

RANDSELECT

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 $\longleftrightarrow$

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- For simplicity, assume no duplicates $\rightarrow$ Easy to handle.
- If necessary shuffle data to make random order.

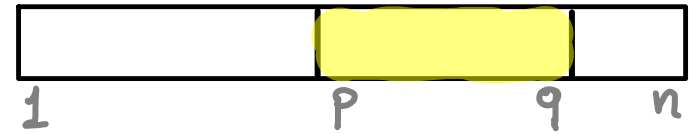
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Recursive function: **RandSelect**(k, p, q)



returns k -th smallest in subarray from index p to index q .

We start with **RandSelect**($r, 1, n$)

// Find k -th smallest within $[p, q]$

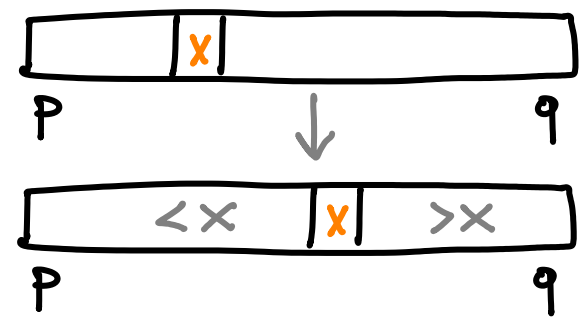
RandSelect(k, p, q)



// Find k-th smallest within $[p, q]$

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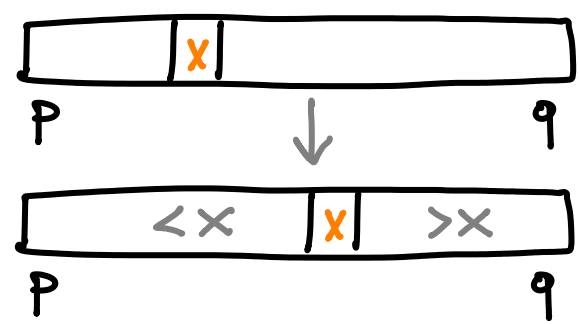
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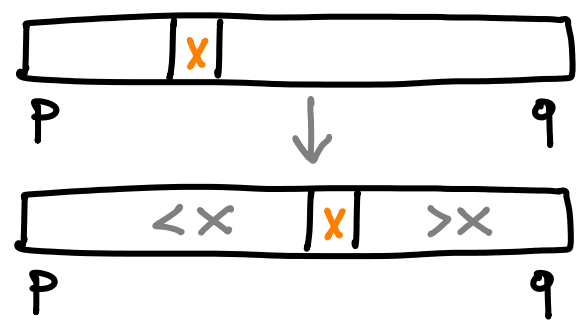
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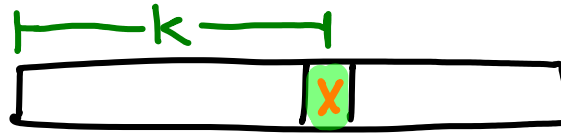
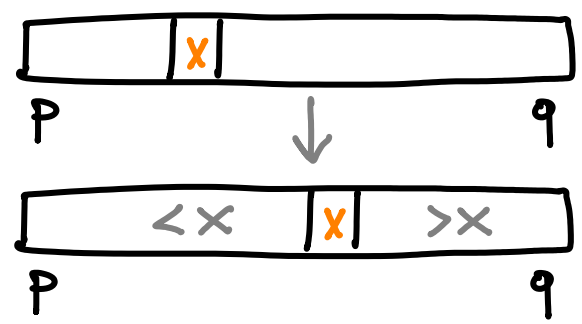
- 1) Use a random pivot x to partition $[p, q]$
- 2) Calculate rank of x within $[p, q]$: $1 + \text{\#elements smaller than } x$



// Find k-th smallest within $[p, q]$

RandSelect(k, p, q)

- 1) Use a random pivot x to partition $[p, q]$
- 2) Calculate rank of x within $[p, q]$
- 3) if $\text{rank}(x) = k$, return x



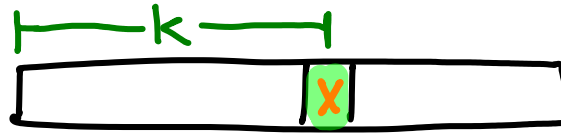
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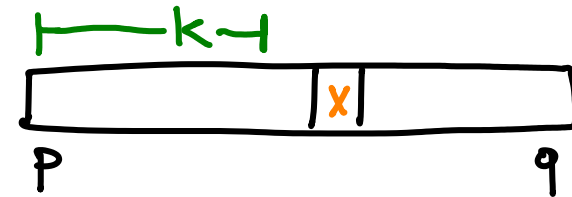
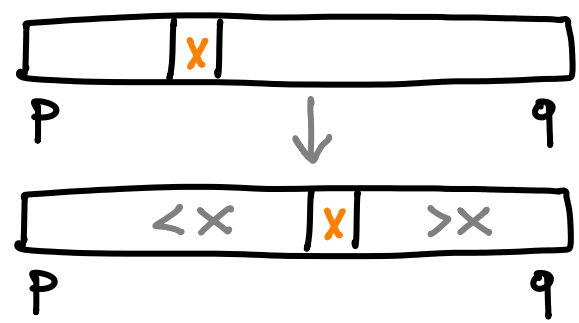
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if $k < \text{rank}(x)$... ?



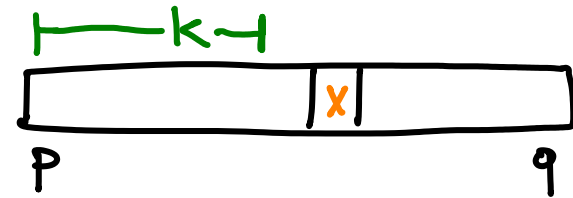
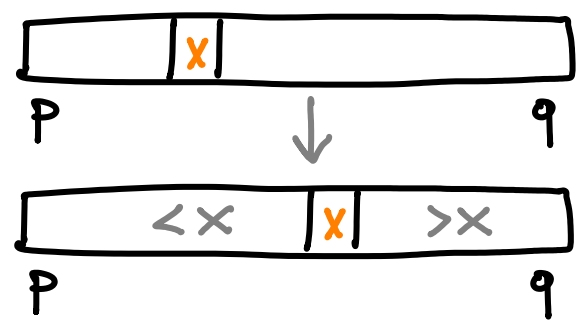
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if $k < \text{rank}(x)$, **RandSelect**($?, ?, ?$)



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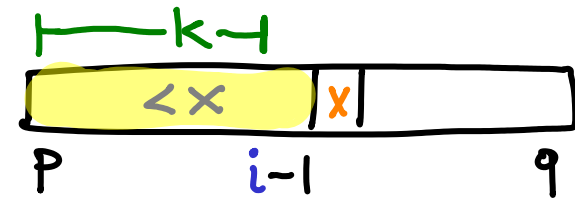
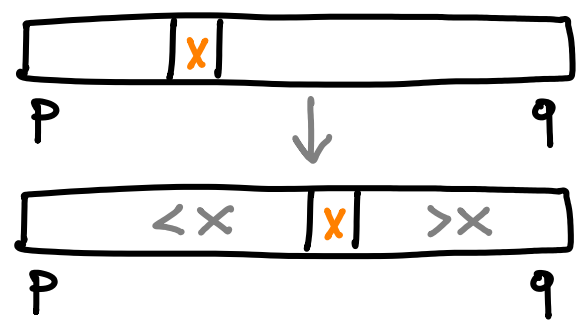
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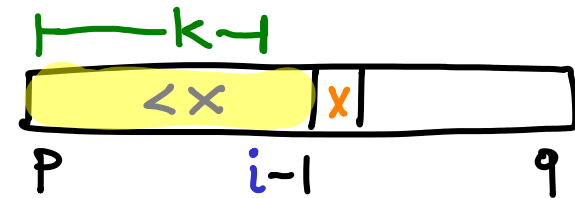
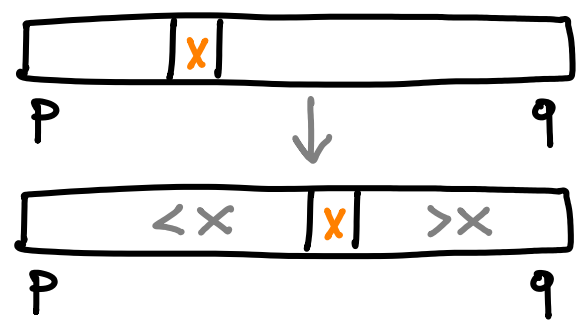
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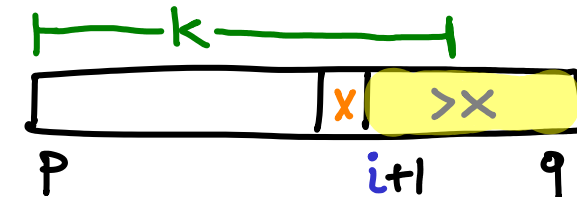
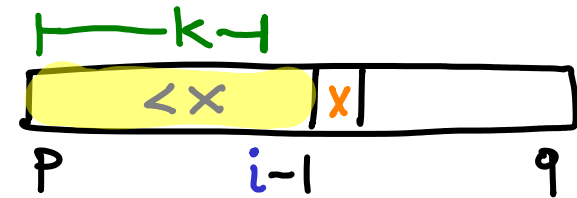
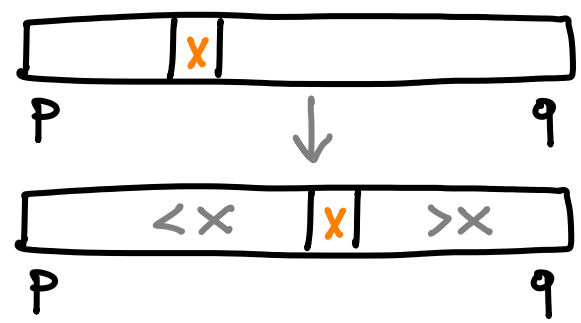
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if $k < \text{rank}(x)$, **RandSelect**($k, p, i-1$)

if $\text{rank}(x) < k$, **RandSelect**($?, i+1, ?$)



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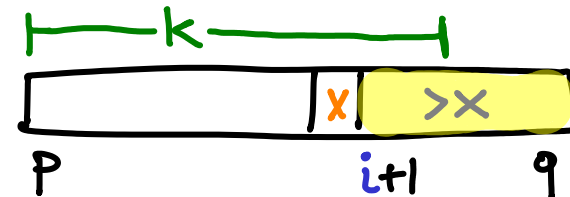
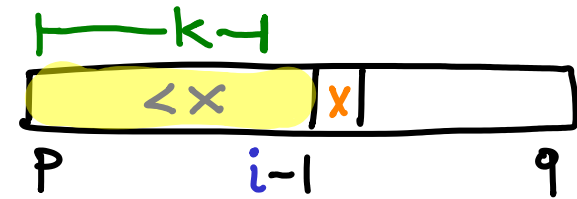
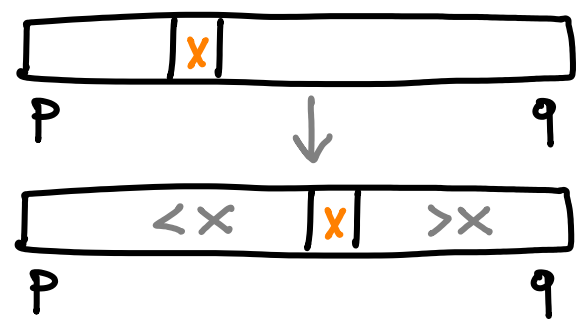
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if $k < \text{rank}(x)$, **RandSelect**($k, p, i-1$)

if $\text{rank}(x) < k$, **RandSelect**($k - \text{rank}(x), i+1, q$)



Example: Find 7th smallest

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↳ Partition: $\text{pivot} = x = 11$

Example: Find 7th smallest
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$\text{RandSelect}(7, 1, 12) \rightarrow$
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A red curved line connects the element 1 at index 4 to the element 13 at index 10. A blue highlight is under indices 2-3, and a pink highlight is under indices 11-12.

Example: Find 7th smallest
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Rank(11) = 9

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$$4 = \text{Rank}(5) < k = 7$$

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$$4 = \text{Rank}(5) < k = 7$$

$\text{RandSelect}(3, 5, 8)$

$$k - \text{Rank}(5) = 3$$

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$$7 = k < \text{Rank}(11) = 9$$

$\text{RandSelect}(7, 1, 8) \rightarrow$
 $\hookrightarrow$ Partition: $x = 5$

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$\text{RandSelect}(3, 5, 8) \rightarrow$
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Example: Find 7th smallest
 $k=r=7$, $p=1$, $q=n=12$

11	10	8	13	9	3	2	6	5	1	12	15
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$\text{RandSelect}(7, 1, 12) \rightarrow$

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return 9

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RANDSELECT

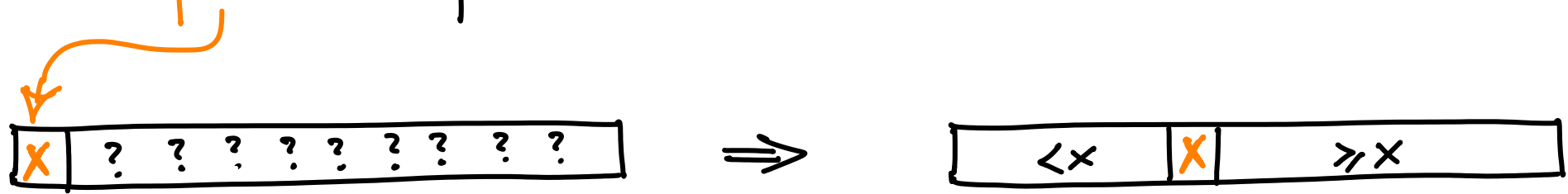
recap

RANDSELECT recap

- If necessary shuffle data to make random order.

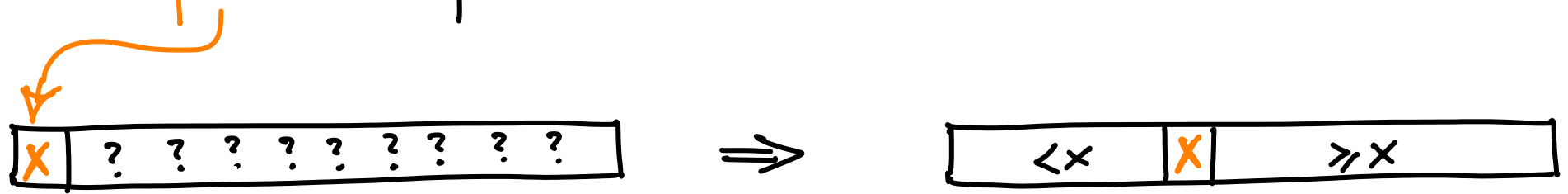
RANDSELECT recap

- If necessary shuffle data to make random order.
- choose a **pivot** & partition.



RANDSELECT recap

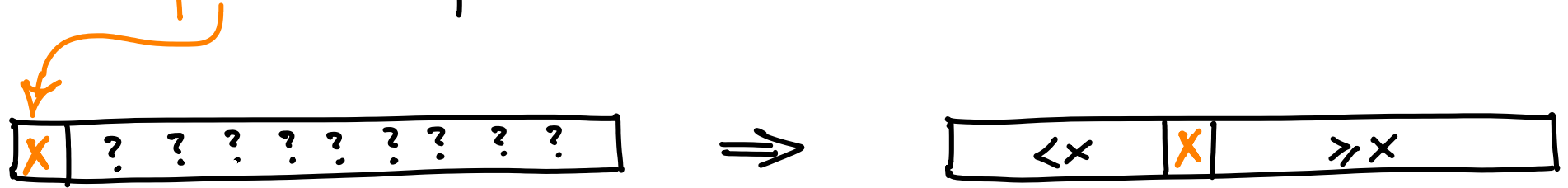
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- in the worst case, **RandSelect** the larger side.

RANDSELECT recap

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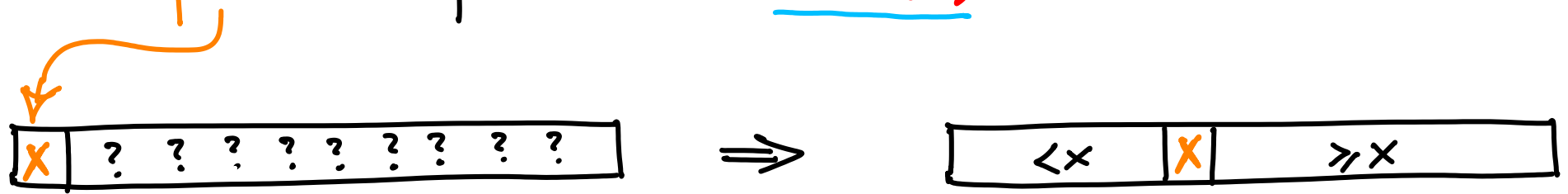
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RANDSELECT recap

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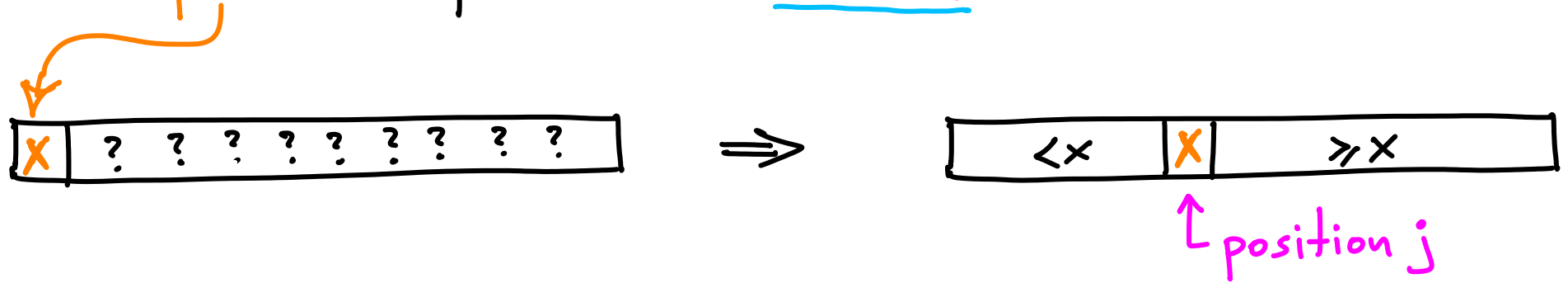


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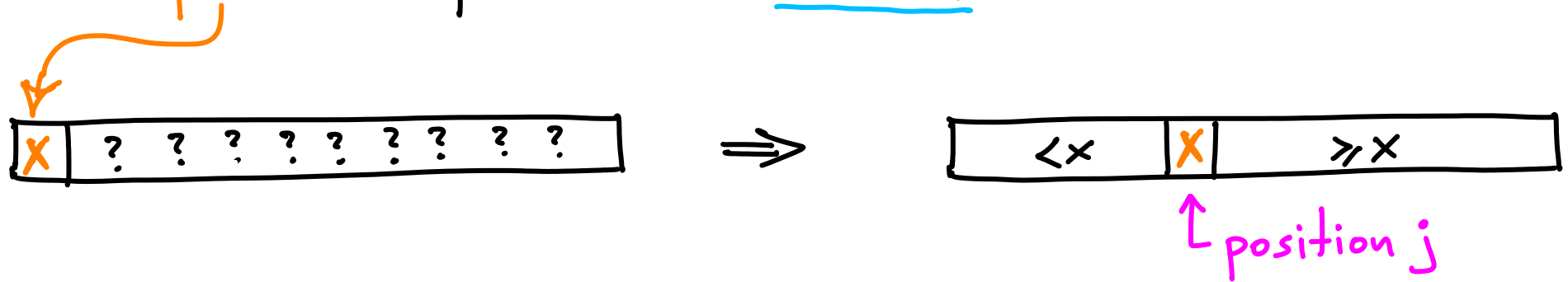
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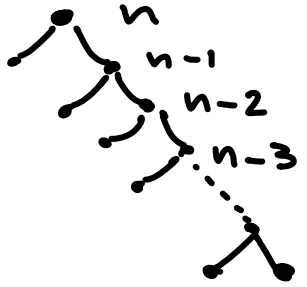
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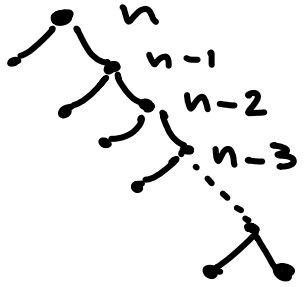


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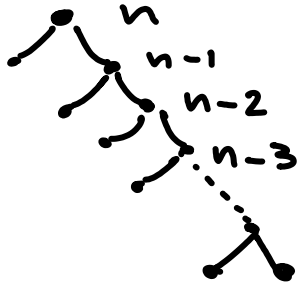
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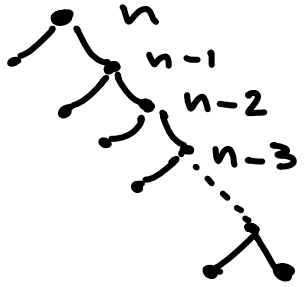
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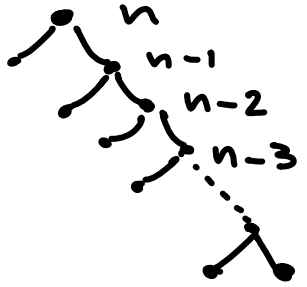
$$T(n) = T\left(\frac{n}{2}\right) + \Theta(n) = ?$$

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$$T(n) = T\left(\frac{n}{2}\right) + \Theta(n) = \Theta(n \log n)$$

What if we always have a "sort-of-balanced" partition?

e.g., $T(n) = T\left(\frac{9n}{10}\right) + \Theta(n) = ?$

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Expected time: call a split balanced if pivot ranks in $[\frac{n}{4} \dots \frac{3n}{4}]$
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Worst case if unbalanced split: $T(n) \leq \dots$

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$$\boxed{2dn \cdot \frac{1}{1 - 3/4} = 8dn}$$