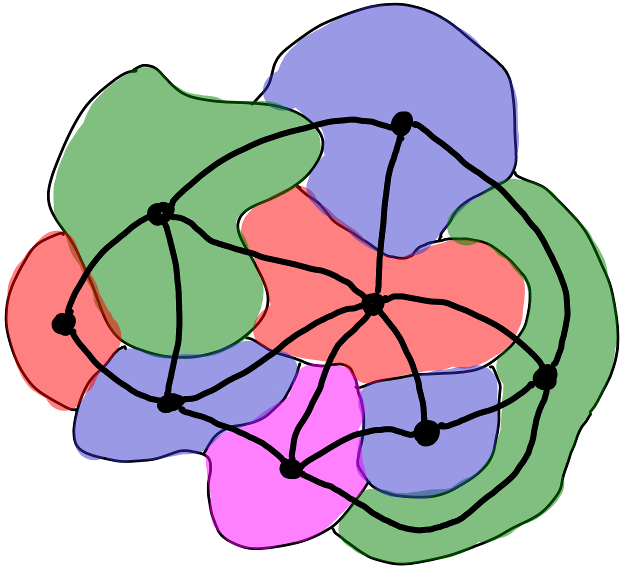
$\chi(G)$: min # colors we can use to color G

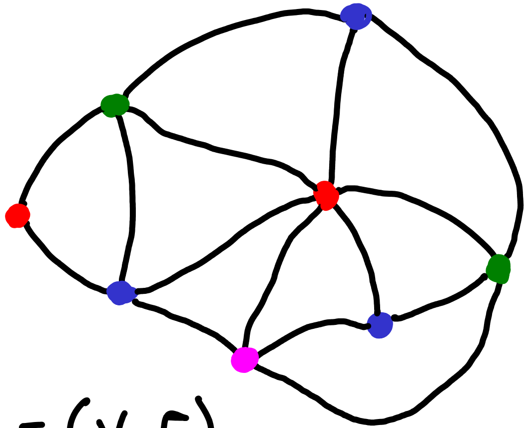
chromatic number

$\chi\rho\tilde{\omega}\mu\alpha$ = color

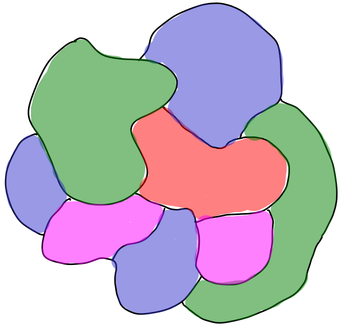
Our map is 4-colorable $\Big\}\ \chi \leq 4$

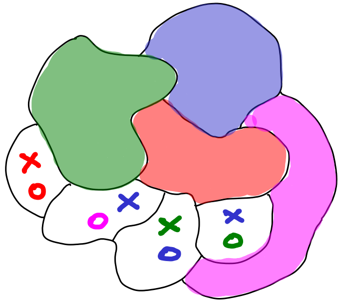
MAP COLORING





$G=(V,E)$





etc

COLORING G → no adjacent vertices get same color

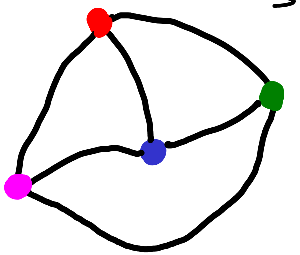
G is k-colorable if we can use $\leq k$ colors

$\chi(G)$: min # colors we can use to color G

chromatic number

χρῶμα = color

Our map is 4-colorable } $\chi \leq 4$



subgraph K_4

so $\chi \geq 4$

...but not 3-colorable

EXAM SCHEDULING

students : s_1 s_2 s_3 s_4 s_5 classes c_1 c_2 c_3 c_4 c_5

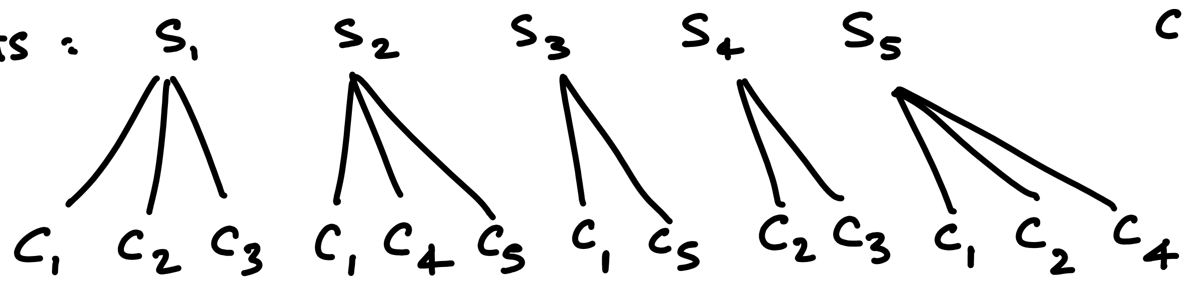
EXAM SCHEDULING

students : s_1 s_2 s_3 s_4 s_5 classes c_1 c_2 c_3 c_4 c_5

c_1 c_2 c_3 c_1 c_4 c_5 c_1 c_5 c_2 c_3 c_1 c_2 c_4

EXAM SCHEDULING

students :



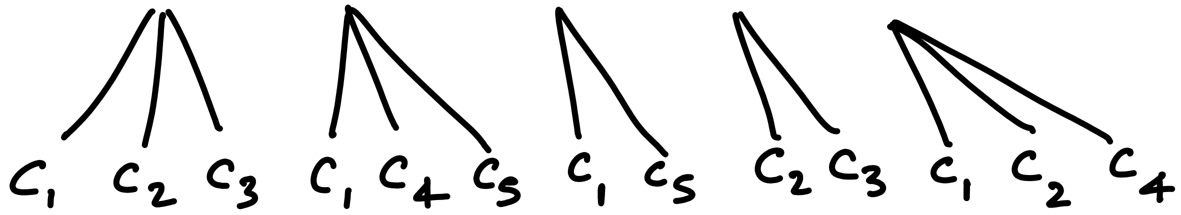
classes c_1 c_2 c_3 c_4 c_5

Can't schedule exam simultaneously for classes taken by s_i

Want to minimize exam slots.

EXAM SCHEDULING

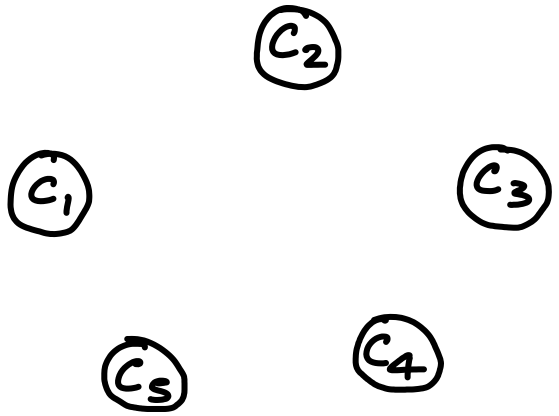
students: s_1 s_2 s_3 s_4 s_5 classes c_1 c_2 c_3 c_4 c_5



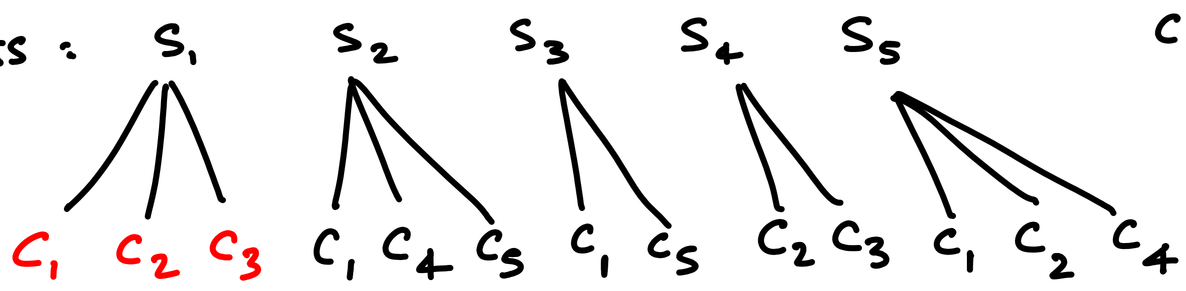
Can't schedule exam simultaneously for classes taken by s_i

Want to minimize exam slots.

Make G: V = classes



EXAM SCHEDULING

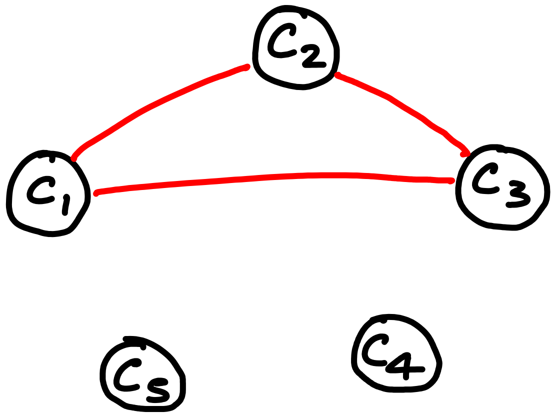
students : 

classes c_1 c_2 c_3 c_4 c_5

Can't schedule exam simultaneously for classes taken by s_i

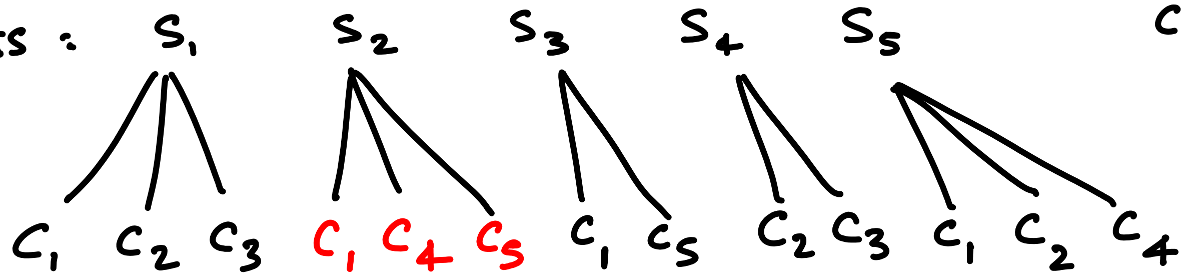
Want to minimize exam slots.

Make G: V = classes E = conflicts



EXAM SCHEDULING

students :



classes c_1 c_2 c_3 c_4 c_5

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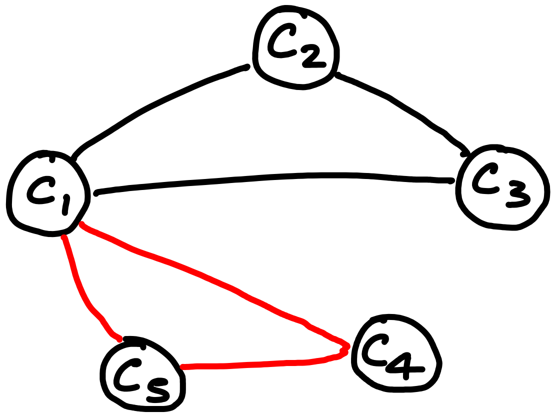
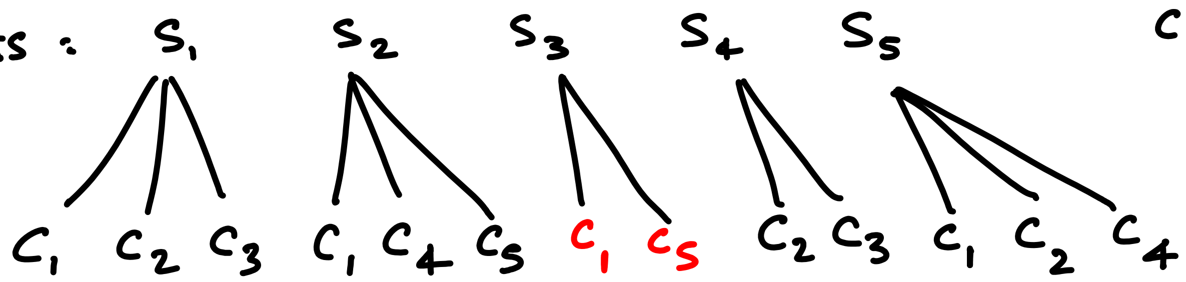
Make G : V = classes E = conflicts



EXAM SCHEDULING

students:

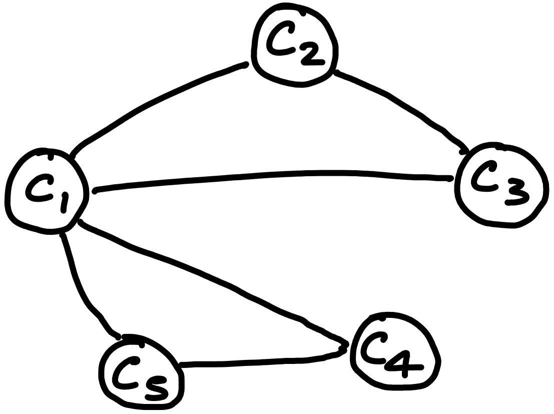


classes c_1 c_2 c_3 c_4 c_5

Can't schedule exam simultaneously for classes taken by s_i

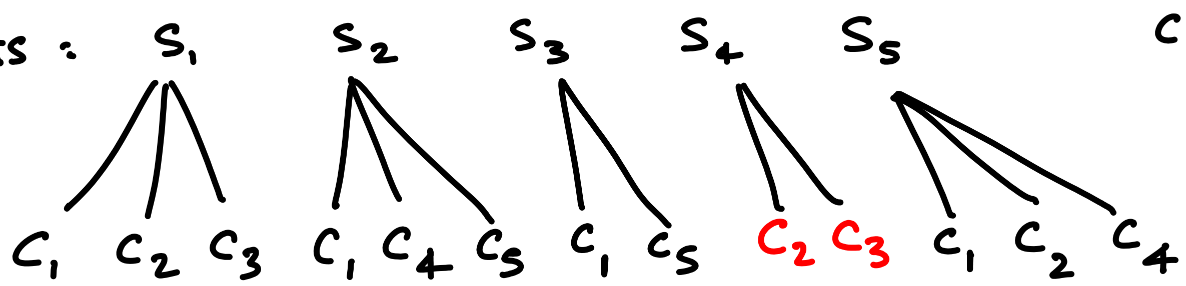
Want to minimize exam slots.

Make G: V = classes E = conflicts



EXAM SCHEDULING

students :



classes c_1 c_2 c_3 c_4 c_5

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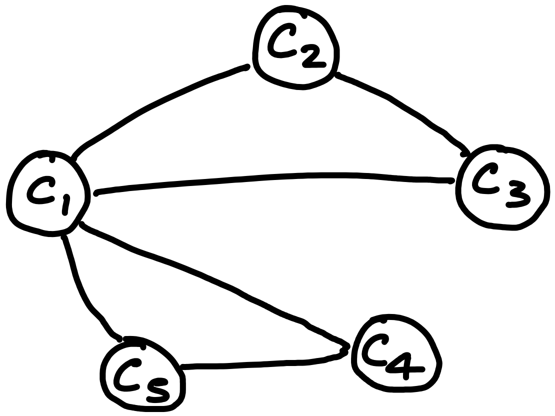
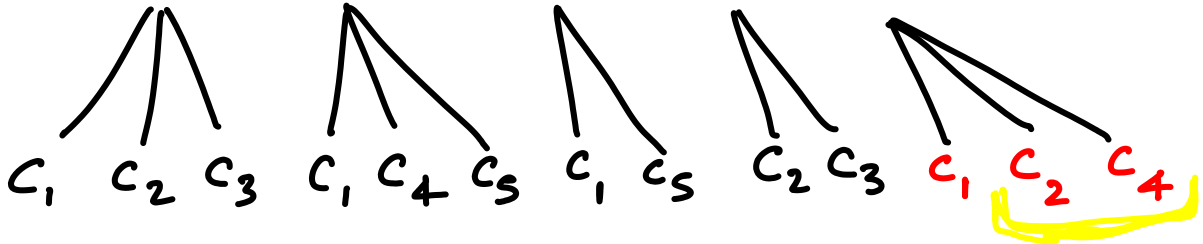


EXAM SCHEDULING

students: s_1 s_2 s_3 s_4 s_5 classes c_1 c_2 c_3 c_4 c_5



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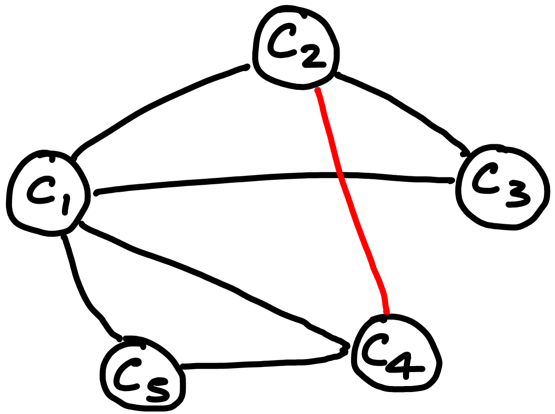
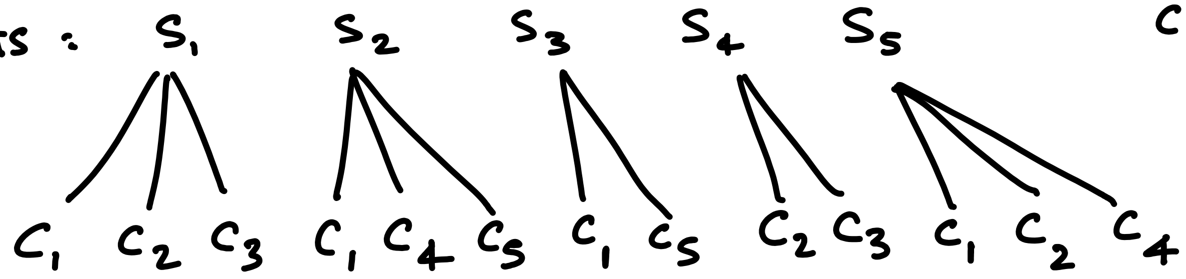


EXAM SCHEDULING

students :



classes c_1 c_2 c_3 c_4 c_5

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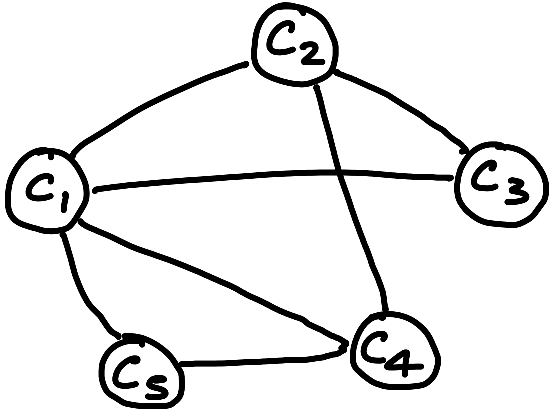


Colors = slots (minimize colors)

If no edge has same color at endpoints, then no 2 classes are in same slot

EXAM SCHEDULING

students: s_1 s_2 s_3 s_4 s_5 classes c_1 c_2 c_3 c_4 c_5

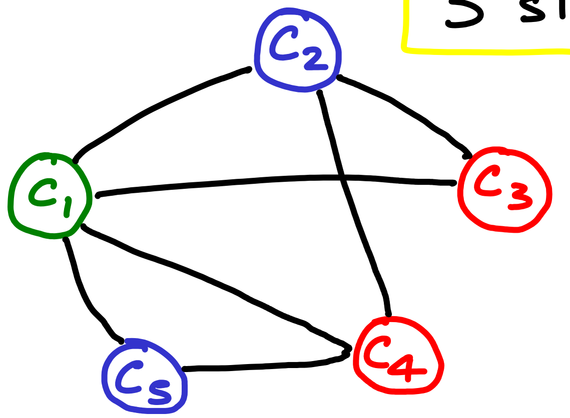
c_1 c_2 c_3 c_1 c_4 c_5 c_1 c_5 c_2 c_3 c_1 c_2 c_4

Can't schedule exam simultaneously for classes taken by s_i

Want to minimize exam slots.

$\boxed{\text{3 slots}}$

Make G: V = classes E = conflicts

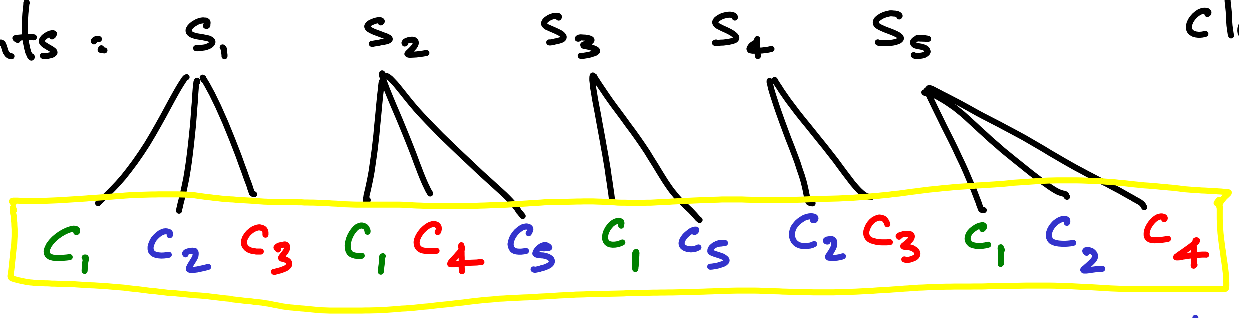


Colors = slots (minimize colors)

If no edge has same color at endpoints,
then no 2 classes are in same slot

EXAM SCHEDULING

students :



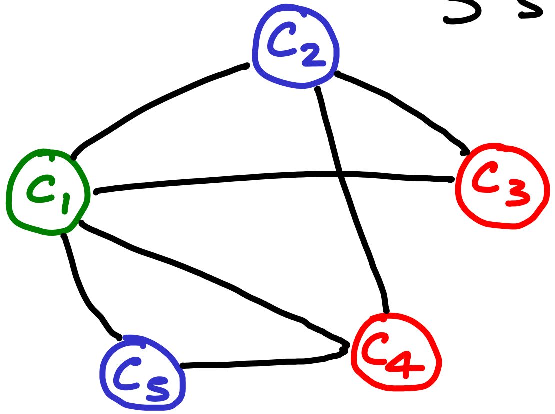
classes c_1 c_2 c_3 c_4 c_5

Can't schedule exam simultaneously for classes taken by s_i

Want to minimize exam slots.

Make G : V = classes E = conflicts

3 slots

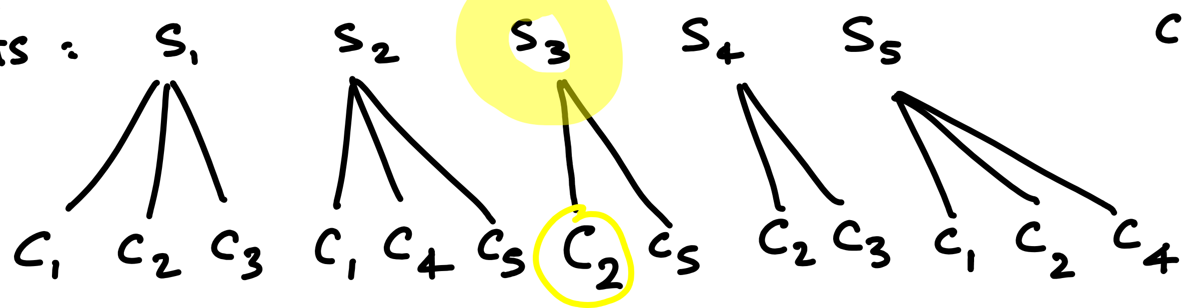


Colors = slots (minimize colors)

If no edge has same color at endpoints, then no 2 classes are in same slot

EXAM SCHEDULING

students :



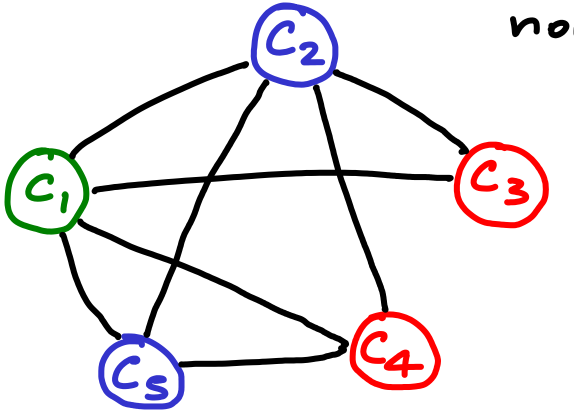
classes c_1 c_2 c_3 c_4 c_5

Can't schedule exam simultaneously for classes taken by s_i

Want to minimize exam slots.

Make G : V = classes E = conflicts

Need 4 now



Colors = slots (minimize colors)

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