if you came to the first lecture, you've seen me before (fact)

(for Spring'14)

if you came to the first lecture, you've seen me before (fact)

if you've never seen me before, you didn't come to the first lecture

CONTRAPOSITIVE

if you came to the first lecture, you've seen me before (fact)

equivalent

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$\updownarrow$ equivalent $\updownarrow$

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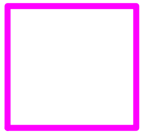
if you are a square, you have corners  (fact)

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? ?

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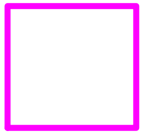
if you are a square, you have corners  (fact)

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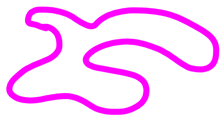
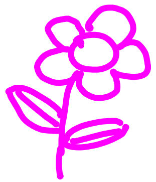
if you don't have corners, you are not a square  

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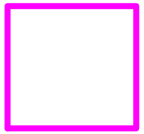
if you are a square, you have corners  (fact)

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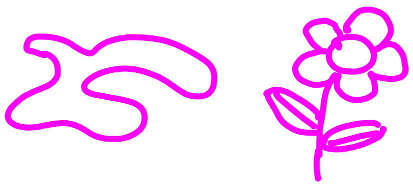
if you don't have corners, you are not a square 

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if A then B = ?

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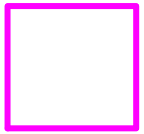
if you are a square, you have corners  (fact)

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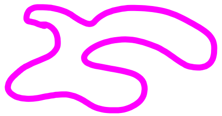
if you don't have corners, you are not a square  

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if A then B = if not B, then not A

CONTRAPOSITIVE: if A then B = if not B, then not A

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$$A \to B = \neg B \to \neg A$$

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What if $\neg A$ holds, but B is still true?

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a	b
T	T
T	F
F	T
F	F

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a	b	$a \to b$ valid?
T	T	?
T	F	
F	T	
F	F	

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a	b	$a \to b$ valid?
T	T	✓
T	F	?
F	T	
F	F	

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a	b	$a \to b$ valid?
T	T	✓
T	F	✗
F	T	?
F	F	?

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T	T	✓
T	F	✗
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don't contradict

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a	b	$a \to b$ valid?	$\neg b$	$\neg a$
T	T	✓	F	F
T	F	✗	T	F
F	T	✓	F	T
F	F	✓	T	T

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$A \to B$ = $\neg B \to \neg A$

What if $\neg A$ holds, but B is still true?

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a	b	$a \to b$ valid?	$\neg b$	$\neg a$	$(\neg b) \to (\neg a)$ valid?
T	T	✓	F	F	
T	F	✗	T	F	
F	T	✓	F	T	
F	F	✓	T	T	?

don't contradict

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T	T	✓	F	F	
T	F	✗	T	F	?
F	T	✓	F	T	
F	F	✓	T	T	✓

don't contradict

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	a	b	$a \to b$ valid?	$\neg b$	$\neg a$	$(\neg b) \to (\neg a)$ valid?
	T	T	✓	F	F	?
	T	F	✗	T	F	✗
don't contradict	F	T	✓	F	T	?
	F	F	✓	T	T	✓

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T	T	✓	F	F	✓
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F	T	✓	F	T	✓
F	F	✓	T	T	✓

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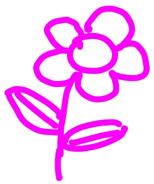
context so far: we know $A \to B$, so if we observe $\neg B$ then we can conclude $\neg A$



$\neg$ corners $\to$ $\neg$ square

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We don't know how to prove $A \to B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

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Prove: if $7x+9$ is even, then x is odd (for $x \in \mathbb{Z}$)

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direct

$7x+9 = 2a$ // a: integer $\to$ $7x+9$: even

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$x = 2(a - 3x - 5) + 1$

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direct	contrapositive
$7x+9 = 2a$ // a: integer $\to 7x+9$: even	
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$x = 2a - 6x - 10 + 1$	
$x = 2(a - 3x - 5) + 1$	
$x = 2b+1$ (odd) $(b = a-3x-5)$ $\square$	

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$$x = 2b + 1 \text{ (odd)} \; (b = a - 3x - 5) \quad \square$$

contrapositive

Suppose ... ?

context so far: we know $A \to B$, so if we observe $\neg B$ then we can conclude $\neg A$

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Prove: if $7x+9$ is even, then x is odd (for $x \in \mathbb{Z}$)

direct	contrapositive
$7x+9 = 2a$ // a: integer $\to 7x+9$: even	Suppose x is not odd: $x = ?$
$x = 2a - 6x - 9$	
$x = 2a - 6x - 10 + 1$	
$x = 2(a - 3x - 5) + 1$	
$x = 2b+1$ (odd) ($b = a - 3x - 5$) $\square$	

context so far: we know $A \to B$, so if we observe $\neg B$
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$$x = 2b + 1 \text{ (odd) } (b = a - 3x - 5) \quad \square$$

contrapositive

Suppose x is not odd: $x = 2c$

... then?

context so far: we know $A \to B$, so if we observe $\neg B$ then we can conclude $\neg A$

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$x = 2b+1$ (odd) $(b = a - 3x - 5)$ $\square$

contrapositive

Suppose x is not odd: $x = 2c$

$7x + 9 = 7 \cdot 2c + 9$

context so far: we know $A \to B$, so if we observe $\neg B$ then we can conclude $\neg A$

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contrapositive

Suppose x is not odd: $x = 2c$

$7x+9 = 7\cdot 2c+9$

$= 14c+8+1$

context so far: we know $A \to B$, so if we observe $\neg B$ then we can conclude $\neg A$

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contrapositive

Suppose x is not odd: $x = 2c$

$7x+9 = 7 \cdot 2c + 9$

$= 14c + 8 + 1$

$= 2 \cdot (7c+4) + 1$

context so far: we know $A \to B$, so if we observe $\neg B$ then we can conclude $\neg A$

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$$7x+9 = 2a \quad // a\text{: integer} \to 7x+9\text{: even}$$
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$$x = 2a-6x-10+1$$
$$x = 2(a-3x-5)+1$$
$$x = 2b+1 \text{ (odd) } (b = a-3x-5) \quad \square$$

contrapositive

Suppose x is not odd: $x = 2c$

$$7x+9 = 7\cdot 2c+9$$
$$= 14c+8+1$$
$$= 2\cdot(7c+4)+1$$
$$= 2\cdot d+1 \quad (d = 7c+4)$$

context so far: we know $A \to B$, so if we observe $\neg B$ then we can conclude $\neg A$

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contrapositive

Suppose x is not odd: $x = 2c$

$7x+9 = 7 \cdot 2c + 9$

$= 14c + 8 + 1$

$= 2 \cdot (7c+4) + 1$

$= 2 \cdot d + 1$ $(d = 7c+4)$

$7x+9 = \text{odd}$ $\square$

PROOF BY CONTRAPOSITIVE

We don't know how to prove $A \to B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $x^2 - 6x + 5$ is even, then x is odd

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phrase mathematically?

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phrase mathematically?

$$(x^2 - 6x + 5 = 2a) \longrightarrow (x = 2b + 1)$$

...assuming?

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Prove: if $x^2 - 6x + 5$ is even, then x is odd

phrase mathematically?

$$(x^2 - 6x + 5 = 2a) \longrightarrow (x = 2b + 1)$$

...assuming x, a, b are integers

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Prove: if $x^2 - 6x + 5$ is even, then x is odd

$x^2 - 6x + 5 = 2a$ direct

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$x^2 - 6x + 5 = 2a$ direct

⋮ ?

$x = 2b + 1$

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Prove: if $x^2 - 6x + 5$ is even, then x is odd

$$x^2 - 6x + 5 = 2a$$

direct

$$x^2 - 6x + (5 - 2a) = 0$$

$$x = \frac{6 \pm \sqrt{36 + 8a - 20}}{2}$$

$$x = 3 \pm \sqrt{4 + 2a}$$

$\vdots$?

$$x = 2b + 1$$

PROOF BY CONTRAPOSITIVE

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direct	contrapositive
$x^2 - 6x + 5 = 2a$ $\vdots$? $x = 2b + 1$	...?

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direct	contrapositive
$x^2 - 6x + 5 = 2a$ $\vdots$? $x = 2b + 1$	Suppose x is not odd: $x = 2c$

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$x^2 - 6x + 5 = 2a$ direct

$\vdots$?

$x = 2b + 1$

contrapositive

Suppose x is not odd: $x = 2c$

$x^2 - 6x + 5 = (2c)^2 - 6 \cdot 2c + 5$

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Prove: if $x^2 - 6x + 5$ is even, then x is odd

direct	contrapositive
$x^2 - 6x + 5 = 2a$ $\vdots$? $x = 2b + 1$	Suppose x is not odd: $x = 2c$ $x^2 - 6x + 5 = (2c)^2 - 6 \cdot 2c + 5$ $= 4c^2 - 12c + 5$

PROOF BY CONTRAPOSITIVE

We don't know how to prove $A \to B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

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$\vdots$?

$x = 2b + 1$

contrapositive

Suppose x is not odd: $x = 2c$

$$\begin{aligned} x^2 - 6x + 5 &= (2c)^2 - 6 \cdot 2c + 5 \\ &= 4c^2 - 12c + 5 \\ &= 4c^2 - 12c + 4 + 1 \end{aligned}$$

because we want to get something odd

PROOF BY CONTRAPOSITIVE

We don't know how to prove $A \to B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $x^2 - 6x + 5$ is even, then x is odd

direct	contrapositive
$x^2 - 6x + 5 = 2a$	Suppose x is not odd: $\quad x = 2c$
$\vdots$?	$x^2 - 6x + 5 = (2c)^2 - 6 \cdot 2c + 5$
$x = 2b + 1$	$= 4c^2 - 12c + 5$
	$= 4c^2 - 12c + 4 + 1$
	$= 2 \cdot (2c^2 - 6c + 2) + 1$

PROOF BY CONTRAPOSITIVE

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$x^2 - 6x + 5 = 2a$ $\vdots$? $x = 2b + 1$	Suppose x is not odd: $x = 2c$ $x^2 - 6x + 5 = (2c)^2 - 6 \cdot 2c + 5$ $= 4c^2 - 12c + 5$ $= 4c^2 - 12c + 4 + 1$ $= 2 \cdot (2c^2 - 6c + 2) + 1$ $= 2 \cdot d + 1$ $(d = 2c^2 - 6c + 2)$

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PROOF BY CONTRAPOSITIVE

We don't know how to prove $A \to B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove : if x is irrational then $\sqrt{x}$ is irrational

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direct

???

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???	Suppose $\sqrt{x}$ is not irrational

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direct	contrapositive
???	Suppose $\sqrt{x}$ is not irrational $\sqrt{x} = \frac{a}{b}$

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direct	contrapositive
???	Suppose $\sqrt{x}$ is not irrational $\sqrt{x} = \frac{a}{b}$ $x = \frac{a^2}{b^2}$

PROOF BY CONTRAPOSITIVE

We don't know how to prove $A \to B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if x is irrational then $\sqrt{x}$ is irrational

direct

???

contrapositive

Suppose $\sqrt{x}$ is not irrational

$$\sqrt{x} = \frac{a}{b}$$

$$x = \frac{a^2}{b^2} \quad : \text{ not irrational} \quad \square$$

PROOF BY CONTRADICTION

a slight generalization of proof by contrapositive

still proving if A then B for now

PROOF BY CONTRADICTION

a slight generalization of proof by contrapositive

still proving **if A then B** for now

$$(a+b)\cdot(a-b) \longrightarrow a^2-ab+ba-b^2 \longrightarrow a^2-b^2$$

You can prove something directly (in one direction)

PROOF BY CONTRADICTION

a slight generalization of proof by contrapositive

---

still proving **if A then B** for now

$$(a+b)\cdot(a-b) \longrightarrow a^2-ab+ba-b^2 \longleftarrow a^2-b^2$$

You can prove something directly (in one direction)
or work in both directions

PROOF BY CONTRADICTION

a slight generalization of proof by contrapositive

still proving if A then B for now

$(a+b)\cdot(a-b) \longleftrightarrow a^2-ab+ba-b^2 \longleftrightarrow a^2-b^2$

You can prove something directly (in one direction)
or work in both directions

contrapositive $\Big\{$ starting w/ $\neg B$ & leading to $\neg A$

contradicts $A \to \neg B$

PROOF BY CONTRADICTION

a slight generalization of proof by contrapositive

still proving if A then B for now

$$(a+b)\cdot(a-b) \longleftrightarrow a^2-ab+ba-b^2 \longleftrightarrow a^2-b^2$$

You can prove something directly (in one direction) or work in both directions

instead of starting w/ $\neg B$ & leading to $\neg A$
(which contradicts $A \to \neg B$)

PROOF BY CONTRADICTION

a slight generalization of proof by contrapositive

still proving if A then B for now

$$(a+b)\cdot(a-b) \longleftrightarrow a^2 - ab + ba - b^2 \longleftrightarrow a^2 - b^2$$

You can prove something directly (in one direction)
or work in both directions

instead of starting w/ $\neg B$ & leading to $\neg A$
(which contradicts $A \to \neg B$)

$\boxed{\text{assume both } A \text{ and } \neg B \text{ are true \& arrive at some contradicting statement}}$

PROOF BY CONTRADICTION

If x is even then x is not odd

PROOF BY CONTRADICTION

If $\underbrace{x \text{ is even}}_{A}$ then $\underbrace{x \text{ is not odd}}_{B}$

PROOF BY CONTRADICTION

If $\underbrace{\text{x is even}}_{A}$ then $\underbrace{\text{x is not odd}}_{B}$

Assume $A \wedge \neg B$: ...

PROOF BY CONTRADICTION

If $\underbrace{\text{x is even}}_{A}$ then $\underbrace{\text{x is not odd}}_{B}$

Assume $A \wedge \neg B$: x is even & x is odd

PROOF BY CONTRADICTION

If $\underbrace{x \text{ is even}}_{A}$ then $\underbrace{x \text{ is not odd}}_{B}$

Assume $A \wedge \neg B$: x is even & x is odd

$\downarrow$? $\qquad \downarrow$?

PROOF BY CONTRADICTION

If $\underbrace{x \text{ is even}}_{A}$ then $\underbrace{x \text{ is not odd}}_{B}$

Assume $A \wedge \neg B$: x is even & x is odd

$$\downarrow \qquad\qquad \downarrow$$

(a: int.) $x = 2a$ $\qquad$ $x = 2b+1$ (b: int.)

PROOF BY CONTRADICTION

If $\underbrace{x \text{ is even}}_{A}$ then $\underbrace{x \text{ is not odd}}_{B}$

Assume $A \wedge \neg B$: x is even & x is odd

$$x = 2a \quad (a: \text{int.}) \qquad x = 2b+1 \quad (b: \text{int.})$$

$$2a = 2b+1$$

PROOF BY CONTRADICTION

If $\underbrace{x \text{ is even}}_{A}$ then $\underbrace{x \text{ is not odd}}_{B}$

Assume $A \wedge \neg B$: x is even & x is odd

$$\downarrow \qquad\qquad \downarrow$$

(a: int.) $x = 2a$ $\qquad$ $x = 2b+1$ (b: int.)

$$2a = 2b+1$$

$$a = b + \frac{1}{2}$$

PROOF BY CONTRADICTION

If $\underbrace{x \text{ is even}}_{A}$ then $\underbrace{x \text{ is not odd}}_{B}$

Assume $A \wedge \neg B$: x is even & x is odd

$$x \text{ is even} \Rightarrow x = 2a \quad (a: \text{int.})$$

$$x \text{ is odd} \Rightarrow x = 2b+1 \quad (b: \text{int.})$$

$$2a = 2b+1$$

$$a = b + \tfrac{1}{2}$$

impossible/absurd/contradiction

PROOF BY CONTRADICTION

If $\underbrace{x \text{ is even}}_{A}$ then $\underbrace{x \text{ is not odd}}_{B}$

Assume $A \wedge \neg B$: x is even & x is odd

$$\downarrow \qquad\qquad \downarrow$$

(a: int.) $x = 2a$ $\qquad$ $x = 2b+1$ (b: int.)

$$2a = 2b+1$$

$$a = b + \frac{1}{2}$$

impossible / absurd / contradiction

Notice we met halfway at an incorrect statement.

PROOF BY CONTRADICTION

For integers $a \neq 0$ & b, there is only one number x s.t. $ax + b = 0$.

state this in IF-THEN form

PROOF BY CONTRADICTION

For integers $a \neq 0$ & b, there is only one number s.t. $ax+b=0$.

(if $ax+b=0$ then for $y \neq x$, $ay+b \neq 0$)

PROOF BY CONTRADICTION

For integers $a \neq 0$ & b, there is only one number s.t. $ax + b = 0$.

(if $\underbrace{ax+b=0}_{A}$ then for $y \neq x$, $\underbrace{ay+b \neq 0}_{B}$)

PROOF BY CONTRADICTION

For integers $a \neq 0$ & b, there is only one number s.t. $ax + b = 0$.

$\left(\text{if } \underbrace{ax+b=0}_{A} \text{ then for } y \neq x,\ \underbrace{ay+b \neq 0}_{B} \right)$

Assume $A \wedge \neg B$: $ax+b=0$ & $ay+b=0$

PROOF BY CONTRADICTION

For integers $a \neq 0$ & b, there is only one number s.t. $ax+b=0$.

$\left(\text{if } \underbrace{ax+b=0}_{A} \text{ then for } y \neq x, \ \underbrace{ay+b \neq 0}_{B} \right)$

Assume $A \wedge \neg B$: $ax+b=0$ & $ay+b=0$

$$ax+b = ay+b$$

PROOF BY CONTRADICTION

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(if $\underbrace{ax+b=0}_{A}$ then for $y \neq x$, $\underbrace{ay+b \neq 0}_{B}$)

Assume $A \wedge \neg B$: $ax+b=0$ & $ay+b=0$

$$ax+b = ay+b$$

$$ax = ay$$

PROOF BY CONTRADICTION

For integers $a \neq 0$ & b, there is only one number s.t. $ax + b = 0$.

$$\text{(if } \underbrace{ax+b=0}_{A} \text{ then for } y \neq x,\ \underbrace{ay+b \neq 0}_{B}\text{)}$$

Assume $A \wedge \neg B$:

$$ax+b=0 \quad \& \quad ay+b=0$$

$$ax+b = ay+b$$

$$ax = ay$$

$$x = y$$

PROOF BY CONTRADICTION

For integers $a \neq 0$ & b, there is only one number s.t. $az + b = 0$.

(if $\underbrace{ax+b=0}_{A}$ then for $y \neq x$, $\underbrace{ay+b \neq 0}_{B}$)

Assume $A \wedge \neg B$: $ax+b=0$ & $ay+b=0$

$$ax+b = ay+b$$

$$ax = ay$$

$$x = y$$ — contradicts

up next:

more cool examples,

<u>not</u> in "if A then B" format

$\sqrt{2}$ IS IRRATIONAL - PROOF BY CONTRADICTION

$\sqrt{2}$ IS IRRATIONAL – PROOF BY CONTRADICTION

1) what does the claim mean?

$\sqrt{2}$ IS IRRATIONAL - PROOF BY CONTRADICTION

1) what does the claim mean?

1) $\nexists$ integers $\{a, b\}$ s.t. $\sqrt{2} = \frac{a}{b}$

$\sqrt{2}$ IS IRRATIONAL – PROOF BY CONTRADICTION

1) what does the claim mean?

2) assume the contrary is true

1) $\nexists$ integers $\{a, b\}$ s.t. $\sqrt{2} = \frac{a}{b}$

2) $\exists$ integers $\{a, b\}$ s.t. $\sqrt{2} = \frac{a}{b}$

$\sqrt{2}$ IS IRRATIONAL – PROOF BY CONTRADICTION

1) what does the claim mean?

2) assume the contrary is true

3) use this to establish something that you know is wrong

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3) if (2) is true, then choose $\{a,b\}$ w/ <u>no common divisor</u>

(simplify)

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By (2), $2 = \frac{a^2}{b^2}$

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By (2), $2 = \frac{a^2}{b^2} \Rightarrow \boxed{2b^2 = a^2}$

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2) $\exists$ integers $\{a,b\}$ s.t. $\sqrt{2} = \frac{a}{b}$

3) if (2) is true, then choose $\{a,b\}$ w/ no common divisor

By (2), $2 = \frac{a^2}{b^2} \Rightarrow \underbrace{2}b^2 = a^2 \Rightarrow a^2$: $\underbrace{\text{even}}$

$\sqrt{2}$ IS IRRATIONAL – PROOF BY CONTRADICTION

1) what does the claim mean?

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3) use this to establish something that you know is wrong

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2) $\exists$ integers $\{a,b\}$ s.t. $\sqrt{2} = \frac{a}{b}$

3) if (2) is true, then choose $\{a,b\}$ w/ no common divisor

By (2), $2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2 \Rightarrow \boxed{\begin{array}{l} a^2 : \text{even} \\ (a : \text{even}) \end{array}}$

why?

homework?

$\sqrt{2}$ IS IRRATIONAL - PROOF BY CONTRADICTION

1) what does the claim mean?

2) assume the contrary is true

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By (2), $2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2 \Rightarrow a^2$: even
(a: even)

$a = 2c$ $\{c: \text{int}\}$

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By (2), $2 = \frac{a^2}{b^2} \Rightarrow \underbrace{2b^2 = a^2} \Rightarrow a^2$: even
(a: even)

$a = 2c$ $\{c: \text{int}\}$ $\Rightarrow 2b^2 = 4c^2$

$\sqrt{2}$ IS IRRATIONAL - PROOF BY CONTRADICTION

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3) if (2) is true, then choose $\{a,b\}$ w/ <u>no common divisor</u>

By (2), $2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2 \Rightarrow a^2$: even

(a: even)

$a = 2c$ $\{c: \text{int}\}$ $\Rightarrow 2b^2 = 4c^2 \Rightarrow b$: even

$(b^2 = 2c^2 \Rightarrow b^2 : \text{even})$

$\sqrt{2}$ IS IRRATIONAL - PROOF BY CONTRADICTION

1) what does the claim mean?

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By (2), $2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2 \Rightarrow a^2$: even
(a: even)

$a = 2c$ $\{c: \text{int}\} \Rightarrow 2b^2 = 4c^2 \Rightarrow b$: even
($b = 2d$)

$\sqrt{2} = \frac{a}{b} = \frac{2c}{2d}$

$\sqrt{2}$ IS IRRATIONAL – PROOF BY CONTRADICTION

1) what does the claim mean?

2) assume the contrary is true

3) use this to establish something that you know is wrong

1) $\nexists$ integers $\{a,b\}$ s.t. $\sqrt{2} = \frac{a}{b}$

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3) if (2) is true, then choose $\{a,b\}$ w/ no common divisor

By (2), $2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2 \Rightarrow a^2$: even (a: even)

$a = 2c$ $\{c: \text{int}\} \Rightarrow 2b^2 = 4c^2 \Rightarrow b$: even

$\sqrt{2} = \frac{a}{b} = \frac{2c}{2d}$ contradiction

$\sqrt{2}$ IS IRRATIONAL – PROOF BY CONTRADICTION

1) what does the claim mean?

2) assume the contrary is true

3) use this to establish something that you know is wrong

4) conclude that (2) is false thus the initial claim is true

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2) $\exists$ integers $\{a, b\}$ s.t. $\sqrt{2} = \frac{a}{b}$

3) if (2) is true, then choose $\{a, b\}$ w/ no common divisor

By (2), $2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2 \Rightarrow a^2$: even
(a: even)

$a = 2c$ $\{c: \text{int}\} \Rightarrow 2b^2 = 4c^2 \Rightarrow b$: even

$\sqrt{2} = \frac{a}{b} = \frac{2c}{2d}$ contradiction

THERE ARE AN INFINITE NUMBER OF PRIMES

(proof by contradiction)

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- Assume that #primes is finite: $p_1, p_2, \ldots, p_n$

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- Assume that #primes is finite: $p_1, p_2, \ldots, p_n$
- Let $t = 1 + \prod_{i=1}^{n} p_i$ (i.e., $1 + p_1 \times p_2 \times \cdots \times p_n$)

THERE ARE AN INFINITE NUMBER OF PRIMES

(proof by contradiction)

- Assume that #primes is finite: $p_1, p_2, \ldots, p_n$
- Let $t = 1 + \prod_{i=1}^{n} p_i$ (i.e., $1 + p_1 \times p_2 \times \cdots \times p_n$)
- Notice $t > p_i$ for all i.

THERE ARE AN INFINITE NUMBER OF PRIMES

(proof by contradiction)

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- Let $t = 1 + \prod_{i=1}^{n} p_i$ (i.e., $1 + p_1 \times p_2 \times \cdots \times p_n$)
- Notice $t > p_i$ for all i. So if t is prime, contradiction.

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(proof by contradiction)

- Assume that #primes is finite: $p_1, p_2, \ldots, p_n$
- Let $t = 1 + \prod_{i=1}^{n} p_i$ (i.e., $1 + p_1 \times p_2 \times \cdots \times p_n$)
- Notice $t > p_i$ for all i. So if t is prime, contradiction.
- If t is not prime then $\exists$ prime factor $q \neq t$ of t

(you've seen this, t is composite)

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(proof by contradiction)

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- Let $t = 1 + \prod_{i=1}^{n} p_i$ (i.e., $1 + p_1 \times p_2 \times \cdots \times p_n$)
- Notice $t > p_i$ for all i. So if t is prime, contradiction.
- If t is not prime then $\exists$ prime factor $q \neq t$ of t (you've seen this, t is composite)
  - if $q \neq p_i$ (for all i) ?

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(proof by contradiction)

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- Let $t = 1 + \prod_{i=1}^{n} p_i$ (i.e., $1 + p_1 \times p_2 \times \cdots \times p_n$)
- Notice $t > p_i$ for all i. So if t is prime, contradiction.
- If t is not prime then $\exists$ prime factor $q \neq t$ of t (you've seen this, t is composite)
  - if $q \neq p_i$ (for all i), contradiction.

THERE ARE AN INFINITE NUMBER OF PRIMES

(proof by contradiction)

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- Let $t = 1 + \prod_{i=1}^{n} p_i$ (i.e., $1 + p_1 \times p_2 \times \cdots \times p_n$)
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  (you've seen this, t is composite)

  - if $q \neq p_i$ (for all i), contradiction.
  - if $q = p_j$?

THERE ARE AN INFINITE NUMBER OF PRIMES

(proof by contradiction)

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- Let $t = 1 + \prod_{i=1}^{n} p_i$ (i.e., $1 + p_1 \times p_2 \times \cdots \times p_n$)
- Notice $t > p_i$ for all i. So if t is prime, contradiction.
- If t is not prime then $\exists$ prime factor $q \neq t$ of t

  (you've seen this, t is composite)

  - if $q \neq p_i$ (for all i), contradiction.
  - if $q = p_j$, we know q divides $\prod_{i=1}^{n} p_i$

THERE ARE AN INFINITE NUMBER OF PRIMES

(proof by contradiction)

- Assume that #primes is finite: $p_1, p_2, \dots, p_n$
- Let $t = 1 + \prod_{i=1}^{n} p_i$ (i.e., $1 + p_1 \times p_2 \times \cdots \times p_n$)
- Notice $t > p_i$ for all i. So if t is prime, contradiction.
- If t is not prime then $\exists$ prime factor $q \neq t$ of t (you've seen this, t is composite)
  - if $q \neq p_i$ (for all i), contradiction.
  - if $q = p_j$, we know q divides $\prod_{i=1}^{n} p_i$

    but then ?

THERE ARE AN INFINITE NUMBER OF PRIMES

(proof by contradiction)

- Assume that #primes is finite: $p_1, p_2, \dots, p_n$
- Let $t = 1 + \prod_{i=1}^{n} p_i$ (i.e., $1 + p_1 \times p_2 \times \cdots \times p_n$)
- Notice $t > p_i$ for all i. So if t is prime, contradiction.
- If t is not prime then $\exists$ prime factor $q \neq t$ of t $\}$ q is prime, divides t, and $1 < q < n$

  (you've seen this, t is composite)
  - if $q \neq p_i$ (for all i), contradiction.
  - if $q = p_j$, we know q divides $\prod_{i=1}^{n} p_i$

    but then it can't also divide $1 + \prod_{i=1}^{n} p_i$ (contr.)