(directed) graph s.t. every vertex can be reached from every vertex.

- note -

\[ \begin{array}{l}
\text{Not necessarily true that any 2 vertices reach each other with a "simple" cycle.}
\end{array} \]
STRONGLY CONNECTED COMPONENTS

- No longer strongly connected
- Chain reaction: several vertices no longer mutually reachable
- We get groups (components) each of which is strongly connected
The new graph must be a DAG
(any cycle would merge components)
RECOGNIZING
STRONGLY CONNECTED COMPONENTS
we know that no $W_i$ can be reached from outside $W$, in $G$.

(W wouldn't be 1st) OR (we'd have a cycle)

No $W_i$ can reach any vertex outside $W$ in $G^T$ (= $G$ w/ reversed edges)

If we run DFS on $G$, some vertex $W_i$ will finish last.

Take $W_i$ and DFS($w_i$) on $G^T \rightarrow$ finds $W$!
Once again, some $w_i$ will finish last, on any DFS run on $G$.

If not $w_i$, then some equivalent vertex in a component that could fit first in topological sort

No diff
Once again, some $w_i$ will finish last, on any DFS run on $G$.

$\Rightarrow$ $W$ may explore other areas which will finish before backtracking (i.e. before $w_i$ finishes).

$\Rightarrow$ no vertex will ever "discover" $w_i$ unless it is some $w_j$.

We don't know what $w_i$ or $W$ is in advance. All we know is that on any DFS on $G$, the vertex with the last finishing time belongs to the first component in topological order.
- DFS on $G$
- List finishing times
- Some vertex will be last. Call it $w_i$
- Construct $G^T$
- DFS starting from $w_i$ on $G^T$. 

$G$

$G^T$
- DFS on G
  - G list finishing times
  - some vertex will be last. Call it \( w_i \).

- Construct \( G^T \)
- DFS starting from \( w_i \) on \( G^T \).
  - will return exactly the vertices in \( W \).
    (same SCC as \( w_i \))

\[ \text{what remains?} \]
\[ \begin{align*}
\begin{cases}
\text{Some unexplored subgraph of } G^T. \text{ All } w_i \text{ marked.} \\
\text{Continue DFS w/ } X_i \ldots \\
\text{...then } Y_i, Z_i
\end{cases}
\]