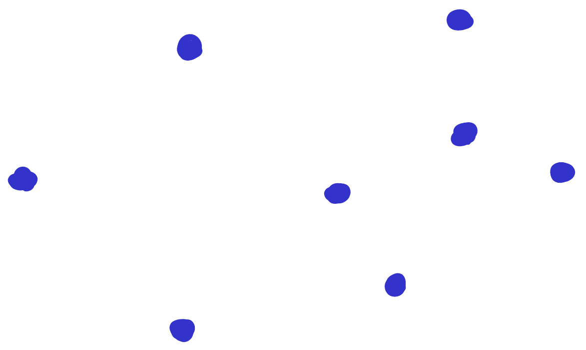
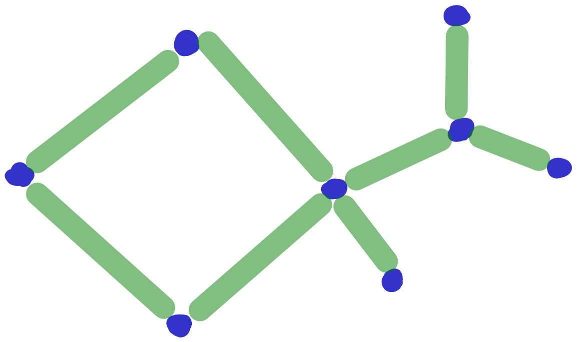
SPANNERS



Given a set of vertices

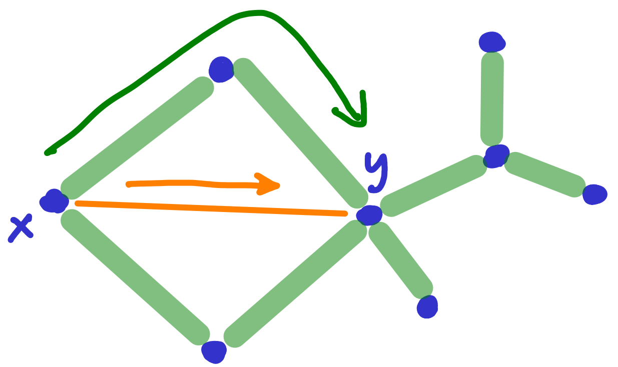
SPANNERS



Objective

Given a set of vertices,
form a graph (place edges)

SPANNERS

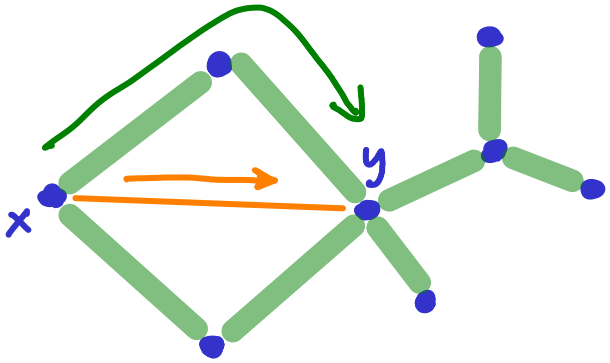


Objective

Given a set of vertices,
form a graph (place edges)
s.t. the spanning ratio (detour) for
any pair of vertices is "low".

$$\text{ratio} = \frac{\text{distance via graph}}{\text{Euclidean dist.}}$$

SPANNERS



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Given a set of vertices,
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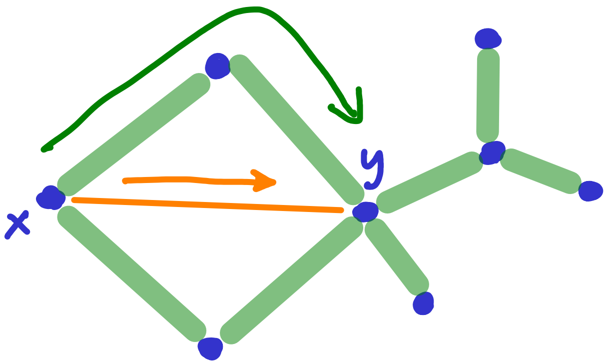
Other desirable properties

- use few edges

SPANNERS



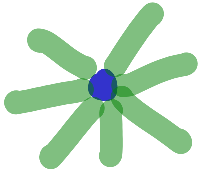
Objective

Given a set of vertices, form a graph (place edges) s.t. the spanning ratio (detour) for any pair of vertices is "low".

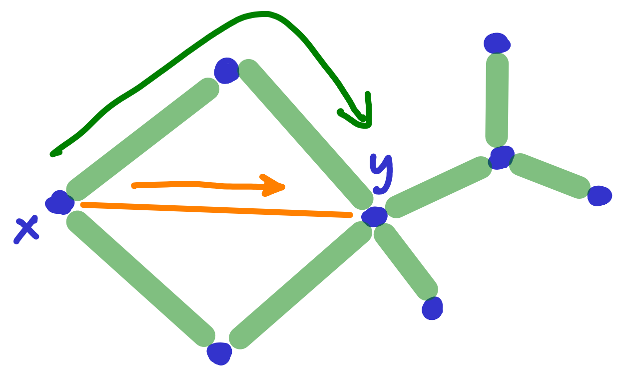
$$\text{ratio} = \frac{\text{distance via graph}}{\text{Euclidean dist.}}$$

Other desirable properties

- use few edges
- minimize degree



SPANNERS



Other desirable properties

- use few edges
- minimize degree
- planarity

Objective

Given a set of vertices, form a graph (place edges) s.t. the spanning ratio (detour) for any pair of vertices is "low".

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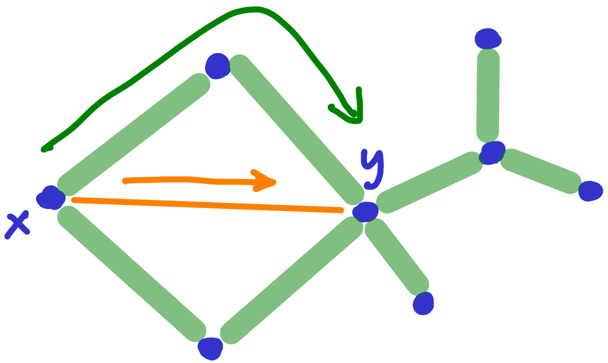
SPANNERS



Other desirable properties

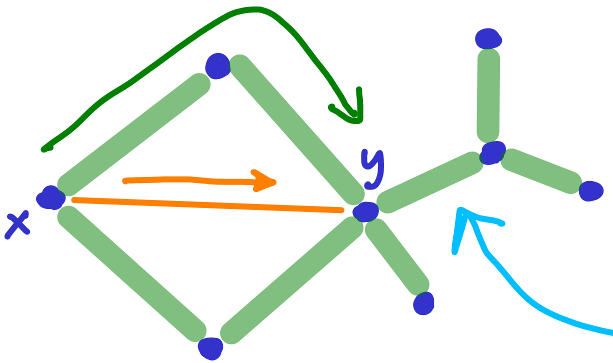
- use few edges
- minimize degree
- planarity
- minimize max. path length

Objective

Given a set of vertices, form a graph (place edges) s.t. the spanning ratio (detour) for any pair of vertices is "low".

$$\text{ratio} = \frac{\text{distance via graph}}{\text{Euclidean dist.}}$$

Spanners



Other desirable properties

- use few edges
- minimize degree
- planarity
- minimize max. path length
- robustness: good connectivity

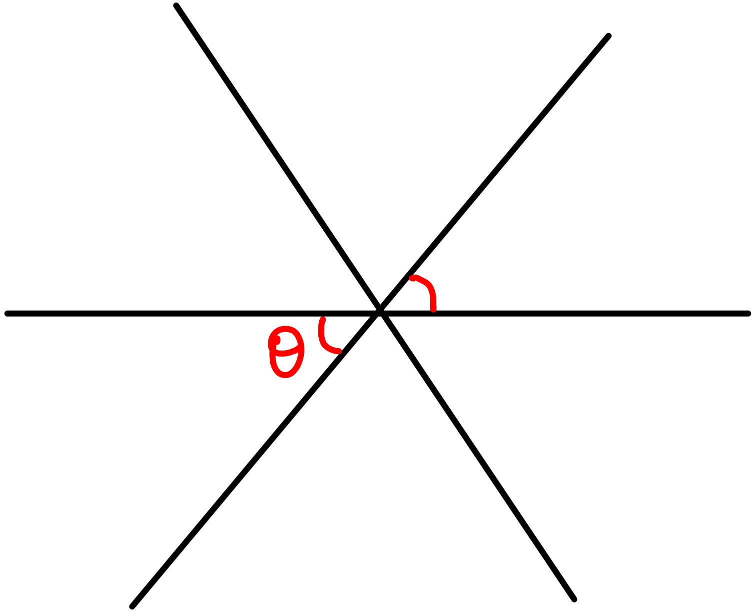
etc

Objective

Given a set of vertices, form a graph (place edges) s.t. the spanning ratio (detour) for any pair of vertices is "low".

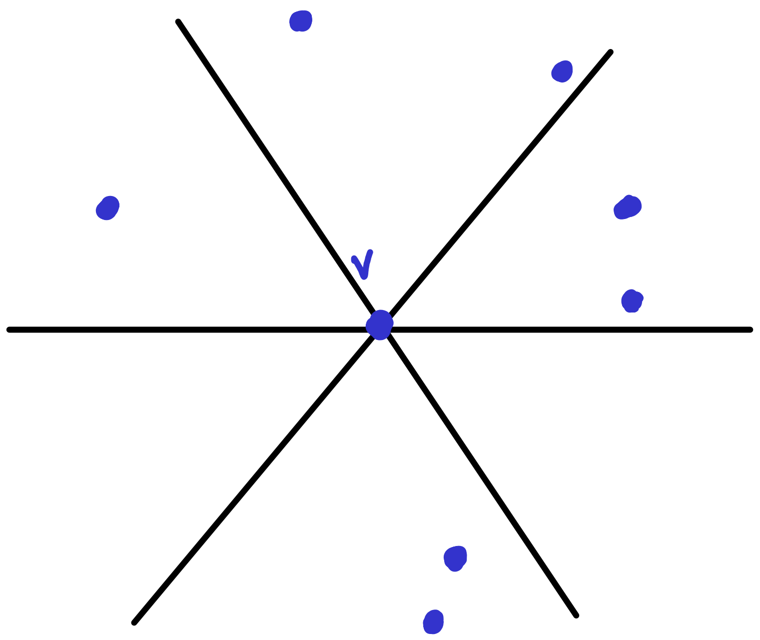
$$\text{ratio} = \frac{\text{distance via graph}}{\text{Euclidean dist.}}$$

the Θ-graph



Form K cones w/ angle $\theta = \frac{2\pi}{K}$

the Θ-graph

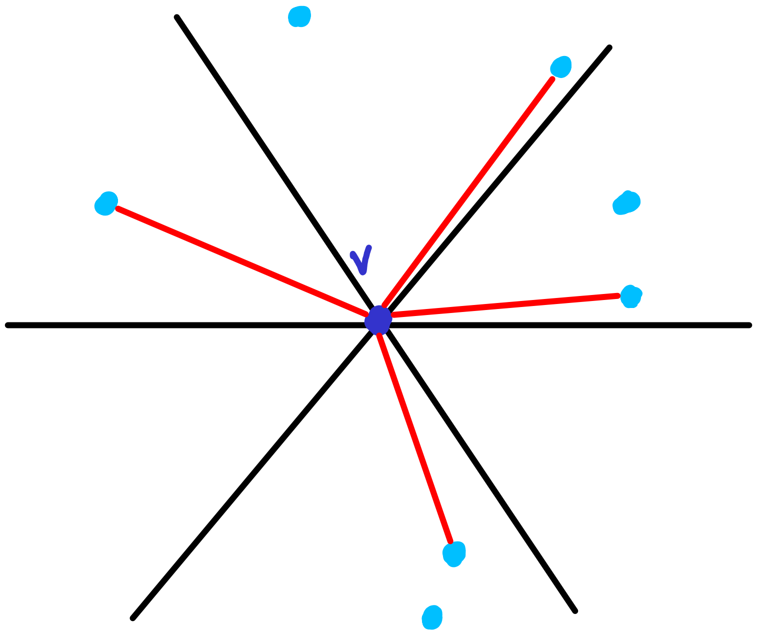


Form k cones w/ angle $\theta = \frac{2\pi}{k}$

For every vertex v

shift cone structure onto v

the Θ-graph



Form k cones w/ angle $\theta = \frac{2\pi}{k}$

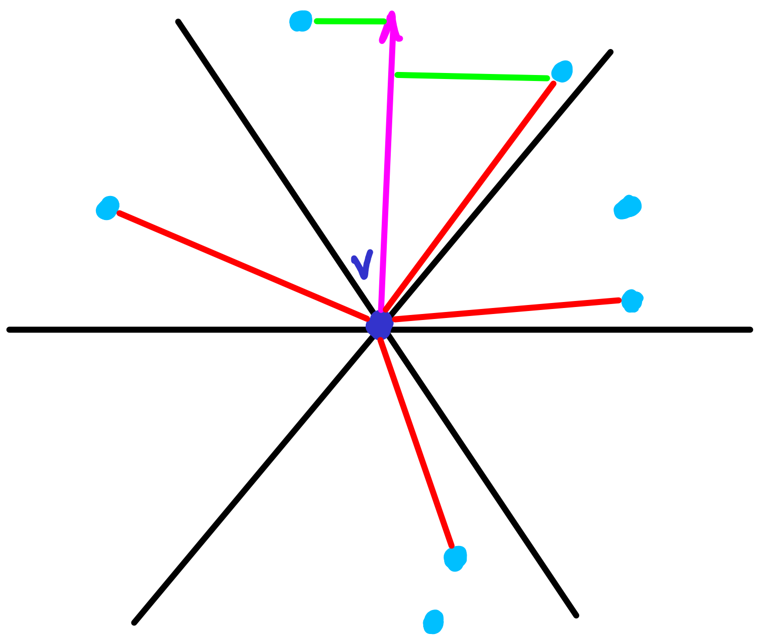
For every vertex v

shift cone structure onto v

& connect v to 1 vertex in each cone

(if non-empty)

the Θ-graph



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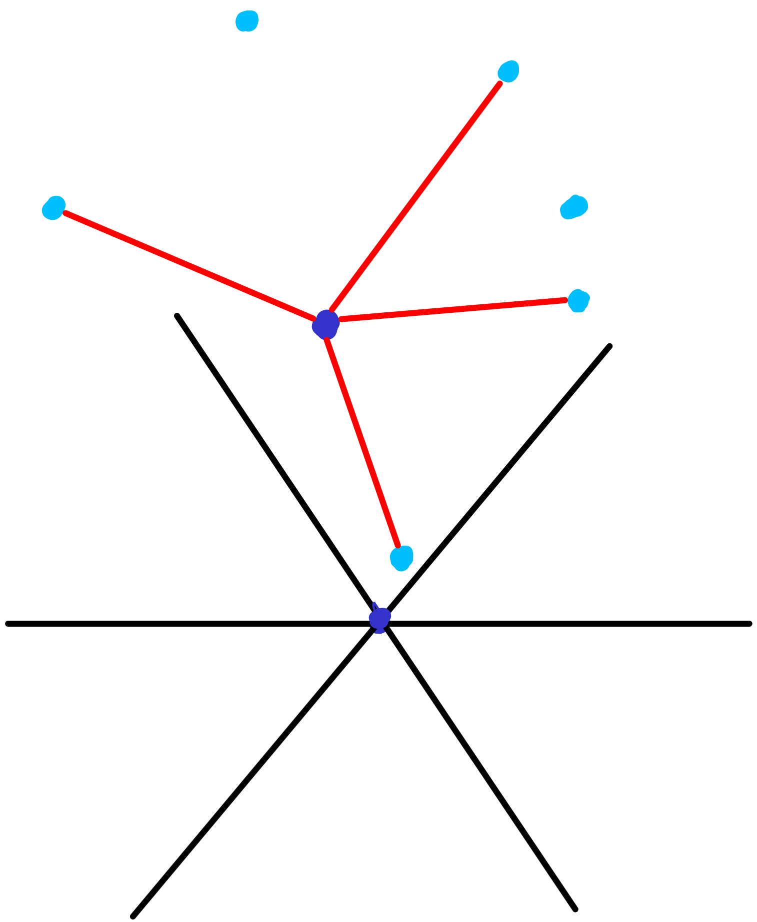
Connection rule:

- project vertices onto cone bisector
- choose vertex with closest projection

the Θ-graph



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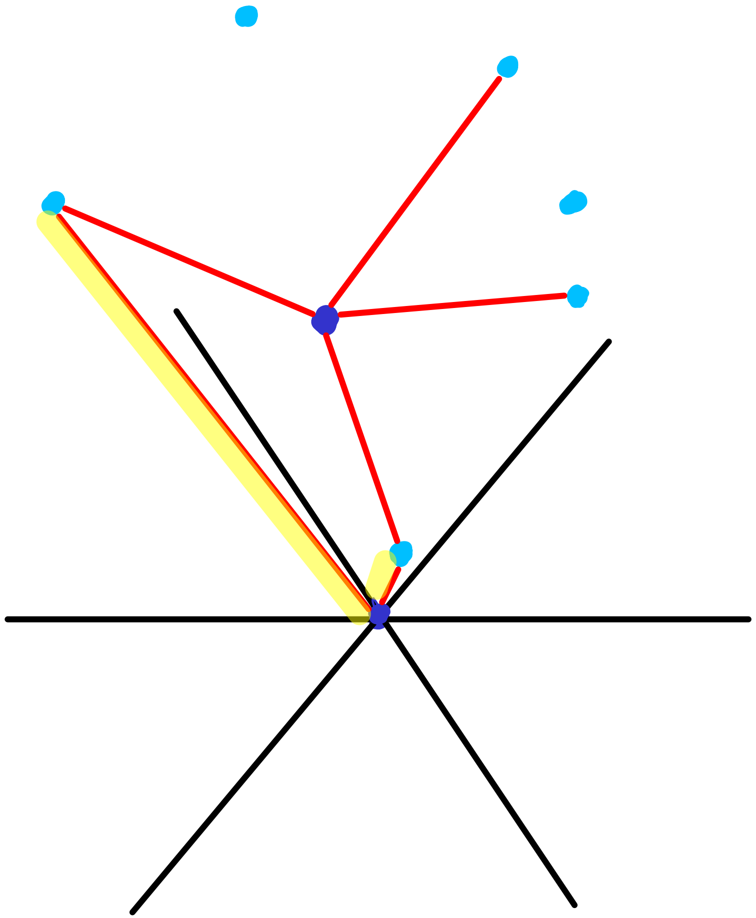

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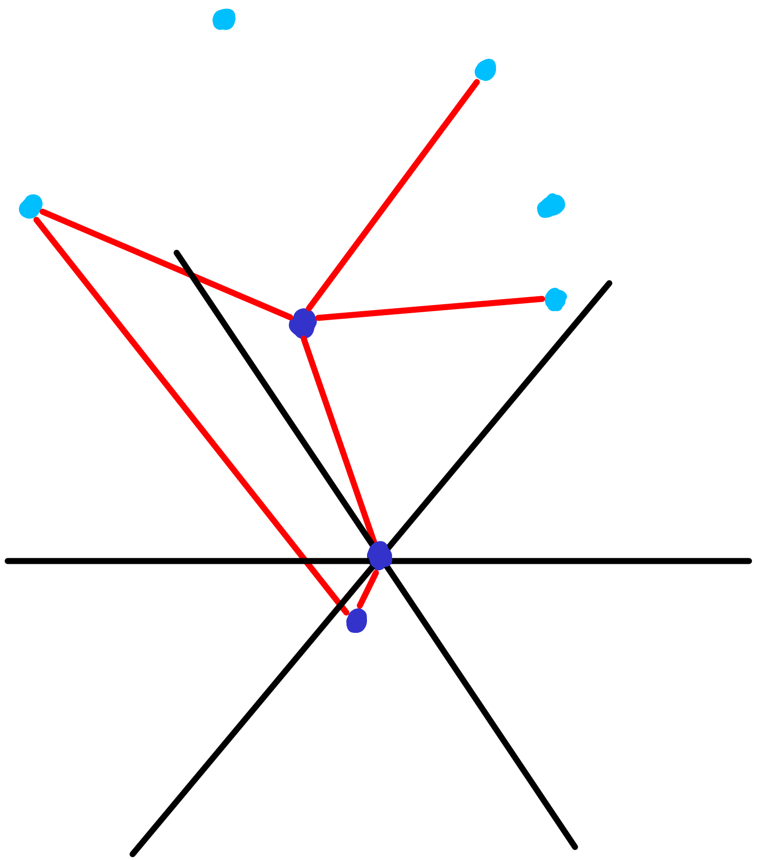

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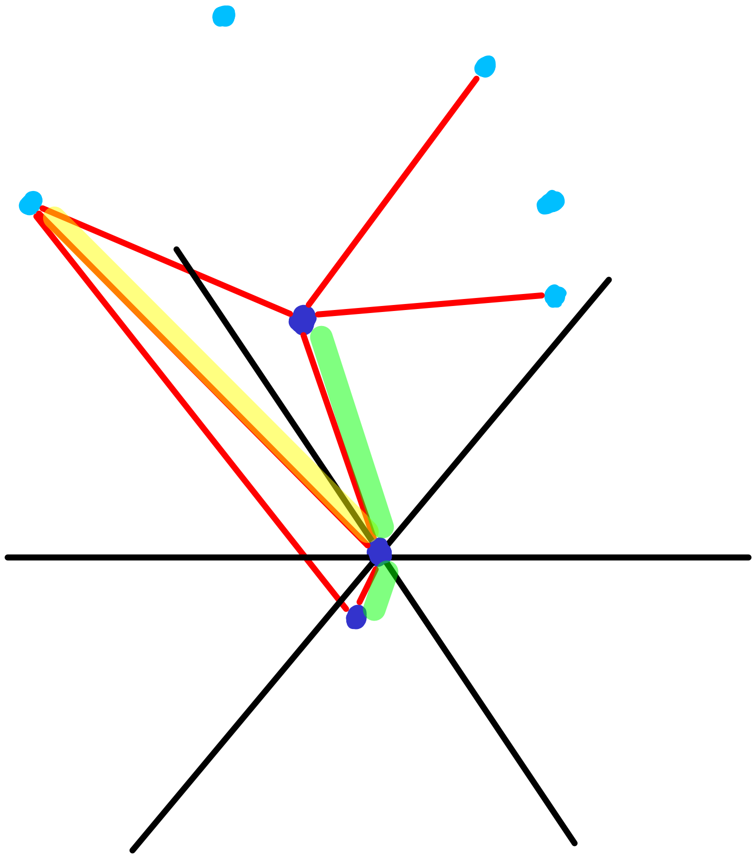

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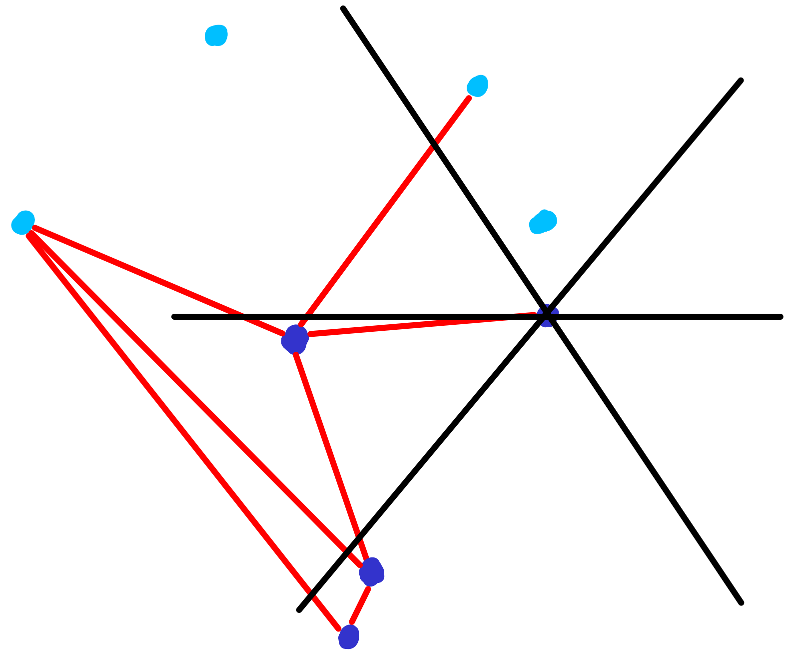

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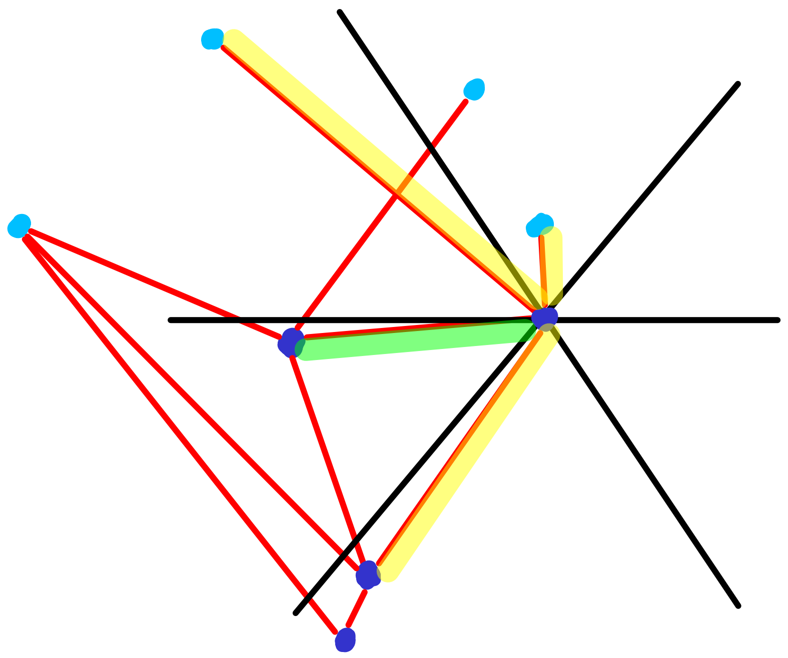

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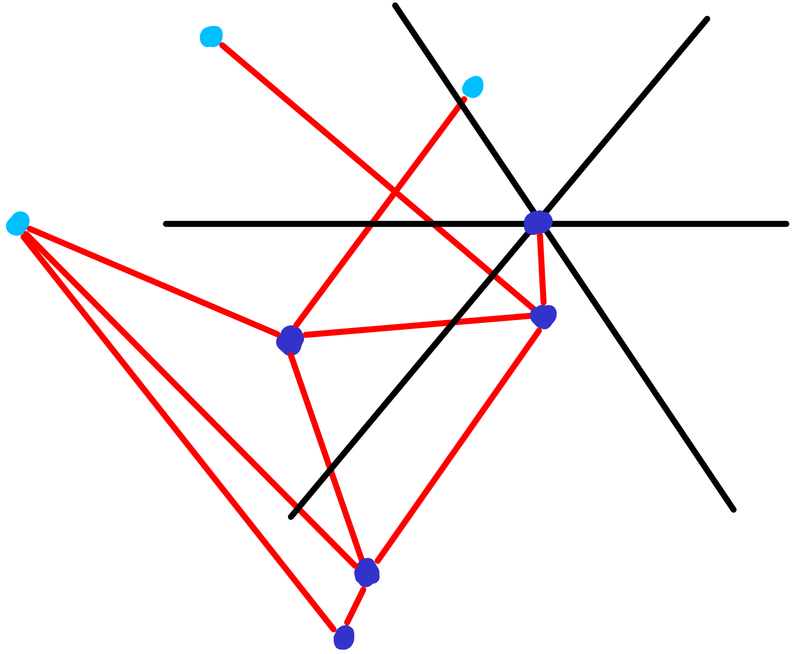

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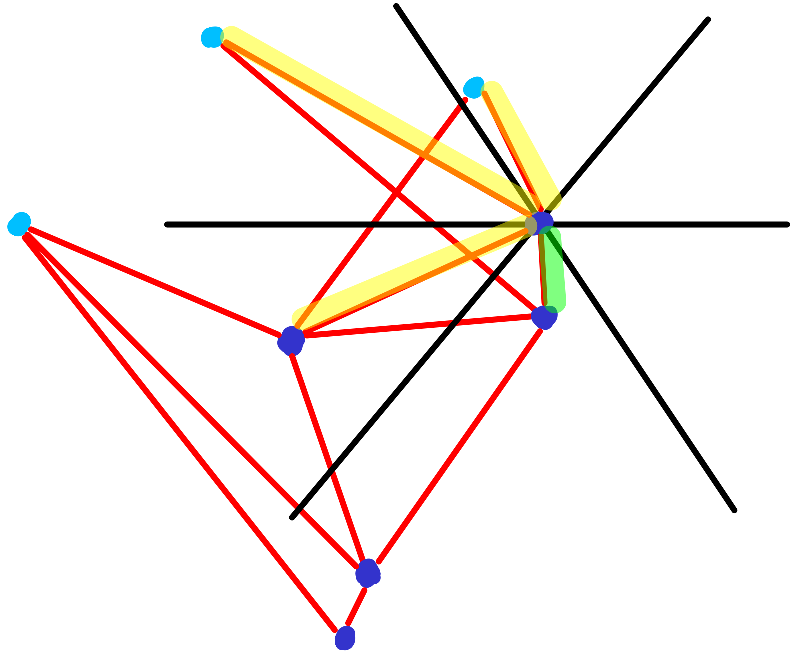

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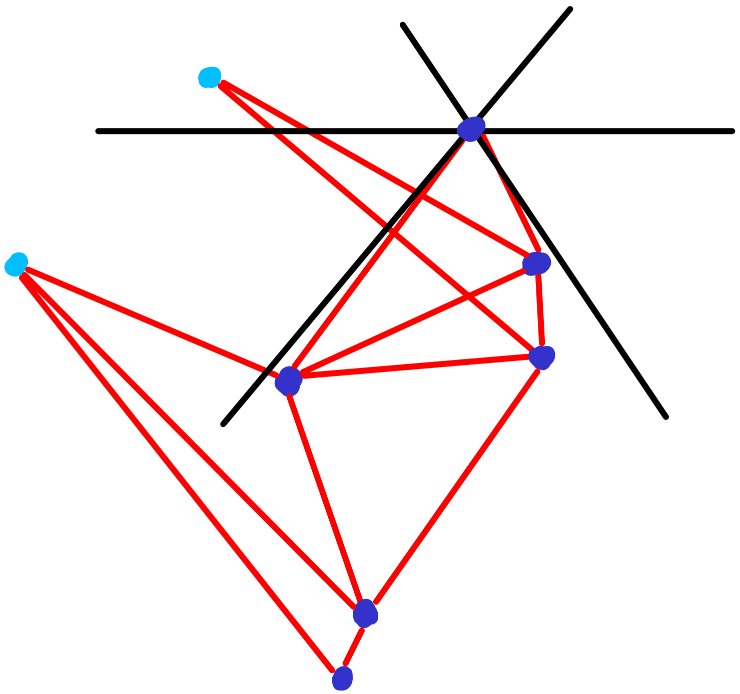

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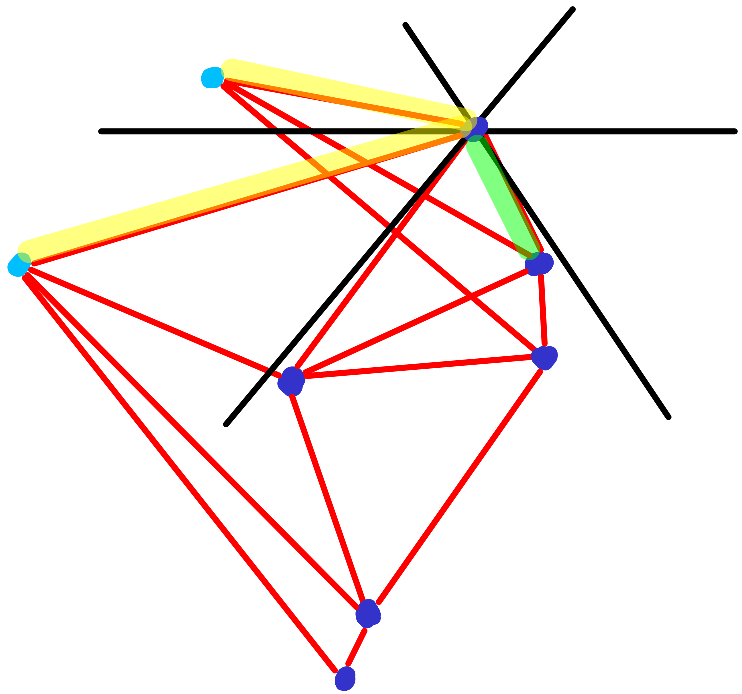

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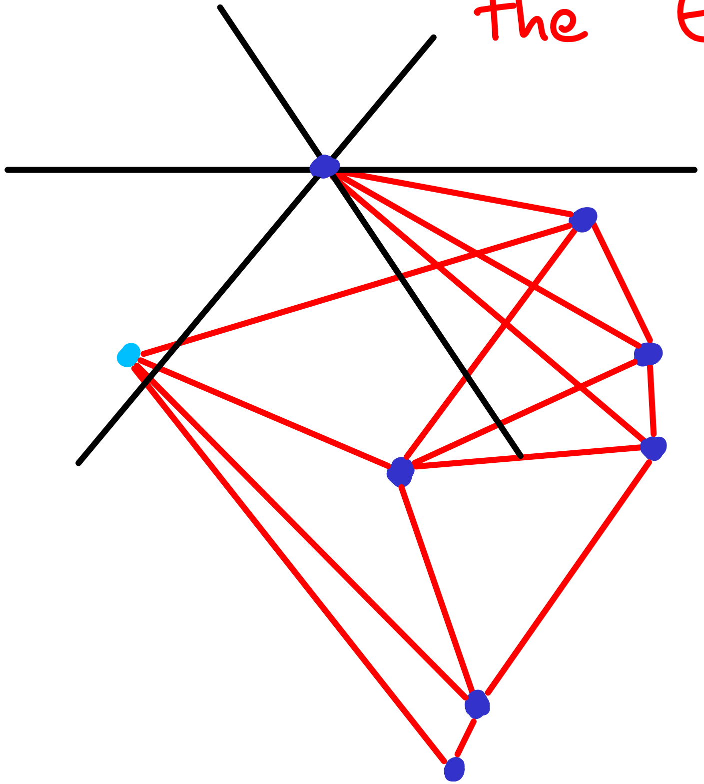

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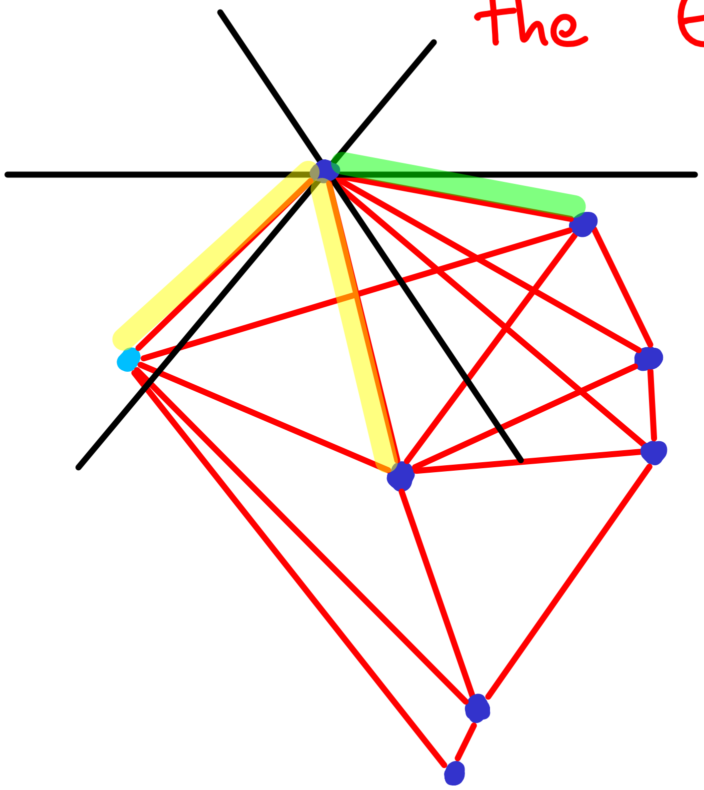

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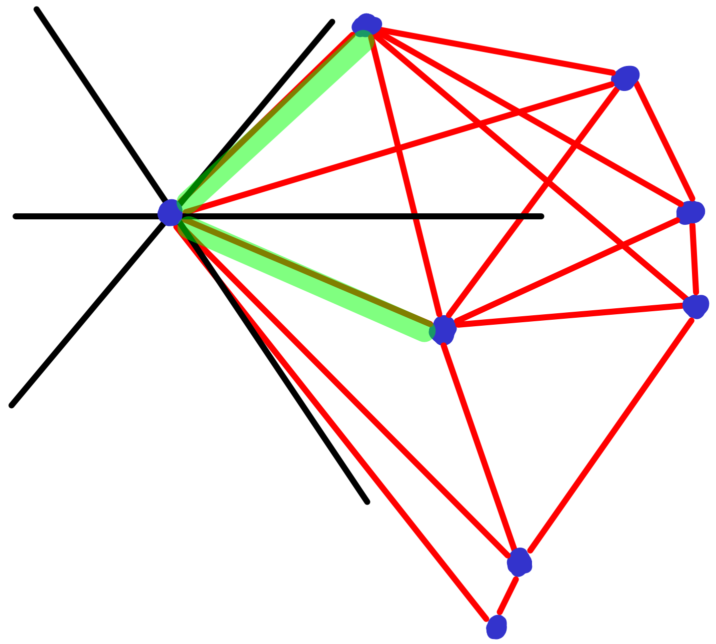

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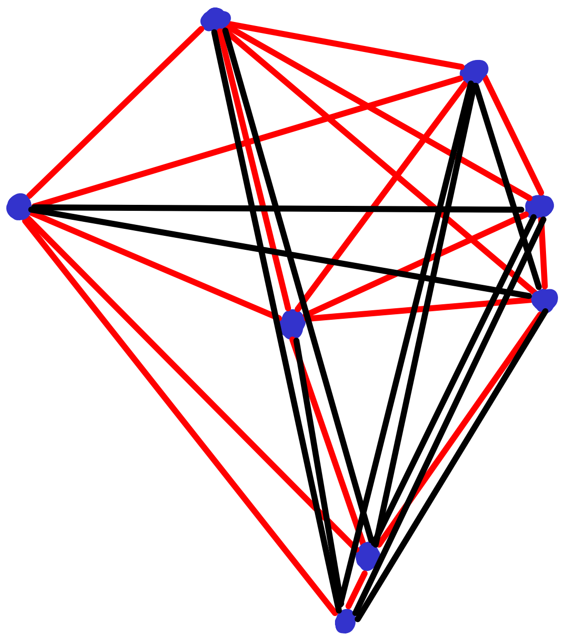

Form K cones w/ angle $\theta = \frac{2\pi}{K}$

For every vertex v

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(if non-empty)

for $k = O(1)$ we keep

$\leq k \cdot V = O(V)$ edges

Connection rule:

- project vertices onto cone bisector
- choose vertex with closest projection

Θ-graph: spanner? Depends on θ.

Θ-graph: good spanner? Depends on θ.

connected for $\underbrace{\theta = 180^\circ}_{\text{trivial}}$, $\underbrace{120^\circ?,\ 90^\circ?}_{\text{try it out}}$

Θ-graph : good spanner?

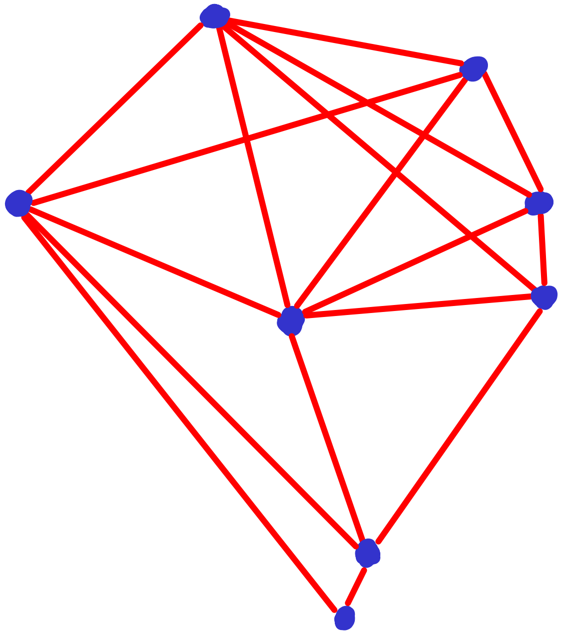
Depends on θ.

connected for $\theta = 180°$, $120°$?, $90°$?

trivial

Here we focus on $k \geqslant 9$ i.e $\theta \leqslant \frac{2\pi}{9}$

even though the illustration has $k=6$



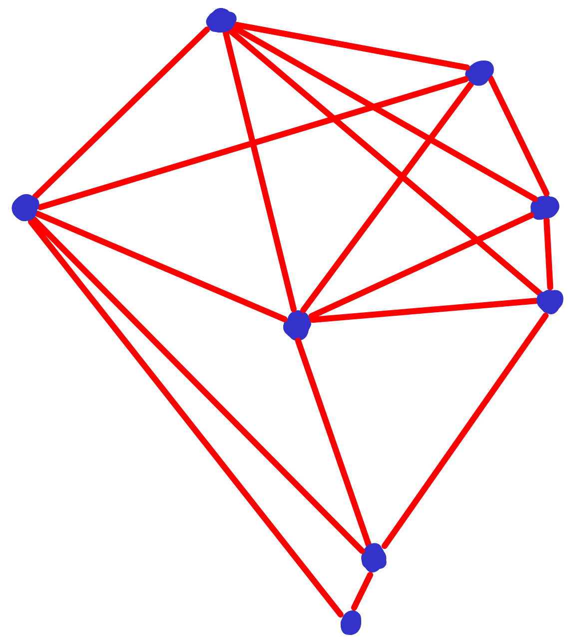
Θ-graph: good spanner?

Depends on θ.

Here we focus on $k \geq 9$ i.e $\theta \leq \frac{2\pi}{9}$

even though the illustration has $k=6$

(create larger example)



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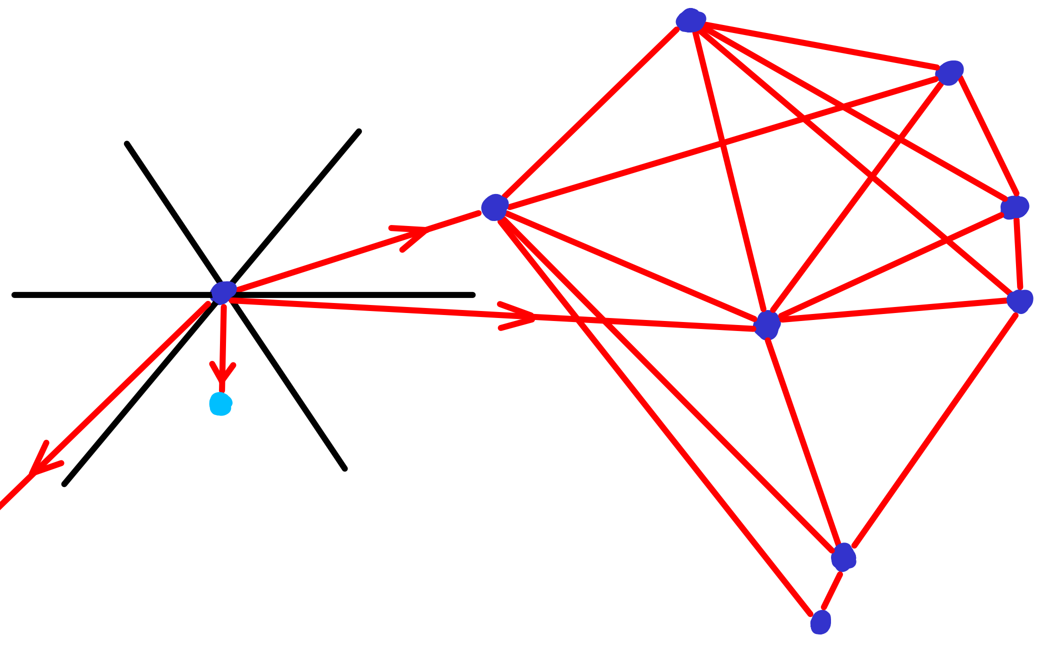

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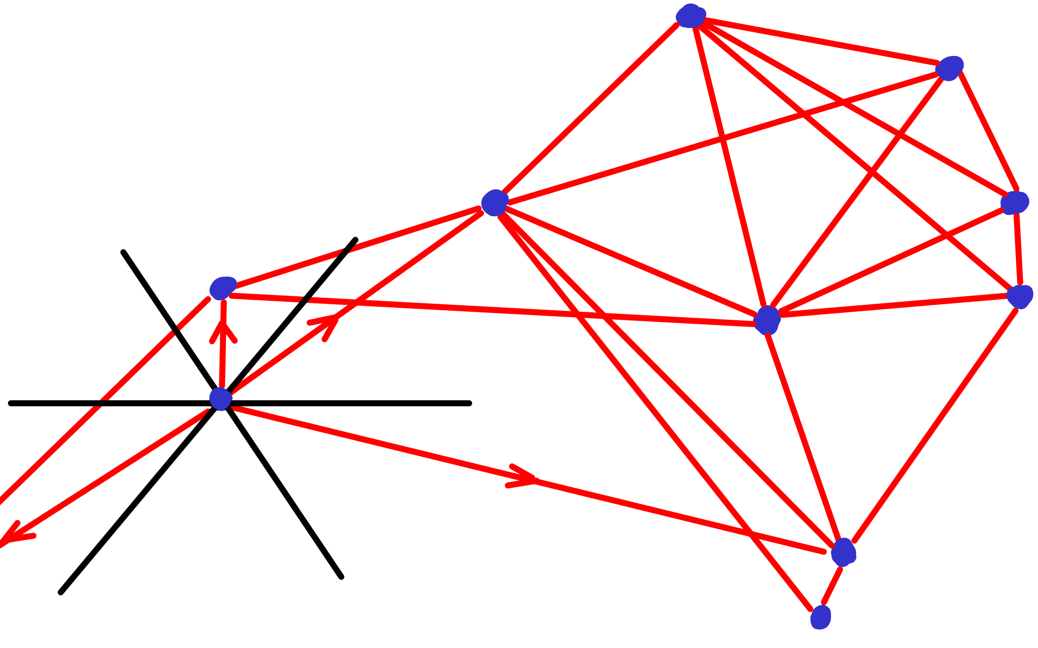

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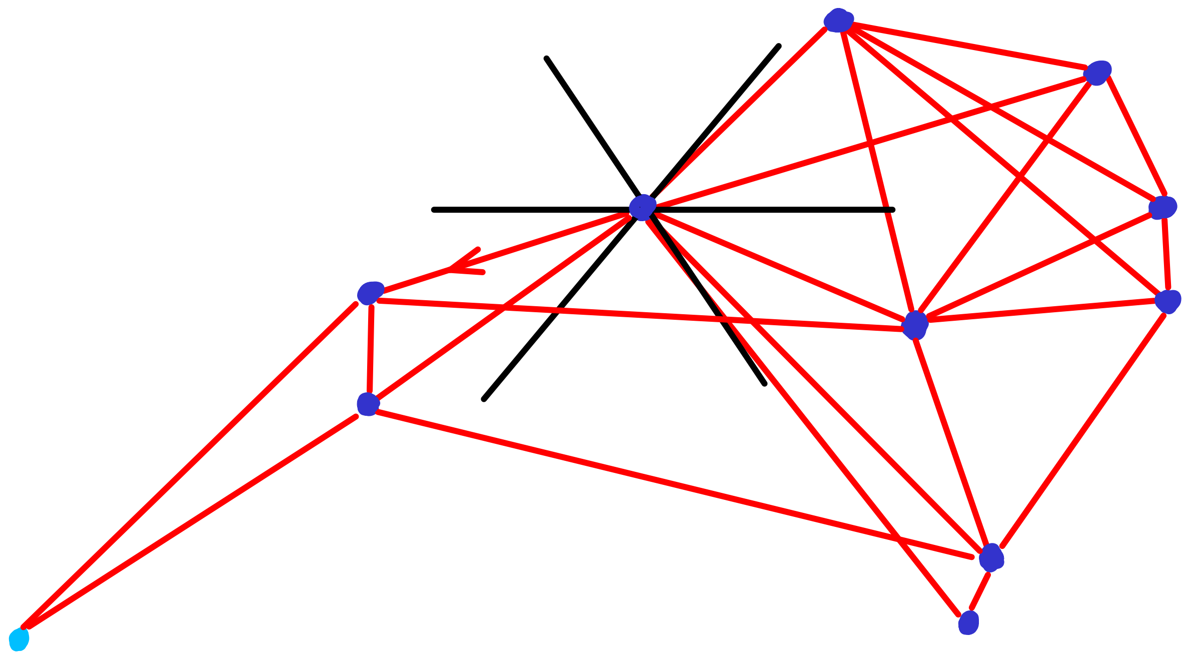

Θ-graph : good spanner? Depends on θ.

Here we focus on $k \geq 9$ i.e $\theta \leq \frac{2\pi}{9}$

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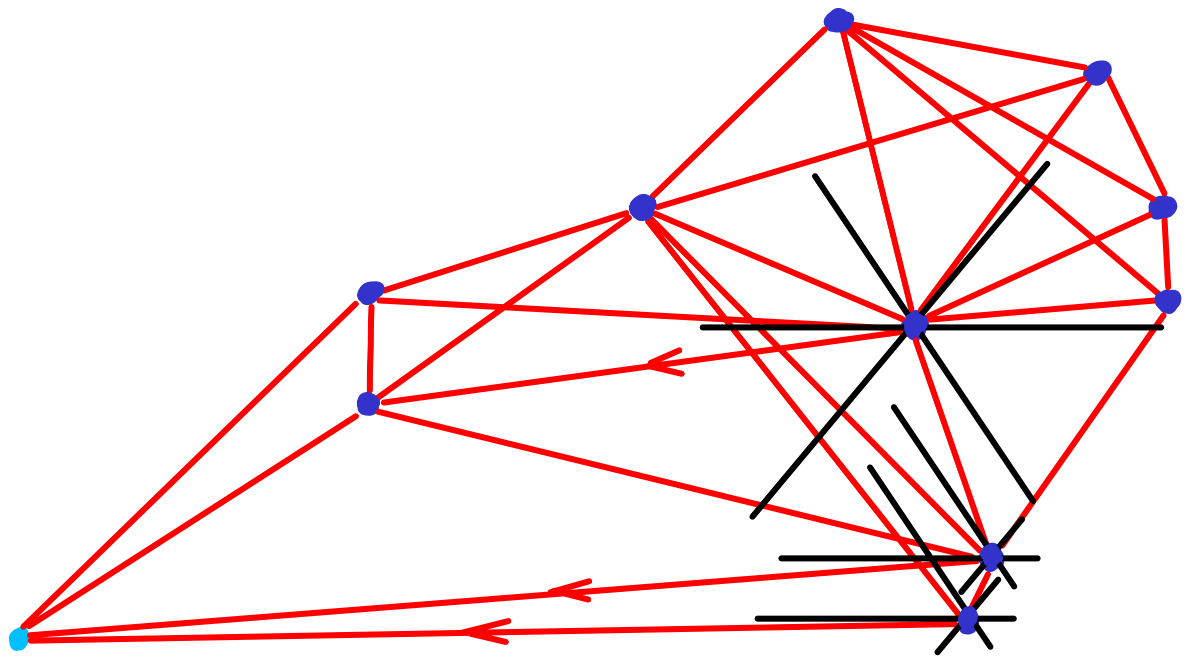

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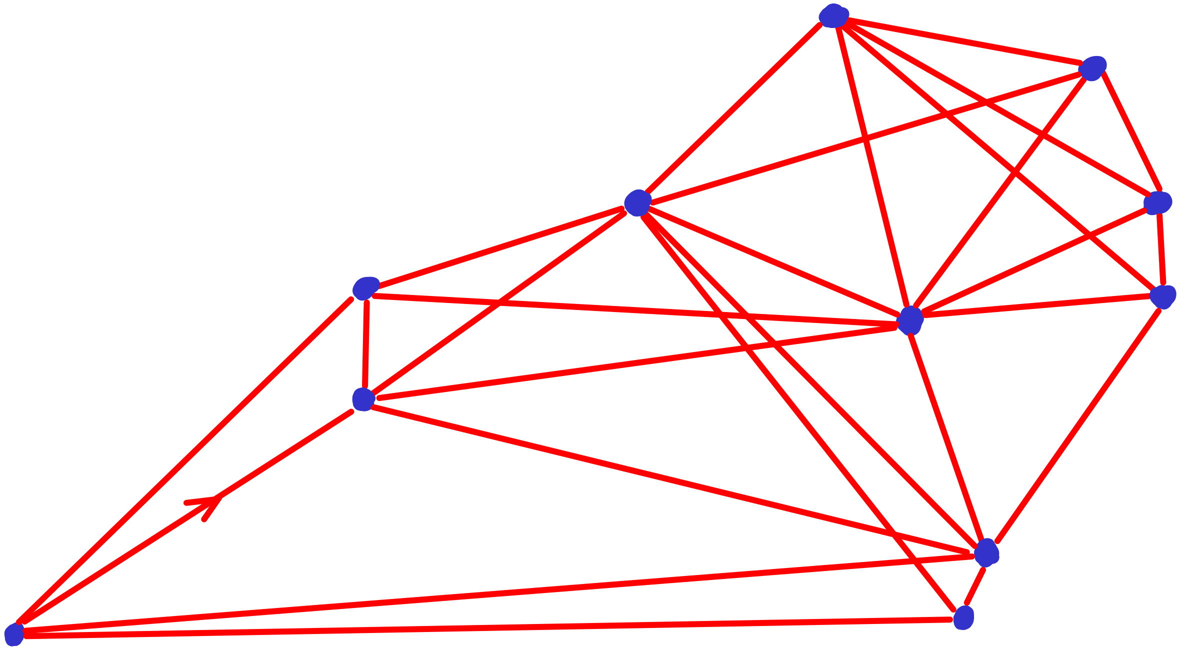

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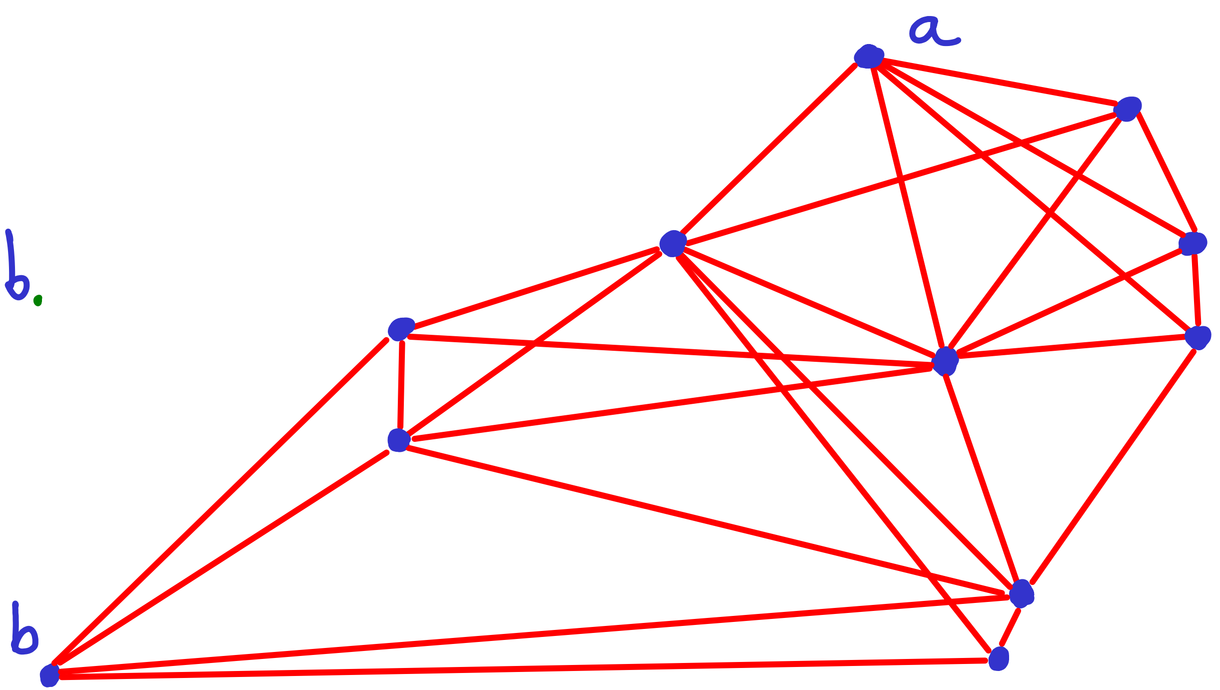
Here we focus on $k \geqslant 9$ i.e $\theta \leqslant \frac{2\pi}{9}$

even though the illustration has $k=6$

Now to define a sufficient path from any a to any b.

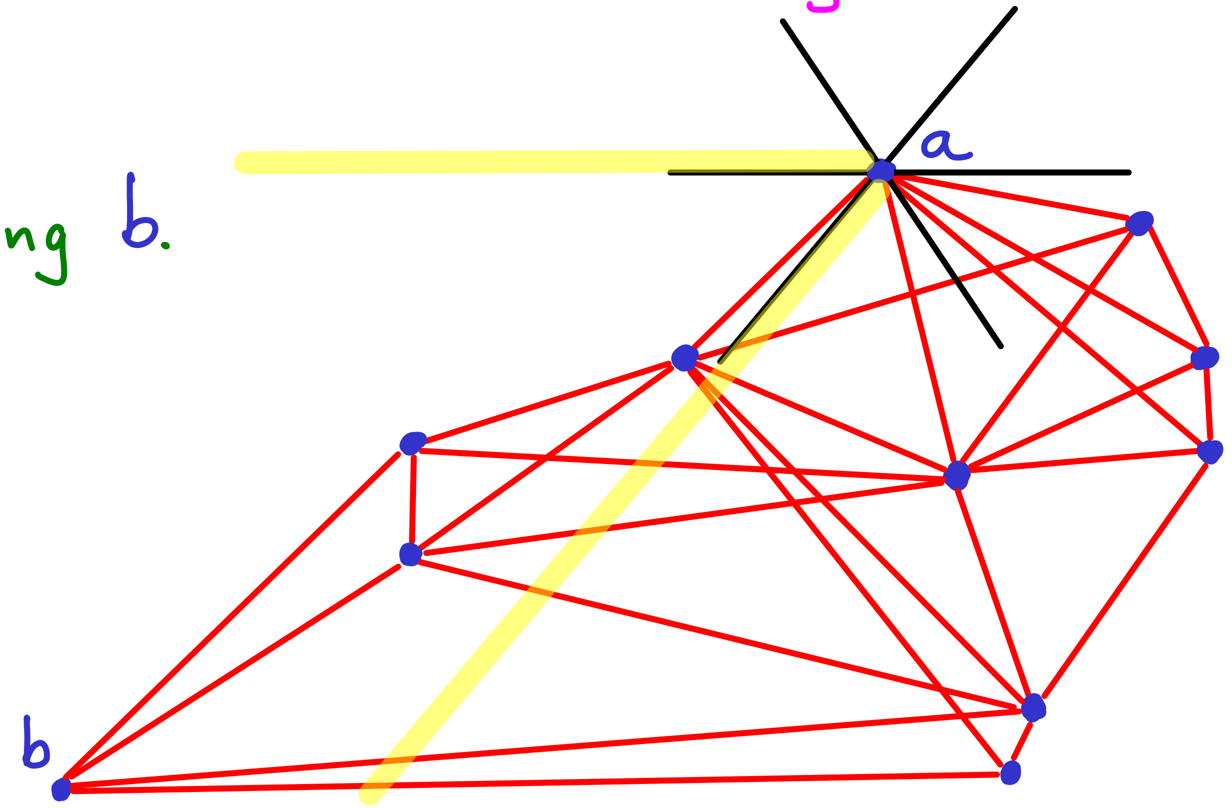


Θ-graph : good spanner? Depends on θ.

Here we focus on $k \geqslant 9$ i.e $\theta \leqslant \frac{2\pi}{9}$

even though the illustration has $k=6$

Starting at a
identify cone containing b.

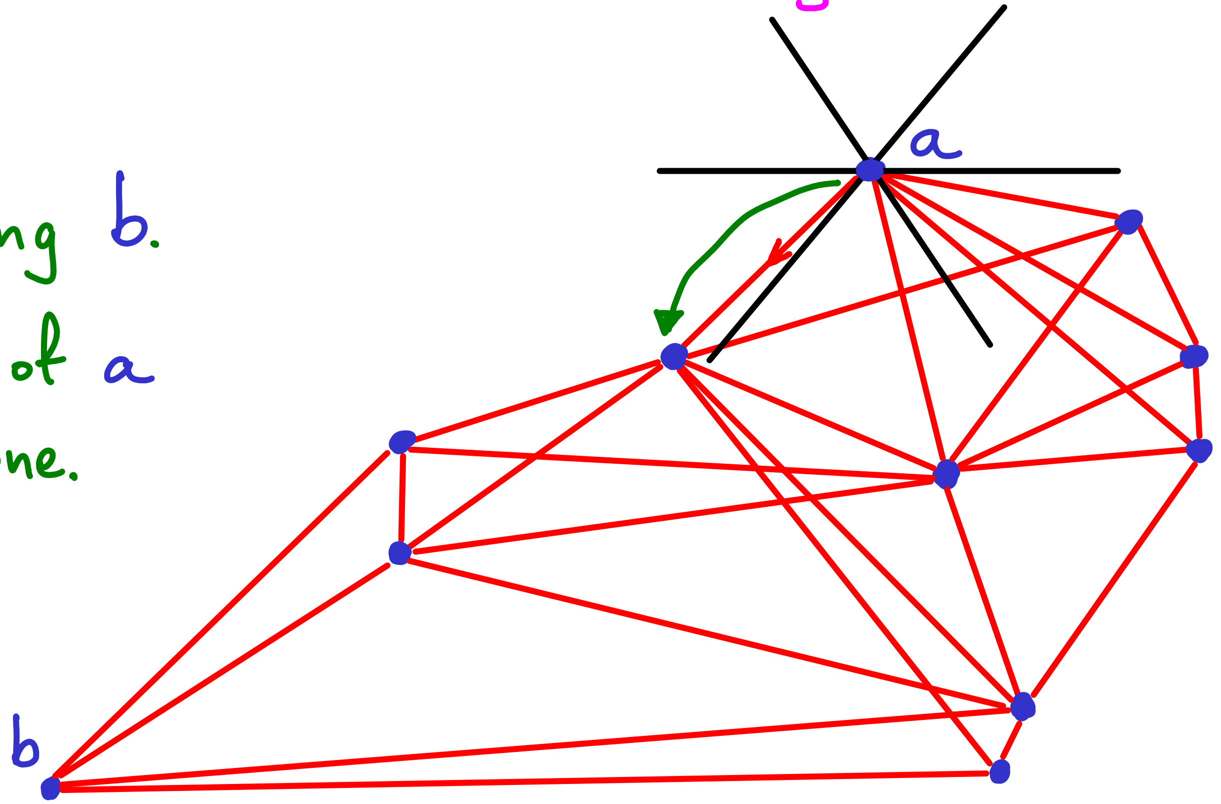


Θ-graph: good spanner? Depends on θ.

Here we focus on $k \geq 9$ i.e $\theta \leq \frac{2\pi}{9}$

even though the illustration has $k=6$

Starting at a
identify cone containing b.
Move to a neighbor of a
in that cone.



Θ-graph : good spanner?   Depends on θ.

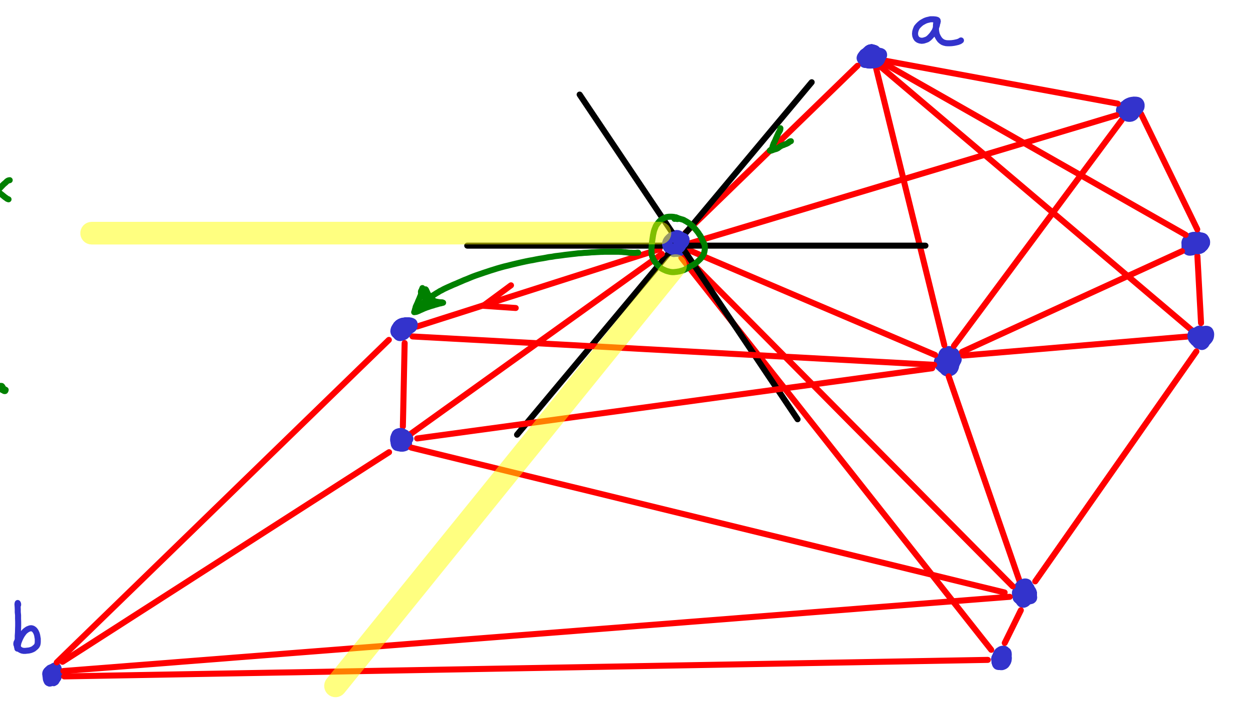
Here we focus on $k \geqslant 9$ i.e $\theta \leqslant \frac{2\pi}{9}$

even though the illustration has $k=6$

Repeat:

From current vertex move to a neighbor in cone containing b.

Any neighbor but why not the outgoing one



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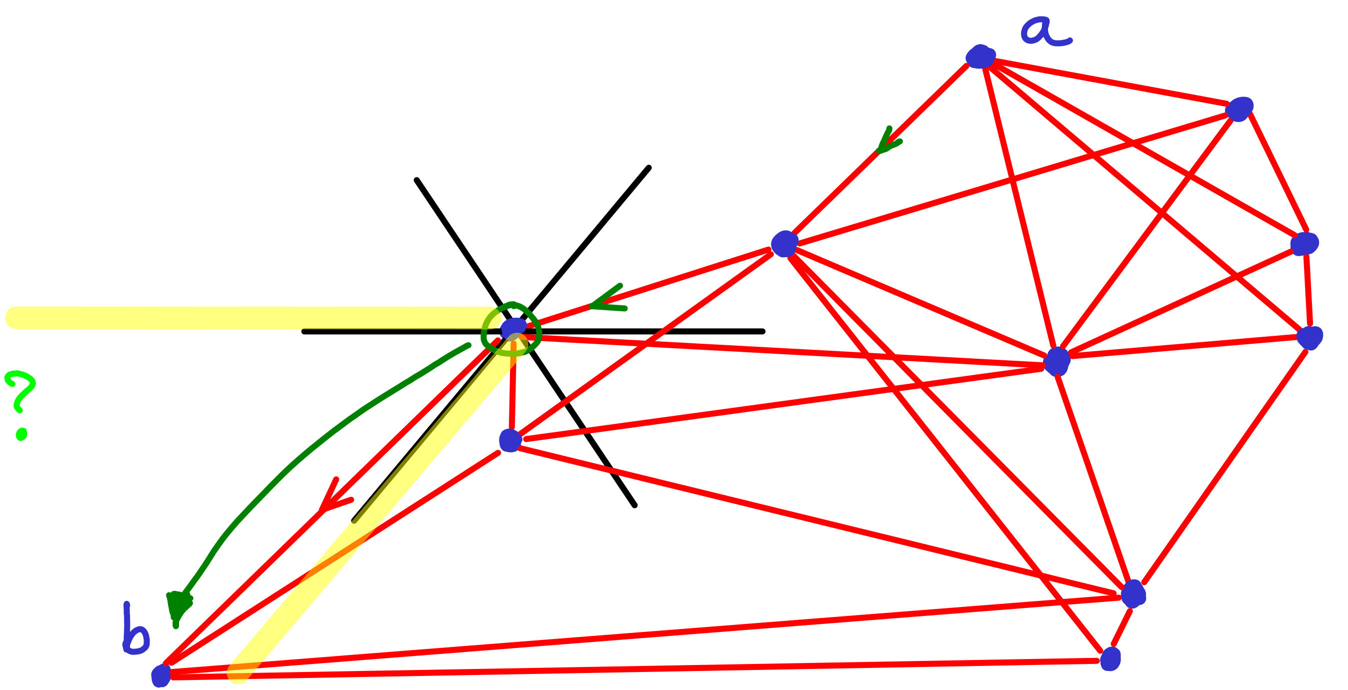
even though the illustration has $k=6$

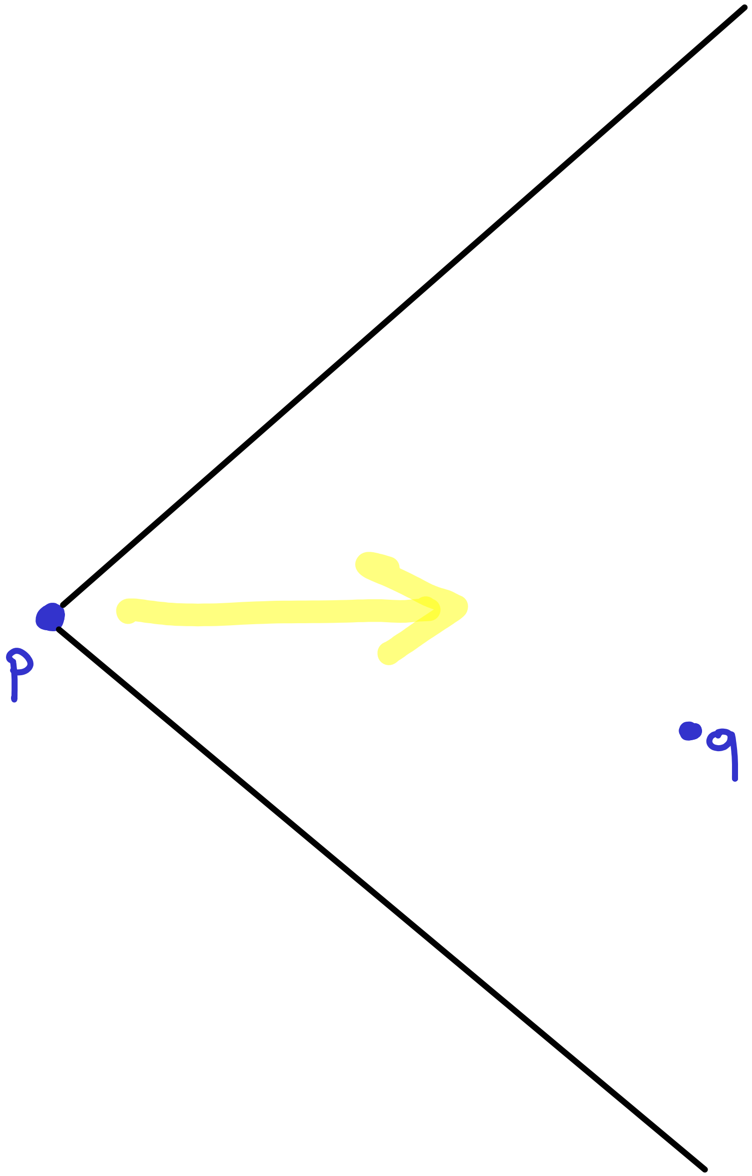
Eventually (ideally) reach b.

$\downarrow$

- can we cycle?
- can we "overshoot" b?

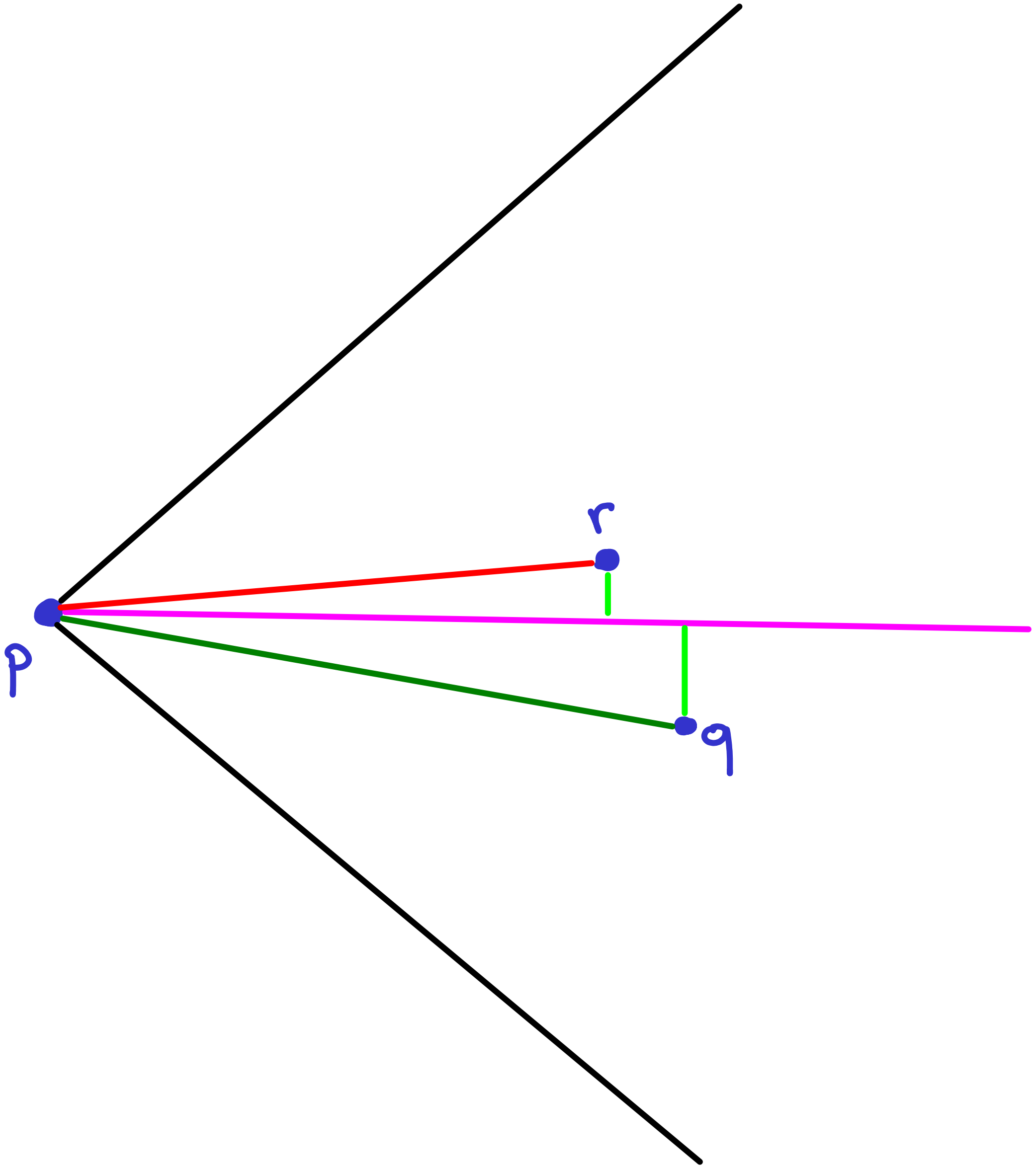


Suppose we are aiming for q



Suppose we are aiming for q
but our algo sends us to r.

Can we overshoot ?

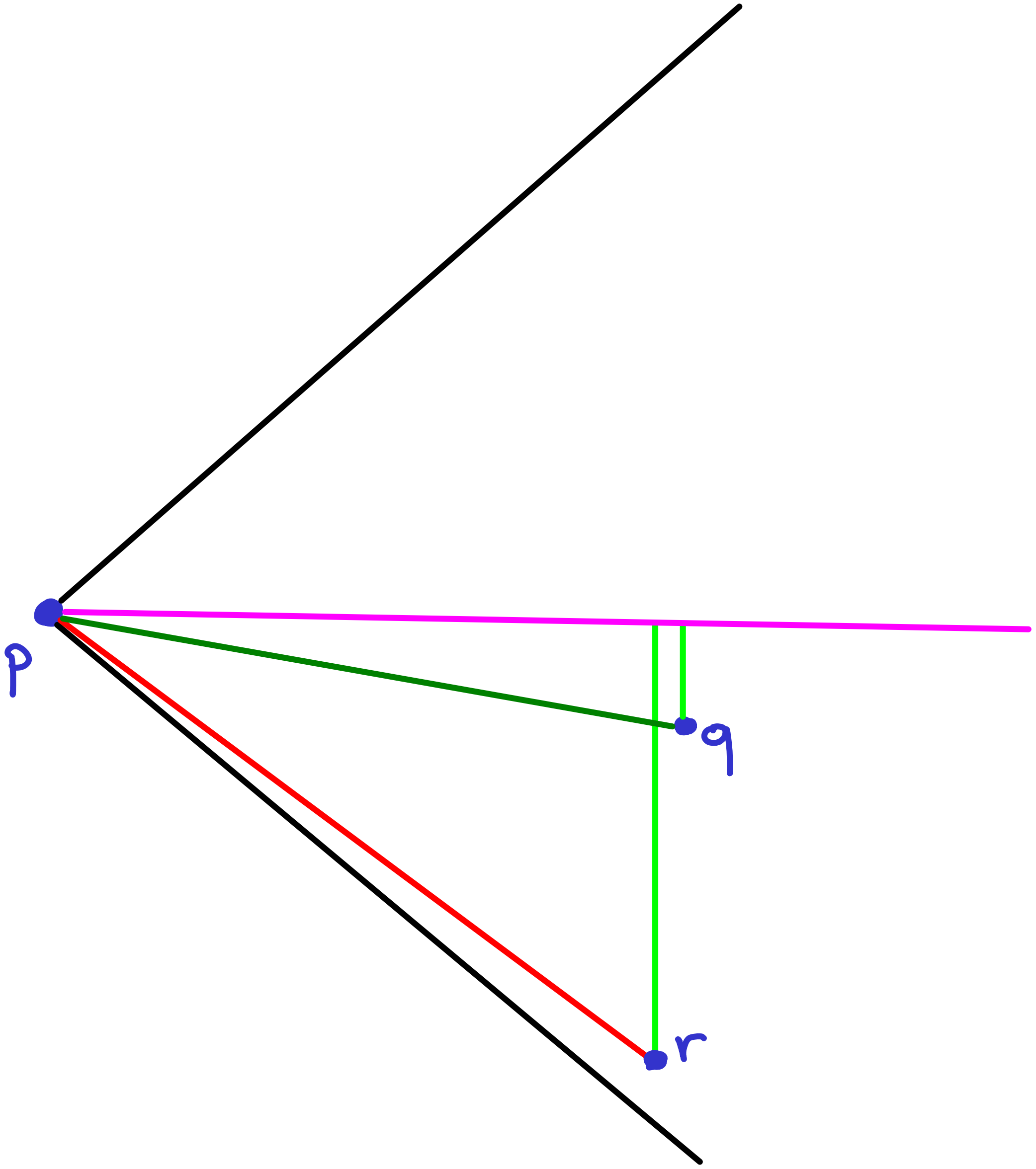


Suppose we are aiming for q but our algo sends us to r.

Can we overshoot ?

YES

$\overline{pr} > \overline{pq}$



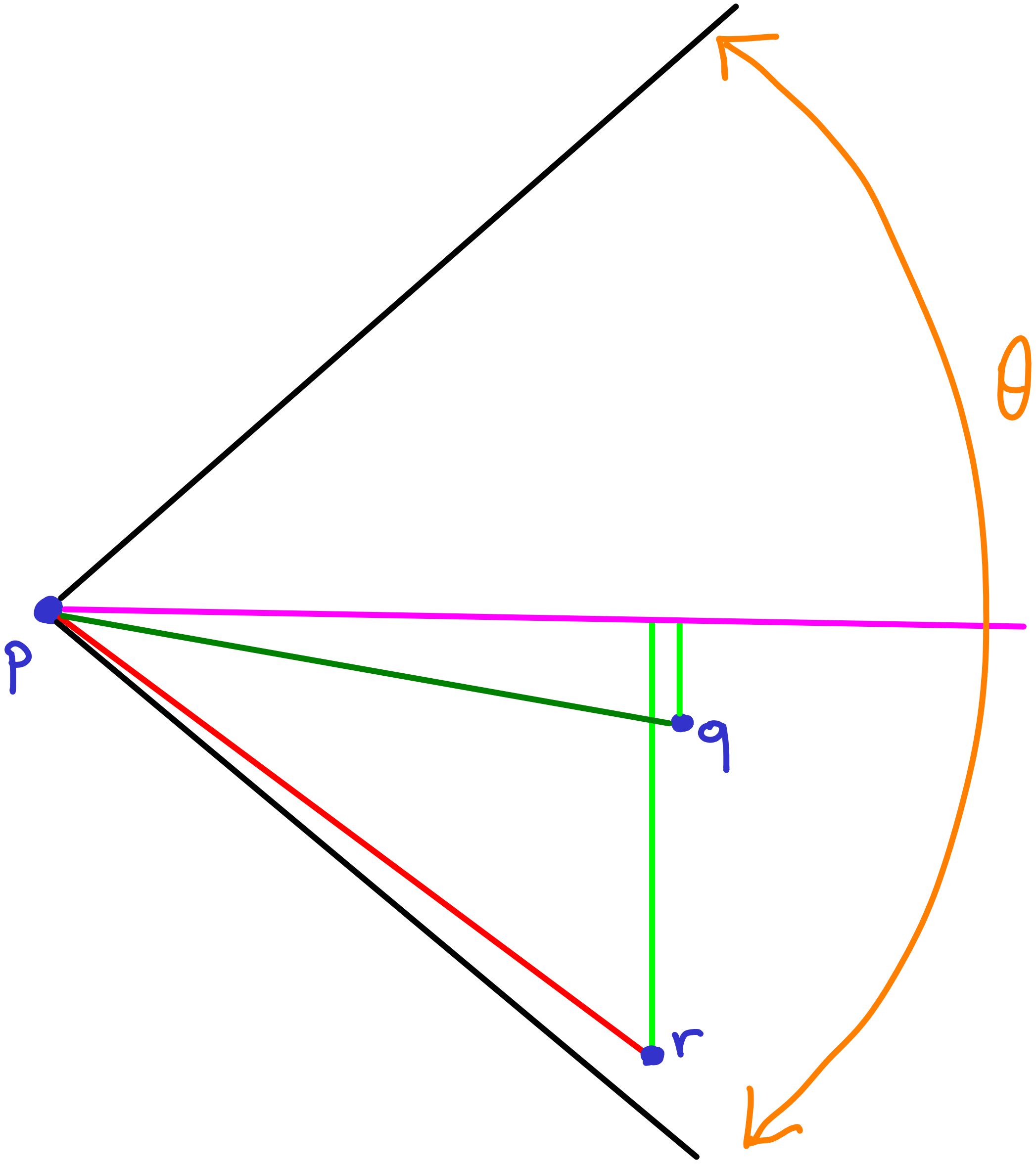
Suppose we are aiming for q
but our algo sends us to r.

Can we overshoot ?

YES

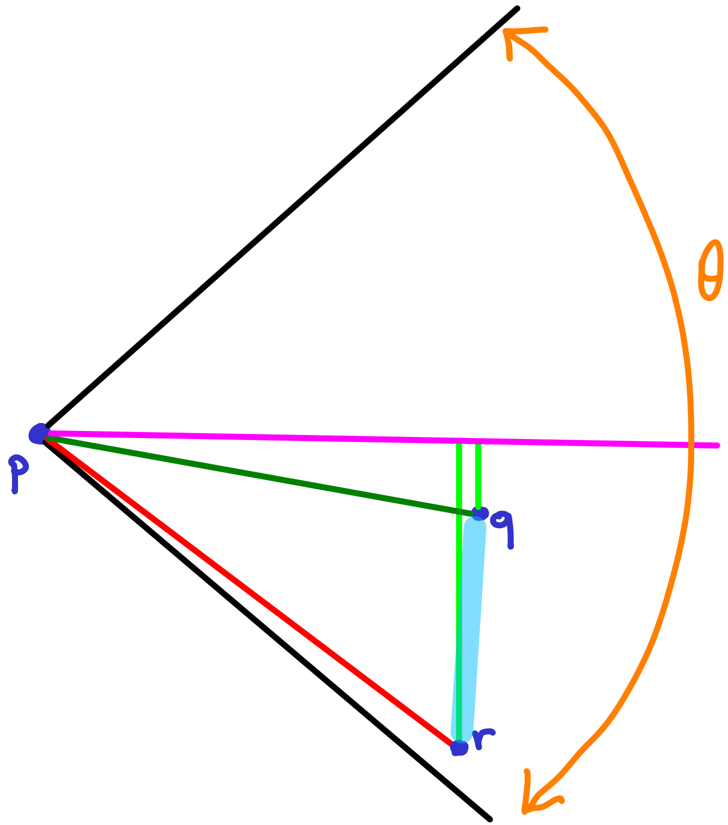
$\overline{pr} > \overline{pq}$

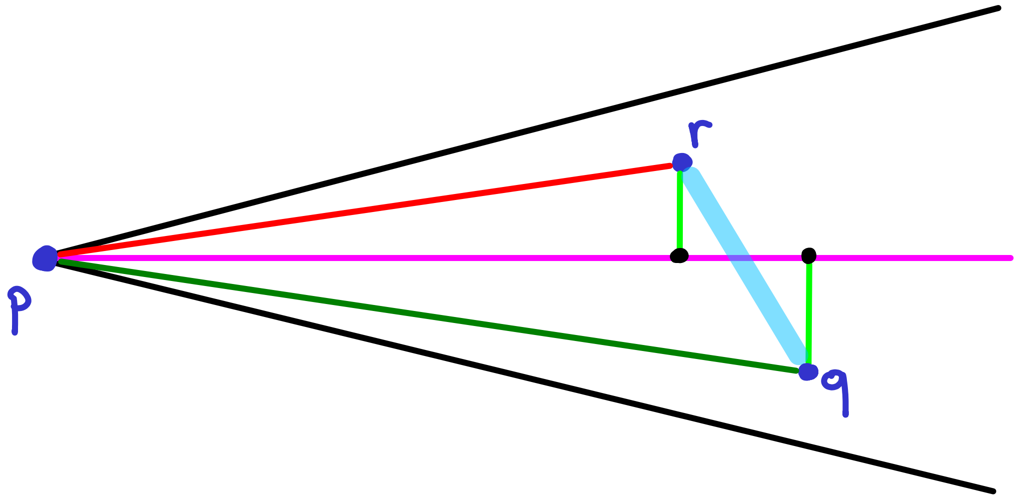
The smaller θ is,
the less we can overshoot



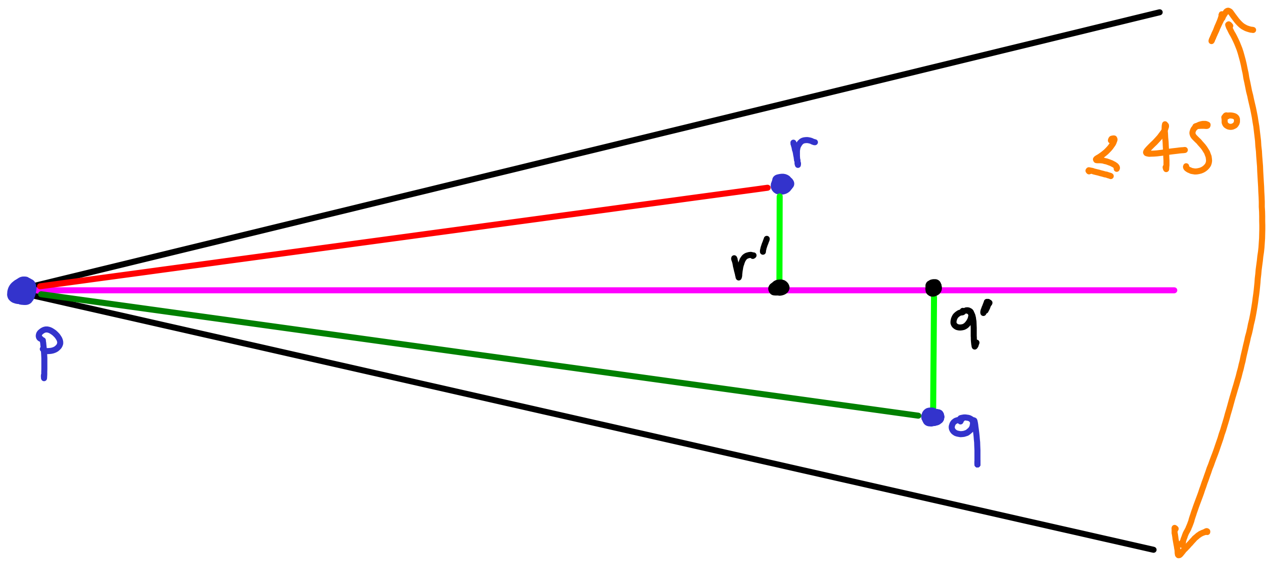
We want to bound leftover distance

If leftover is smaller than original then the algorithm won't cycle





Given a cone w/ $\theta \leq \frac{2\pi}{8}$ at p: suppose r projects no further than q (i.e., $pq' \geq pr'$)



Given a cone w/ $\theta \leq \frac{2\pi}{8}$ at p: suppose r projects no further than q

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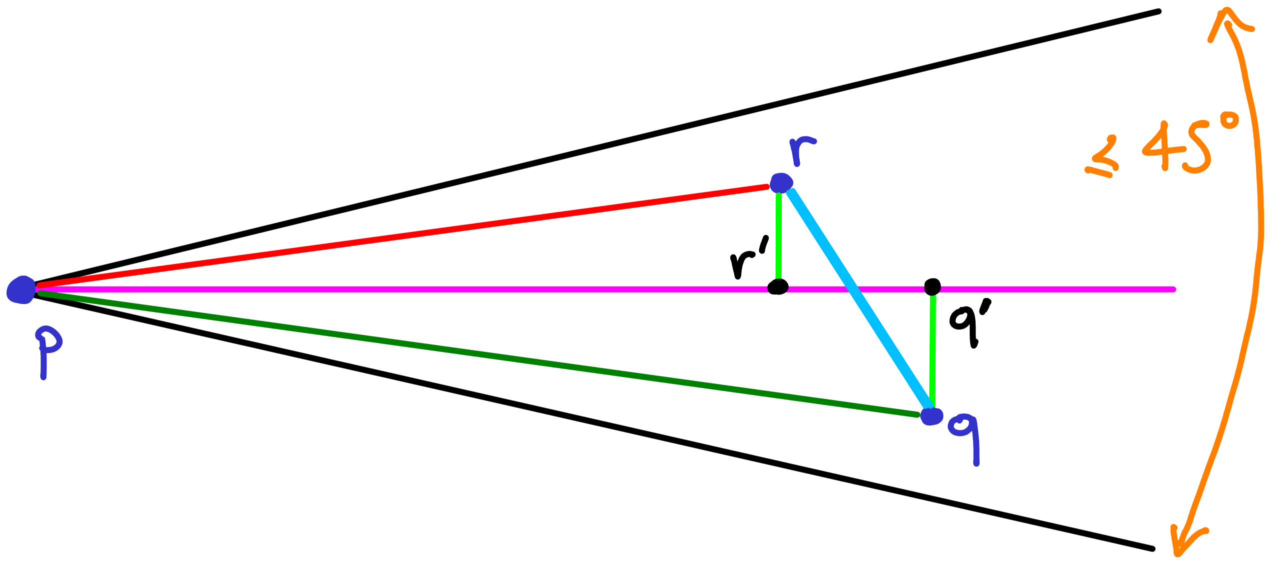
Claim: $rq \leq pq + (\sin\theta - \cos\theta) \cdot pr$



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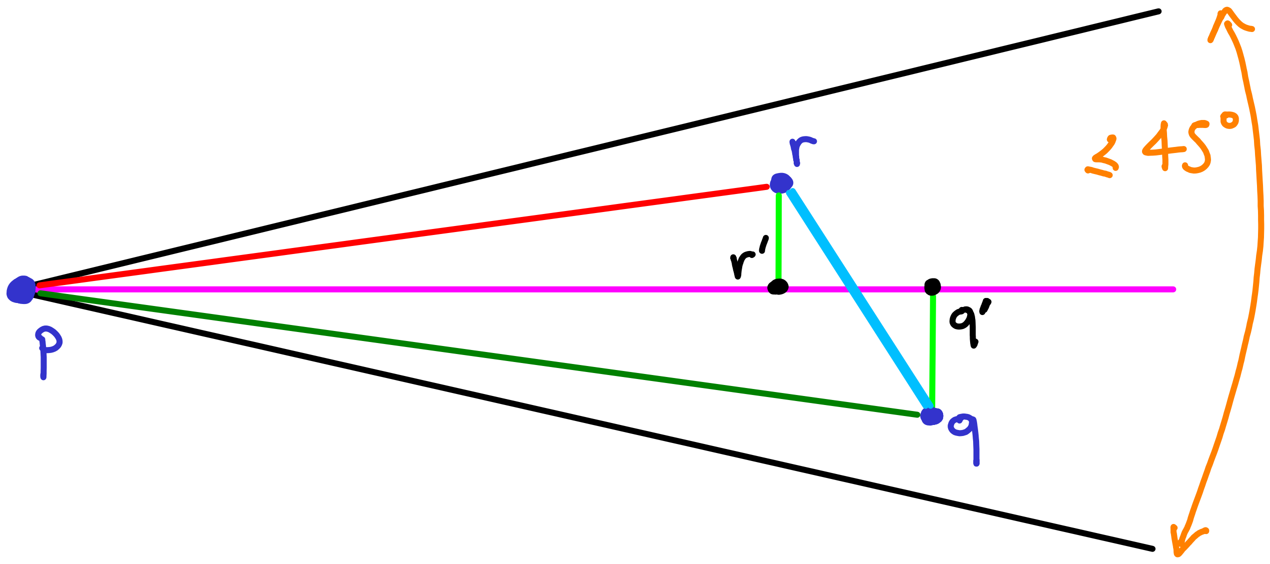
Claim: $rq \leq pq + (\sin\theta - \cos\theta) \cdot pr$

$\boxed{\text{Proof: trig. } \theta \text{ matters}}$



Given a cone w/ $\theta \leq \frac{2\pi}{8}$ at p: suppose r projects no further than q
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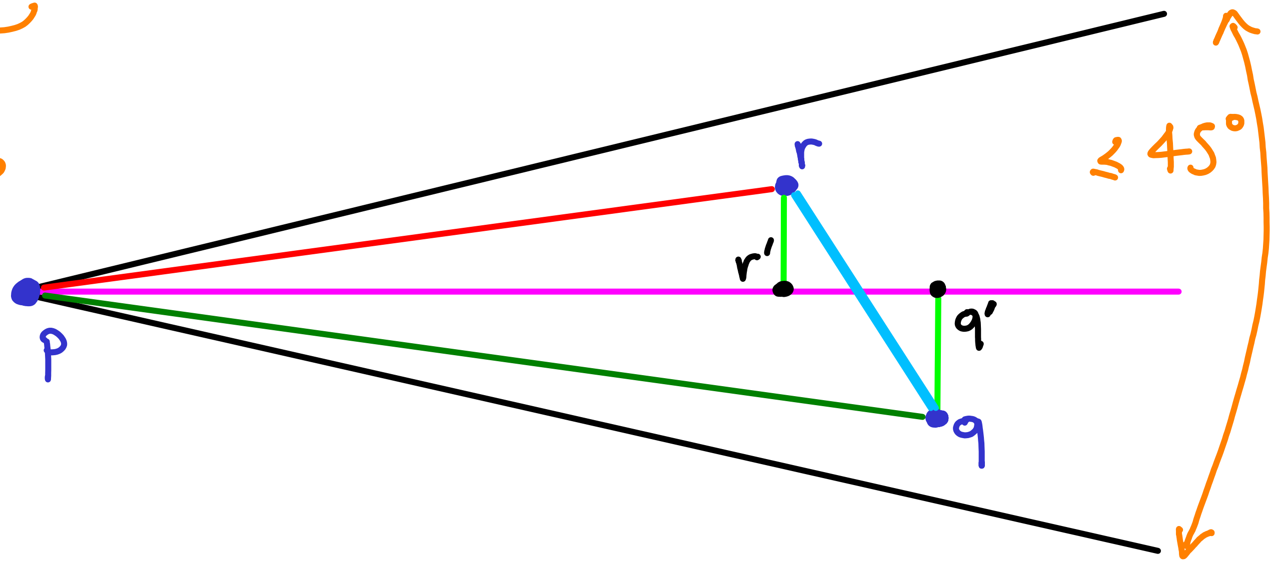
Claim: $rq \leq pq + \underbrace{(\sin\theta - \cos\theta)}_{<0 \text{ for } \theta < 45^\circ} \cdot pr$

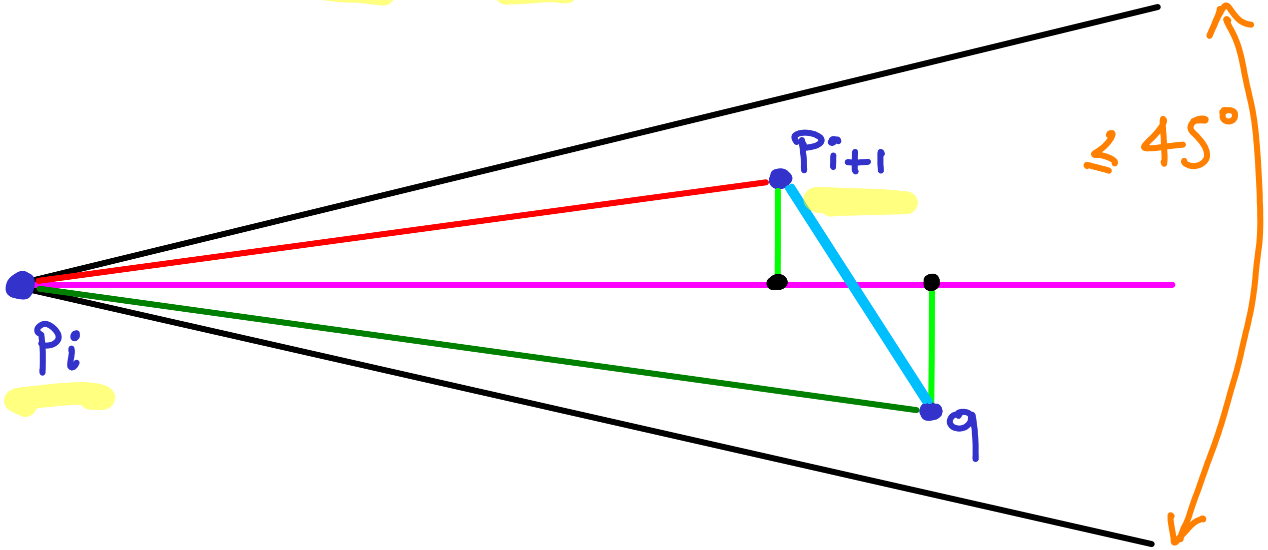
$\boxed{\text{Proof: trig. } \theta \text{ matters}}$



New dist < old dist ⇒ no cycles, search is finite, graph is connected

Given a cone w/ $\theta \leq \frac{2\pi}{8}$ at p_i: suppose p_{i+1} projects no further than q

Proved : $p_{i+1}q \leq p_iq + (\sin\theta - \cos\theta) \cdot p_ip_{i+1} < p_iq$



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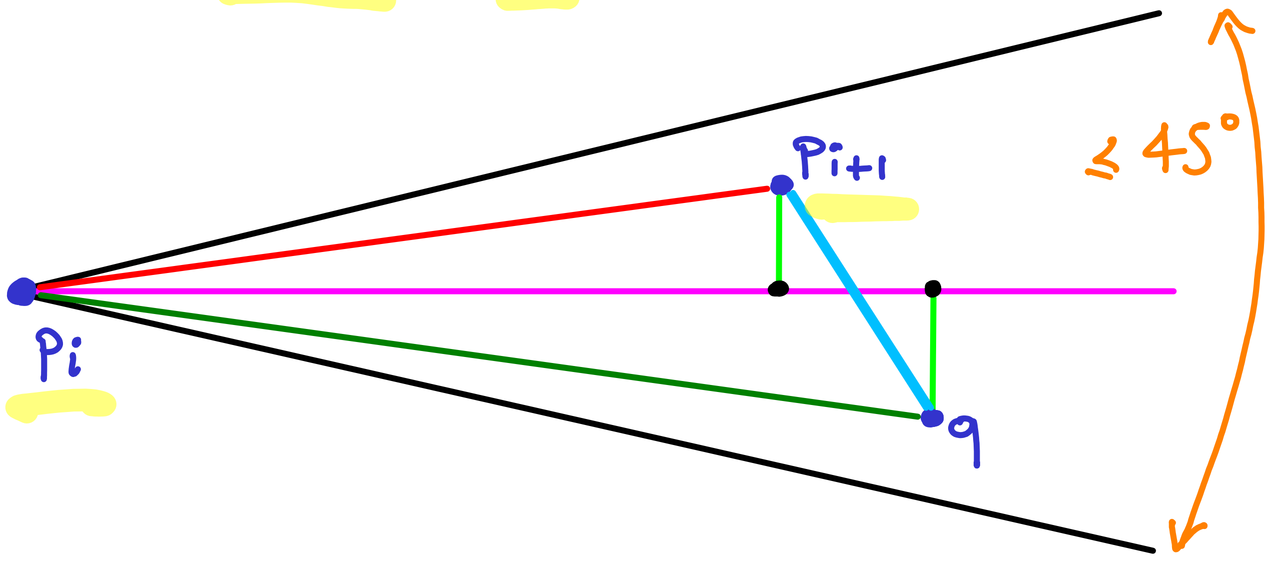
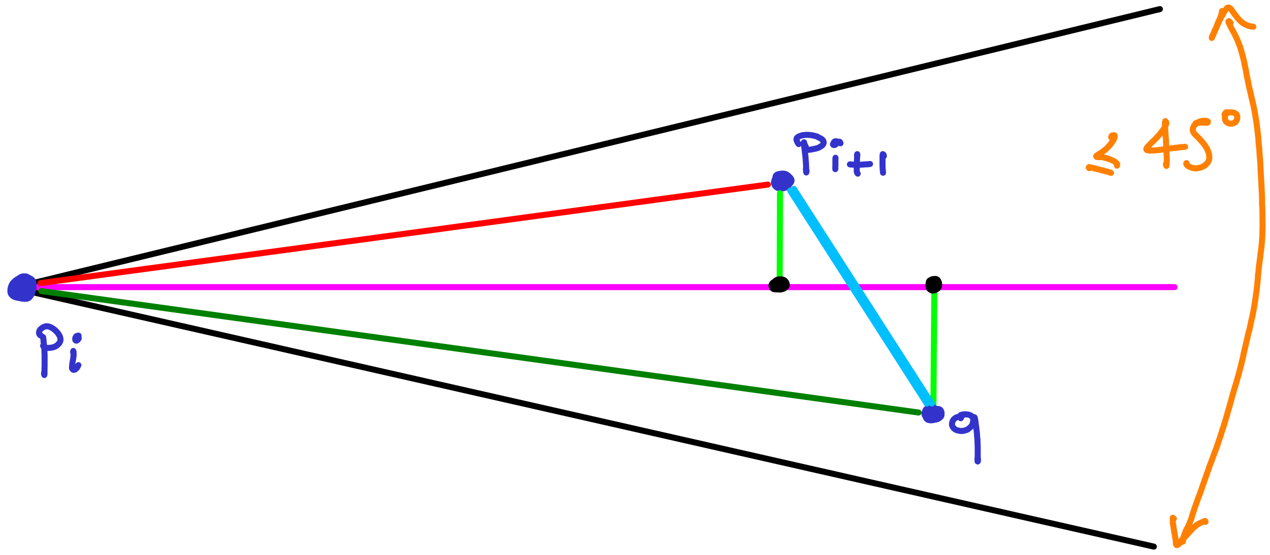


Derive upper bound on total path length?

Given a cone w/ $\theta \le \frac{2\pi}{8}$ at p_i: suppose p_{i+1} projects no further than q

Proved : $\quad p_{i+1}q \le p_i q + (\sin\theta - \cos\theta)\cdot p_i p_{i+1} < p_i q$



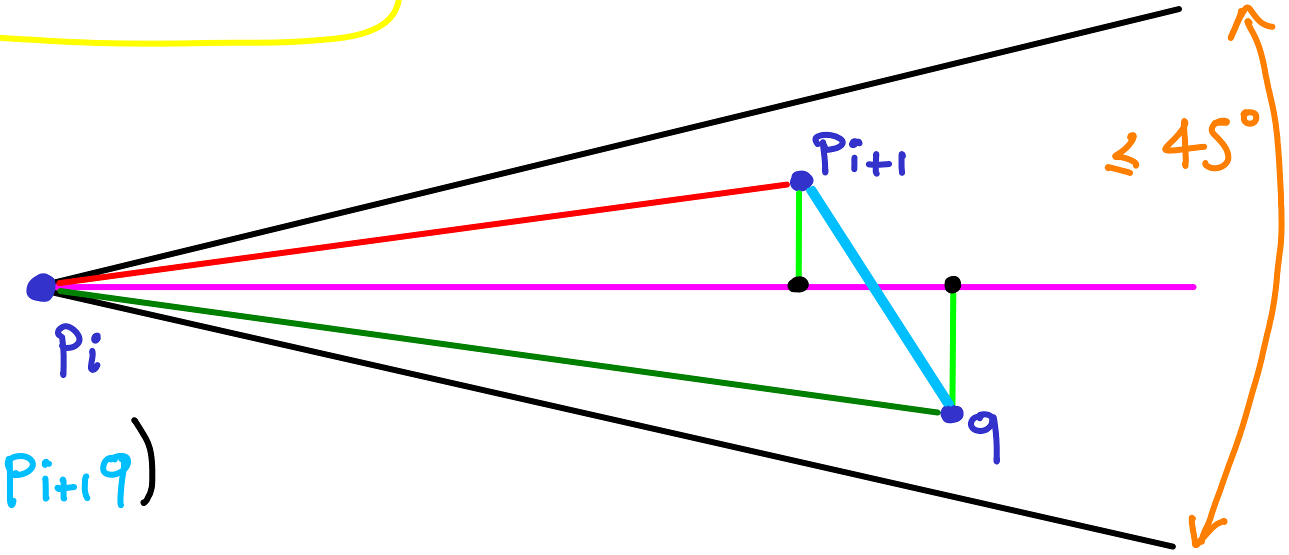
Total path = $\sum_{i=0}^{m-1} p_i p_{i+1}$

$a \longrightarrow q$

$a = p_0$, $\quad q = p_m$

Given a cone w/ $\theta \leq \frac{2\pi}{8}$ at p_i: suppose p_{i+1} projects no further than q

Proved : $\underbrace{p_{i+1}q \leq p_i q + (\sin\theta - \cos\theta)\cdot p_i p_{i+1}}$ $< p_i q$



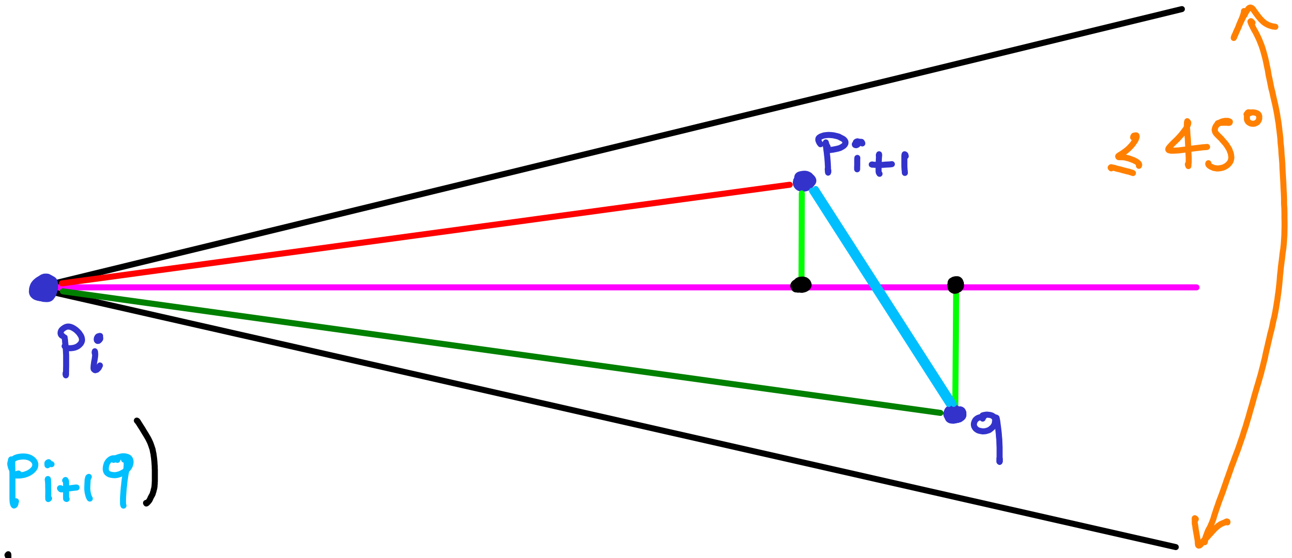
$$p_i p_{i+1} \leq \frac{1}{\cos\theta - \sin\theta}\left(p_i q - p_{i+1} q\right)$$

Total path $= \sum_{i=0}^{m-1} p_i p_{i+1}$

$\underset{p_0}{a} \longrightarrow \underset{p_m}{q}$

Given a cone w/ $\theta \leq \frac{2\pi}{8}$ at p_i: suppose p_{i+1} projects no further than q

Proved : $\quad p_{i+1}q \leq p_iq + (\sin\theta - \cos\theta) \cdot p_ip_{i+1} < p_iq$



$$p_ip_{i+1} \leq \frac{1}{\cos\theta - \sin\theta}(p_iq - p_{i+1}q)$$

$$\text{Total path} = \sum_{i=0}^{m-1} p_ip_{i+1} \leq \frac{1}{\cos\theta - \sin\theta}\sum_{i=0}^{m-1}(p_iq - p_{i+1}q)$$

$\underbrace{a}_{p_0} \rightarrow \underbrace{q}_{p_m}$

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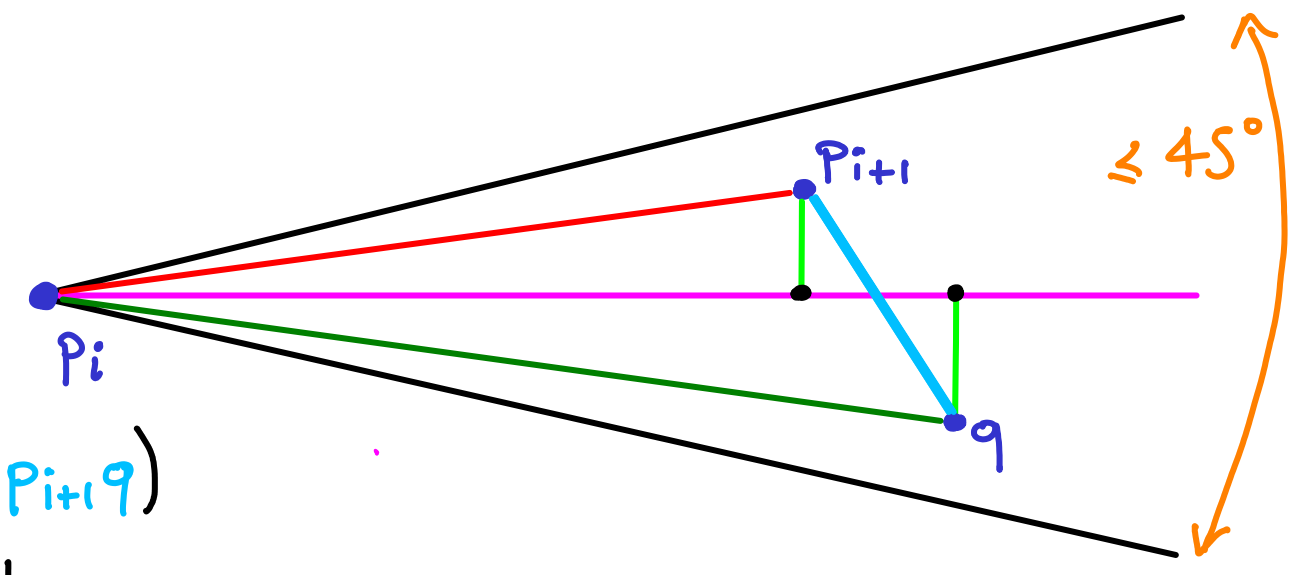

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$a \rightarrow q$

p_0 p_m

Given a cone w/ $\theta \leq \frac{2\pi}{8}$ at p_i: suppose p_{i+1} projects no further than q

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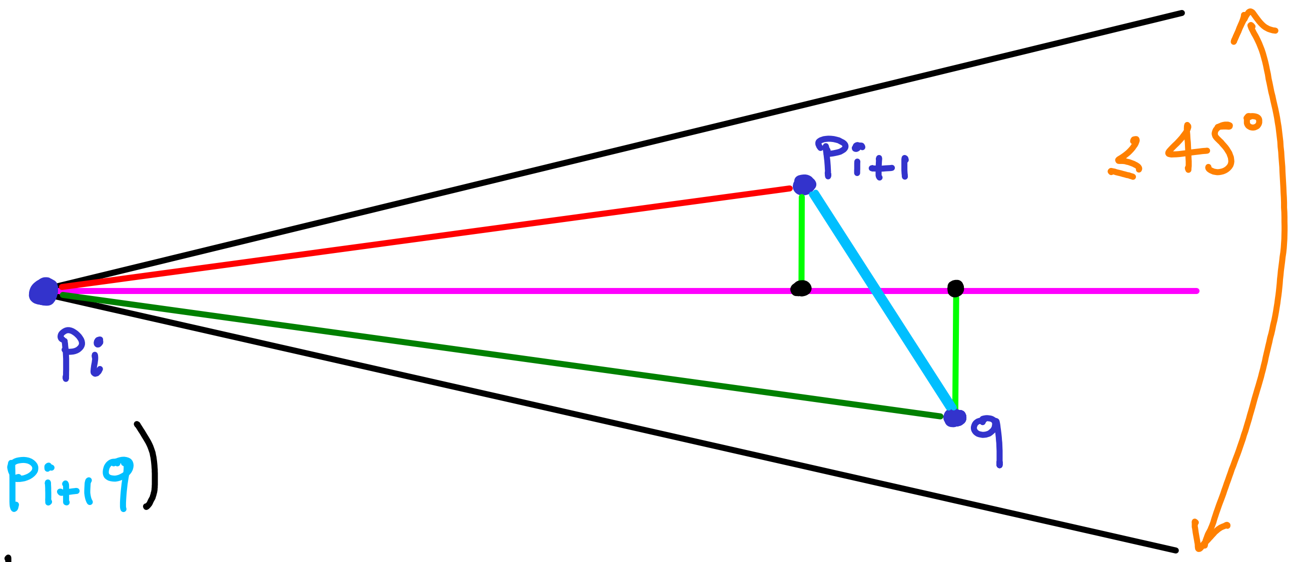

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$$= \frac{1}{\cos\theta - \sin\theta}\cdot aq$$

$a \rightarrow q$

p_0 p_m

Summary

For $\theta < 45^\circ$, i.e. w/ $k \geq 9$ cones,

a θ-graph is a t-spanner, where $t \leq \dfrac{1}{\cos\theta - \sin\theta}$

Also, it uses $\leq k \cdot V$ edges

Summary

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k	θ	t
9	40°	8.1
12	30°	2.7
24	15°	1.4
120	3°	1.06

Summary

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k	θ	t
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PROJECTS

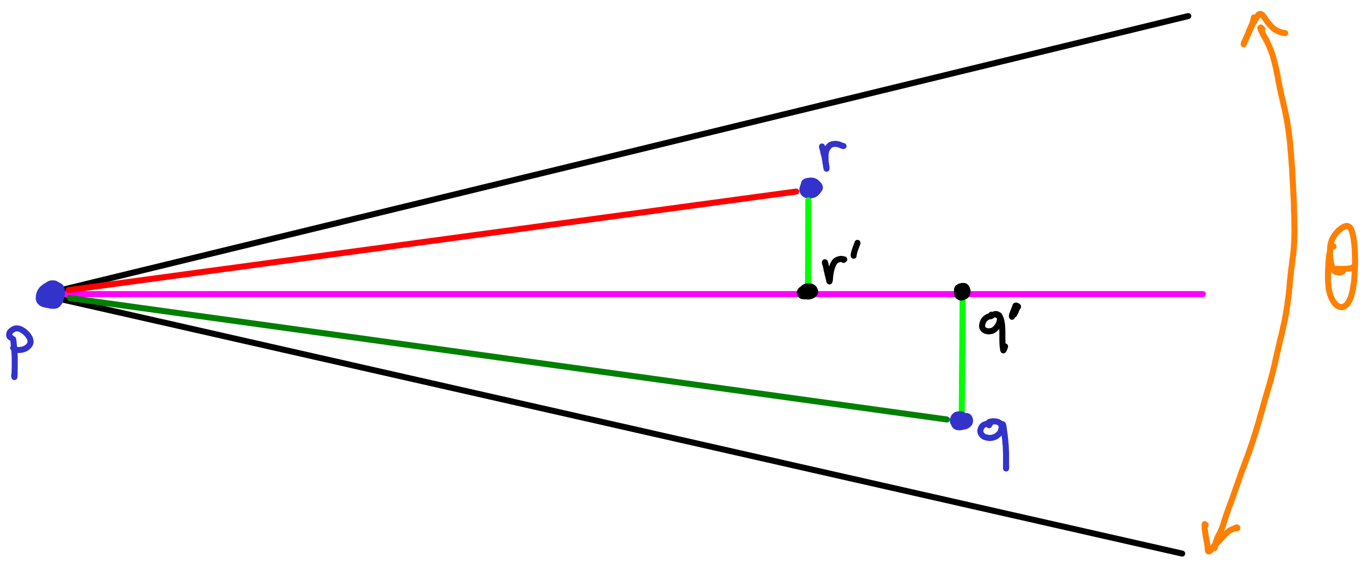
Construction of θ-graph
Getting bounded degree : see links (textbook on spanners)

Trig proof of claim

Given a cone at p: suppose r projects no further than q

(i.e., $pq' \geq pr'$)

Claim: $pq \geq pr \cdot \cos\theta$



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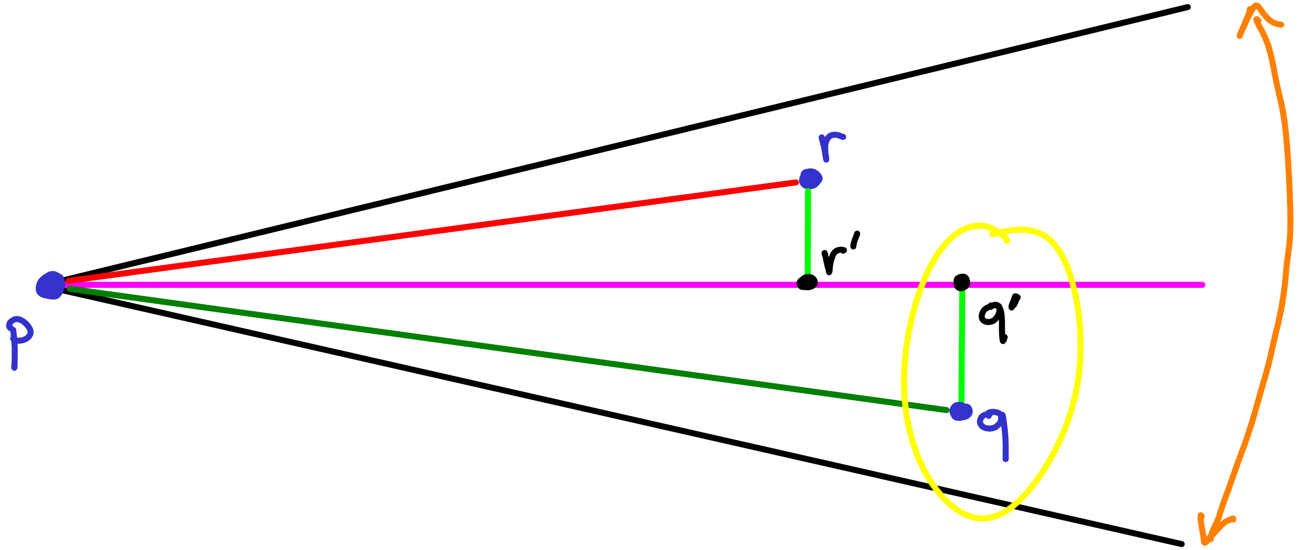
$pq \geqslant pq'$ // trivially



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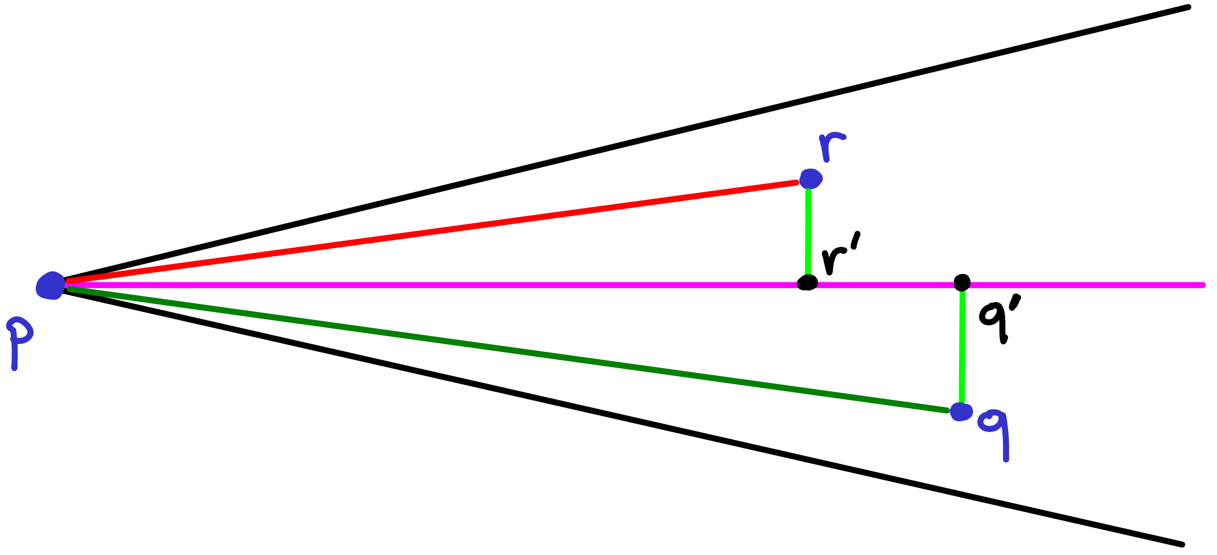
$\geq pr'$ // by assumption



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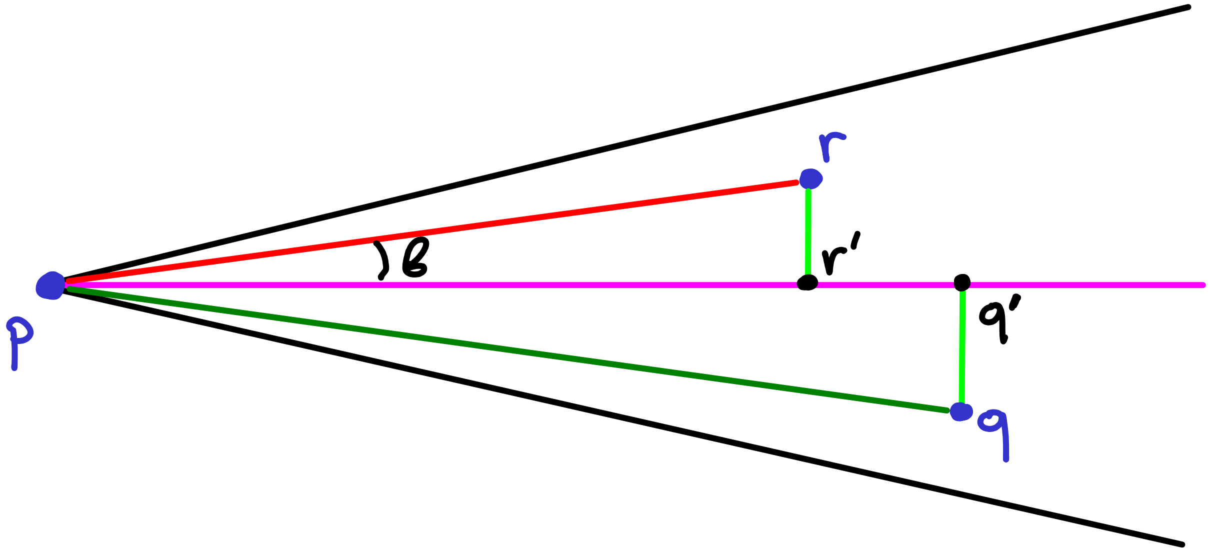
$pr' = pr \cdot \cos\theta$



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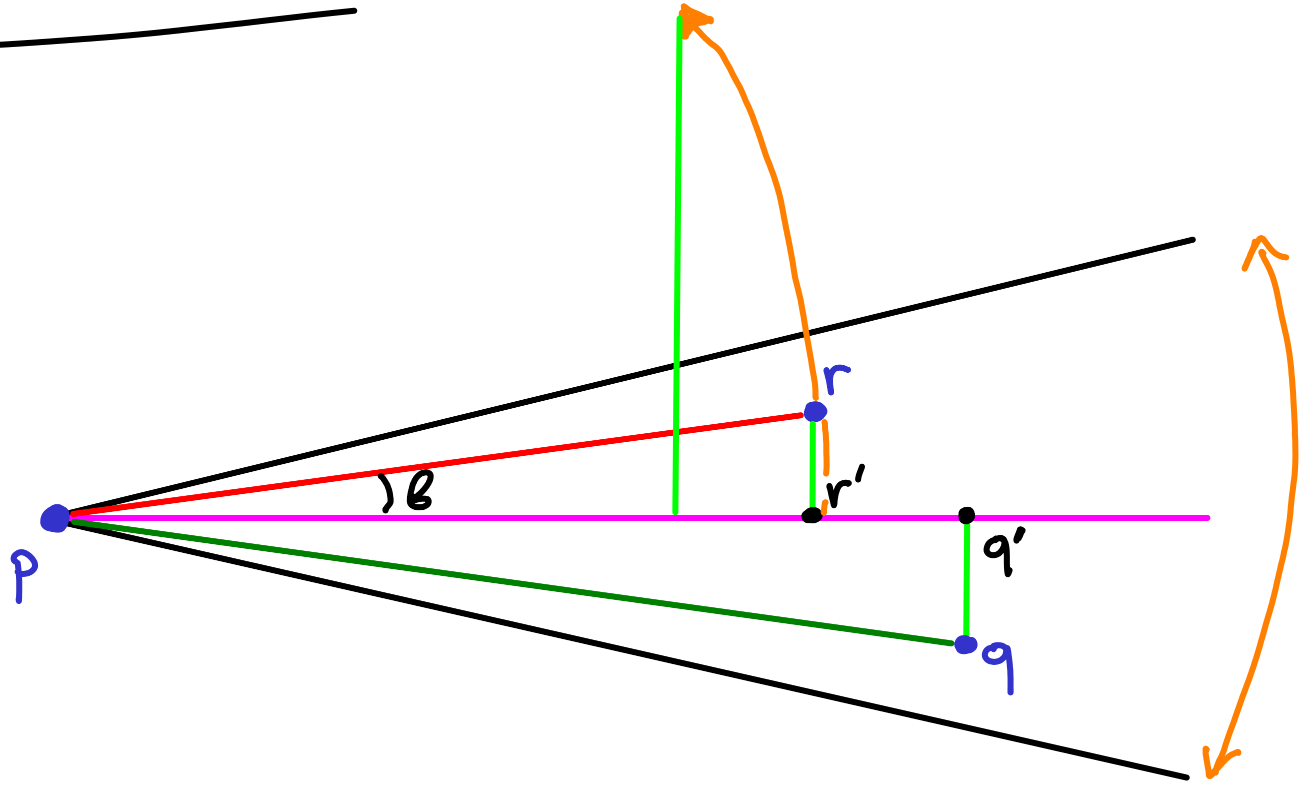
$\geq pr \cdot \cos\theta$



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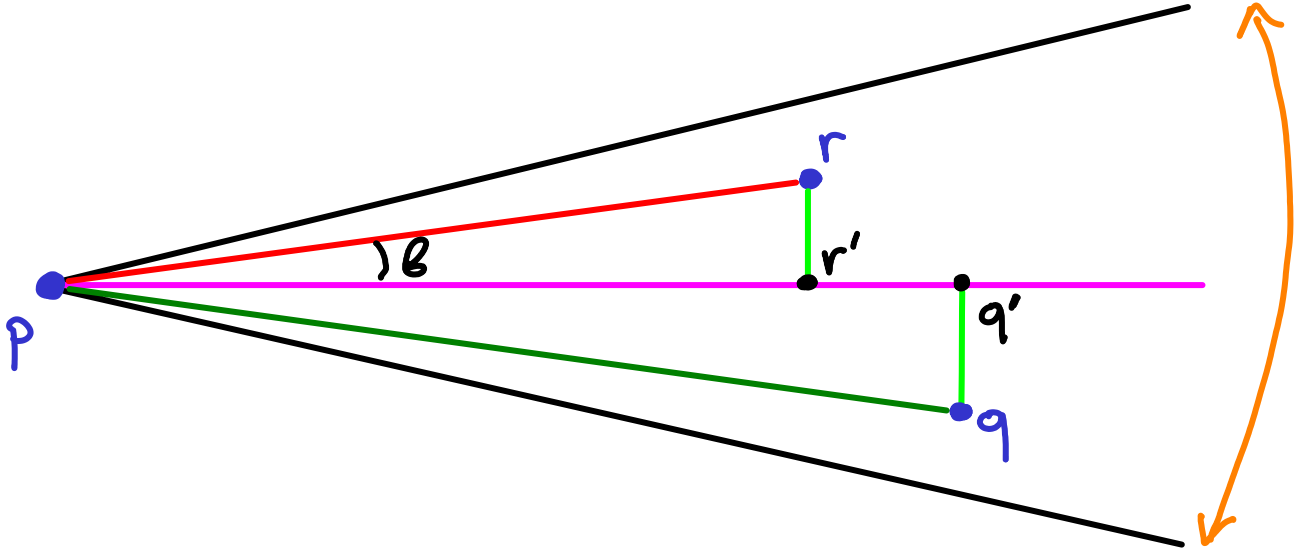

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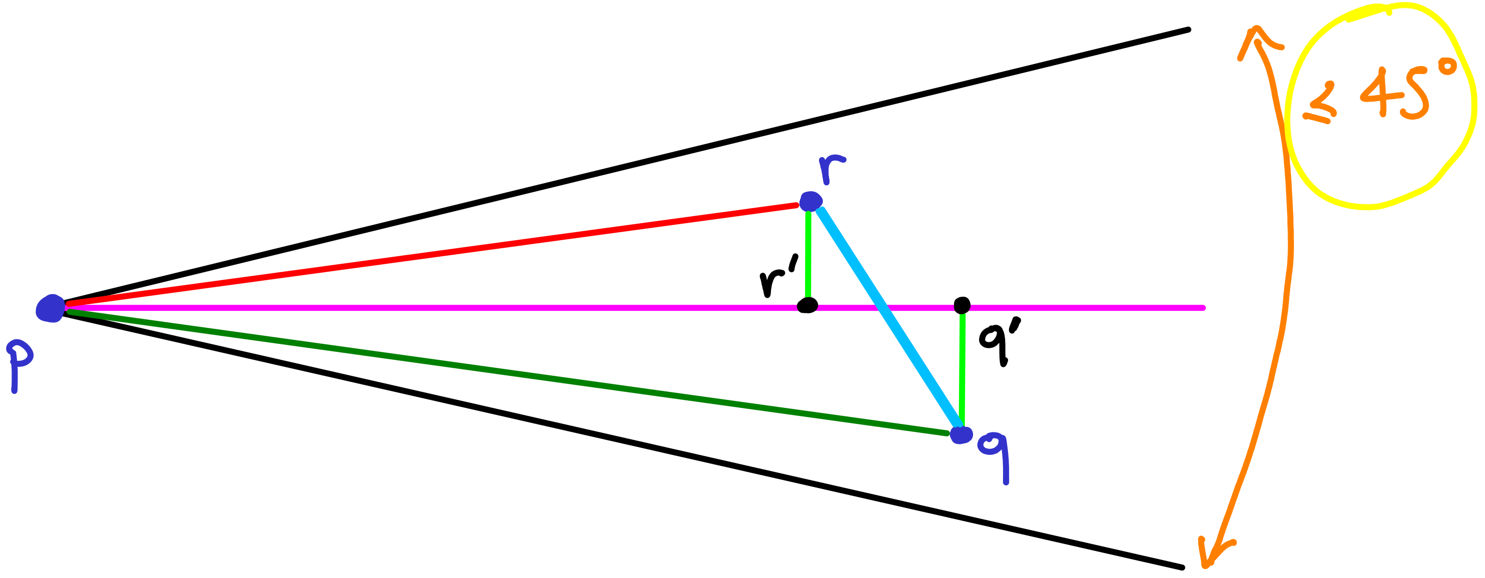
New claim: $rq \leq pq + (\sin\theta - \cos\theta) \cdot pr$



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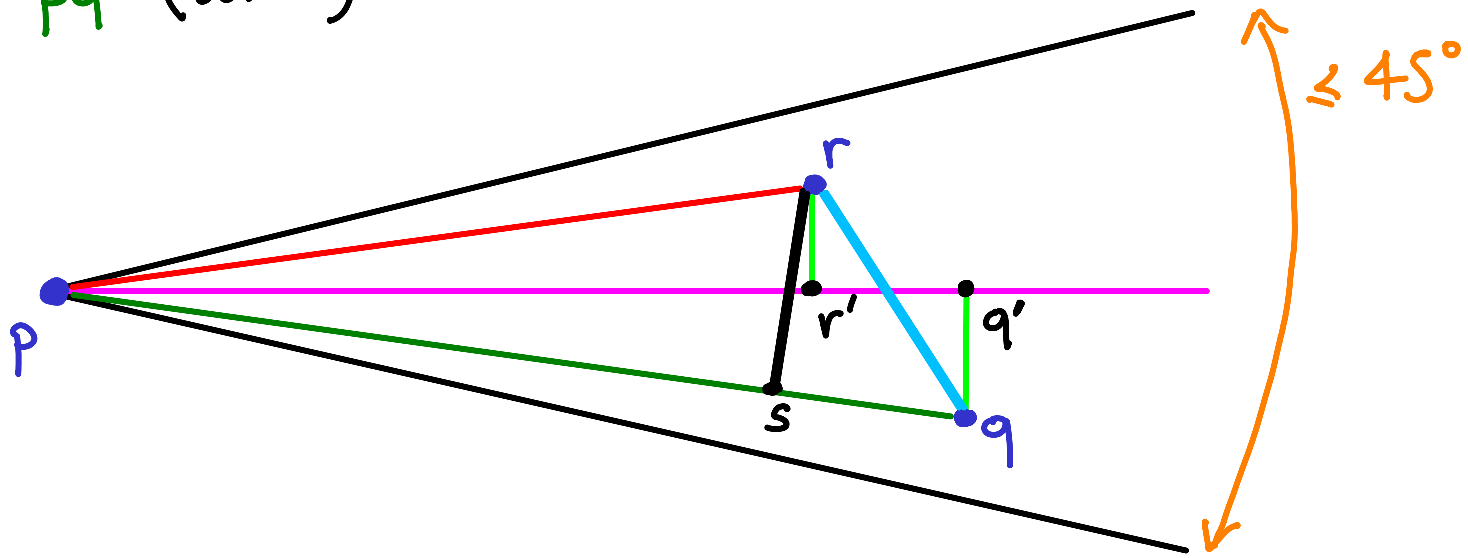
Suppose r projects onto pq (at s)



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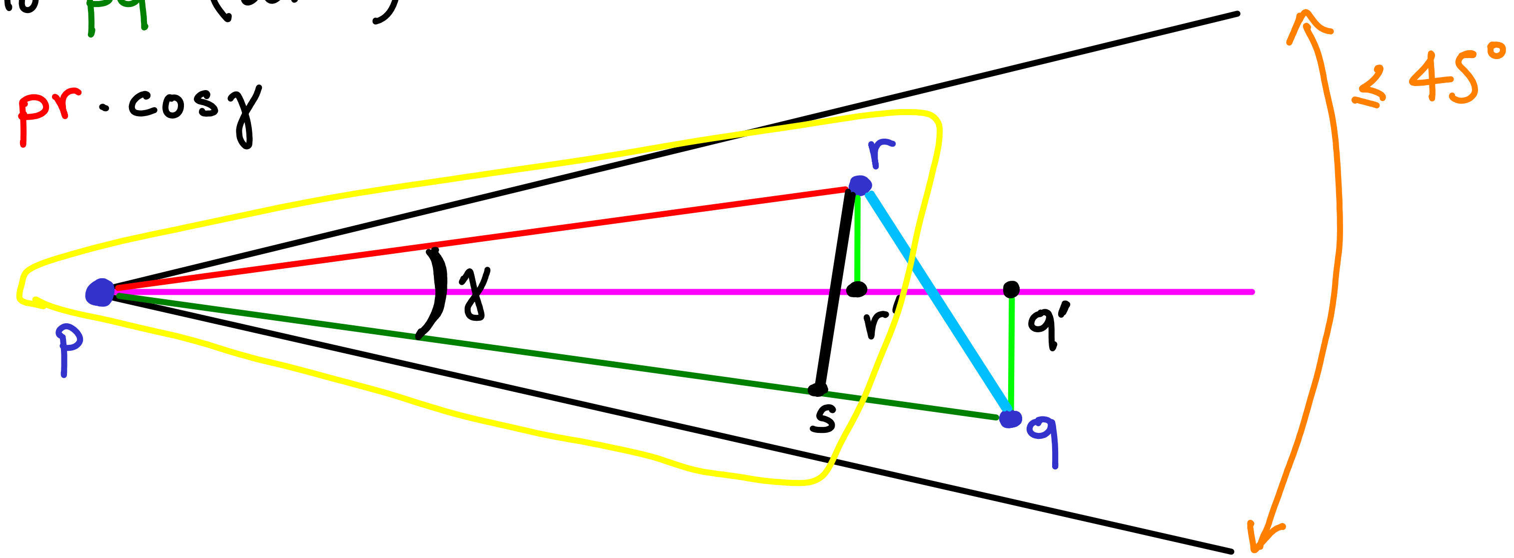
$rs = pr \cdot \sin\gamma$ $\qquad$ $ps = pr \cdot \cos\gamma$



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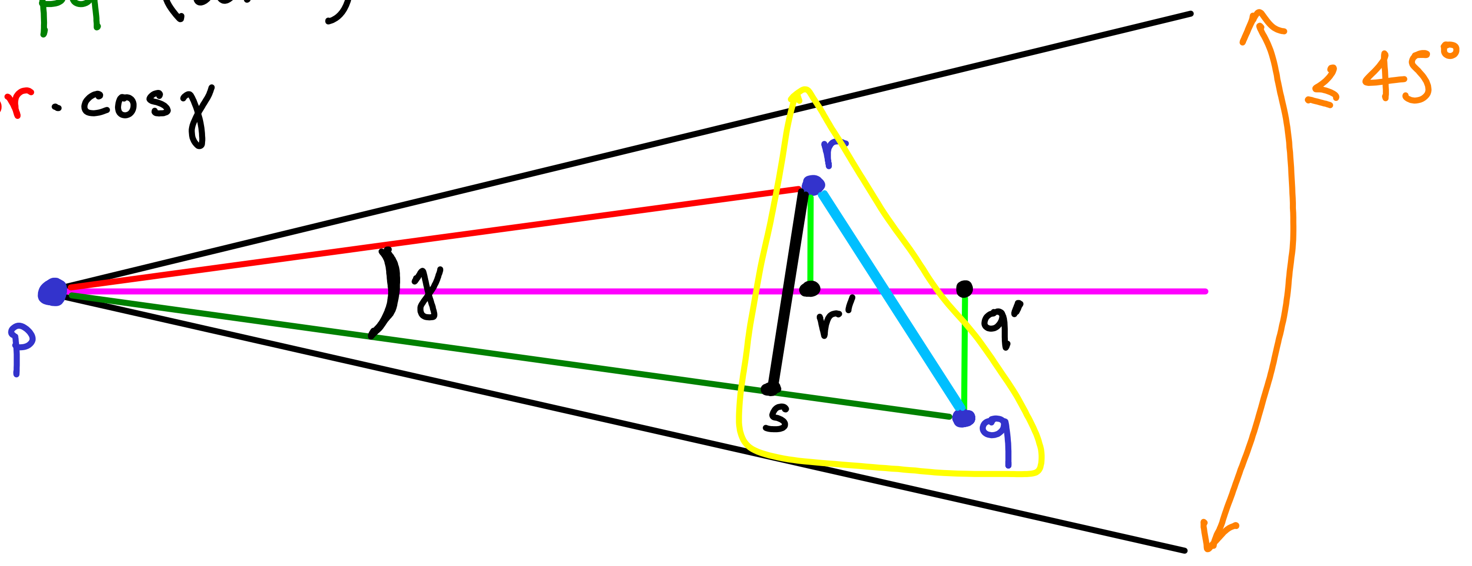
$rs = pr \cdot \sin\gamma$ $\qquad$ $ps = pr \cdot \cos\gamma$

$rq \leq rs + sq$ // triangle



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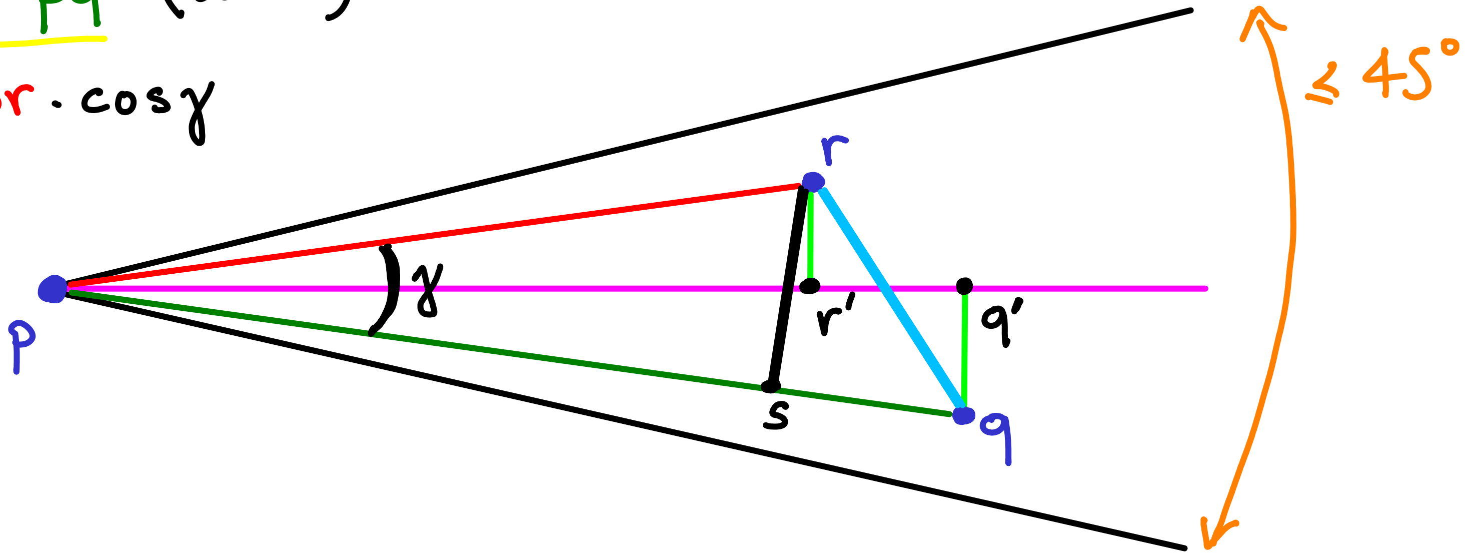
Suppose r projects onto pq (at s)

$rs = pr \cdot \sin\gamma$ $\qquad$ $ps = pr \cdot \cos\gamma$

$rq \leq rs + sq$

$= rs + pq - ps$



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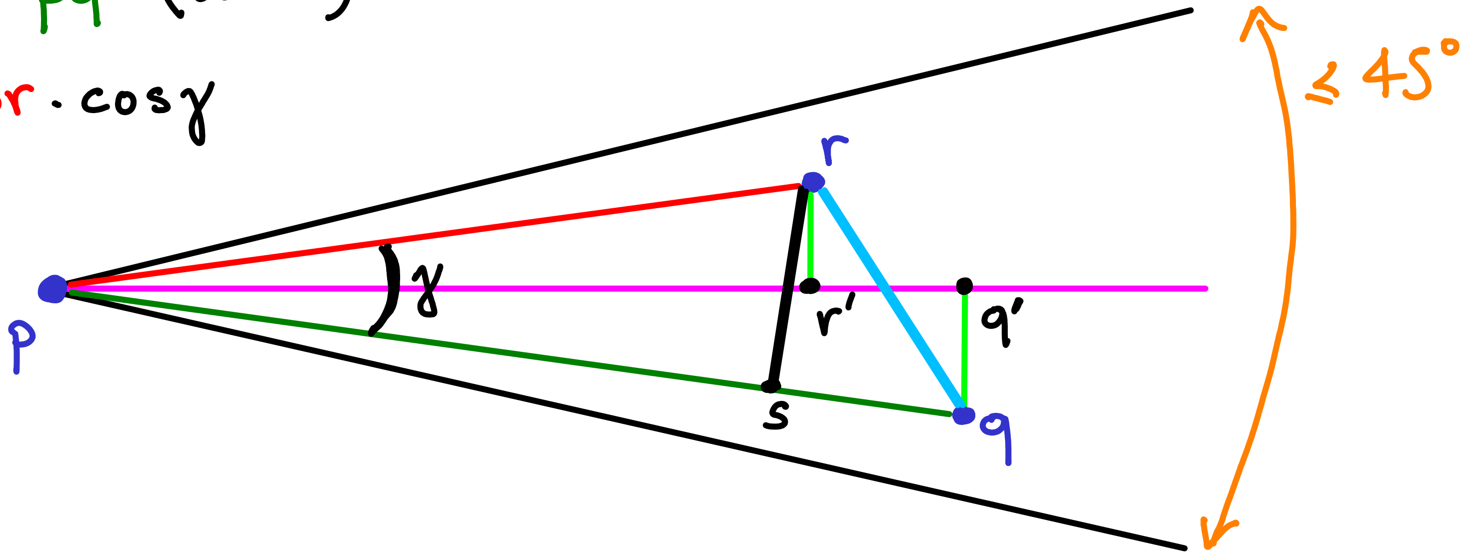
Suppose r projects onto pq (at s)

$rs = pr \cdot \sin\gamma$ $\qquad$ $ps = pr \cdot \cos\gamma$

$$\begin{aligned} rq &\leq rs + sq \\ &= rs + pq - ps \\ &= pr \cdot (\sin\gamma - \cos\gamma) + pq \end{aligned}$$



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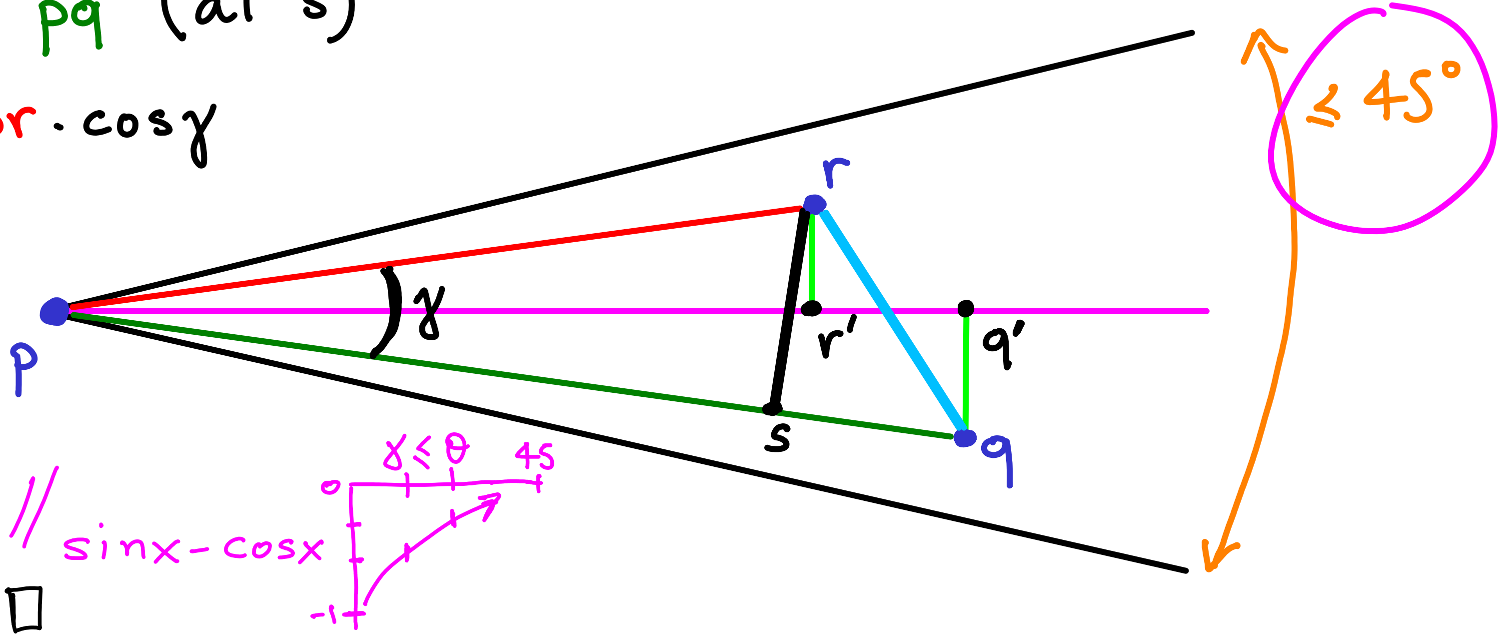
∎



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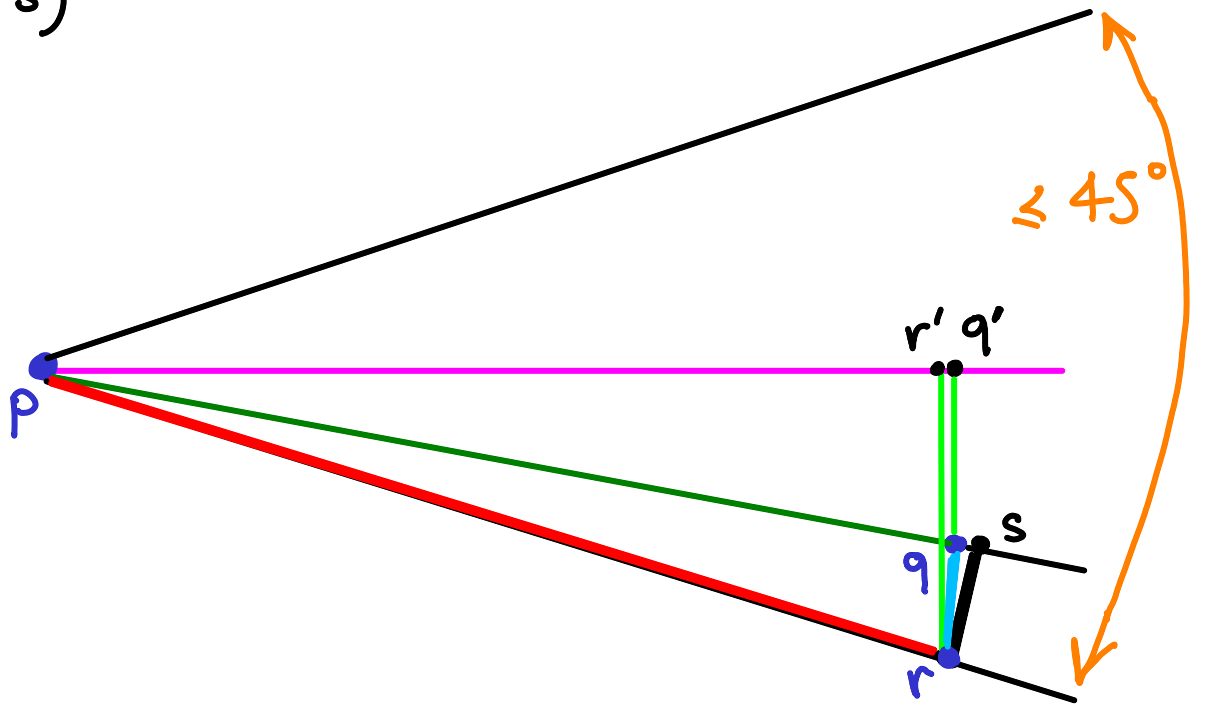
Suppose r projects $\boxed{\text{OFF}}$ pq (at s)



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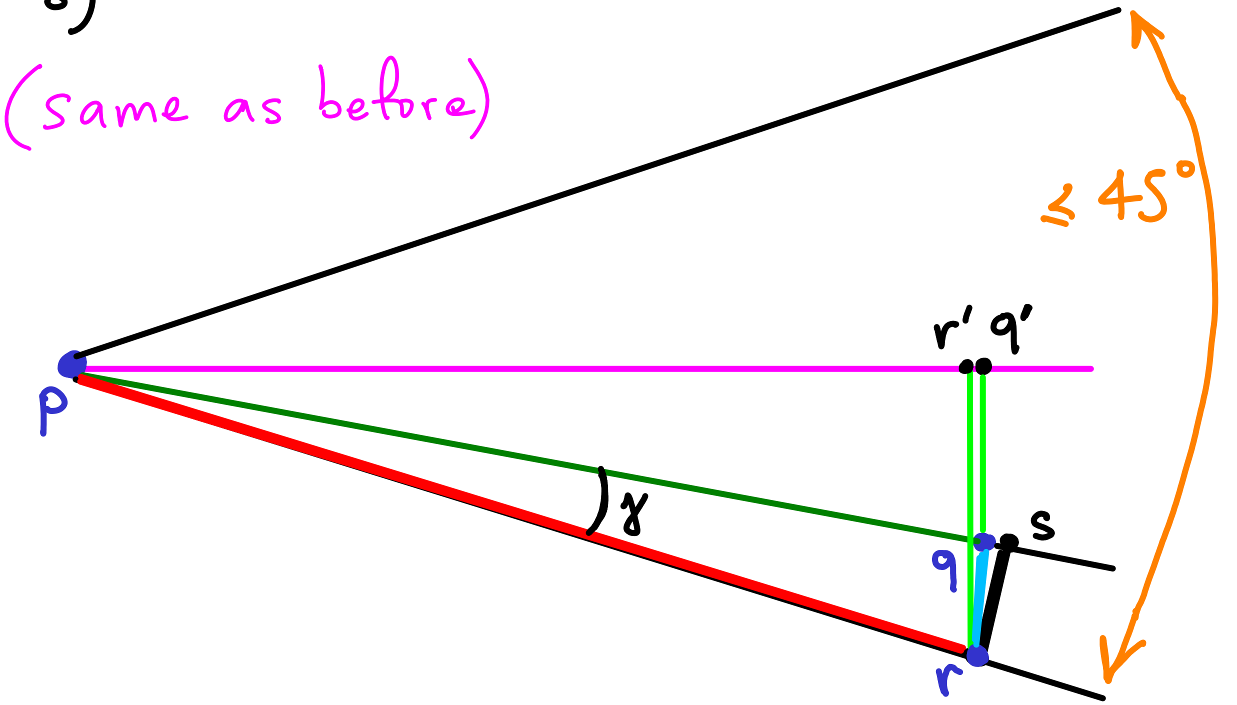
Proved: $pq \geq pr \cdot \cos\theta$ (i.e., $pq' \geq pr'$)

New claim: $rq \leq pq + (\sin\theta - \cos\theta) \cdot pr$

Suppose r projects OFF pq (at s)

$rs = pr \cdot \sin\gamma \quad ps = pr \cdot \cos\gamma$ (same as before)



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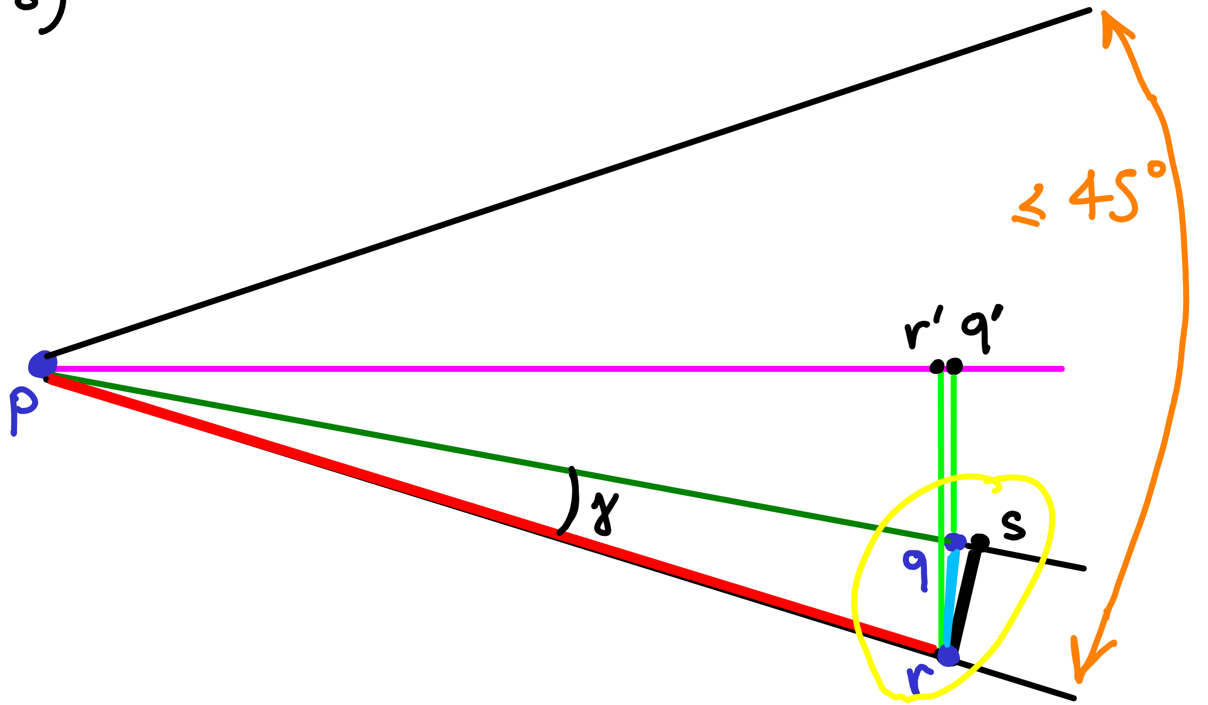
Suppose r projects OFF pq (at s)

$rs = pr \cdot \sin\gamma$ $\quad$ $ps = pr \cdot \cos\gamma$

$rq \leq rs + sq$ // triangle, as before



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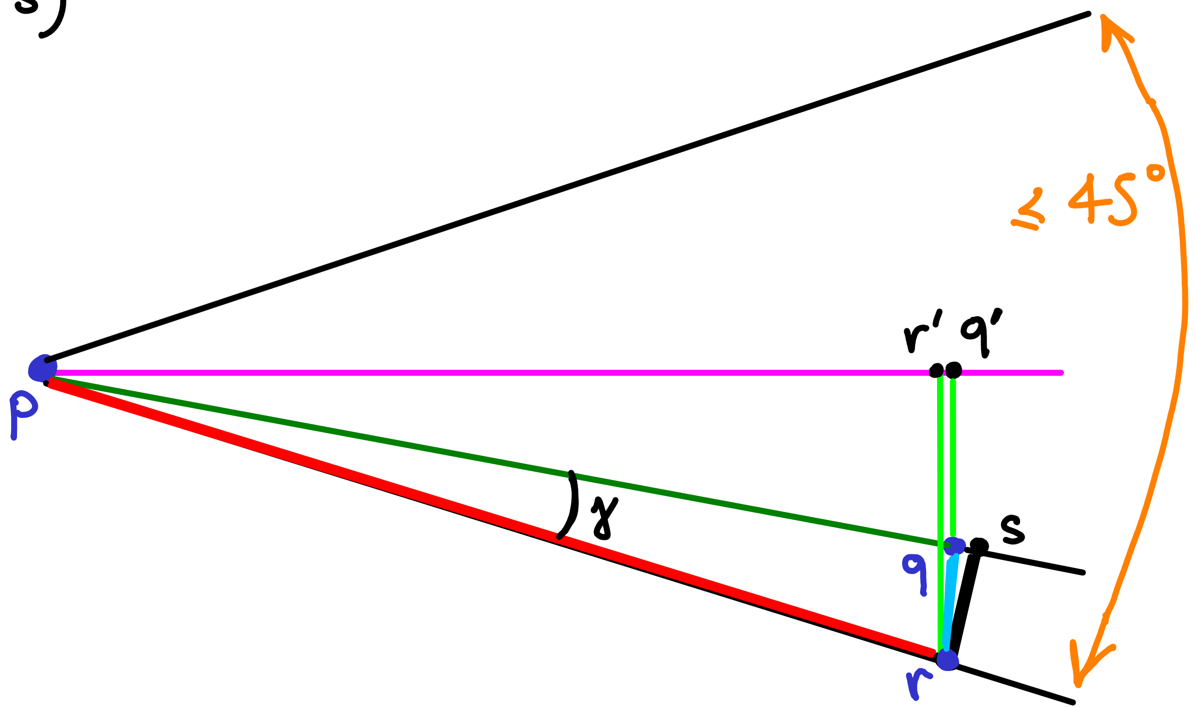
$$rq \leq rs + \underline{sq}$$
$$= rs + \underline{ps - pq}$$



Given a cone w/ $\theta \le \frac{2\pi}{8}$ at p: suppose r projects no further than q

(i.e., $pq' \geqslant pr'$)

Proved : $pq \geqslant pr \cdot \cos\theta$

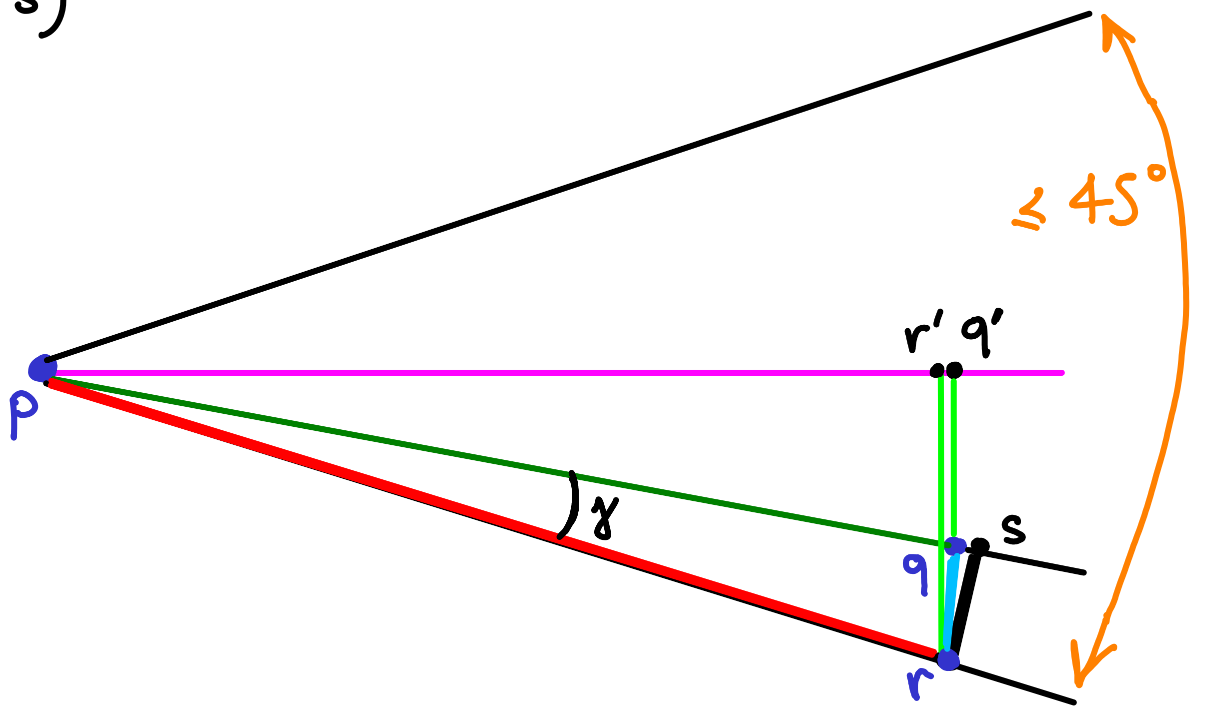
New claim : $rq \le pq + (\sin\theta - \cos\theta) \cdot pr$

Suppose r projects OFF pq (at s)

$rs = pr \cdot \sin\gamma$ $\qquad$ $ps = pr \cdot \cos\gamma$

$$\begin{aligned} rq &\le rs + sq \\ &= rs + ps - pq \\ &= pr \cdot (\sin\gamma + \cos\gamma) - pq \end{aligned}$$

like before but signs changed



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Suppose r projects OFF pq (at s)

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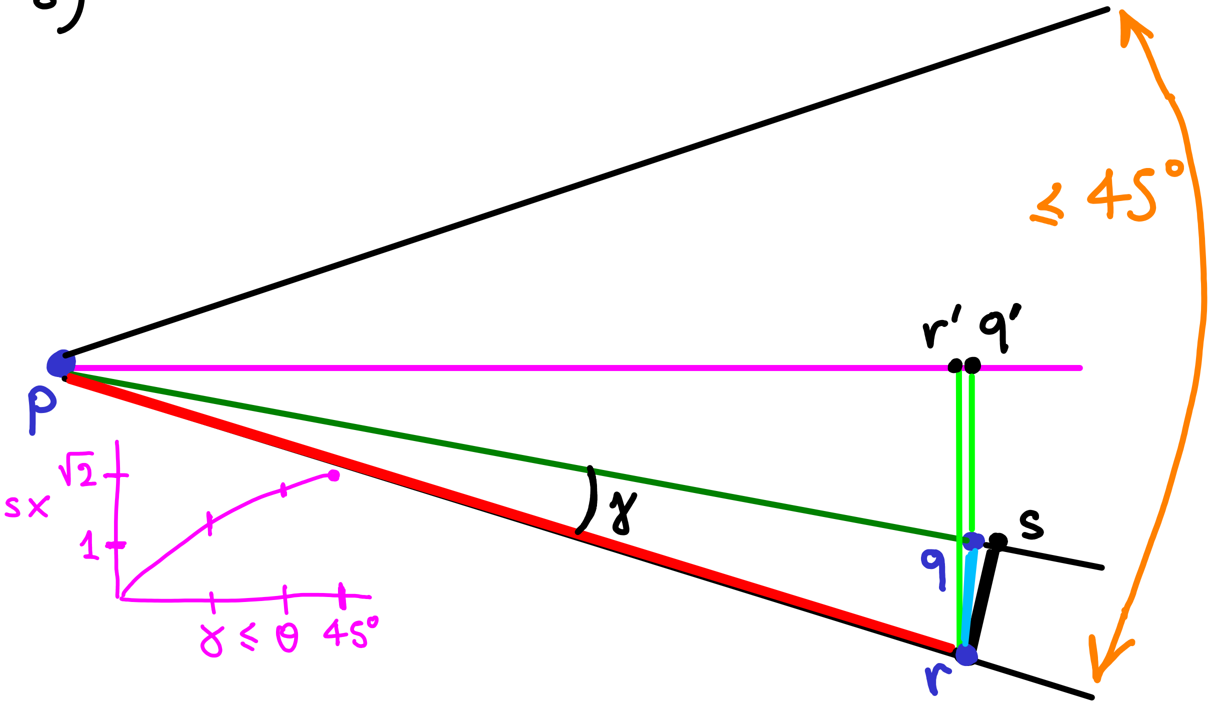
$$\begin{aligned} rq &\leq rs + sq \\ &= rs + ps - pq \\ &= pr \cdot (\sin\gamma + \cos\gamma) - pq \\ &\leq pr \cdot (\sin\theta + \cos\theta) - pq \end{aligned}$$

// $\sin x + \cos x$



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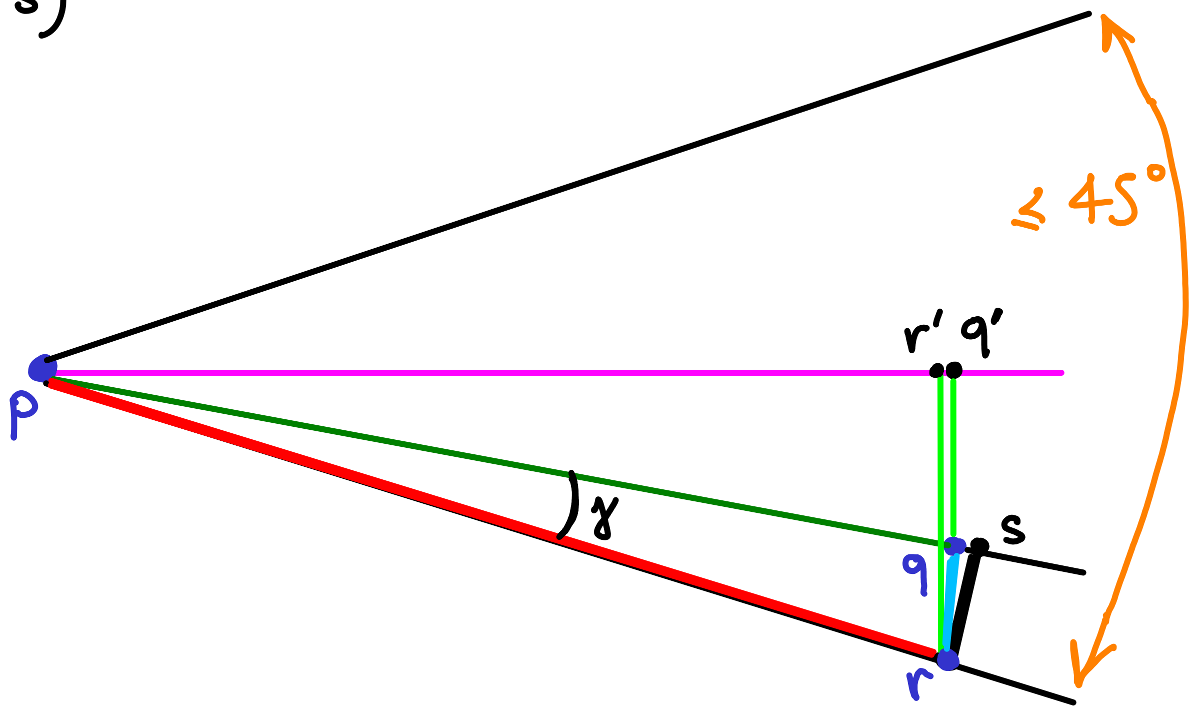
rq &\leq rs + sq \\
&= rs + ps - pq \\
&= pr \cdot (\sin\gamma + \cos\gamma) - pq \\
&\leq pr \cdot (\sin\theta + \cos\theta) - pq \\
&\leq pq + pr\sin\theta - pq \quad *
\end{aligned}$$



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Suppose r projects OFF pq (at s)

$rs = pr \cdot \sin\gamma \qquad ps = pr \cdot \cos\gamma$

$$\begin{aligned} rq &\le rs + sq \\ &= rs + ps - pq \\ &= pr \cdot (\sin\gamma + \cos\gamma) - pq \\ &\le pr \cdot (\sin\theta + \cos\theta) - pq \\ &\le pq + pr\sin\theta - \underline{pq} \quad * \\ &\le pq + pr\sin\theta - (\underline{pr\cos\theta}) \quad * \end{aligned}$$

$\square$