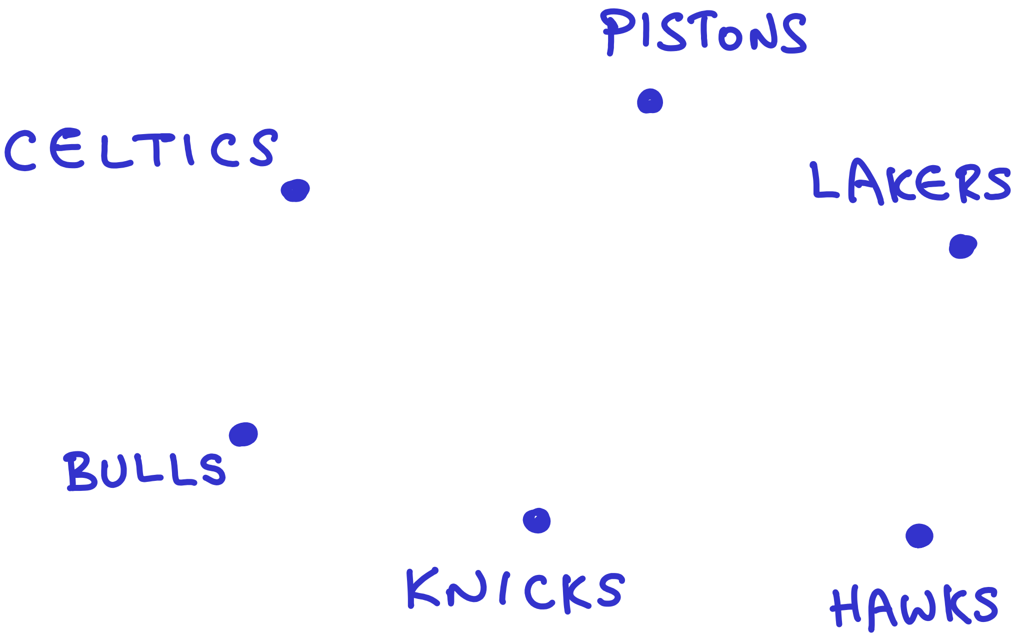
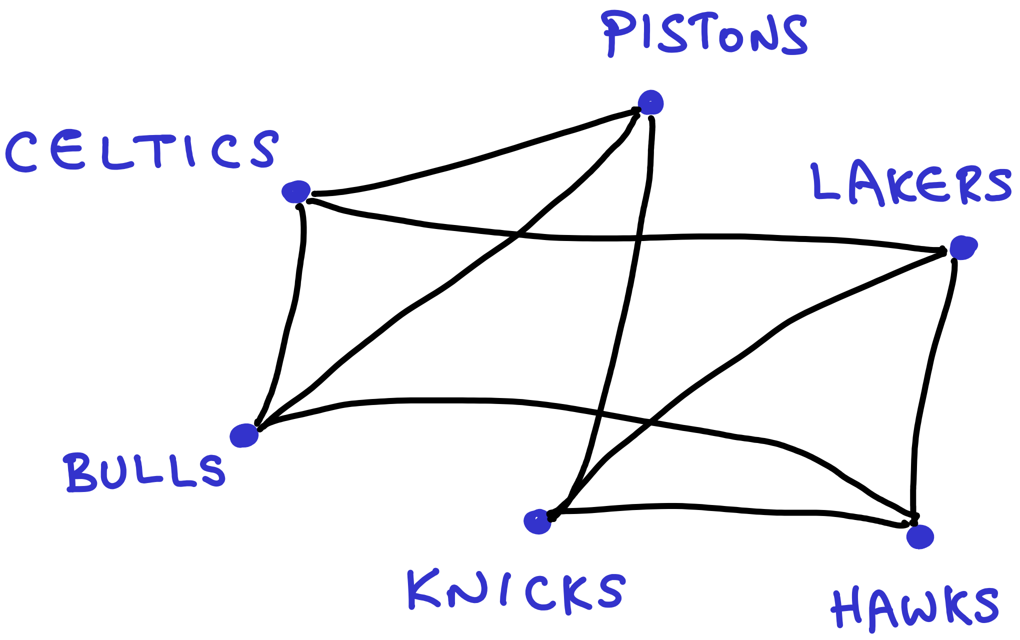
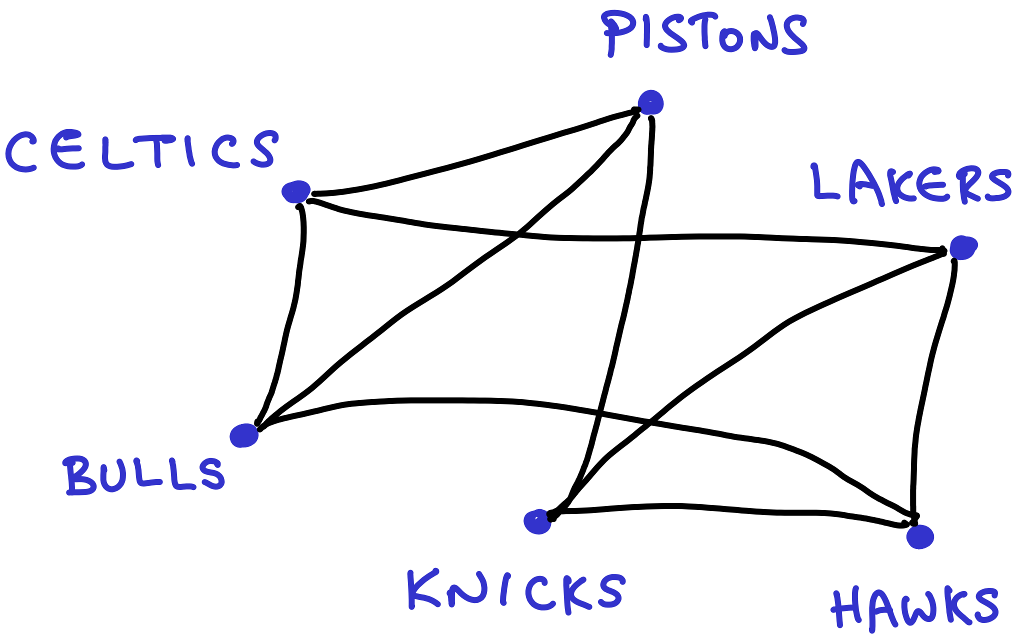


6 teams



6 teams

each plays 3 others



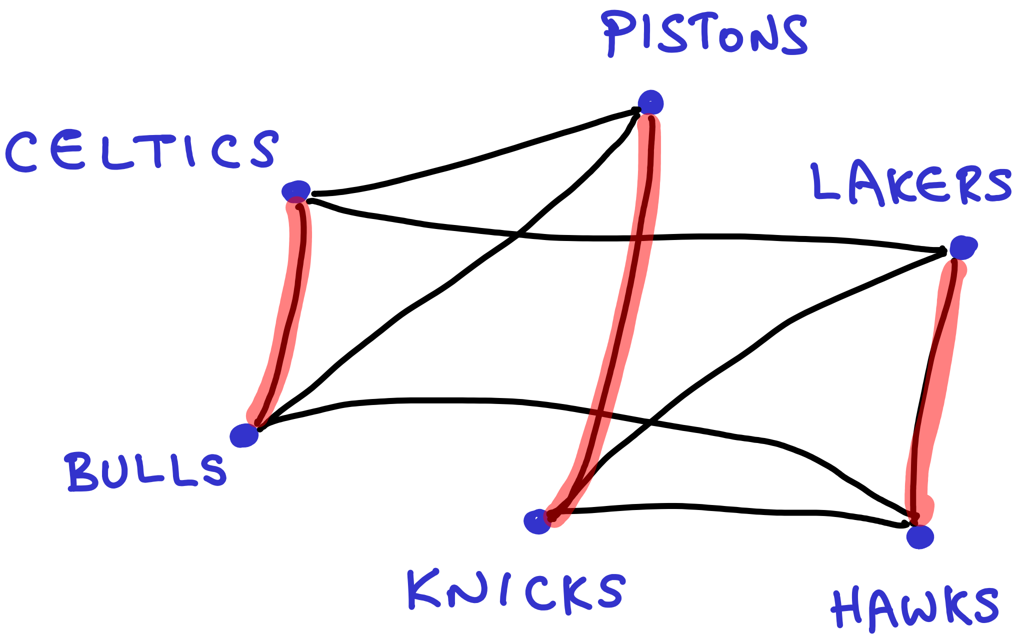
6 teams

each plays 3 others

Games can be played in parallel

Want to finish ASAP

How many rounds required?



Round 1

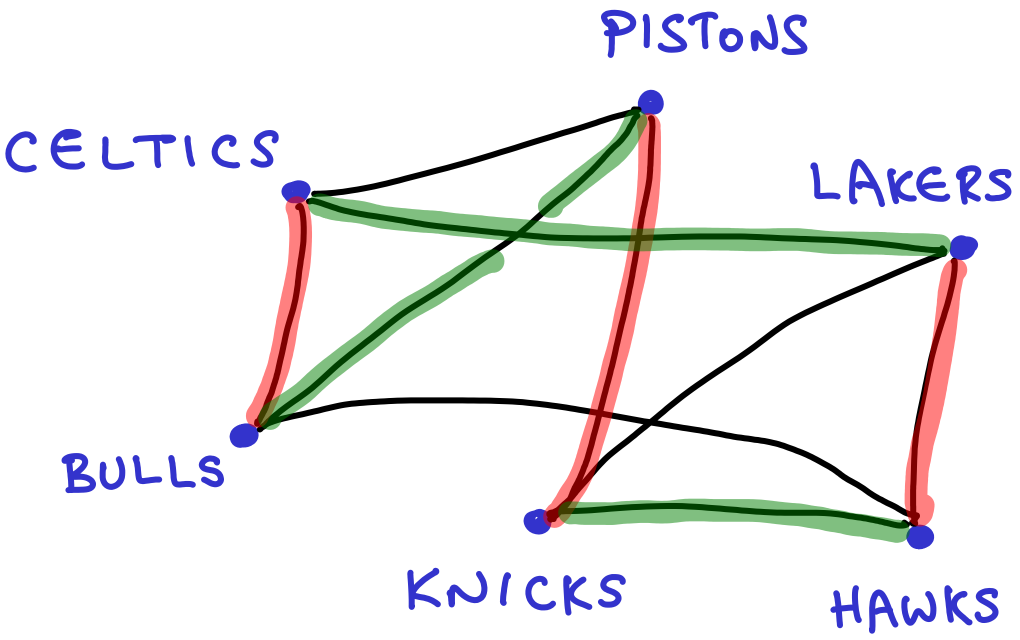
6 teams

each plays 3 others

Games can be played in parallel

Want to finish ASAP

How many rounds required?

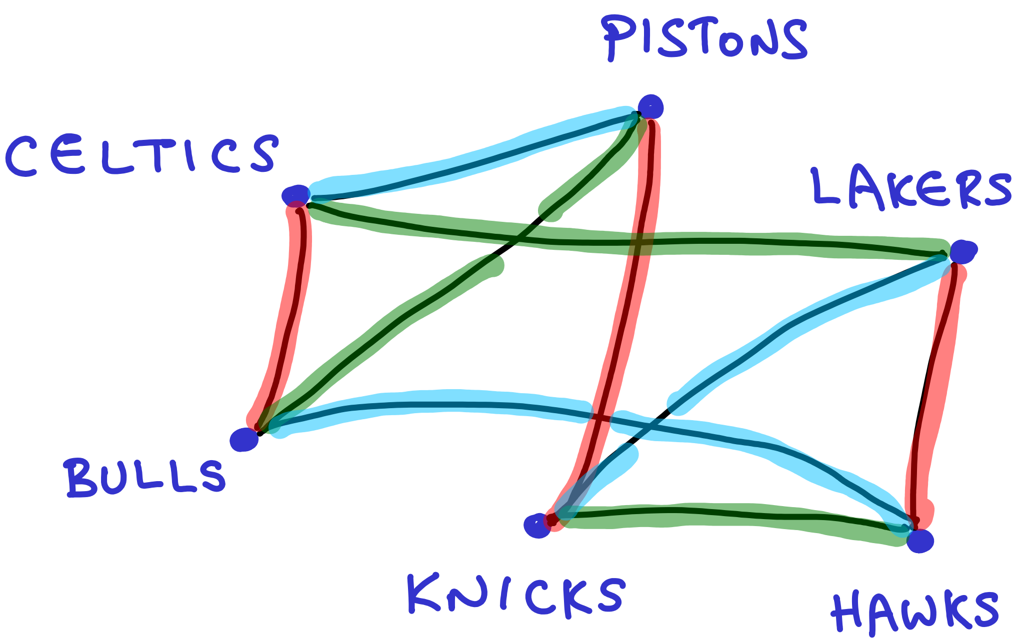


Round 1
Round 2

6 teams
each plays 3 others

Games can be played in parallel
Want to finish ASAP

How many rounds required?

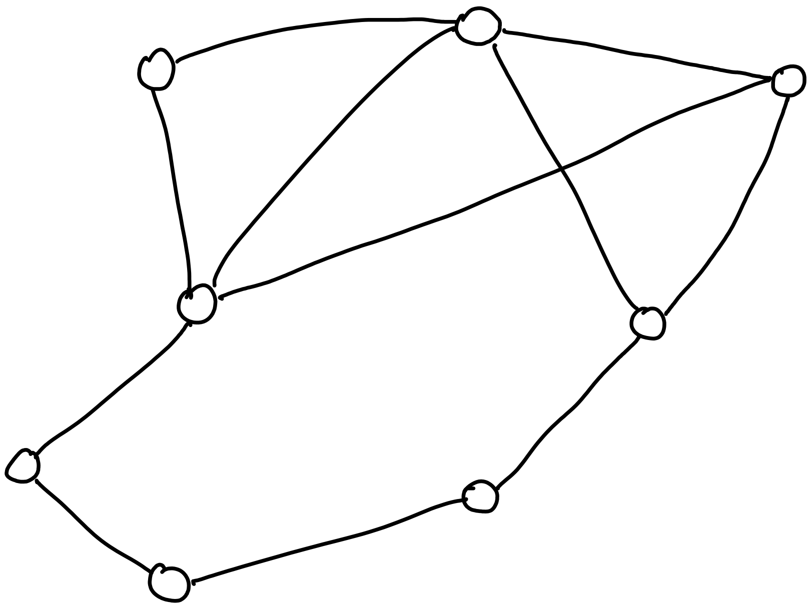


Round 1
Round 2
Round 3

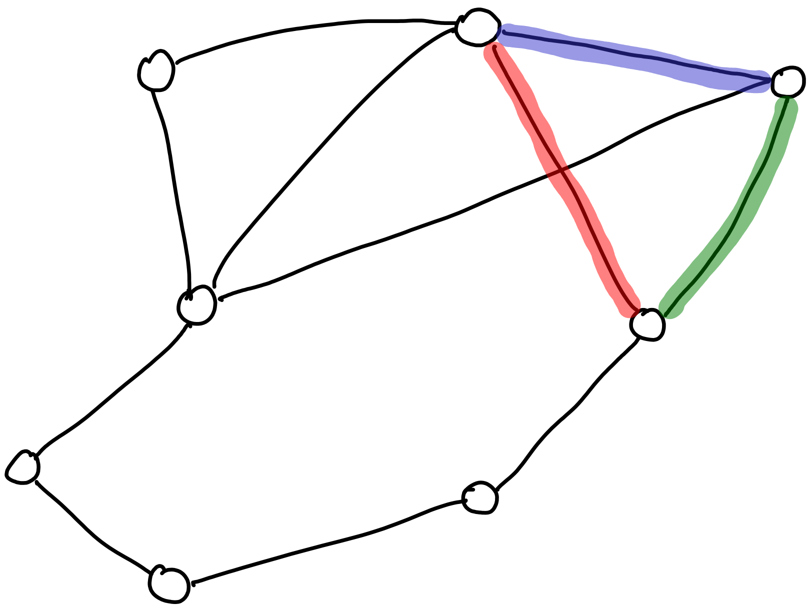
6 teams
each plays 3 others

Games can be played in parallel
Want to finish ASAP
How many rounds required?

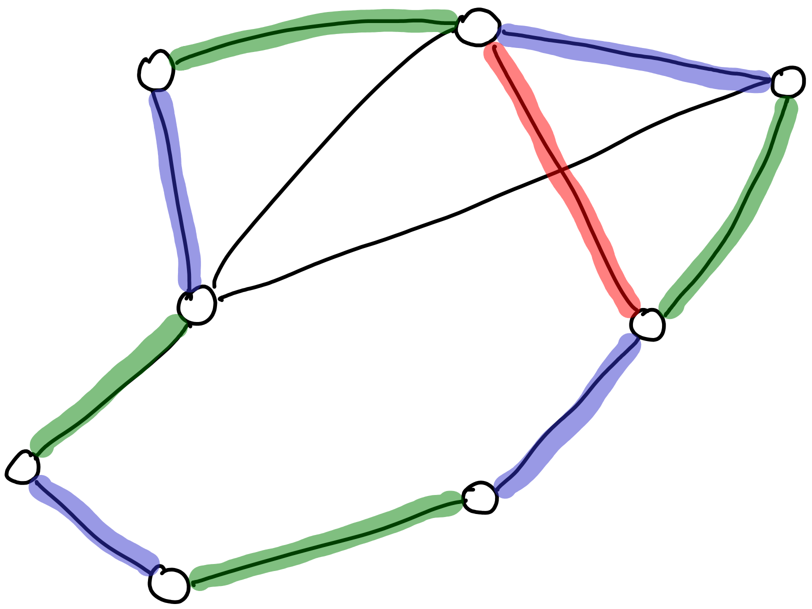
EDGE COLORING - no adjacent edges w same color



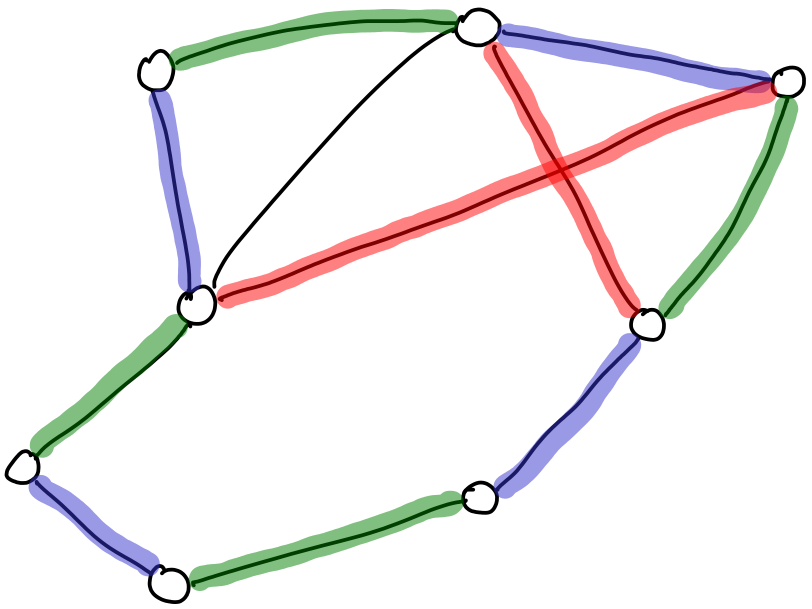
EDGE COLORING - no adjacent edges w same color



EDGE COLORING - no adjacent edges w same color

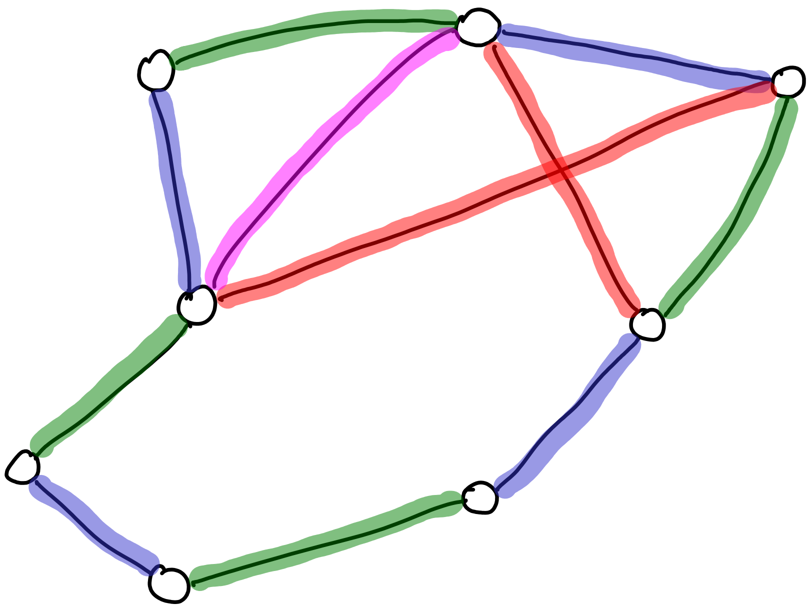


EDGE COLORING - no adjacent edges w same color



EDGE COLORING - no adjacent edges w same color

$\chi'(G)$: min # colors : edge-chromatic #

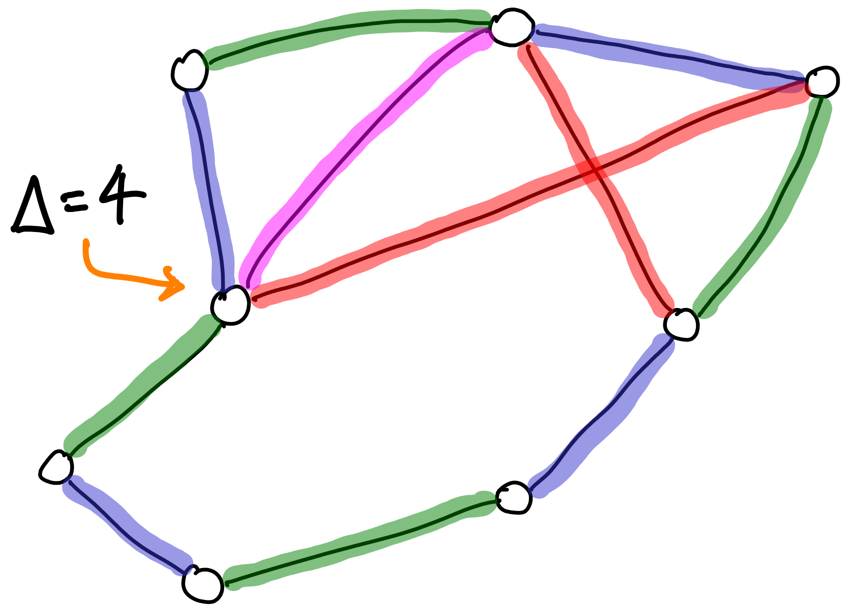


$\chi' \leq 4$

can we do better?

EDGE COLORING - no adjacent edges w same color

$\chi'(G)$: min # colors : edge-chromatic #



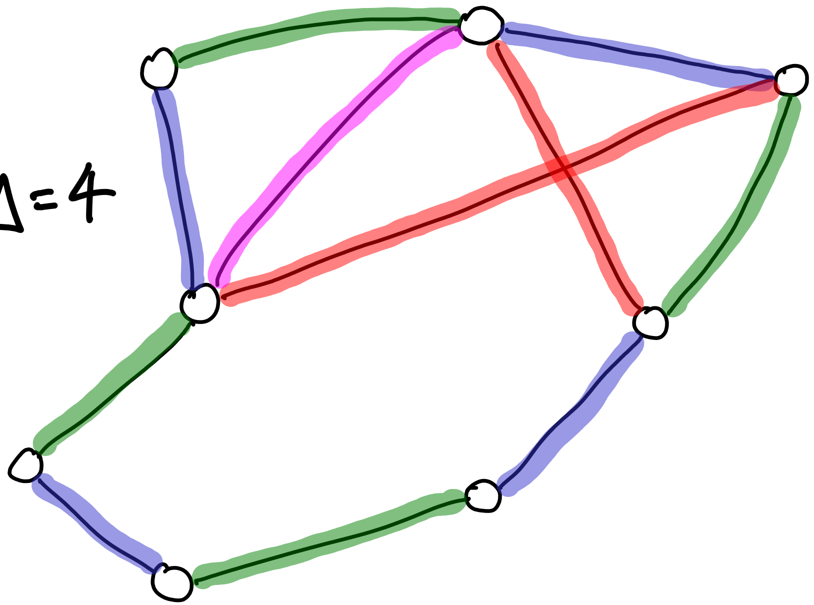
$\chi' = 4$

$\geq \Delta$ {max degree}

EDGE COLORING - no adjacent edges w same color

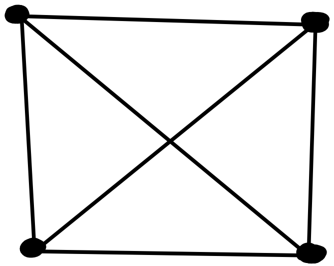
$\chi'(G)$: min # colors : edge-chromatic #

$\Delta = 4$



$\chi' = 4$

$\geq \Delta$ {max degree}

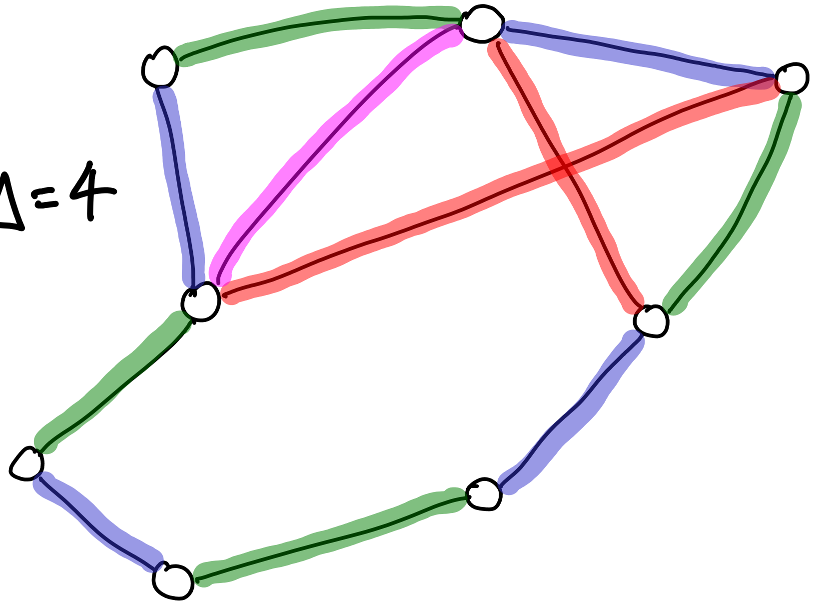


$\chi' = ?$

EDGE COLORING – no adjacent edges w same color

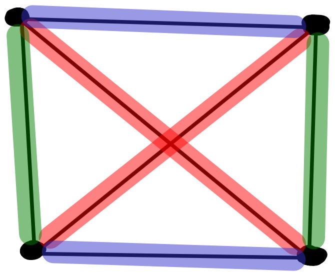
$\chi'(G)$: min # colors : edge-chromatic #

$\Delta = 4$



$\chi' = 4$

$\geq \Delta$ {max degree}



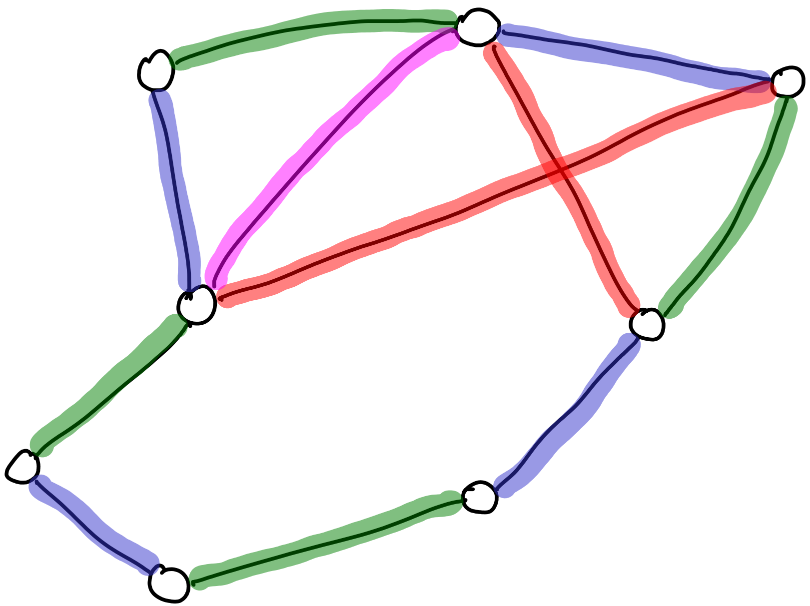
K_4

$\Delta = 3$

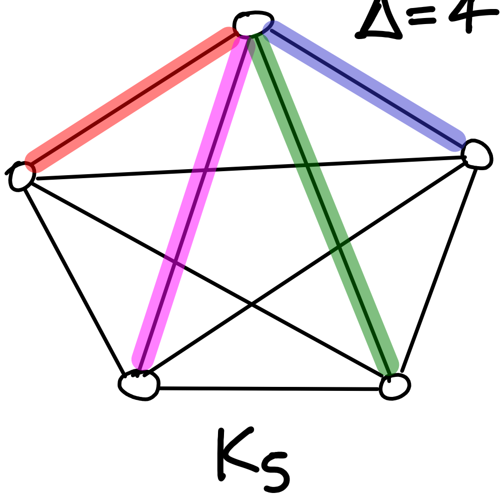
$\chi' = 3$

EDGE COLORING - no adjacent edges w same color

$\chi'(G)$: min # colors : edge-chromatic #

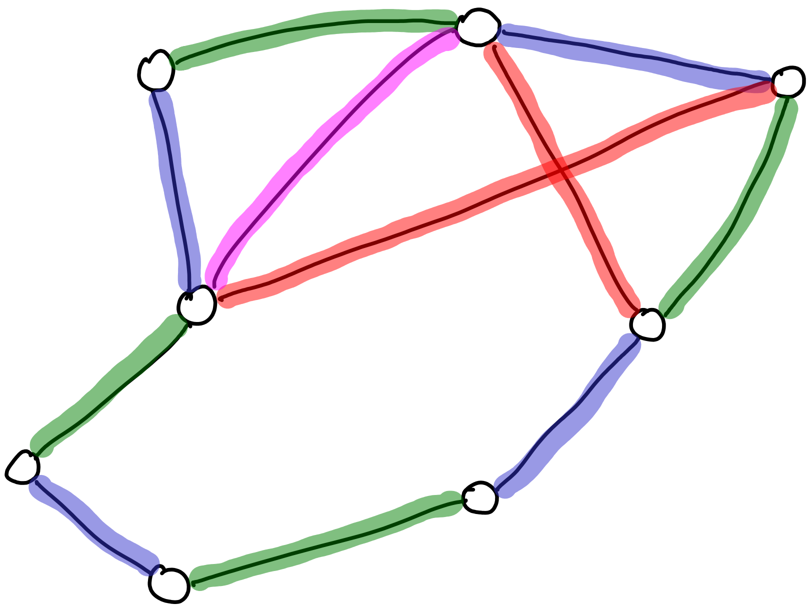


$\chi' = 4$
$\geq \Delta$

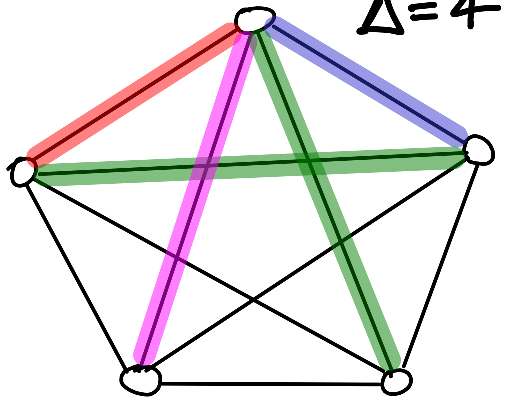


EDGE COLORING - no adjacent edges w same color

$\chi'(G)$: min # colors : edge-chromatic #



$\Delta = 4$



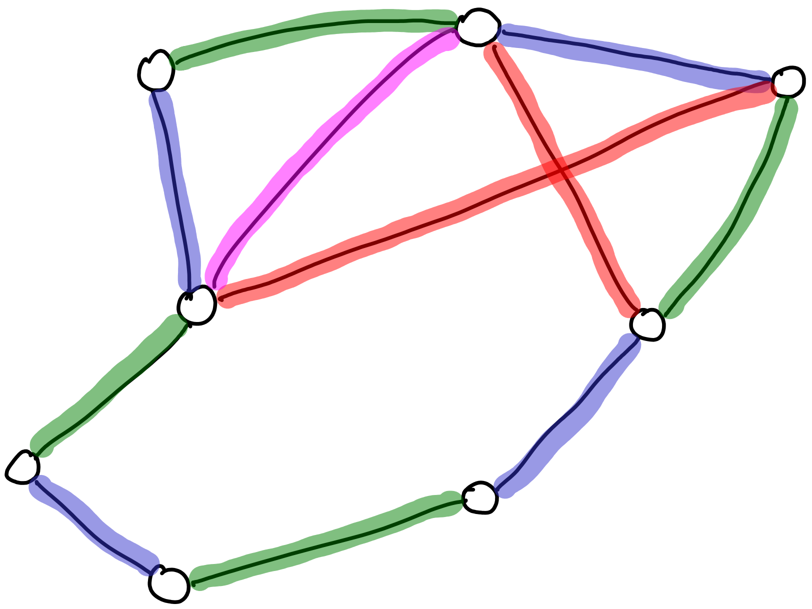
WLOG

$\chi' = 4$

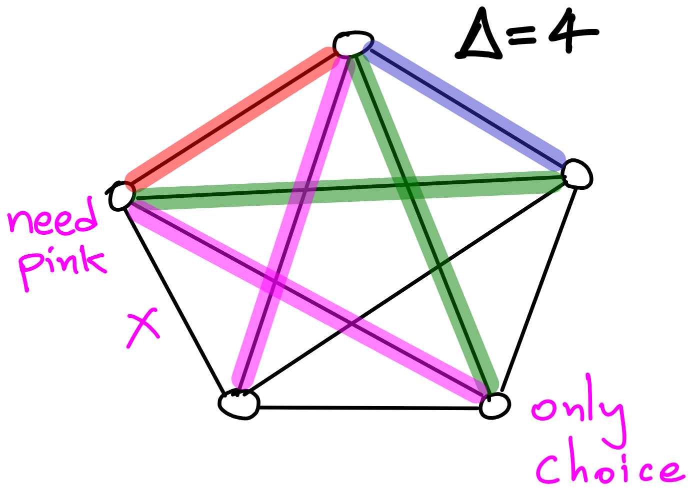
$\geq \Delta$

EDGE COLORING - no adjacent edges w same color

$\chi'(G)$: min # colors : edge-chromatic #

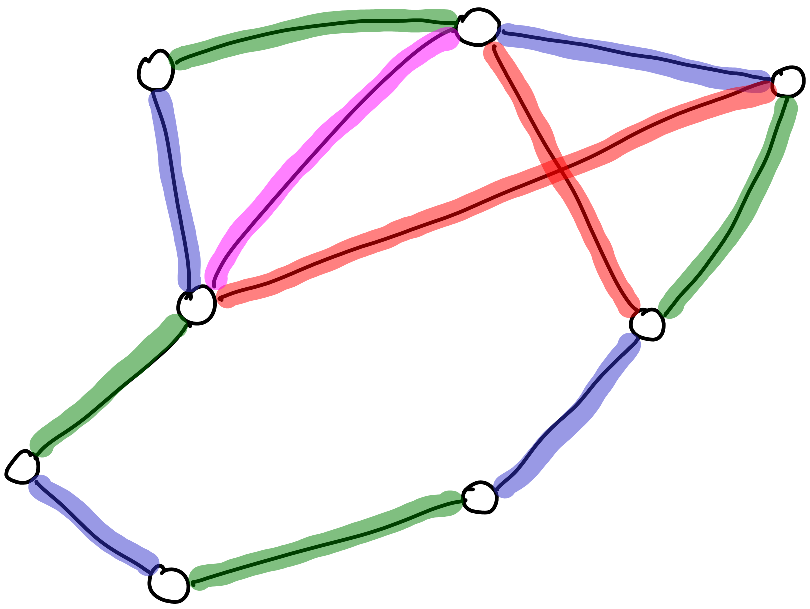


$\chi' = 4$
$\geq \Delta$

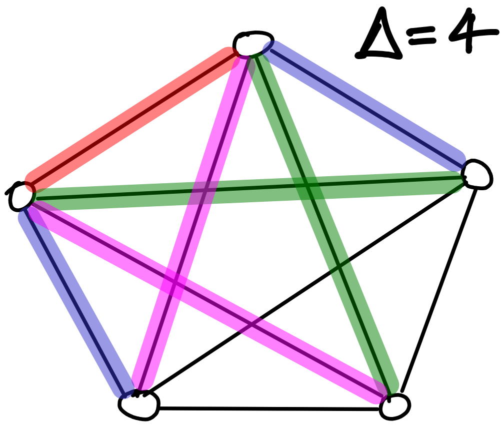


EDGE COLORING - no adjacent edges w same color

$\chi'(G)$: min # colors : edge-chromatic #



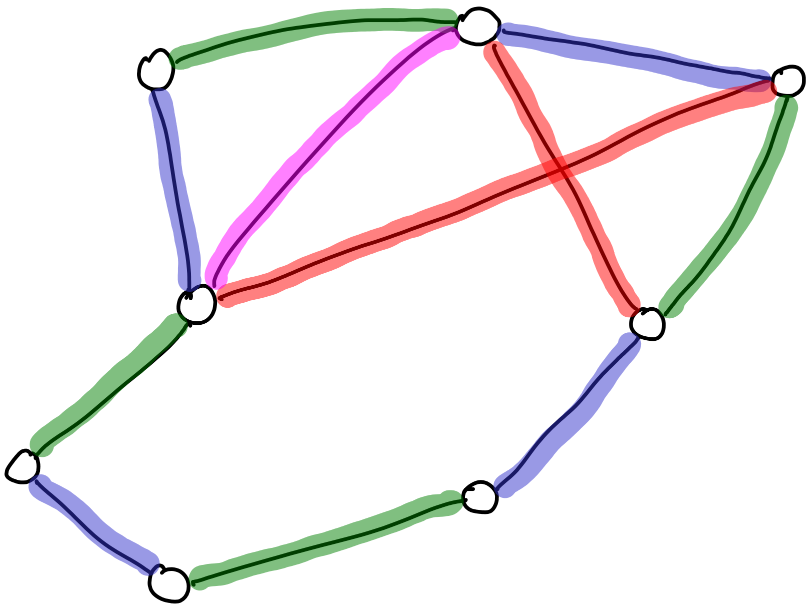
$\chi' = 4$
$\geq \Delta$



- Can't add more blue, pink, green
- Can add one red & will need 2 more colors

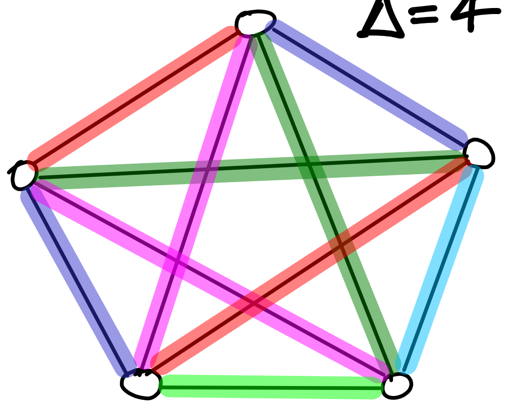
EDGE COLORING - no adjacent edges w same color

$\chi'(G)$: min # colors : edge-chromatic #



$\chi' = 4$
$\geq \Delta$

$\Delta = 4$



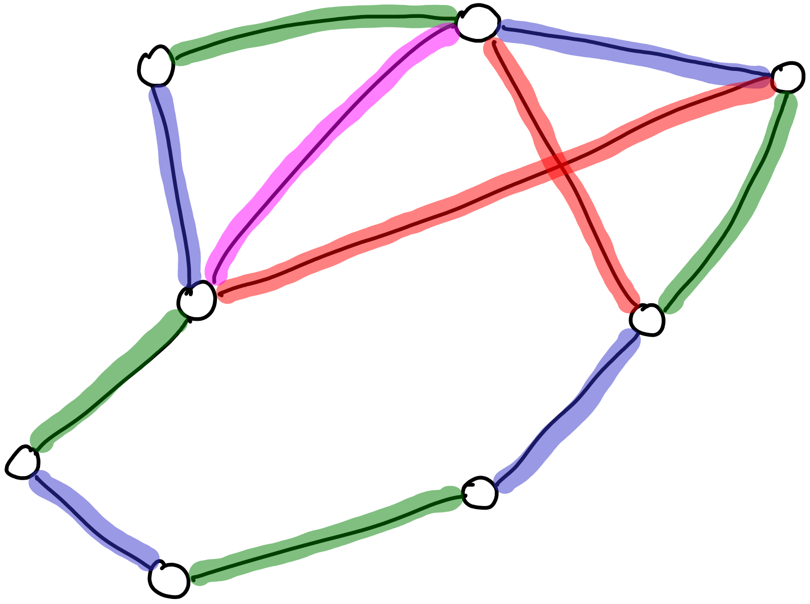
- Can't add more
- Can add one red & will need 2 more colors

$4 \leq \chi' \leq 6$

Can $\chi' = 4$?

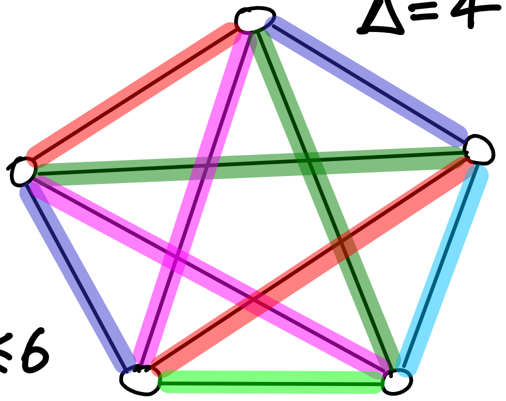
EDGE COLORING - no adjacent edges w same color

$\chi'(G)$: min # colors : edge-chromatic #



$\chi' = 4$
$\geq \Delta$

$\Delta = 4$



$\chi' \leq 6$

Can $\chi' = 4$?

10 edges → average #edges per color = 2.5

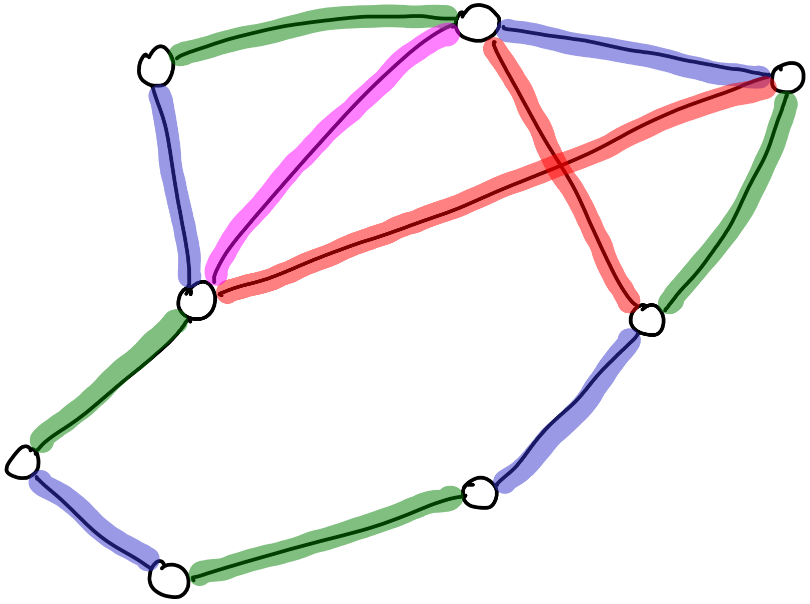
So some color goes on 3 edges : PROBLEM

↳ $\chi' > 4$
$> \Delta$

can't always achieve Δ

EDGE COLORING - no adjacent edges w same color

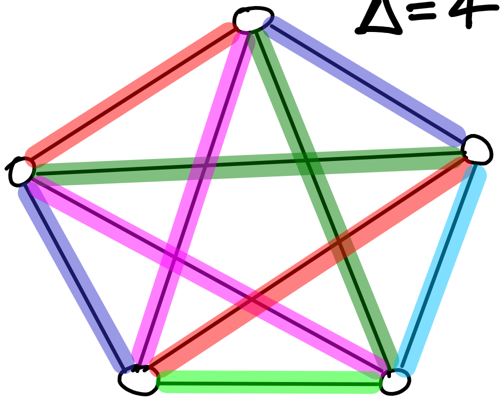
$\chi'(G)$: min # colors : edge-chromatic #



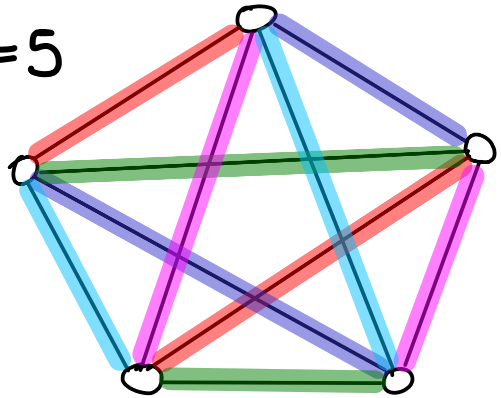
$\chi' = 4$

$\geq \Delta$

$\Delta = 4$

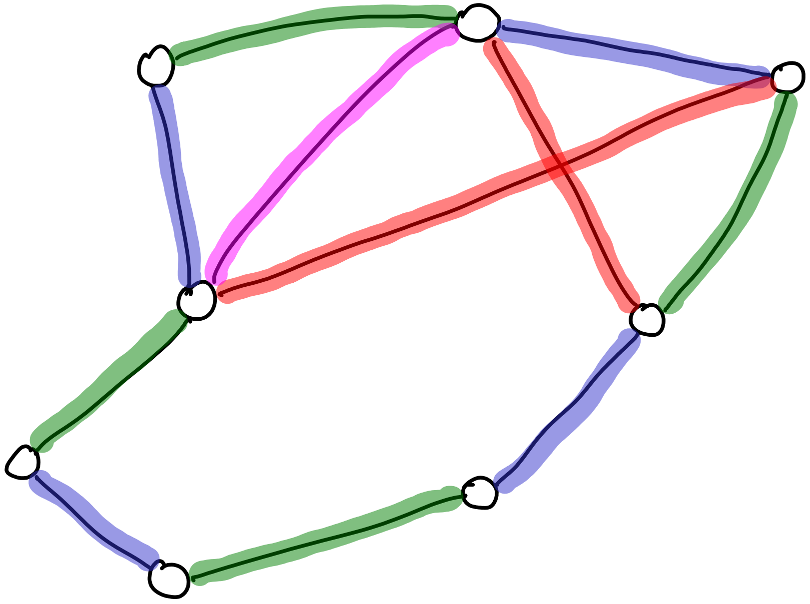


$\chi' = 5$



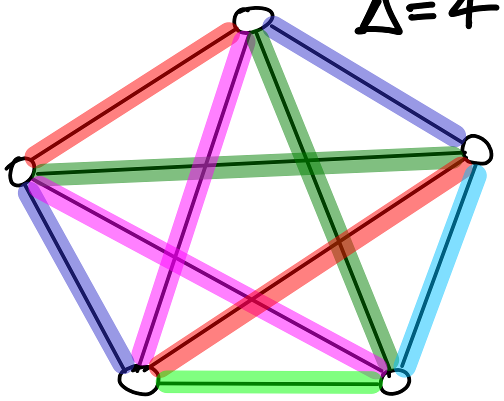
EDGE COLORING - no adjacent edges w same color

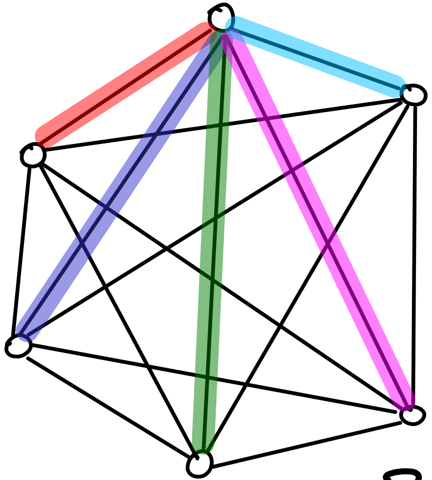
$\chi'(G)$: min # colors : edge-chromatic #



$\chi' = 4$
$\geq \Delta$

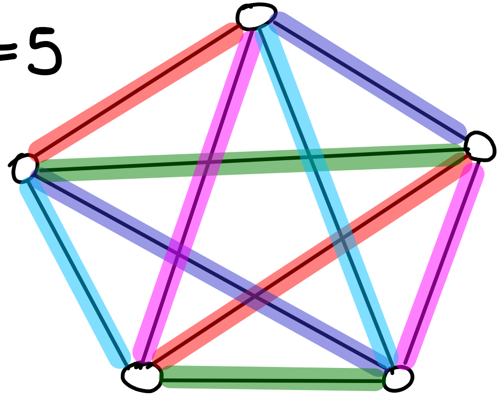
$\Delta = 4$





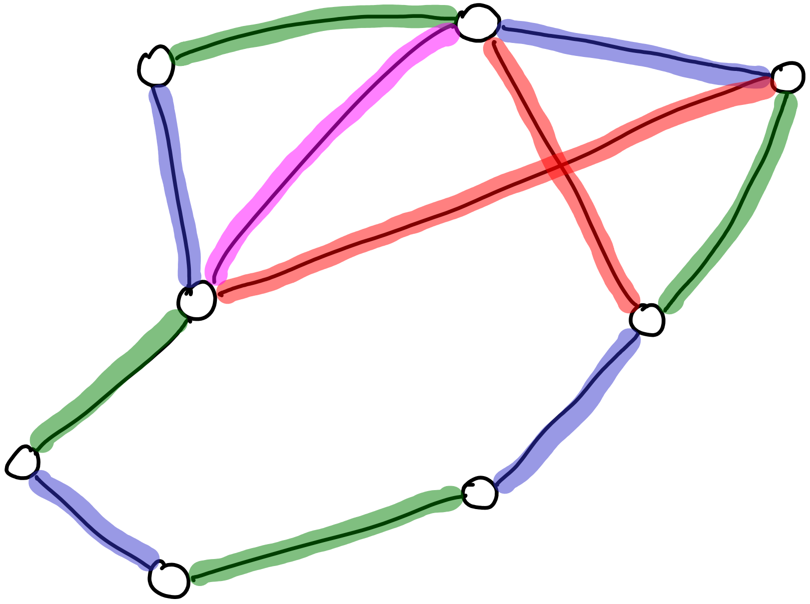
?

$\chi' = 5$



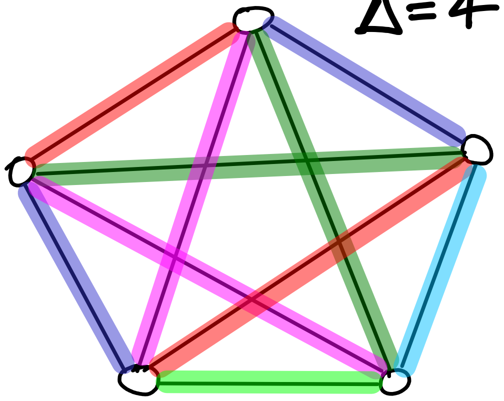
EDGE COLORING - no adjacent edges w same color

$\chi'(G)$: min # colors : edge-chromatic #

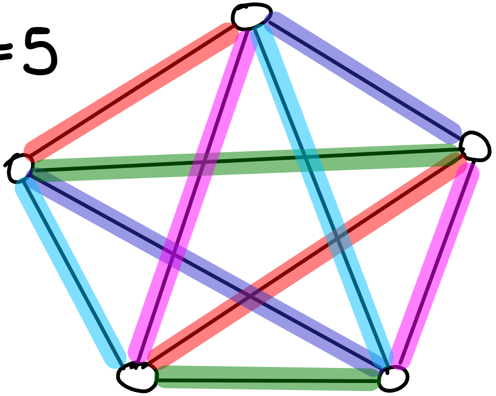


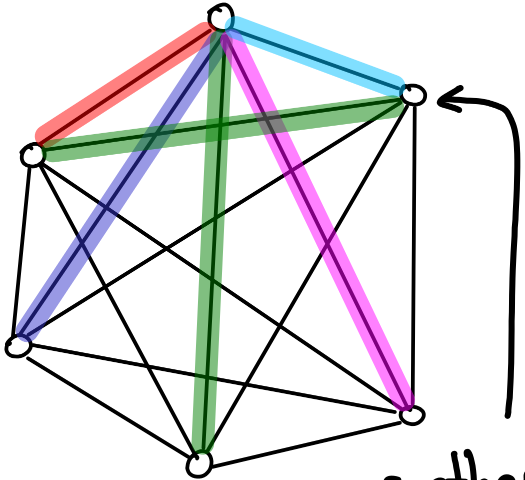
$\chi' = 4 \geq \Delta$

$\Delta = 4$



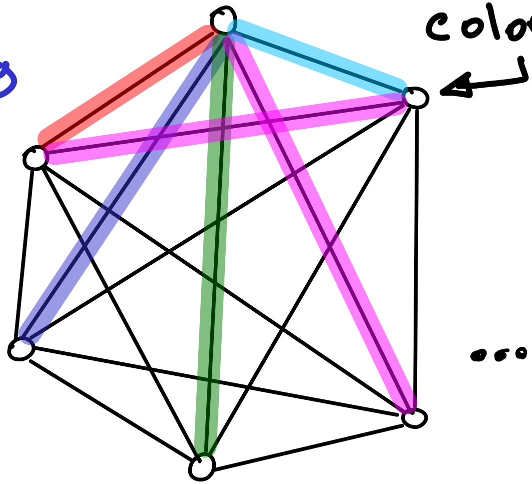
$\chi' = 5$





or other color ?

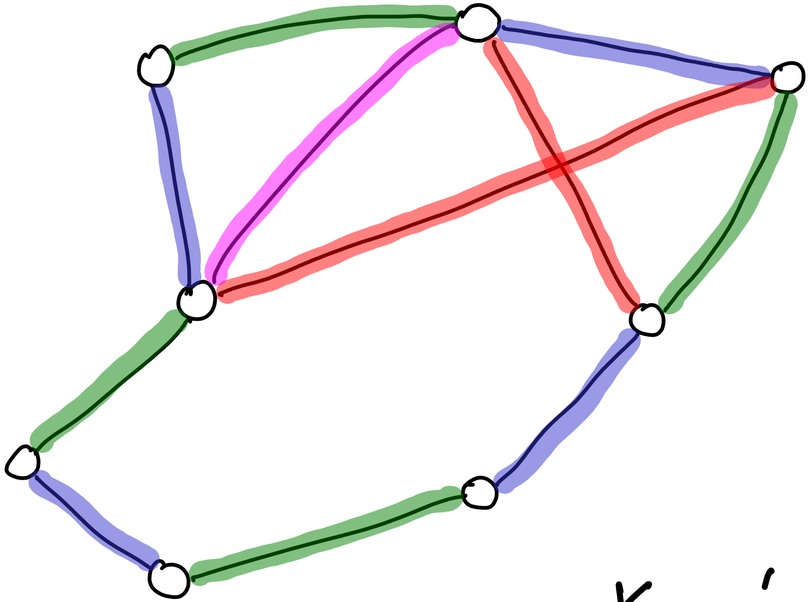
wlog



...?

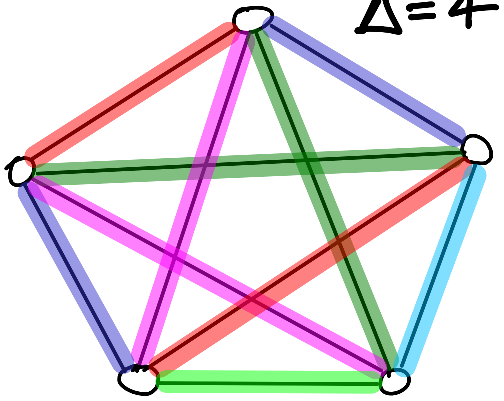
EDGE COLORING - no adjacent edges w same color

$\chi'(G)$: min # colors : edge-chromatic #



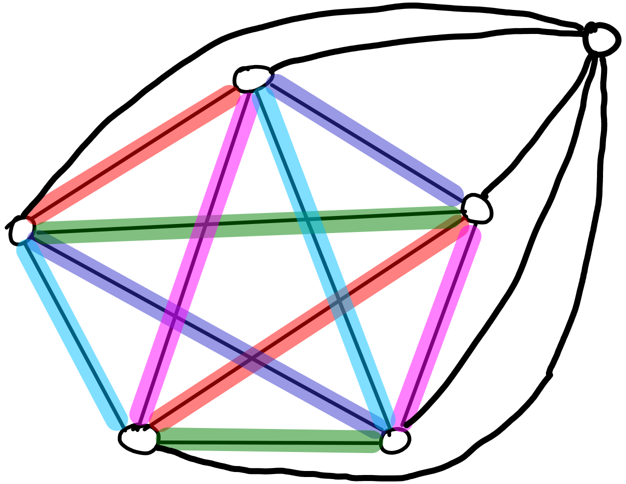
$\chi' = 4$
$\geq \Delta$

$\Delta = 4$



$K_5 : \chi' = 5$

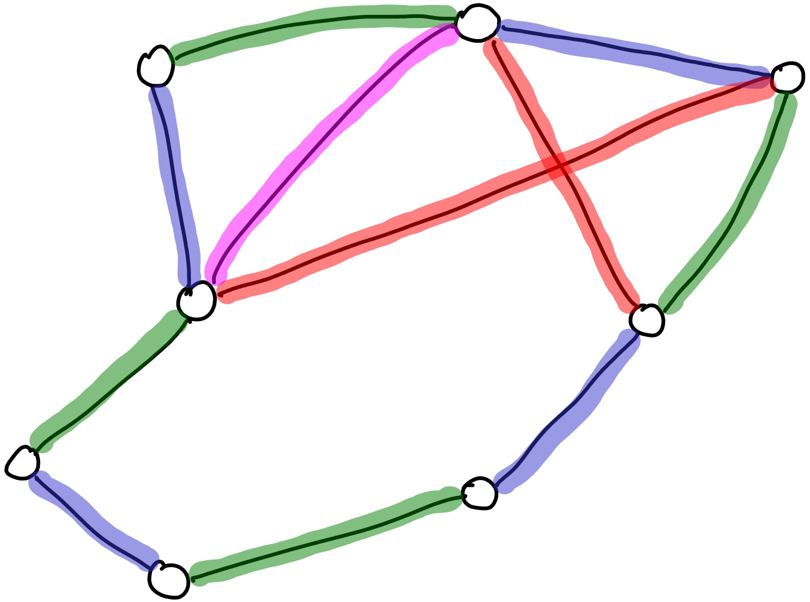
Easy trick:
extend



K_6

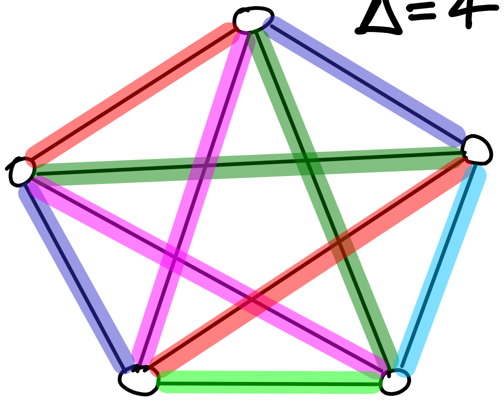
EDGE COLORING - no adjacent edges w same color

$\chi'(G)$: min # colors: edge-chromatic #



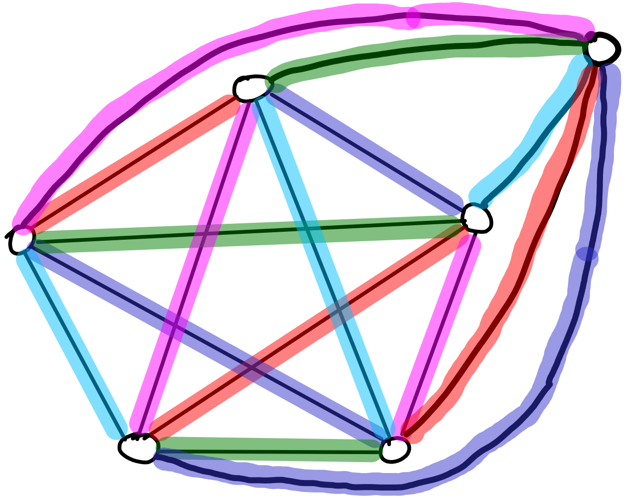
$\chi' = 4$

$\geq \Delta$



$\Delta = 4$

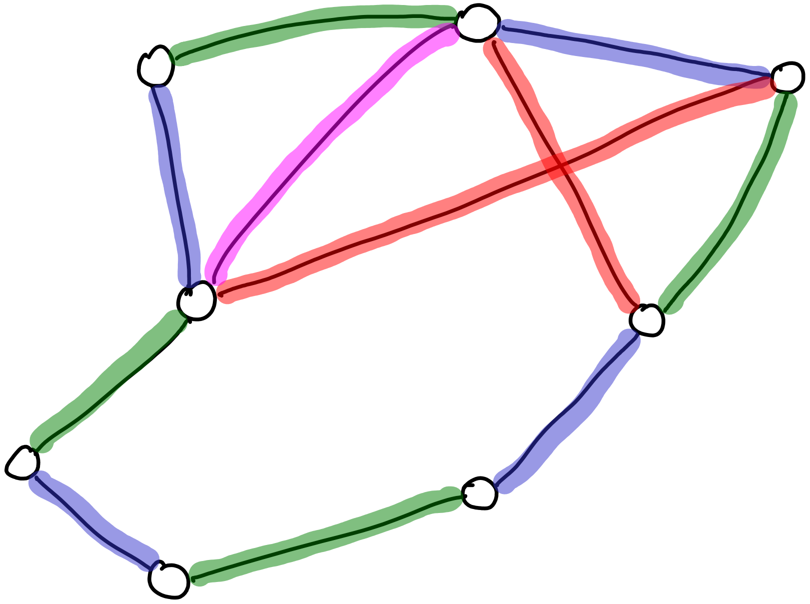
K_3	$\Delta = 2$	$\chi' = 3$
K_4	$\Delta = 3$	$\chi' = 4$
K_5	$\Delta = 4$	$\chi' = 5$
K_6	$\Delta = 5$	$\chi' = 5$!



$\chi' = 5$ remains

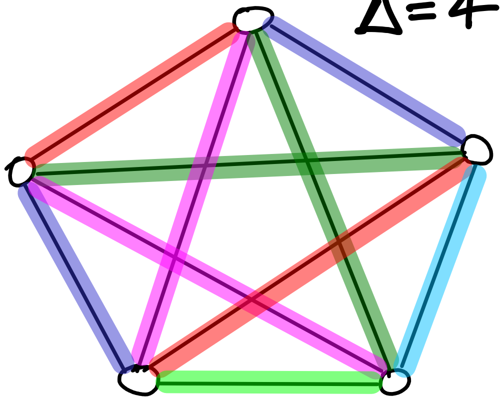
EDGE COLORING – no adjacent edges w same color

$\chi'(G)$: min # colors : edge-chromatic #

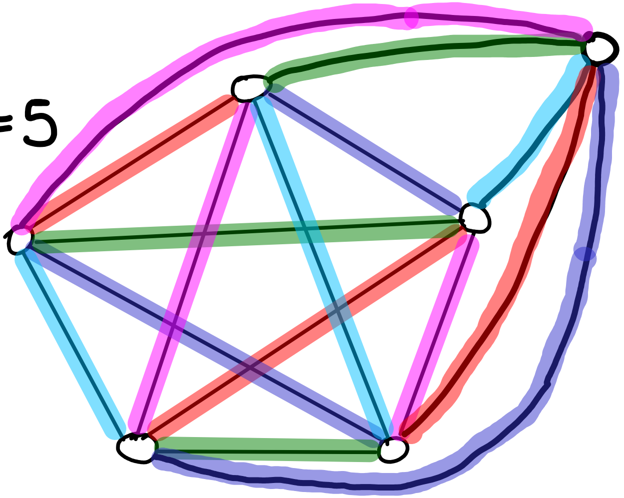


$\chi' = 4$
$\geq \Delta$

$\Delta = 4$



$\chi' = 5$



K_3	$\Delta = 2$	$\chi' = 3$
K_4	$\Delta = 3$	$\chi' = 4$
K_5	$\Delta = 4$	$\chi' = 5$
K_6	$\Delta = 5$	$\chi' = 5$!

K_7 can't continue extension

K_n : $f(n)$ and/or $f(\Delta)$?

Vizing's Thm: for all graphs, $\chi' \leq$ ... drum roll...

Vizing's Thm: for all graphs, $\chi' \leq \Delta + 1$ (close to trivial lower bound)

Vizing's Thm: for all graphs, $\chi' \leq \Delta + 1$ (close to trivial lower bound)

There are proofs by induction on E, V and Δ

most common

Equivalent to algorithm that incrementally extends G.

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most common

Equivalent to algorithm that incrementally extends G.

Observation : If we are willing to use $\Delta+1$ colors, then every vertex has ≥ 1 "missing" color. $d(v) \leq \Delta < \Delta+1$

no incident edge has that color

Vizing's Thm: for all graphs, $\chi' \leq \Delta + 1$ (close to trivial lower bound)

There are proofs by induction on E, V and Δ

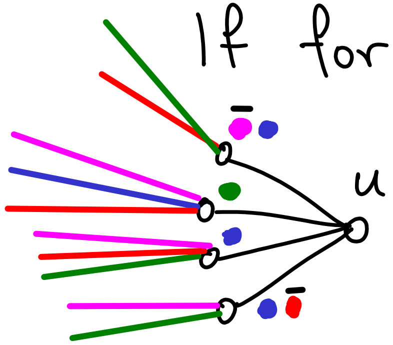
$E \downarrow$ most common

Equivalent to algorithm that incrementally extends G.

Observation: If we are willing to use $\Delta+1$ colors, then every vertex has ≥ 1 "missing" color. $d(v) \leq \Delta < \Delta+1$

↳ no incident edge has that color

implies

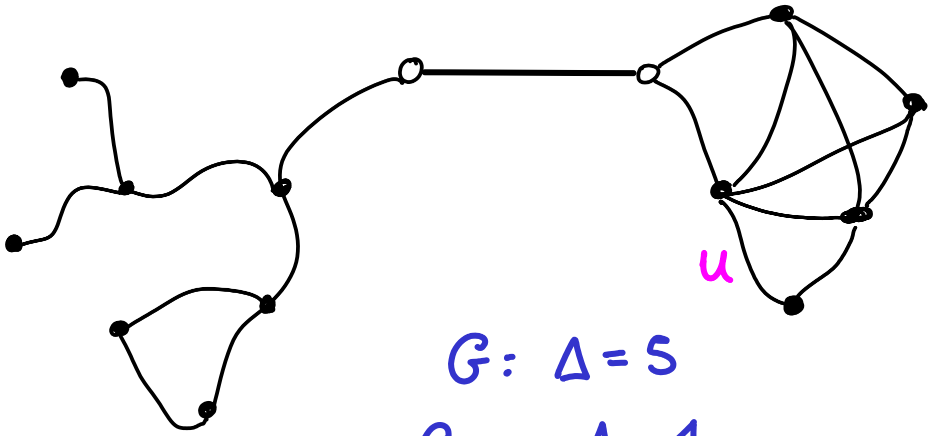


If for any $G' = G - u$, each neighbor of u in G can contribute one of its missing colors to a list of size $\geq d(u)$ then **DONE**.

list(u):

(induction on V)

Observations for induction on E.



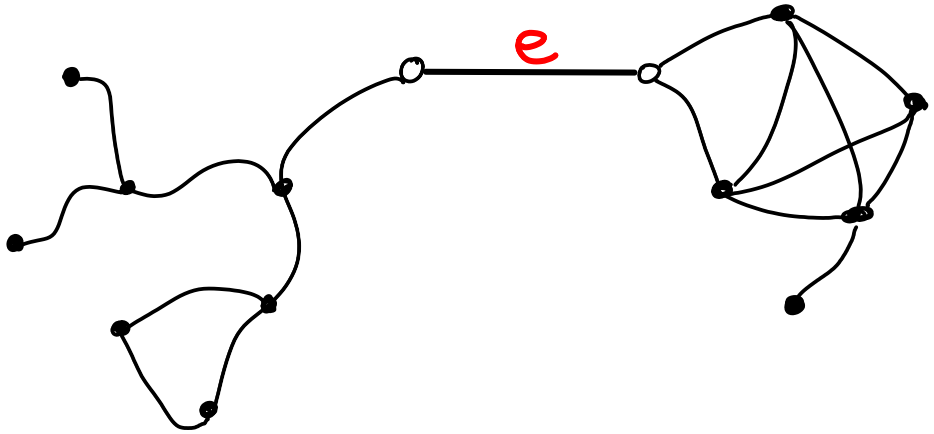
G: $\Delta = 5$

G-u: $\Delta = 4$

- If G-u has $\Delta - 1$, it uses $\le \Delta$ colors. Pick a new color for u.

  -DONE

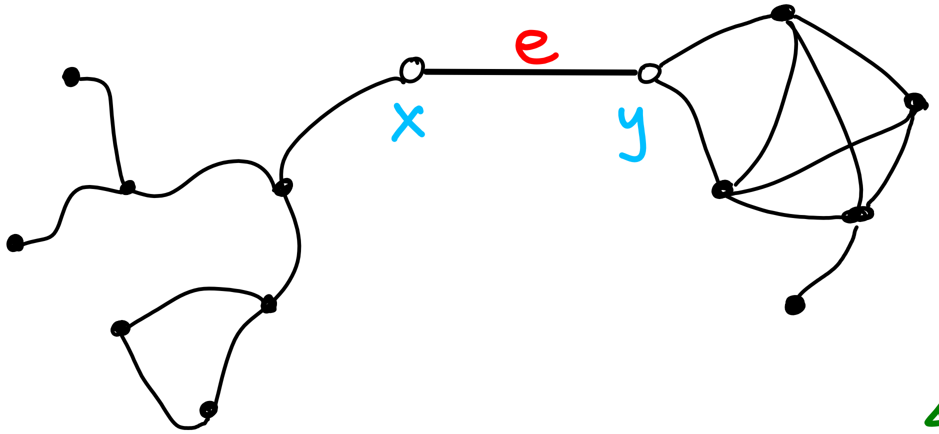
Observations for induction on E.



- If $G-u$ has $\Delta-1$, it uses $\leq \Delta$ colors. Pick a new color for u.
  –DONE

- If $G-e$ is disconnected, then remove e & color the 2 components by induction.

Observations for induction on E.

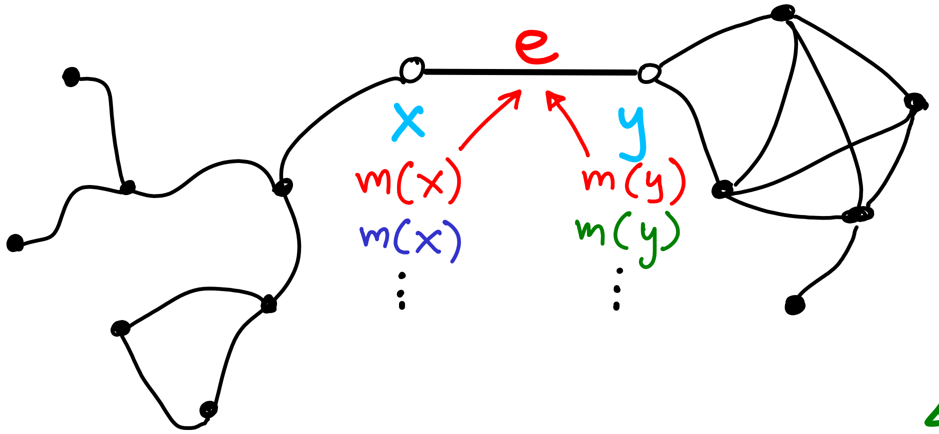


- If $G-u$ has $\Delta-1$, it uses $\leq \Delta$ colors. Pick a new color for u.
  -DONE

- If $G-e$ is disconnected, then remove e & color the 2 components by induction.

Even if components have Δ, using $\Delta+1$ colors,
x & y have degree $< \Delta$ $\therefore$ have ≥ 2 missing colors

Observations for induction on E.



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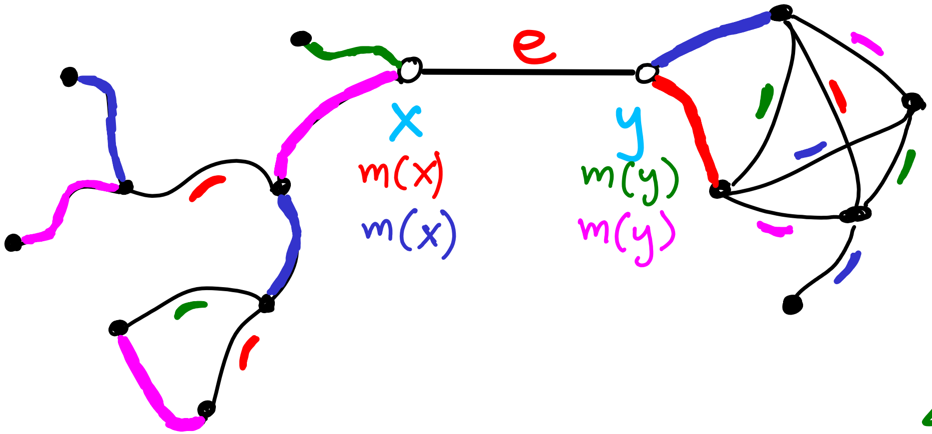
- If $G-e$ is disconnected, then remove e & color the 2 components by induction.

Even if components have Δ, using $\Delta+1$ colors,
x & y have degree $< \Delta$: have ≥ 2 missing colors

If $\exists$ missing color $m(x) = m(y)$, use this for e.

Observations for induction on E.



- If $G-u$ has $\Delta-1$, it uses $\leq \Delta$ colors. Pick a new color for u.
  -DONE

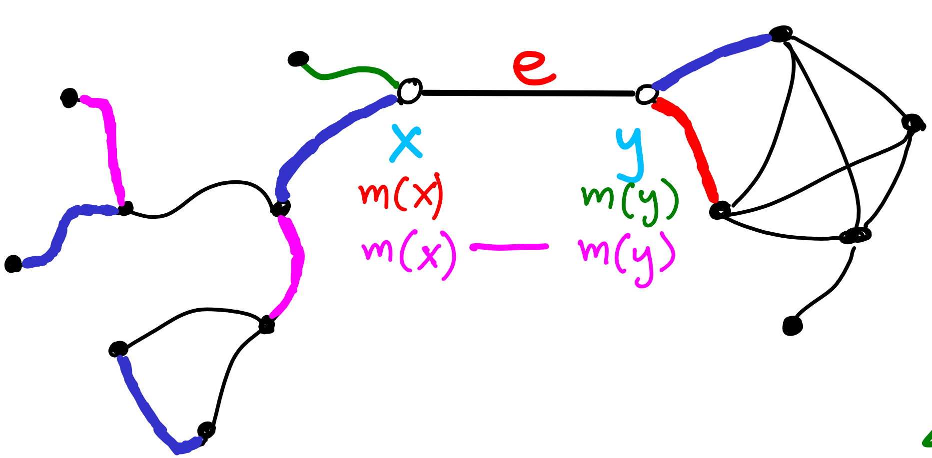
- If $G-e$ is disconnected, then remove e & color the $\underline{\underline{2}}$ components by induction.

Even if components have Δ, using $\Delta+1$ colors,

x & y have degree $< \Delta$: have ≥ 2 missing colors

$\begin{cases} \text{If } \exists \text{ missing color } m(x) = m(y), \text{ use this for } e. \\ \text{Else,} \end{cases}$

Observations for induction on E.



- If $G-u$ has $\Delta-1$, it uses $\leq \Delta$ colors. Pick a new color for u.
  -DONE

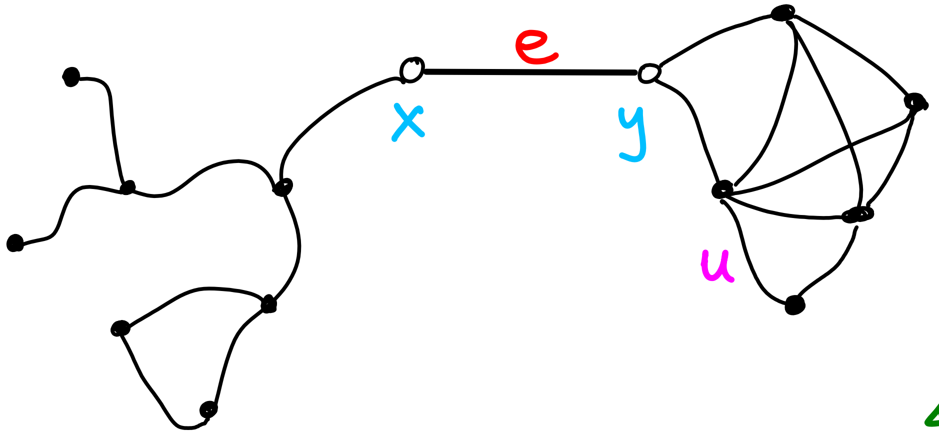
- If $G-e$ is disconnected, then remove e & color the $\underline{\underline{2}}$ components by induction.

Even if components have Δ, using $\Delta+1$ colors,
x & y have degree $< \Delta$: have ≥ 2 missing colors

$\left\{ \begin{array}{l} \text{If } \exists \text{ missing color } m(x) = m(y), \text{ use this for } e. \\ \text{Else, remap colors in one component s.t } m(x) = m(y) \end{array} \right.$

(permute)

Observations for induction on E.



- If $G-u$ has $\Delta-1$, it uses $\le \Delta$ colors. Pick a new color for u.
  -DONE

- If $G-e$ is disconnected, then remove e & color the 2 components by induction.

Even if components have Δ, using $\Delta+1$ colors,
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$\begin{cases} \text{If } \exists \text{ missing color } m(x) = m(y), \text{ use this for } e. \\ \text{Else, remap colors in one component s.t } m(x) = m(y) \end{cases}$

Conclusion: Induction on edges: trivial if any edge not on a cycle, or uniquely increases Δ