if you came to the first lecture, you've seen me before  (fact)

(for Spring'14)
if you've never seen me before, you didn't come to the first lecture
if you came to the first lecture, you've seen me before (fact)

\[ \text{equivalent} \]

if you've never seen me before, you didn't come to the first lecture
CONTRAPOSITIVE

if you came to the first lecture, you’ve seen me before (fact)

if you’ve never seen me before, you didn’t come to the first lecture

if you are a square, you have corners (fact)

? equivalent ?
**CONTRAPOSITIVE**

If you came to the first lecture, you've seen me before \( \text{fact} \)

\[ \text{equivalent} \]

If you've never seen me before, you didn't come to the first lecture

If you are a square, you have corners \( \text{fact} \)

\[ \text{equivalent} \]

If you don't have corners, you are not a square
CONTRAPOSITIVE

if you came to the first lecture, you've seen me before (fact)

if you've never seen me before, you didn't come to the first lecture

if you are a square, you have corners (fact)

if you don't have corners, you are not a square

if A then B = ?
CONTRAPOSITIVE

if you came to the first lecture, you've seen me before \( \text{ (fact)} \)

if you've never seen me before, you didn't come to the first lecture

if you are a square, you have corners \( \text{ (fact)} \)

if you don't have corners, you are not a square

if \( A \) then \( B \) = if not \( B \), then not \( A \)
CONTRAPOSITIVE: \[ \text{if } A \text{ then } B \; \Rightarrow \; \text{if not } B, \text{ then not } A \]
CONTRAPOSITIVE: \[ \text{if } A \text{ then } B \quad = \quad \text{if not } B, \text{ then not } A \]

\[ A \rightarrow B \quad = \quad \neg B \rightarrow \neg A \]
CONTRAPOSITIVE:  \( \text{if } A \text{ then } B = \text{ if not } B, \text{ then not } A \)

\( A \rightarrow B = \neg B \rightarrow \neg A \)

What if \( \neg A \) holds, but \( B \) is still true?
CONTRAPOSITIVE: \( \text{if } A \text{ then } B = \text{if not } B, \text{ then not } A \)

\[ A \rightarrow B = \neg B \rightarrow \neg A \]

What if \( \neg A \) holds, but \( B \) is still true?

\( \iff \text{That's ok; doesn't contradict the above. It's not IFF} \)
CONTRAPOSITIVE: \[ \text{if } A \text{ then } B = \text{if not } B, \text{ then not } A \]
\[ A \rightarrow B = \neg B \rightarrow \neg A \]

What if \( \neg A \) holds, but \( B \) is still true?

\[ \iff \text{That's ok; doesn't contradict the above. It's not IFF} \]

\[
\begin{array}{c|c|c}
  a & b & a \rightarrow b \\
  
  T & T & T \\
  T & F & F \\
  F & T & F \\
  F & F & F \\
\end{array}
\]
CONTRAPOSITIVE: \[ \text{if } A \text{ then } B = \text{ if not } B, \text{ then not } A \]
\[ A \rightarrow B = \neg B \rightarrow \neg A \]

What if \( \neg A \) holds, but \( B \) is still true?

\[ \rightarrow \text{That's ok; doesn't contradict the above. It's not IFF} \]

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>( a \rightarrow b ) valid?</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>?</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>
CONTRAPOSITIVE: \[ \text{if } A \text{ then } B = \text{ if not } B, \text{ then not } A \]

\[ A \rightarrow B = \neg B \rightarrow \neg A \]

What if \( \neg A \) holds, but \( B \) is still true?

\( \neg A \rightarrow \neg B \rightarrow \neg A \)

That's ok; doesn't contradict the above. It's not IFF

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>a→b valid?</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>✓</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>?</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>
CONTRAPOSITIVE:  \[\text{if } A \text{ then } B = \text{if not } B, \text{ then not } A\]
\[A \rightarrow B = \neg B \rightarrow \neg A\]

What if \(\neg A\) holds, but \(B\) is still true?
\[\leftrightarrow\text{ That's ok; doesn't contradict the above. It's not IFF}\]

\[
\begin{array}{c|c|c|c}
 a & b & a \rightarrow b & \text{valid?} \\
 T & T & \checkmark & \\
 T & F & \times & \\
 F & T & ? & \\
 F & F & ? & \\
\end{array}
\]
CONTRAPOSITIVE: \(\text{if } A \text{ then } B \equiv \text{if not } B, \text{ then not } A\)

\(A \rightarrow B \equiv \neg B \rightarrow \neg A\)

What if \(\neg A\) holds, but \(B\) is still true?

\(\neg\neg\) That's ok; doesn't contradict the above. It's not IFF

<table>
<thead>
<tr>
<th>(a)</th>
<th>(b)</th>
<th>(a \rightarrow b)</th>
<th>valid?</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>✔</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>✘</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>✗</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>✔</td>
<td></td>
</tr>
</tbody>
</table>

\(\) don't contradict \(\{\)
**CONTRAPOSITIVE:** \[ \text{if } A \text{ then } B = \text{if not } B, \text{ then not } A \]
\[ A \rightarrow B = \neg B \rightarrow \neg A \]

What if \( \neg A \) holds, but B is still true?

\[ \Rightarrow \text{That's OK; doesn't contradict the above. It's not IFF} \]

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>a→b valid?</th>
<th>\neg b</th>
<th>\neg a</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>✓</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>✗</td>
<td>T</td>
<td>F</td>
</tr>
</tbody>
</table>

*don't contradict*

\[ \begin{align*}
\{ & F, T \\
\{ & F, F \\
\end{align*} \]

\[ \begin{align*}
\checkmark & | F, T \\
\checkmark & | T, T \\
\]
CONTRAPOSITIVE: \( \text{if } A \text{ then } B = \text{ if not } B, \text{ then not } A \)

\[ A \rightarrow B = \neg B \rightarrow \neg A \]

What if \( \neg A \) holds, but \( B \) is still true?

\[ \neg A \rightarrow \text{not true} \]

\[ \neg B \rightarrow \neg A \]

\[ \neg b \rightarrow \neg a \]

\[ (\neg b) \rightarrow (\neg a) \]

\[ \text{valid?} \]

<table>
<thead>
<tr>
<th>( a )</th>
<th>( b )</th>
<th>( a \rightarrow b )</th>
<th>( \neg b )</th>
<th>( \neg a )</th>
<th>( (\neg b) \rightarrow (\neg a) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>✓</td>
<td>F</td>
<td>F</td>
<td>✓</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>✗</td>
<td>T</td>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>

\( \neg a \) is valid because \( a \rightarrow b \) and \( b \) is true.

\( \neg a \) is valid because \( a \rightarrow b \) and \( b \) is true.

\( \neg a \) is valid because \( a \rightarrow b \) and \( b \) is true.

\( \neg a \) is valid because \( a \rightarrow b \) and \( b \) is true.

\( \neg a \) is valid because \( a \rightarrow b \) and \( b \) is true.

\( \neg a \) is valid because \( a \rightarrow b \) and \( b \) is true.
**CONTRAPOSITIVE:** \[ \text{if } A \text{ then } B = \text{if not } B, \text{ then not } A \]
\[ A \rightarrow B = \neg B \rightarrow \neg A \]

What if \( \neg A \) holds, but \( B \) is still true?

\( \leftrightarrow \) That's OK; doesn't contradict the above. It's not IFF

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>( a\rightarrow b ) valid?</th>
<th>( \neg b )</th>
<th>( \neg a )</th>
<th>(( \neg b \rightarrow \neg a )) valid?</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>✓</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>×</td>
<td>T</td>
<td>F</td>
<td>?</td>
</tr>
</tbody>
</table>

\text{don't contradict} \{ \begin{align*}
\text{F} & \text{T} \quad \checkmark \\
\text{F} & \text{F} \quad \checkmark \\
\end{align*} \}  \\

\text{F} \text{ T} \quad \text{T} \text{ T} \quad \checkmark
**CONTRAPOSITIVE:** \[ \text{if } A \text{ then } B = \text{if not } B, \text{ then not } A \]
\[ A \rightarrow B = \neg B \rightarrow \neg A \]

What if \( \neg A \) holds, but \( B \) is still true?

\( \neg A \rightarrow \neg B \) doesn't contradict the above. It's not IFF

<table>
<thead>
<tr>
<th>( a )</th>
<th>( b )</th>
<th>( a \rightarrow b ) valid?</th>
<th>( \neg b )</th>
<th>( \neg a )</th>
<th>( (\neg b) \rightarrow (\neg a) ) valid?</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>✓</td>
<td>F</td>
<td>F</td>
<td>?</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>×</td>
<td>T</td>
<td>F</td>
<td>×</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>✓</td>
<td>F</td>
<td>T</td>
<td>?</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>✓</td>
<td>T</td>
<td>T</td>
<td>✓</td>
</tr>
</tbody>
</table>

\{ don't contradict \}
CONTRAPOSITIVE: \[ \text{if } A \text{ then } B = \text{if not } B, \text{ then not } A \]
\[ A \rightarrow B = \neg B \rightarrow \neg A \]

What if \( \neg A \) holds, but \( B \) is still true?

\[ \Leftrightarrow \text{That's ok; doesn't contradict the above. It's not IFF} \]

<table>
<thead>
<tr>
<th>( a )</th>
<th>( b )</th>
<th>( a \rightarrow b ) valid?</th>
<th>( \neg b )</th>
<th>( \neg a )</th>
<th>( (\neg b) \rightarrow (\neg a) ) valid?</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>✓</td>
<td>F</td>
<td>F</td>
<td>✓</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>✗</td>
<td>T</td>
<td>F</td>
<td>✗</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>✓</td>
<td>F</td>
<td>T</td>
<td>✓</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>✓</td>
<td>T</td>
<td>T</td>
<td>✓</td>
</tr>
</tbody>
</table>

don't contradict \{ F, T \}
**CONTRAPOSITIVE:**  
if $A$ then $B$  
$A \rightarrow B$  
if not $B$, then not $A$  
$\neg B \rightarrow \neg A$

What if $\neg A$ holds, but $B$ is still true?

$\neg A$ holds, but $B$ is still true?

That's ok; doesn't contradict the above. It's not IFF.

<table>
<thead>
<tr>
<th>$a$</th>
<th>$b$</th>
<th>$a \rightarrow b$ valid?</th>
<th>$\neg b$</th>
<th>$\neg a$</th>
<th>$(\neg b) \rightarrow (\neg a)$ valid?</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>✓</td>
<td>F</td>
<td>F</td>
<td>✓</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>✗</td>
<td>T</td>
<td>F</td>
<td>✗</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>✓</td>
<td>F</td>
<td>T</td>
<td>✓</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>✓</td>
<td>T</td>
<td>T</td>
<td>✓</td>
</tr>
</tbody>
</table>
context so far: we know $A \rightarrow B$, so if we observe $\neg B$
then we can conclude $\neg A$

$\neg$ corners $\rightarrow \neg$ square
context so far: we know $A \rightarrow B$, so if we observe $\neg B$
then we can conclude $\neg A$

**PROOF BY CONTRAPOSITIVE**

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.
context so far: we know \( A \to B \), so if we observe \( \neg B \)
then we can conclude \( \neg A \)

**PROOF BY CONTRAPOSITIVE**

We don't know how to prove \( A \to B \) (easily), so we try to start by assuming \( \neg B \). If we conclude \( \neg A \), we are done.

Prove: if \( 7x + 9 \) is even, then \( x \) is odd \((\text{for } x \in \mathbb{Z})\)
context so far: we know $A \rightarrow B$, so if we observe $\neg B$
then we can conclude $\neg A$

**Proof by Contrapositive**

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $7x + 9$ is even, then $x$ is odd (for $x \in \mathbb{Z}$)

$7x + 9 = 2a$  // $a: \text{integer} \Rightarrow 7x + 9: \text{even}$
Context so far: we know $A \rightarrow B$, so if we observe $\neg B$ then we can conclude $\neg A$.

**Proof by Contrapositive**

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $7x + 9$ is even, then $x$ is odd \((\text{for } x \in \mathbb{Z})\)

\[
\begin{align*}
7x + 9 &= 2a \quad \// a : \text{integer } \rightarrow 7x + 9 : \text{even} \\
x &= 2a - 6x - 9
\end{align*}
\]
context so far: we know $A \rightarrow B$, so if we observe $\neg B$ then we can conclude $\neg A$

**PROOF BY CONTRAPOSITIVE**

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

**Prove:** if $7x + 9$ is even, then $x$ is odd  
(for $x \in \mathbb{Z}$)

\[
\begin{align*}
7x + 9 &= 2a \quad // a: \text{integer} \rightarrow 7x + 9: \text{even} \\
x &= 2a - 6x - 9 \\
x &= 2a - 6x - 10 + 1
\end{align*}
\]
context so far: we know $A \rightarrow B$, so if we observe $\neg B$
then we can conclude $\neg A$

---

**Proof by Contrapositive**

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $7x + 9$ is even, then $x$ is odd (for $x \in \mathbb{Z}$)

\[
7x + 9 = 2a \quad // a: integer \rightarrow 7x + 9: even
\]

\[
x = 2a - 6x - 9
\]

\[
x = 2a - 6x - 10 + 1
\]

\[
x = 2(a - 3x - 5) + 1
\]
context so far: we know $A \rightarrow B$, so if we observe $\neg B$
then we can conclude $\neg A$

**Proof by Contrapositive**

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $7x + 9$ is even, then $x$ is odd (for $x \in \mathbb{Z}$)

$$7x + 9 = 2a \quad // a: integer \Rightarrow 7x + 9 : even$$

\[
x = 2a - 6x - 9 \\
x = 2a - 6x - 10 + 1 \\
x = 2(a - 3x - 5) + 1 \\
x = 2b + 1 \quad (b = a - 3x - 5)
\]
context so far: we know \( A \rightarrow B \), so if we observe \( \neg B \)
then we can conclude \( \neg A \)

**Proof by Contrapositive**

We don't know how to prove \( A \rightarrow B \) (easily), so we try to start by assuming \( \neg B \). If we conclude \( \neg A \), we are done.

Prove: if \( 7x+9 \) is even, then \( x \) is odd (for \( x \in \mathbb{Z} \))

\[
7x + 9 = 2a \quad // \text{a: integer} \rightarrow 7x + 9 \text{: even}
\]

\[
x = 2a - 6x - 9
\]

\[
x = 2a - 6x - 10 + 1
\]

\[
x = 2(a - 3x - 5) + 1
\]

\[
x = 2b + 1 \quad \text{(odd)} \quad (b = a - 3x - 5)
\]
context so far: we know \( A \rightarrow B \), so if we observe \( \neg B \) then we can conclude \( \neg A \)

**Proof by Contrapositive**

We don't know how to prove \( A \rightarrow B \) (easily), so we try to start by assuming \( \neg B \). If we conclude \( \neg A \), we are done.

Prove: if \( 7x + 9 \) is even, then \( x \) is odd \hspace{1cm} (\text{for } x \in \mathbb{Z})

\[
\begin{align*}
7x + 9 &= 2a \quad \text{// } a: \text{integer } \rightarrow 7x + 9: \text{even} \\
x &= 2a - 6x - 9 \\
x &= 2a - 6x - 10 + 1 \\
x &= 2(a - 3x - 5) + 1 \\
x &= 2b + 1 \quad \text{(odd)} \quad (b = a - 3x - 5)
\end{align*}
\]
Proof: if \( 7x + 9 \) is even, then \( x \) is odd (for \( x \in \mathbb{Z} \))

\[
\begin{align*}
7x + 9 &= 2a \quad \text{//} a \in \text{integer} \Rightarrow 7x + 9 \text{ is even} \\
x &= 2a - 6x - 9 \\
x &= 2a - 6x - 10 + 1 \\
x &= 2(a - 3x - 5) + 1 \\
x &= 2b + 1 \quad \text{(odd)} \quad (b = a - 3x - 5)
\end{align*}
\]
context so far: we know $A \rightarrow B$, so if we observe $\neg B$ then we can conclude $\neg A$ 

**PROOF BY CONTRAPOSITION**

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

---

**Prove:** if $7x+9$ is even, then $x$ is odd (for $x \in \mathbb{Z}$)

<table>
<thead>
<tr>
<th>Direct</th>
<th>Contrapositive</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7x + 9 = 2a \quad // a: integer \rightarrow 7x + 9: even$</td>
<td>$\neg x$ is not odd: $x = ?$</td>
</tr>
<tr>
<td>$x = 2a - 6x - 9$</td>
<td>Suppose $x$ is not odd: $x = ?$</td>
</tr>
<tr>
<td>$x = 2a - 6x - 10 + 1$</td>
<td></td>
</tr>
<tr>
<td>$x = 2(a - 3x - 5) + 1$</td>
<td></td>
</tr>
<tr>
<td>$x = 2b + 1 \quad \text{(odd)} \quad (b = a - 3x - 5)$</td>
<td></td>
</tr>
</tbody>
</table>
context so far: we know $A \rightarrow B$, so if we observe $\neg B$ then we can conclude $\neg A$

**PROOF BY CONTRAPOSITIVE**

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

---

**Prove:** if $7x+9$ is even, then $x$ is odd (for $x \in \mathbb{Z}$)

**Direct:***

\[
7x + 9 = 2a \quad // a: \text{integer} \rightarrow 7x+9: \text{even} \\
x = 2a - 6x - 9 \\
x = 2a - 6x - 10 + 1 \\
x = 2(a - 3x - 5) + 1 \\
x = 2b+1 \quad (\text{odd}) \quad (b = a - 3x - 5)
\]

**Contrapositive:**

Suppose $x$ is not odd: $x = 2c$ ... then?
context so far: we know \( A \rightarrow B \), so if we observe \( \neg B \) then we can conclude \( \neg A \)

**Proof by Contrapositive**

We don't know how to prove \( A \rightarrow B \) (easily), so we try to start by assuming \( \neg B \). If we conclude \( \neg A \), we are done.

---

**Prove:** if \( 7x+9 \) is even, then \( x \) is odd (for \( x \in \mathbb{Z} \))

- **Direct**
  
  \[
  7x + 9 = 2a \quad \text{// } a : \text{integer} \rightarrow 7x + 9 : \text{even}
  \]
  
  \[
  x = 2a - 6x - 9
  \]
  
  \[
  x = 2a - 6x - 10 + 1
  \]
  
  \[
  x = 2(a - 3x - 5) + 1
  \]
  
  \[
  x = 2b + 1 \text{ (odd)} \quad (b = a - 3x - 5)
  \]

- **Contrapositive**

  Suppose \( x \) is not odd: \( x = 2c \)
  
  \[
  7x + 9 = 7 \cdot 2c + 9
  \]
context so far: we know $A \rightarrow B$, so if we observe $\neg B$ then we can conclude $\neg A$.

**Proof by Contrapositive**

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $7x + 9$ is even, then $x$ is odd \(\text{for} \ x \in \mathbb{Z}\)

\[
\begin{align*}
7x + 9 &= 2a \quad \text{//}a: \text{integer} \rightarrow 7x + 9: \text{even} \\
x &= 2a - 6x - 9 \\
x &= 2a - 6x - 10 + 1 \\
x &= 2(a - 3x - 5) + 1 \\
x &= 2b + 1 \quad \text{(odd)} (b = a - 3x - 5)
\end{align*}
\]

contrapositive

Suppose $x$ is not odd: $x = 2c$

\[
\begin{align*}
7x + 9 &= 7 \cdot 2c + 9 \\
&= 14c + 8 + 1
\end{align*}
\]
context so far: we know $A \rightarrow B$, so if we observe $\neg B$
then we can conclude $\neg A$

PROOF BY CONTRAPOSITIVE

We don't know how to prove $A \rightarrow B$ (easily), so we try to
start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $7x+9$ is even, then $x$ is odd \hspace{1cm} (for $x \in \mathbb{Z}$)

\[
\begin{align*}
7x + 9 &= 2a \quad \text{// } a: \text{integer} \rightarrow 7x+9: \text{even} \\
    x &= 2a - 6x - 9 \\
    x &= 2a - 6x - 10 + 1 \\
    x &= 2(a - 3x - 5) + 1 \\
    x &= 2b + 1 \, \text{(odd)} \quad (b = a - 3x - 5)
\end{align*}
\]

direct

\[
\begin{align*}
\text{Suppose } x \text{ is not odd: } &= x = 2c \\
7x + 9 &= 7 \cdot 2c + 9 \\
    &= 14c + 8 + 1 \\
    &= 2 \cdot (7c + 4) + 1
\end{align*}
\]

ccontrapositive
context so far: we know \( A \rightarrow B \), so if we observe \( \neg B \), then we can conclude \( \neg A \)

---

**PROOF BY CONTRAPOSITIVE**

We don't know how to prove \( A \rightarrow B \) (easily), so we try to start by assuming \( \neg B \). If we conclude \( \neg A \), we are done.

---

Prove: if \( 7x+9 \) is even, then \( x \) is odd (for \( x \in \mathbb{Z} \))

\[
\begin{align*}
7x + 9 &= 2a \quad \text{\( \text{direct} \)} \\
\quad &\quad \text{\( a : \text{integer} \rightarrow 7x+9 : \text{even} \)} \\
x &= 2a - 6x - 9 \\
x &= 2a - 6x - 10 + 1 \\
x &= 2(a - 3x - 5) + 1 \\
x &= 2b + 1 \quad \text{(odd)} \quad \text{\( b = a - 3x - 5 \)}
\end{align*}
\]

\[
\begin{align*}
\text{contrapositive} \\
\text{Suppose } x \text{ is not odd: } \quad &x = 2c \\
7x + 9 &= 7 \cdot 2c + 9 \\
&= 14c + 8 + 1 \\
&= 2 \cdot (7c + 4) + 1 \\
&= 2 \cdot d + 1 \quad \text{(d=7c+4)}
\end{align*}
\]
context so far: we know \( A \rightarrow B \), so if we observe \( \neg B \)
then we can conclude \( \neg A \)

**PROOF BY CONTRAPOSITIVE**

We don't know how to prove \( A \rightarrow B \) (easily), so we try to start by assuming \( \neg B \). If we conclude \( \neg A \), we are done.

Prove: if \( 7x + 9 \) is even, then \( x \) is odd (for \( x \in \mathbb{Z} \))

\[
\begin{align*}
7x + 9 &= 2a \quad \text{// } a \text{: integer } \rightarrow 7x + 9 \text{ : even} \\
\text{direct} \\
7x + 9 &= 2a - 6x - 9 \\
x &= 2a - 6x - 10 + 1 \\
x &= 2(a - 3x - 5) + 1 \\
x &= 2b + 1 \quad \text{(odd)} \quad (b = a - 3x - 5) \quad \square
\end{align*}
\]

\[
\begin{align*}
\text{contrapositive} \\
\text{Suppose } x \text{ is not odd: } x = 2c \\
7x + 9 &= 7 \cdot 2c + 9 \\
&= 14c + 8 + 1 \\
&= 2 \cdot (7c + 4) + 1 \\
&= 2 \cdot d + 1 \quad (d = 7c + 4) \\
7x + 9 &= \text{ odd} \quad \square
\end{align*}
\]
We don’t know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $x^2 - 6x + 5$ is even, then $x$ is odd
PROOF BY CONTRAPOSITIVE

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $x^2 - 6x + 5$ is even, then $x$ is odd

phrase mathematically?
PROOF BY CONTRAPPOSITIVE

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $x^2 - 6x + 5$ is even, then $x$ is odd

phrase mathematically?

$(x^2 - 6x + 5 = 2a) \rightarrow (x = 2b + 1)$

...assuming?
PROOF BY CONTRAPOSITIVE

We don't know how to prove $A \implies B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $x^2 - 6x + 5$ is even, then $x$ is odd

phrase mathematically?

$$(x^2 - 6x + 5 = 2a) \implies (x = 2b + 1)$$

...assuming $x, a, b$ are integers
PROOF BY CONTRAPOSITIVE

We don't know how to prove \( A \rightarrow B \) (easily), so we try to start by assuming \( \neg B \). If we conclude \( \neg A \), we are done.

Prove: if \( x^2 - 6x + 5 \) is even, then \( x \) is odd

\[
x^2 - 6x + 5 = 2a \quad \text{direct}
\]
PROOF BY CONTRAPOSITIVE

We don't know how to prove \( A \to B \) (easily), so we try to start by assuming \( \neg B \). If we conclude \( \neg A \), we are done.

**Prove:** if \( x^2 - 6x + 5 \) is even, then \( x \) is odd

\[
x^2 - 6x + 5 = 2a \quad \text{direct}
\]

\[
\therefore \quad \text{?}
\]

\[
\downarrow
\]

\[
x = 2b + 1
\]
PROOF BY CONTRAPOSITIVE

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $x^2 - 6x + 5$ is even, then $x$ is odd

$x^2 - 6x + 5 = 2a$  \hspace{1cm} \text{direct}

$x^2 - 6x + (5 - 2a) = 0$

$x = \frac{6 \pm \sqrt{36 + 8a - 20}}{2}$

$x = 3 \pm \sqrt{4 + 2a} \quad \therefore \quad x = 2b + 1$
PROOF BY CONTRAPOSITIVE

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $x^2 - 6x + 5$ is even, then $x$ is odd

\[ x^2 - 6x + 5 = 2a \]

direct

\[ x = 2b + 1 \]

contrapositive

\[ \ldots ? \]
PROOF BY CONTRAPOSITIVE

We don't know how to prove \( A \rightarrow B \) (easily), so we try to start by assuming \( \neg B \). If we conclude \( \neg A \), we are done.

Prove: if \( x^2 - 6x + 5 \) is even, then \( x \) is odd

\[
\begin{align*}
x^2 - 6x + 5 &= 2a \\
\text{direct} & \quad \text{contrapositive} \\
\text{Suppose } x \text{ is not odd: } x = 2c
\end{align*}
\]

\[
x = 2b + 1
\]
PROOF BY CONTRAPOSITIVE

We don’t know how to prove $A \implies B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $x^2 - 6x + 5$ is even, then $x$ is odd

$x^2 - 6x + 5 = 2a$  direct

contrapositive

Suppose $x$ is not odd: $x = 2c$

$x^2 - 6x + 5 = (2c)^2 - 6 \cdot 2c + 5$

$x = 2b + 1$
PROOF BY CONTRAPOSITIVE

We don't know how to prove \( A \rightarrow B \) (easily), so we try to start by assuming \( \neg B \). If we conclude \( \neg A \), we are done.

Prove: if \( x^2 - 6x + 5 \) is even, then \( x \) is odd

\[ x^2 - 6x + 5 = 2a \]

\begin{align*}
\text{direct} & \\
\text{contrapositive} & \\
\text{Suppose } x \text{ is not odd: } x = 2c & \\
\Rightarrow x^2 - 6x + 5 = (2c)^2 - 6 \cdot 2c + 5 & \\
& = 4c^2 - 12c + 5 & \\
\Rightarrow x = 2b + 1 &
\end{align*}
**PROOF BY CONTRAPOSITIVE**

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

**Prove:** if $x^2 - 6x + 5$ is even, then $x$ is odd

\[
x^2 - 6x + 5 = 2a
\]

**Direct**

\[
x = 2b + 1
\]

**Contrapositive**

Suppose $x$ is not odd: $x = 2c$

\[
x^2 - 6x + 5 = (2c)^2 - 6 \cdot 2c + 5
\]

\[
= 4c^2 - 12c + 5
\]

\[
= 4c^2 - 12c + 4 + 1
\]

because we want to get something odd
PROOF BY CONTRAPOSITIVE

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $x^2 - 6x + 5$ is even, then $x$ is odd.

$x^2 - 6x + 5 = 2a$ \hspace{1cm} \text{direct}

contrapositive

Suppose $x$ is not odd: $x = 2c$

$x^2 - 6x + 5 = (2c)^2 - 6 \cdot 2c + 5$
$= 4c^2 - 12c + 5$
$= 4c^2 - 12c + 4 + 1$
$= 2 \cdot (2c^2 - 6c + 2) + 1$

\vdots 

\vdots 

x = 2b + 1
**Proof by Contrapositive**

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

**Prove:** if $x^2 - 6x + 5$ is even, then $x$ is odd

\[x^2 - 6x + 5 = 2a\]  \quad \text{direct}  \\
\text{Suppose } x \text{ is not odd: } x = 2c \\
\begin{align*}
x^2 - 6x + 5 &= (2c)^2 - 6 \cdot 2c + 5 \\
&= 4c^2 - 12c + 5 \\
&= 4c^2 - 12c + 4 + 1 \\
&= 2 \cdot (2c^2 - 6c + 2) + 1 \\
&= 2 \cdot d + 1 \quad (d = 2c^2 - 6c + 2)
\end{align*}

\[x = 2b + 1\]  \quad \text{contrapositive}
PROOF BY CONTRAPOSITIVE

We don't know how to prove \( A \rightarrow B \) (easily), so we try to start by assuming \( \neg B \). If we conclude \( \neg A \), we are done.

Prove: if \( x^2 - 6x + 5 \) is even, then \( x \) is odd

\[
x^2 - 6x + 5 = 2a
\]

\[
\text{direct}
\]

\[
\text{contrapositive}
\]

Suppose \( x \) is not odd: \( x = 2c \)

\[
x^2 - 6x + 5 = (2c)^2 - 6 \cdot 2c + 5
\]

\[
= 4c^2 - 12c + 5
\]

\[
= 4c^2 - 12c + 4 + 1
\]

\[
= 2 \cdot (2c^2 - 6c + 2) + 1
\]

\[
= 2 \cdot d + 1 \quad \text{(d = 2c^2 - 6c + 2)}
\]

\[
= \text{not even} \quad \Box
\]

\[
x = 2b + 1
\]
PROOF BY CONTRAPOSITIVE

We don’t know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $x$ is irrational then $\sqrt{x}$ is irrational.
PROOF BY CONTRAPOSITIVE

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $x$ is irrational then $\sqrt{x}$ is irrational

direct

???
PROOF BY CONTRAPOSITVE

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $x$ is irrational then $\sqrt{x}$ is irrational

direct

contrapositive
PROOF BY CONTRAPOSITIVE

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $x$ is irrational then $\sqrt{x}$ is irrational

direct

contrapositive

Suppose $\sqrt{x}$ is not irrational
Proof by Contrapositive

We don’t know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $x$ is irrational then $\sqrt{x}$ is irrational

<table>
<thead>
<tr>
<th>direct</th>
<th>contrapositive</th>
</tr>
</thead>
<tbody>
<tr>
<td>???</td>
<td>Suppose $\sqrt{x}$ is not irrational</td>
</tr>
</tbody>
</table>

$\sqrt{x} = \frac{a}{b}$
PROOF BY CONTRAPOSITIVE

We don't know how to prove $A \rightarrow B$ (easily), so we try to start by assuming $\neg B$. If we conclude $\neg A$, we are done.

Prove: if $x$ is irrational then $\sqrt{x}$ is irrational

direct

contrapositive

Suppose $\sqrt{x}$ is not irrational

$$\sqrt{x} = \frac{a}{b}$$

$$x = \frac{a^2}{b^2}$$
PROOF BY CONTRAPOSITIVE

We don't know how to prove \( A \rightarrow B \) (easily), so we try to start by assuming \( \neg B \). If we conclude \( \neg A \), we are done.

Prove: if \( x \) is irrational then \( \sqrt{x} \) is irrational

direct

contrapositive

Suppose \( \sqrt{x} \) is not irrational

\[
\sqrt{x} = \frac{a}{b}
\]

\[
x = \frac{a^2}{b^2} \quad : \text{not irrational} \quad \square
\]
PROOF BY CONTRADICTION

a slight generalization of proof by contrapositive

still proving if $A$ then $B$ for now
PROOF BY CONTRADICTION

a slight generalization of proof by contrapositive

still proving \( \text{if } A \text{ then } B \) for now

\[
(a+b)(a-b) \rightarrow a^2 - ab + ba - b^2 \rightarrow a^2 - b^2
\]

You can prove something directly (in one direction)
PROOF BY CONTRADICTION

a slight generalization of proof by contrapositive

Still proving if $A$ then $B$ for now

$$(a+b)(a-b) \rightarrow a^2 - ab + ba - b^2 \leftarrow a^2 - b^2$$

You can prove something directly (in one direction) or work in both directions
PROOF BY CONTRADICTION

a slight generalization of proof by contrapositive

still proving if $A$ then $B$ for now

$$(a+b)(a-b) \iff a^2 - ab + ba - b^2 \iff a^2 - b^2$$

You can prove something directly (in one direction) or work in both directions

contrapositive \(\iff\) starting w/ \(\neg B\) & leading to \(\neg A\) contradicts \(A \rightarrow \neg B\)
PROOF BY CONTRADICTION

a slight generalization of proof by contrapositive

still proving \( \text{if } A \text{ then } B \) for now

\[(a+b)(a-b) \iff a^2-ab+ba-b^2 \iff a^2-b^2\]

You can prove something directly (in one direction)
or work in both directions

instead of starting w/ \( \neg B \) & leading to \( \neg A \)
(which contradicts \( A \rightarrow \neg B \))
Proof by Contradiction

A slight generalization of proof by contrapositive

still proving \( \text{if } A \text{ then } B \) for now

\[
(a+b) \cdot (a-b) \iff a^2 - ab + ba - b^2 \iff a^2 - b^2
\]

You can prove something directly (in one direction) or work in both directions

instead of starting w/ \( \neg B \) & leading to \( \neg A \) (which contradicts \( A \rightarrow \neg B \))

Assume both \( A \) and \( \neg B \) are true & arrive at some contradicting statement
If $x$ is even then $x$ is not odd
PROOF BY CONTRADICTION

If $x$ is even, then $x$ is not odd

A

B
Proof by contradiction

If $x$ is even, then $x$ is not odd

$A$, $B$

Assume $A \land \neg B$: ...
PROOF BY CONTRADICTION

If $x$ is even then $x$ is not odd

$\overline{A}$

$\overline{B}$

Assume $A \land \neg B$: $x$ is even  & $x$ is odd
PROOF BY CONTRADICTION

If $x$ is even, then $x$ is not odd

\[ A \rightarrow B \]

Assume $A \land \neg B$: $x$ is even $\land$ $x$ is odd

\[ \neg A \land \neg B \]

\[ \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \neg \n
Proof by Contradiction

If $x$ is even, then $x$ is not odd

A

B

Assume $A \land \neg B$: $x$ is even and $x$ is odd

\[
\begin{align*}
(a: \text{int}) & \quad x = 2a \\
(b: \text{int}) & \quad x = 2b + 1
\end{align*}
\]
PROOF BY CONTRADICTION

If \( x \) is even, then \( x \) is not odd

\[
A \quad \text{B}
\]

Assume \( A \land \neg B \): \( x \) is even \& \( x \) is odd

\[
(a: \text{int.}) \quad x = 2a \quad x = 2b + 1 \quad (b: \text{int.})
\]

\[
2a = 2b + 1
\]
PROOF BY CONTRADICTION

If $x$ is even, then $x$ is not odd

A

B

Assume $A \land \neg B$: $x$ is even \& $x$ is odd

$(a: \text{int.})

x = 2a

\downarrow

x = 2b + 1

(a: \text{int.})

2a = 2b + 1

\Rightarrow

a = b + \frac{1}{2}$
Proof by Contradiction

If \( x \) is even, then \( x \) is not odd

\( A \) \hspace{2cm} \( B \)

Assume \( A \land \neg B \): \( x \) is even \& \( x \) is odd

\( (a: \text{int.}) \)

\( x = 2a \)

\( x = 2b+1 \) \hspace{2cm} (\( b: \text{int.} \))

\( 2a = 2b+1 \)

\( a = b + \frac{1}{2} \)

impossible / absurd / contradiction
If $x$ is even then $x$ is not odd

Assume $A \land \neg B$: $x$ is even $\& x$ is odd

$(a: \text{int.})$

$x = 2a$

$x = 2b + 1$ (b: int.)

$2a = 2b + 1$

$a = b + \frac{1}{2}$

Notice we met halfway at an incorrect statement.

impossible / absurd / contradiction
PROOF BY CONTRADICTION

For integers $a \neq 0$ & $b$, there is only one number $x$ s.t. $ax + b = 0$.

State this in IF-THEN form.
PROOF BY CONTRADICTION

For integers $a \neq 0$ & $b$, there is only one number s.t. $ax+b = 0$.

(it if $ax+b = 0$ then for $y \neq x$, $ay+b \neq 0$)
For integers $a \neq 0$ & $b$, there is only one number s.t. $ax + b = 0$. 

$(\text{if } ax+b=0 \text{ then for } y \neq x, ay+b \neq 0)$
PROOF BY CONTRADICTION

For integers $a \neq 0$ & $b$, there is only one number s.t. $ax + b = 0$.

(if $ax + b = 0$ then for $y \neq x$, $ay + b \neq 0$)

Assume $A \land \neg B$: $ax + b = 0$ & $ay + b = 0$
For integers $a \neq 0$ & $b$, there is only one number s.t. $ax+b = 0$.

(If $ax+b = 0$ then for $y \neq x$, $ay+b \neq 0$)

Assume $A \land \neg B$: 

$$ax+b = 0 \quad \& \quad ay+b = 0$$

$$ax+b = ay+b$$
For integers $a \neq 0$ & $b$, there is only one number s.t. $ax + b = 0$.

(If $ax + b = 0$ then for $y \neq x$, $ay + b \neq 0$)

Assume $A \land \neg B$:

$a x + b = 0$ & $a y + b = 0$

$a x + b = a y + b$

$a x = a y$
PROOF BY CONTRADICITION

For integers \( a \neq 0 \) \& \( b \), there is only one number s.t. \( ax + b = 0 \).

(i) \( ax + b = 0 \) then for \( y \neq x \), \( ay + b \neq 0 \) \( \text{A} \)

Assume \( A \wedge \neg B \):

\[
ax + b = 0 \quad \& \quad ay + b = 0
\]

\[
ax + b = ay + b
\]

\[
ax = ay
\]

\[
x = y
\]
Proof by Contradiction

For integers $a \neq 0$ & $b$, there is only one number s.t. $az + b = 0$.

\[ \text{(if } ax + b = 0 \text{ then for } y \neq x, ay + b \neq 0) \]

Assume $A \land \neg B$:

$ax + b = 0$ & $ay + b = 0$

\[ ax + b = ay + b \]
\[ ax = ay \]
\[ x = y \quad \text{contradicts} \]
up next:

more cool examples,

not in "if A then B" format
\[ \sqrt{2} \] is irrational - proof by contradiction
\[ \sqrt{2} \text{ is irrational} \ - \text{ Proof by Contradiction} \]

1) What does the claim mean?
$\sqrt{2}$ is irrational - Proof by contradiction

1) what does the claim mean?

1) A integers \{a, b\} s.t. $\sqrt{2} = \frac{a}{b}$
\[ \sqrt{2} \text{ is irrational} - \text{ Proof by contradiction} \]

1) what does the claim mean?

2) assume the contrary is true

1) \( A \) integers \( \{a, b\} \) s.t. \( \sqrt{2} = \frac{a}{b} \)

2) \( \exists \) integers \( \{a, b\} \) s.t. \( \sqrt{2} = \frac{a}{b} \)
\[ \sqrt{2} \text{ is IRRATIONAL} - \text{PROOF BY CONTRADICTION} \]

1) what does the claim mean?

2) assume the contrary is true

3) use this to establish something that you know is wrong

1) \( \exists \) integers \( \{a, b\} \) s.t. \( \sqrt{2} = \frac{a}{b} \)

2) \( \exists \) integers \( \{a, b\} \) s.t. \( \sqrt{2} = \frac{a}{b} \)
\( \sqrt{2} \) IS IRRATIONAL - PROOF BY CONTRADICTION

1) what does the claim mean?
2) assume the contrary is true
3) use this to establish something that you know is wrong

1) \( \exists \) integers \( \{a, b\} \) s.t. \( \sqrt{2} = \frac{a}{b} \)
2) \( \exists \) integers \( \{a, b\} \) s.t. \( \sqrt{2} = \frac{a}{b} \)
3) if (2) is true, then choose \( \{a, b\} \) w/ no common divisor

(simplify)
<table>
<thead>
<tr>
<th>1) what does the claim mean?</th>
</tr>
</thead>
<tbody>
<tr>
<td>2) assume the contrary is true</td>
</tr>
<tr>
<td>3) use this to establish something that you know is wrong</td>
</tr>
</tbody>
</table>

| 1) \( \text{Let } \{a, b\} \text{ s.t. } \sqrt{2} = \frac{a}{b} \) |
| 2) \( \exists \text{ integers } \{a, b\} \text{ s.t. } \sqrt{2} = \frac{a}{b} \) |
| 3) if (2) is true, then choose \( \{a, b\} \) with no common divisor |

\( \text{By (2), } 2 = \frac{a^2}{b^2} \)
\( \sqrt{2} \) is irrational - Proof by contradiction

1) what does the claim mean?
2) assume the contrary is true
3) use this to establish something that you know is wrong

1) \( \exists \) integers \( \{a, b\} \) s.t. \( \sqrt{2} = \frac{a}{b} \)
2) \( \exists \) integers \( \{a, b\} \) s.t. \( \sqrt{2} = \frac{a}{b} \)
3) if (2) is true, then choose \( \{a, b\} \) with no common divisor

By (2), \( 2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2 \)
$\sqrt{2}$ is IRRATIONAL - PROOF BY CONTRADICTION

1) what does the claim mean?

2) assume the contrary is true

3) use this to establish something that you know is wrong

1) $\exists$ integers $\{a, b\}$ s.t. $\sqrt{2} = \frac{a}{b}$

2) $\exists$ integers $\{a, b\}$ s.t. $\sqrt{2} = \frac{a}{b}$

3) if (2) is true, then choose $\{a, b\}$ with no common divisor

By (2), $2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2 \Rightarrow a^2$: even
\[ \sqrt{2} \text{ is IRRATIONAL - PROOF BY CONTRADICTION} \]

1) what does the claim mean?
2) assume the contrary is true
3) use this to establish something that you know is wrong

1) \( \exists \text{ integers } \{a, b\} \text{ s.t. } \sqrt{2} = \frac{a}{b} \)
2) \( \exists \text{ integers } \{a, b\} \text{ s.t. } \sqrt{2} = \frac{a}{b} \)
3) if (2) is true, then choose \( \{a, b\} \) with no common divisor

By (2), \( 2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2 \Rightarrow a^2 \text{ is even} \) (a: even)

\[ \Rightarrow \text{why?} \]

homework?
\[ \sqrt{2} \] is irrational - Proof by Contradiction

1) what does the claim mean?

2) assume the contrary is true

3) use this to establish something that you know is wrong

1) \( \exists \) integers \( \{a, b\} \) s.t. \( \sqrt{2} = \frac{a}{b} \)

2) \( \exists \) integers \( \{a, b\} \) s.t. \( \sqrt{2} = \frac{a}{b} \)

3) if (2) is true, then choose \( \{a, b\} \) with no common divisor

By (2), \( 2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2 \Rightarrow a^2 : \text{even (a : even)} \)

\( a = 2c \ \{c : \text{int.}\} \)
\( \sqrt{2} \text{ is irrational} \) - Proof by contradiction

1) what does the claim mean?

2) assume the contrary is true

3) use this to establish something that you know is wrong

| 1) Any integers \( \{a, b\} \) s.t. \( \sqrt{2} = \frac{a}{b} \) |
| 2) \exists \text{ integers } \{a, b\} \text{ s.t. } \sqrt{2} = \frac{a}{b} |
| 3) if (2) is true, then choose \( \{a, b\} \) with no common divisor |

By (2), \( 2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2 \Rightarrow \) \( a \): even (\( a \): even)

\[ a = 2c \quad \{c: \text{int.}\} \Rightarrow 2b^2 = 4c^2 \]
$\sqrt{2}$ is IRRATIONAL - Proof by CONTRADICTION

1) what does the claim mean?

2) assume the contrary is true

3) use this to establish something that you know is wrong

1) \exists integers \{a, b\} s.t. \sqrt{2} = \frac{a}{b}

2) \exists integers \{a, b\} s.t. \sqrt{2} = \frac{a}{b}

3) if (2) is true, then choose \{a, b\} with no common divisor

By (2), $2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2 \Rightarrow a^2: even$

(a: even)

$\Rightarrow a = 2c \ (c: int.) \Rightarrow 2b^2 = 4c^2 \Rightarrow b: even$

($b^2 = 2c^2 \Rightarrow b^2: even$)
\[ \sqrt{2} \text{ is IRATIONAL} - \text{PROOF BY CONTRADICTION} \]

1) what does the claim mean?
2) assume the contrary is true
3) use this to establish something that you know is wrong

1) \( \exists \text{ integers } \{a, b\} \text{ s.t. } \sqrt{2} = \frac{a}{b} \)
2) \( \exists \text{ integers } \{a, b\} \text{ s.t. } \sqrt{2} = \frac{a}{b} \)
3) if (2) is true, then choose \( \{a, b\} \text{ w/ no common divisor} \)

By (2), \( 2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2 \Rightarrow a^2: \text{even} \)
\( (a: \text{even}) \)
\( \Rightarrow a = 2c \) \( \{c: \text{int.}\} \Rightarrow 2b^2 = 4c^2 \Rightarrow b: \text{even} \)
\( (b = 2d) \)

\[ \sqrt{2} = \frac{a}{b} = \frac{2c}{2d} \]
$\sqrt{2}$ is irrational - Proof by contradiction

1) what does the claim mean?

2) assume the contrary is true

3) use this to establish something that you know is wrong

$\sqrt{2}$ is irrational - Proof by contradiction

1) A integers \{a, b\} s.t. \( \sqrt{2} = \frac{a}{b} \)

2) \exists integers \{a, b\} s.t. \( \sqrt{2} = \frac{a}{b} \)

3) if (2) is true, then choose \{a, b\} with no common divisor

\[ \text{By (2), } 2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2 \Rightarrow a: \text{even} \]
\[ (a: \text{even}) \]
\[ a = 2c \ (c: \text{int}) \Rightarrow 2b^2 = 4c^2 \Rightarrow b: \text{even} \]
\[ \sqrt{2} = \frac{a}{b} = \frac{2c}{2d} \text{ contradiction} \]
\[ \sqrt{2} \text{ is IRRATIONAL} - \text{ PROOF BY CONTRADICTION} \]

1) what does the claim mean?
2) assume the contrary is true
3) use this to establish something that you know is wrong
4) conclude that (2) is false thus the initial claim is true

1) \( \exists \text{ integers } \{a, b\} \text{ s.t. } \sqrt{2} = \frac{a}{b} \)
2) \( \exists \text{ integers } \{a, b\} \text{ s.t. } \sqrt{2} = \frac{a}{b} \)
3) if (2) is true, then choose \( \{a, b\} \) with no common divisor
4) conclude that (2) is false thus the initial claim is true

By (2), \( 2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2 \Rightarrow a^2 \text{ even} \) 
\( a = 2c \) \( \{c: \text{int}\} \Rightarrow 2b^2 = 4c^2 \Rightarrow b \text{ even} \)

\( \sqrt{2} = \frac{a}{b} = \frac{2c}{2d} \) contradiction
There are an infinite number of primes
(proof by contradiction)
THERE ARE AN INFINITE NUMBER OF PRIMES
(proof by contradiction)

• Assume that #primes is finite: $p_1, p_2, \ldots, p_n$
THERE ARE AN INFINITE NUMBER OF PRIMES
(proof by contradiction)

• Assume that \#primes is finite: p_1, p_2, \ldots, p_n

• Let \( t = 1 + \prod_{i=1}^{n} p_i \) (i.e., \( 1+ p_1 \times p_2 \times \ldots \times p_n \))
THERE ARE AN INFINITE NUMBER OF PRIMES
(proof by contradiction)

• Assume that \#primes is finite: \( p_1, p_2, \ldots, p_n \)

• Let \( t = 1 + \prod_{i=1}^{n} p_i \) (i.e., \( 1 + p_1 \times p_2 \times \cdots \times p_n \))

• Notice \( t > p_i \) for all \( i \).
THERE ARE AN INFINITE NUMBER OF PRIMES
(proof by contradiction)

• Assume that \#primes is finite: \( P_1, P_2, \ldots, P_n \)

• Let \( t = 1 + \prod_{i=1}^{n} p_i \) (i.e., \( 1 + P_1 \times P_2 \times \cdots \times P_n \))

• Notice \( t > P_i \) for all \( i \). So if \( t \) is prime, contradiction.
THERE ARE AN INFINITE NUMBER OF PRIMES
(proof by contradiction)

• Assume that \#primes is finite: \( P_1, P_2, \ldots, P_n \)
• Let \( t = 1 + \prod_{i=1}^{n} p_i \) (i.e., \( 1 + P_1 \times P_2 \times \cdots \times P_n \))
• Notice \( t > p_i \) for all \( i \). So if \( t \) is prime, contradiction.
• If \( t \) is not prime then \( \exists \) prime factor \( q \neq t \) of \( t \)
  (you've seen this, \( t \) is composite)
THERE ARE AN INFINITE NUMBER OF PRIMES
(proof by contradiction)

• Assume that \#primes is finite: \( p_1, p_2, \ldots, p_n \)

• Let \( t = 1 + \prod_{i=1}^{n} p_i \) (i.e., \( 1 + p_1 \times p_2 \times \ldots \times p_n \))

• Notice \( t > p_i \) for all \( i \). So if \( t \) is prime, contradiction.

• If \( t \) is not prime then \exists \text{ prime factor} \ q \neq t \ of \ t

- if \( q \neq p_i \) (for all \( i \)) ?
  (you've seen this, \( t \) is composite)
THERE ARE AN INFINITE NUMBER OF PRIMES
(proof by contradiction)

• Assume that \#primes is finite: \( P_1, P_2, \ldots, P_n \)

• Let \( t = 1 + \prod_{i=1}^{n} p_i \) (i.e., \( 1 + P_1 \times P_2 \times \cdots \times P_n \))

• Notice \( t > p_i \) for all \( i \). So if \( t \) is prime, contradiction.

• If \( t \) is not prime then \( \exists \) prime factor \( q \neq t \) of \( t \)
  - if \( q \neq p_i \) (for all \( i \)), contradiction. (you've seen this, \( t \) is composite)
THERE ARE AN INFINITE NUMBER OF PRIMES
(proof by contradiction)

• Assume that \#primes is finite: \( P_1, P_2, \ldots, P_n \)

• Let \( t = 1 + \prod_{i=1}^{n} P_i \) (i.e., \( 1 + P_1 \times P_2 \times \cdots \times P_n \))

• Notice \( t > P_i \) for all \( i \). So if \( t \) is prime, contradiction.

• If \( t \) is not prime then \( \exists \) prime factor \( q \neq t \) of \( t \)
  - if \( q \neq P_i \) (for all \( i \)), contradiction.
  - if \( q = P_j \)?

(you've seen this, \( t \) is composite)
THERE ARE AN INFINITE NUMBER OF PRIMES
(proof by contradiction)

• Assume that $\#$primes is finite: $P_1, P_2, \ldots, P_n$

• Let $t = 1 + \prod_{i=1}^{n} P_i$ (i.e., $1 + P_1 \times P_2 \times \cdots \times P_n$)

• Notice $t > P_i$ for all $i$. So if $t$ is prime, contradiction.

• If $t$ is not prime then $\exists$ prime factor $q \neq t$ of $t$
  - if $q \neq P_i$ (for all $i$), contradiction.
  - if $q = P_j$, we know $q$ divides $\prod_{i=1}^{n} P_i$
THERE ARE AN INFINITE NUMBER OF PRIMES
(proof by contradiction)

- Assume that \#primes is finite: \( p_1, p_2, \ldots, p_n \)
- Let \( t = 1 + \prod_{i=1}^{n} p_i \) (i.e., \( 1 + p_1 \times p_2 \times \cdots \times p_n \))
- Notice \( t > p_i \) for all \( i \). So if \( t \) is prime, contradiction.
- If \( t \) is not prime then \( \exists \) prime factor \( q \neq t \) of \( t \)
  - if \( q \neq p_i \) (for all \( i \)), contradiction.
  - if \( q = p_j \), we know \( q \) divides \( \prod_{i=1}^{n} p_i \)
    but then ?

(you've seen this, \( t \) is composite)
THERE ARE AN INFINITE NUMBER OF PRIMES
(proof by contradiction)

• Assume that \#primes is finite: \( p_1, p_2, \ldots, p_n \)

• Let \( t = 1 + \prod_{i=1}^{n} p_i \) (i.e., \( 1 + p_1 \times p_2 \times \cdots \times p_n \))

• Notice \( t > p_i \) for all \( i \). So if \( t \) is prime, contradiction.

• If \( t \) is not prime then \( \exists \) prime factor \( q \neq t \) of \( t \)
  - if \( q \neq p_i \) (for all \( i \)), contradiction.
  - if \( q = p_j \), we know \( q \) divides \( \prod_{i=1}^{n} p_i \)
    but then it can't also divide \( 1 + \prod_{i=1}^{n} p_i \) (contr.)