

OPTIMALITY OF BST (ALGORITHMS)

model: every op starts at root,
can move  or rotate current node

Typical extensions: more pointers allowed, or
can start elsewhere

operations known in advance?

op. type

Yes

No

static
(read only)

dynamic programming $\rightarrow$ optimal tree
(Knuth)

Update: splay trees should be here, achieving $O(OPT)$ = static optimality theorem =

- not interesting for static tree
 $\hookrightarrow$ build balanced, only $O(\log n \cdot OPT)$
- dynamic tree: splay gets $O(OPT)$

dynamic
(insert/delete)

- splay tree conjectured $O(OPT)$ = dynamic optimality conjecture = **
- result on geometry of BST characterizes $O(OPT)$ but we don't have a poly-time algo yet (likely hard)
 $\hookrightarrow$ greedy algo conjectured $O(OPT)$

- splay tree conjectured $O(OPT)$
 - tango tree gets $O(\log \log n \cdot OPT)$
 - AVL / RB / etc gets $O(\log n \cdot OPT)$
- $\rightarrow$ result carries over (split trees, etc)

** I don't think it makes
a difference if ops are
known in advance or not

OPT = performance of any custom-made algo that knows op sequence
e.g. if read-only then OPT = Knuth's tree