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\[ \text{so what we care about is } \frac{1}{2}n^2 + \frac{1}{2}n \sim n^2 \]
Most of us would rather not deal with "minor" details such as
- whether it takes 2, 3, 7, or 53 instructions/operations
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- the size of non-leading terms  e.g. \( \frac{1}{2}n^2 + \frac{1}{2}n \)

For theory, the fun (and easier) part is to deal with large \( n \)
so what we care about is \( \frac{1}{2}n^2 + \frac{1}{2}n \approx n^2 \)

This leads to \( \Theta \)-notation
$F(n) = 53n^2 + 107n - 6 = \Theta(n^2)$  just look at the dominating term.
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Whatever $b$ an $c$ are, eventually $n$ will grow large enough that $bn + c \ll an^2$
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The same goes for \( a \): it is a constant and thus overshadowed by any function of \( n \) (that grows \( \to \infty \))
$\Theta$-notation

$f(n) = 53n^2 + 107n - 6 = \Theta(n^2)$

just look at the dominating term.

$a n^2 + b n + c$

Whatever $b$ and $c$ are, eventually $n$ will grow large enough that $bn + c \ll an^2$

The same goes for $a$: it is a constant and thus overshadowed by any function of $n$ that grows $\rightarrow \infty$.

$f(n) = n + 100$

$g(n) = \frac{1}{8} n^2$
Θ-notation can also be called "big-O" notation. Formally, \( f(n) = O(g(n)) \) or \( f(n) \in O(g(n)) \)
Θ-notation can also be called "big-O" notation.

Formally, \( f(n) = O(g(n)) \) or \( f(n) \in O(g(n)) \)

\( \text{there is a constant } c \)
\( \text{such that for all } n > n_0 \quad 0 \leq f(n) \leq c \cdot g(n) \)
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$g(n)$ is an asymptotic upper bound for $f(n)$. upper bound.
Θ-notation can also be called “big-O” notation.

Formally, \( f(n) = O(g(n)) \) or \( f(n) \in O(g(n)) \)

there is a constant \( c \) such that for all \( n > n_0 \) \( 0 \leq f(n) \leq c \cdot g(n) \)

\( g(n) \) is an asymptotic upper bound for \( f(n) \).

Usage: given a complicated \( f(n) \), find a simple \( g(n) \) s.t. \( f(n) = O(g(n)) \)
\( f(n) = O(g(n)) \quad \rightarrow \quad f(n) \leq c \cdot g(n). \)
\[ f(n) = O(g(n)) \quad \rightarrow \quad f(n) \leq c \cdot g(n). \]

\( \Omega \) gives a lower bound: \[ f(n) = \Omega(g(n)) \quad \rightarrow \quad f(n) \geq c \cdot g(n). \]

Again, exists some \( c \) for all \( n > n_0 \). ...
\[ f(n) = \Theta(g(n)) \quad \Rightarrow \quad f(n) = c \cdot g(n) \]

\[ \Omega \] gives a lower bound:  \[ f(n) = \Omega(g(n)) \quad \Rightarrow \quad f(n) \geq c \cdot g(n) \]

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\[ \Theta \] is the combination of \( O \) & \( \Omega \)
\( f(n) = O(g(n)) \implies f(n) \leq c \cdot g(n) \).

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\( \Theta \) is the combination of \( O \) & \( \Omega \)

\( f(n) = \Theta(g(n)) \) if \( f(n) = O(g(n)) \) and \( f(n) = \Omega(g(n)) \)
\[ f(n) = O(g(n)) \quad \Rightarrow \quad f(n) \leq c \cdot g(n). \]

\( \Omega \) gives a lower bound:

\[ f(n) = \Omega(g(n)) \quad \Rightarrow \quad f(n) \geq c \cdot g(n) \]

Again, exists some \( c \) for all \( n > n_0 \) ...

\( \Theta \) is the combination of \( O \) & \( \Omega \)

\[ f(n) = \Theta(g(n)) \quad \text{if} \quad f(n) = O(g(n)) \quad \text{AND} \quad f(n) = \Omega(g(n)) \]

i.e. \( 0 \leq c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n) \)

for all \( n > n_0 \) ...
$f(n) = O(g(n))$
$f(n) = \Theta(h(n))$
\[ f(n) = \Omega(h(n)) \]
If you want a better upper bound, it will only be valid for larger $n$.  

$c \cdot g(n)$  

$c_2 \cdot g(n)$  

$f(n)$  

$c_2 < c$  

$f(n) = O(g(n))$
$f(n) = \Theta(g(n))$
Prove \( \frac{1}{2} n^2 - 3n = \Theta(n^2) \)
Prove $\frac{1}{2}n^2 - 3n = \Theta(n^2)$

Find $c_1, c_2, n_0$ s.t.
Prove $\frac{1}{2}n^2 - 3n = \Theta(n^2)$

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\[
c_1 \leq \frac{1}{2} - \frac{3}{n} \leq c_2
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Prove \( \frac{1}{2}n^2 - 3n = \Theta(n^2) \)

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\[
c_1 \leq \frac{1}{2} - \frac{3}{n} \leq c_2
\]

For \( n \geq 1 : c_2 \geq \frac{1}{2} \) works. \( \rightarrow \frac{1}{2}n^2 - 3n \leq \frac{1}{2}n^2 \) easy upper bound
Prove $\frac{1}{2}n^2 - 3n = \Theta(n^2)$

Find $c_1, c_2, n_0$ s.t. $c_1 n^2 \leq \frac{1}{2}n^2 - 3n \leq c_2 n^2$ for all $n > n_0$.

$c_1 \leq \frac{1}{2} - \frac{3}{n} \leq c_2$

For $n > 1$ : $c_2 > \frac{1}{2}$ works.

For $n > 7$ : $c_1 \leq \frac{1}{14}$  \rightarrow  $\frac{1}{2}n^2 - 3n \geq \frac{1}{14}n^2$
Prove $\frac{1}{2}n^2 - 3n = \Theta(n^2)$

Find $c_1, c_2, n_0$ s.t. $c_1n^2 \leq \frac{1}{2}n^2 - 3n \leq c_2n^2$ for all $n \geq n_0$

$c_1 \leq \frac{1}{2} - \frac{3}{n} \leq c_2$

For $n \gg 1$ : $c_2 \gg \frac{1}{2}$ works.

For $n \gg 7$ : $c_1 \leq \frac{1}{14}$ (or, for $n \gg 30$ : $c_1 \leq 0.4$)

(as $n_0 \to \infty$, $c_1 \to \frac{1}{2}$)
Prove $\frac{1}{2}n^2 - 3n = \Theta(n^2)$

5. Find $c_1, c_2, n_0$ s.t. $c_1 n^2 \leq \frac{1}{2}n^2 - 3n \leq c_2 n^2$ for all $n > n_0$

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Prove $6n^3 = \Theta(n^2)$ ?!
Prove \( \frac{1}{2} n^2 - 3n = \Theta(n^2) \)

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---

Prove \( 6n^3 = \Theta(n^2) \)

5. find \( c_1, c_2, n_0 \) s.t. \( c_1 n^2 \leq 6n^3 \leq c_2 n^2 \)

trivially true for \( n \gg 1 \) & \( c_1 \leq 6 \) \( \Rightarrow 6n^3 = \Omega(n^2) \)
Prove \( \frac{1}{2}n^2 - 3n = \Theta(n^2) \)

For \( n \gg n_0 \):

\[ c_1 n^2 \leq \frac{1}{2}n^2 - 3n \leq c_2 n^2 \]

\[ c_1 \leq \frac{1}{2} - \frac{3}{n} \leq c_2 \]

For \( n \gg 1 \) : \( c_2 \gg \frac{1}{2} \) works.

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\( \text{as } n_0 \to \infty \ , \ c_1 \to \frac{1}{2} \)

Prove \( 6n^3 = \Theta(n^2) \)

For \( n \gg 1 \) : \( c_1 n^2 \leq 6n^3 \leq c_2 n^2 \)

R.h.s. : \( 6n \leq c_2 \Rightarrow n \leq \frac{c_2}{6} \)
Prove \( \frac{1}{2} n^2 - 3n = \Theta(n^2) \)

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\[
\begin{align*}
    c_1 &\leq \frac{1}{2} - \frac{3}{n} \leq c_2 \\
\end{align*}
\]

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For \( n > 7 \) : \( c_1 \leq \frac{1}{14} \) (or, for \( n > 30 \) : \( c_1 \leq 0.4 \))

(as \( n_0 \to \infty \) , \( c_1 \to \frac{1}{2} \))

Prove \( 6n^3 = \Theta(n^2) \)

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trivially true for \( n > 1 \) & \( c_1 \leq 6 \)

R.h.s. : \( 6n \leq c_2 \Rightarrow n \leq \frac{c_2}{6} \) : not true for any constant \( c_2 \)
Is $50 \cdot n = O(n^2)$?
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$50 \cdot n \leq c \cdot n^2$

$50 \leq c \cdot n$
Is \( 50 \cdot n = O(n^2) \)?

\[
\begin{align*}
50 \cdot n & \leq c \cdot n^2 \\
50 & \leq c \cdot n
\end{align*}
\]

\( n > 1 \) \( c = 50 \) \( (n > 50 \ c = 1) \)
Is \( 50 \cdot n = O(n^2) \)?

\[
\begin{align*}
50 \cdot n &\leq c \cdot n^2 \\
50 &< c \cdot n \\
\{ & \quad n > 1 \\
| & \quad n \geq 50 \\
& \quad c = 1
\end{align*}
\]

Yes
Is \( 50 \cdot n = O(n^2) \)?

Yes, but also \( 50 \cdot n = O(n) \)

\[
\begin{align*}
50 \cdot n &\leq c \cdot n^2 \quad \text{for } n \geq 1 \quad c = 50 \\
50 &\leq c \cdot n \quad \text{for } n > 50 \quad c = 1
\end{align*}
\]
Is $50 \cdot n = O(n^2)$? If yes, then $50 \cdot n \leq c \cdot n^2$ for $n \geq 1$ and $c = 50$. If no, then $50 \cdot n \leq c \cdot n$ for all $n$, so $c = 50$.

Yes, but also $50 \cdot n = O(n)$, because $50 \cdot n \leq c \cdot n$ for all $n$.

$50 \cdot n = O(n^2)$ is not wrong; it's just not very accurate.
Some basic functions: \( \log n, \sqrt{n}, n, n \log n, n^2, n^3, 2^n, n! \)

\[ \log n = {?} (\sqrt{n}) \quad n^2 = {?} (n \log n) \]
Some basic functions: $\log n, \sqrt{n}, n, n \log n, n^2, n^3, 2^n, n!$

\[ \log n = O(\sqrt{n}) \quad n^2 = \Omega(n \log n) \]
Some basic functions: \( \log n, \sqrt{n}, n, n \log n, n^2, n^3, 2^n, n! \)

\[
\log n = O(\sqrt{n}) \quad n^2 = \Omega(n \log n)
\]

Try \( n^{1-\varepsilon}, 3^{n-\varepsilon}, \log(n^2), \log^2 n, 2^{\log_2 n}, 4^{\log_2 n}, n, n^2, n^n, n^2 \)

(for \( \varepsilon > 0 \))

compare \( 2^n \) v. \( 3^{n-\varepsilon} \)

\( \log n \) v. \( \log(n^2) \)

\( \log(n^2) \) v. \( \log^2 n \)

\( 2^{\log_2 n} \) v. \( n^k \) (k=?)

etc
Recall that we determined that Insertion sort costs $\sum_{j=1}^{n} j$
This is $\Theta(\cdot)$
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This is $\Theta(n^2)$

Does it always cost this?
Recall that we determined that Insertion sort costs \( \sum_{j=1}^{n} j \).

This is \( \Theta(n^2) \).

Does it always cost this? No. This is the worst case.
Recall that we determined that Insertion sort costs $\sum_{j=1}^{n} j$

This is $\Theta(n^2)$

Does it always cost this? NO. This is the worst case.

We could also discuss the “best case”: that is less interesting.

Any algo can be expanded to handle some input $I$ in $O(|I|)$ time.
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Anyway, Insertion sort would take $O(n)$ time

if ...?
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Any algo can be expanded to handle some input $I$ in $O(|I|)$ time.

Anyway, Insertion sort would take $O(n)$ time if the input was already sorted.
Recall that we determined that Insertion sort costs $\sum_{j=1}^{n} j$

This is $\Theta(n^2)$

Does it always cost this?  No. This is the worst case.

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Any algo can be expanded to handle some input $I$ in $O(|I|)$ time.

Anyway, Insertion sort would take $O(n)$ time

if the input was already sorted.

Insertion sort takes $\Omega(n)$ time

but the $O(n^2)$-time bound is "tight". // The worst-case time complexity of insertion sort is $\Theta(n^2)$
Sometimes big-O is used to describe lower order terms.

\[ 2n^2 + 3n + 1 = 2n^2 + O(n). \]
More asymptotic notation although less common.
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Recall that
\[ \Theta \colon = \]
\[ O \colon \leq \]
\[ \Omega \colon \geq \]
More asymptotic notation although less common.

Recall that

\[ \Theta : = \begin{align*}
\mathcal{O} & : = \\
\Omega & : = \end{align*} \]

\text{little-} o \rightarrow < \quad \text{e.g. } f(n) = o(g(n)) \quad \text{if for any constant } c \text{ there exists large enough } n_0 \\
\text{s.t. } & \quad 0 \leq f(n) < c \cdot g(n) \quad \text{for all } n \geq n_0 \\
& \quad \left[ 0 \leq \frac{1}{c} f(n) < g(n) \right] \]
More asymptotic notation although less common.

Recall that

\[\Theta : = \\]
\[O : = \leq \]
\[\Omega : = \geq\]

little-o \rightarrow < \quad \text{e.g. } f(n) = o(g(n)) \quad \text{if for any constant } c \text{ there exists large enough } n_0 \text{ s.t. } 0 \leq f(n) < c \cdot g(n) \quad \text{for all } n \geq n_0
More asymptotic notation although less common.

Recall that

\[ \Theta : = \]
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\text{little-o} \rightarrow < \quad f(n) = o(g(n)) \quad \text{if for any constant } c \text{ there exists large enough } n_0 \text{ s.t. } 0 < f(n) < c \cdot g(n) \quad \text{for all } n > n_0

\text{little-o} \rightarrow >

In other words, once again in the world of very large n, f(n) is not asymptotically equivalent to g(n).

No matter what constant we multiply f(n) with, it won't exceed g(n)
Example of little-o

$5n = o(n^2)$
Example of little-o

\[ 5n = o(n^2) \] for any constant \( c \)

\[ 5c \cdot n < n^2 \] when \( n > 5c \)
Example of little-o

\[ 5n = o(n^2) \]  
\[ \exists \text{ for any constant } c \]
\[ 5 \cdot c \cdot n < n^2 \text{ when } n > 5 \cdot c \]

In some sense \( f(n) = o(g(n)) \) means \( \lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 \)
Example of little-o

\[ 5n = o(n^2) \] \{ For any constant \( c \)

\[ 5 \cdot c \cdot n < n^2 \] when \( n > 5 \cdot c \)

In some sense \( f(n) = o(g(n)) \) means \( \lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 \)

\( w \) is similar \( \frac{1}{10} n^3 = w(n^2) \) \{ For any \( c \), there is some \( n_0 \) s.t.

\[ \frac{1}{10} n^3 > c \cdot n^2 \] for all \( n > n_0 \)
Example of little-o

\[ 5n = o(n^2) \]  \quad \text{for any constant } c

\[ 5cn < n^2 \quad \text{when } n > 5c \]

In some sense \( f(n) = o(g(n)) \) means \( \lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 \)

\( \omega \) is similar \( \frac{1}{10}n^3 = \omega(n^2) \)  \quad \text{for any } c, \text{ there is some } n_0 \text{ s.t.} \n
\[ \frac{1}{10}n^3 > cn^2 \quad \text{for all } n > n_0 \]

given \( c \), choose \( n > 10c \)
Example of little-o

\[ 5n = o(n^2) \quad \text{if for any constant } c \]

\[ 5 \cdot c \cdot n < n^2 \quad \text{when } n > 5 \cdot c \]

In some sense \( f(n) = o(g(n)) \) means \( \lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 \)

\( \frac{1}{10}n^3 = o(n^2) \)

For any \( c \), there is some \( n_0 \) s.t.

\[ \frac{1}{10}n^3 > c \cdot n^2 \quad \text{for all } n > n_0 \]

Given \( c \), choose \( n > 10 \cdot c \)

If \( f(n) = \omega(g(n)) \), \( \lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty \)
Example of little-o

\[ 5n = o(n^2) \quad \{ \text{For any constant } c \}
\]

\[ 5c \cdot n < n^2 \quad \text{when } n > 5c \]

In some sense \( f(n) = o(g(n)) \) means \( \lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 \)

\( \frac{1}{10}n^3 = o(n^2) \quad \{ \text{For any } c, \text{ there is some } n_0 \text{ s.t.} \}
\]

\[ \frac{1}{10}n^3 > c \cdot n^2 \quad \text{for all } n > n_0 \]

given \( c \), choose \( n > 10c \)

If \( f(n) = o(g(n)) \), \( \lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 \)

Notice \( f(n) = o(g(n)) \iff g(n) = \omega(f(n)) \)
see CLRS 51 for intuitive transitivity rules.

So, what do we accomplish with $\Theta$-notation?
see CLRS 51 for intuitive transitivity rules.

So, what do we accomplish with Θ-notation?

- we can understand behavior of algorithms for large input. (same for complexity of problems)
see CLRS 51 for intuitive transitivity rules.

So, what do we accomplish with Θ-notation?

- we can understand behavior of algorithms for large input.
  (same for complexity of problems)
- we can analyze algorithms in a hardware/implementation-independent way.