GEOMETRY OF BINARY SEARCH TREES (& ALGORITHMS)

AND BST OPTIMALITY

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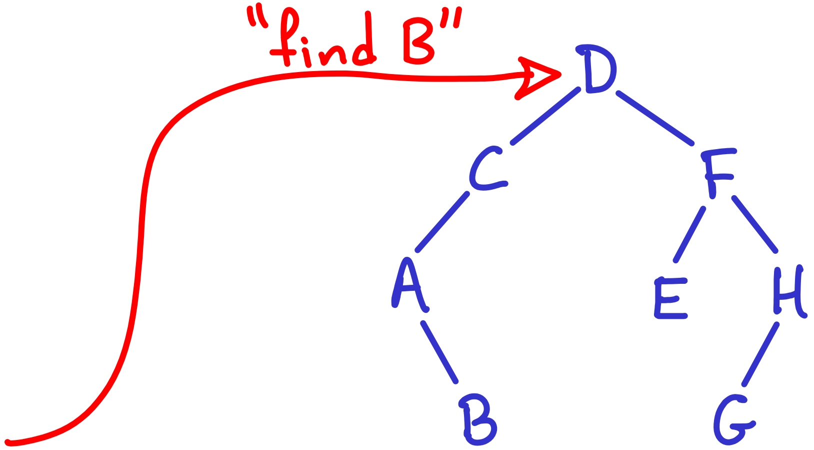

Model:

- every operation starts at the root

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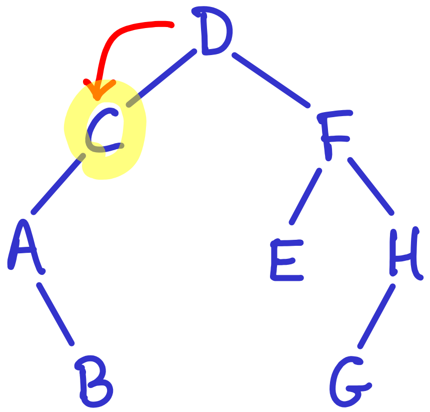

Model:

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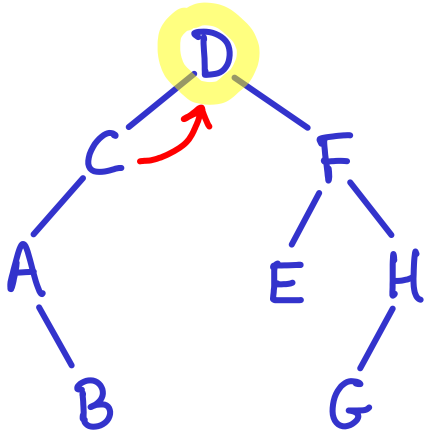

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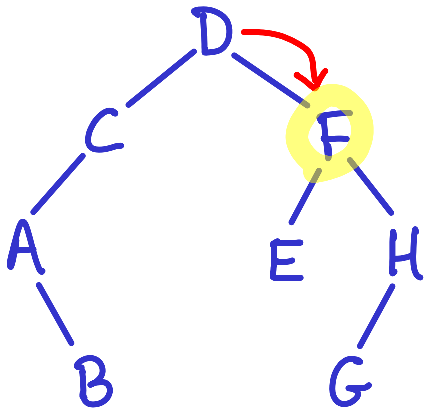

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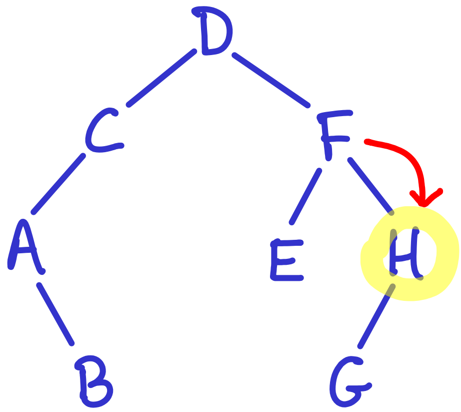

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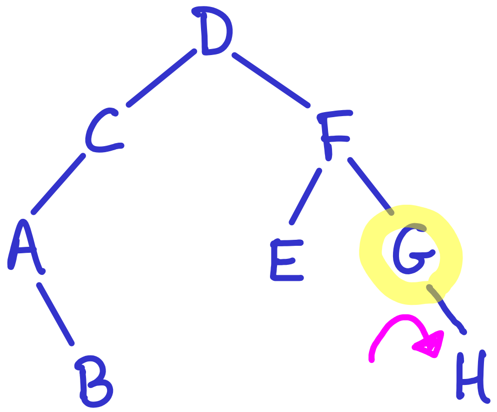

Model:

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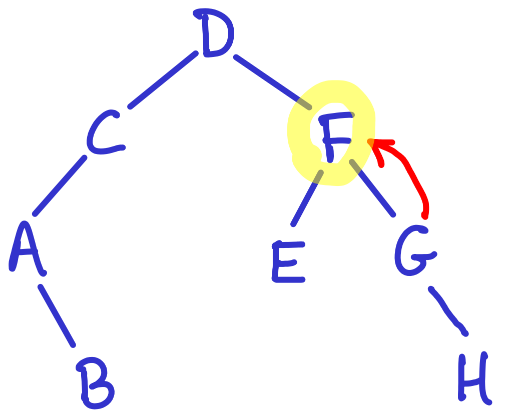

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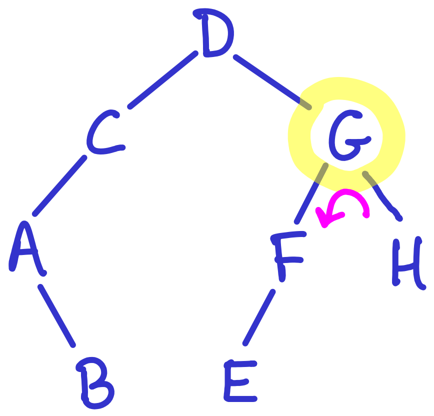

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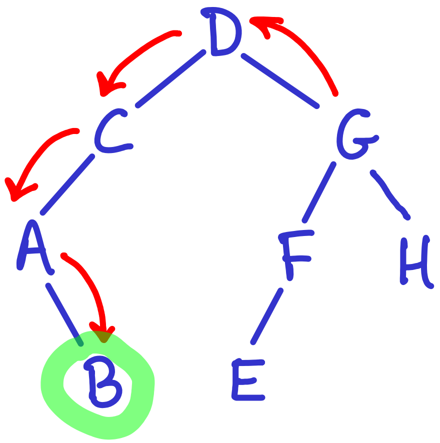

Model:

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- at some point during operation we must access (find/insert/delete) given target

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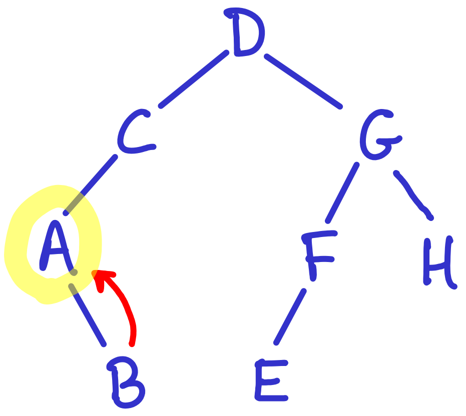

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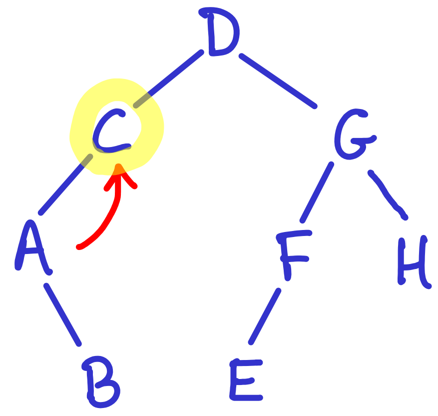

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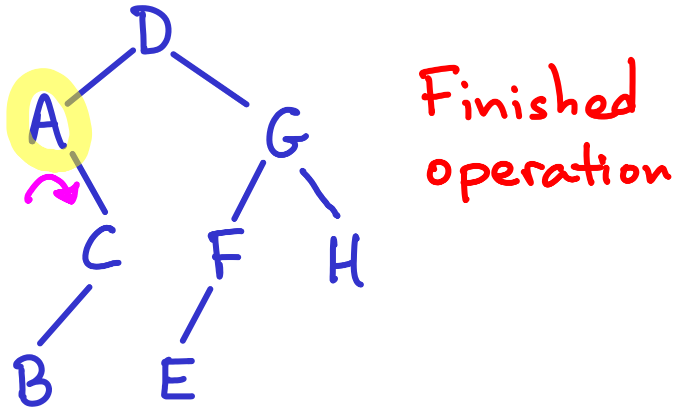

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Task: sequence of operations on n keys (one at a time)

t_1 : insert(3)

t_2 : insert(5)

t_3 : insert(1)

t_4 : insert(n)

t_5 : insert(4)

t_6 : search(5)

t_7 : delete(5)

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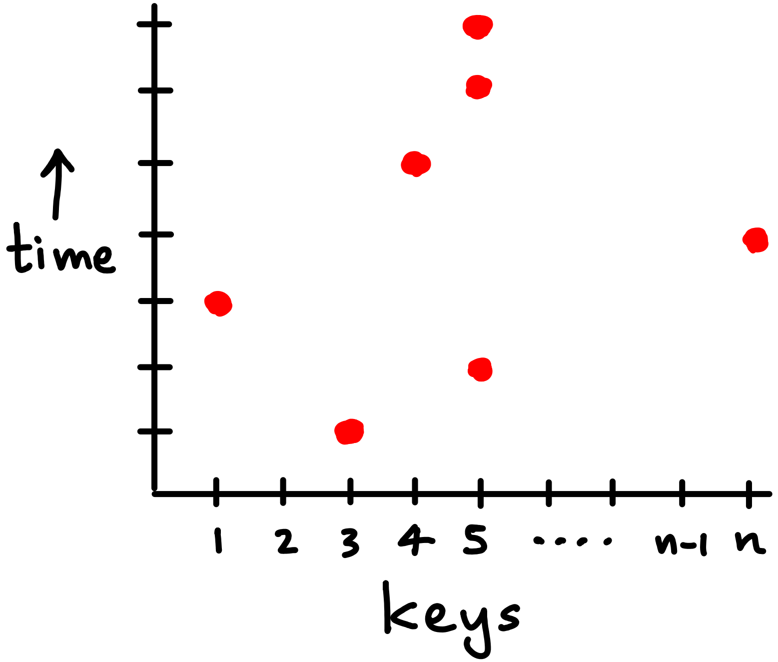

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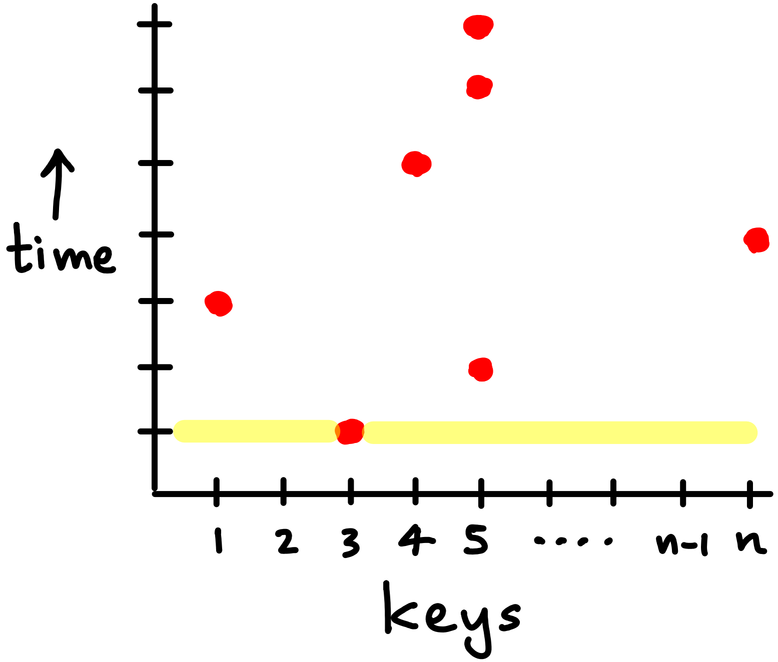

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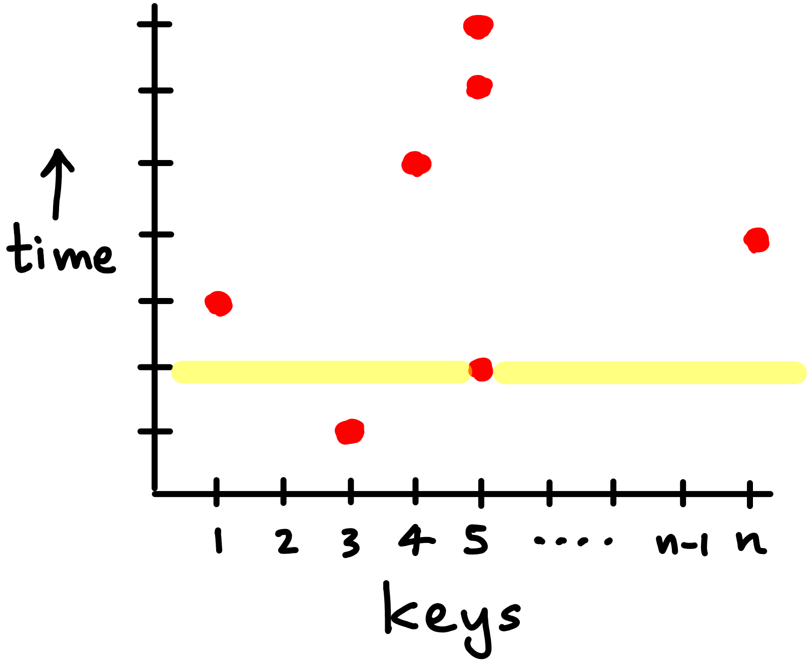
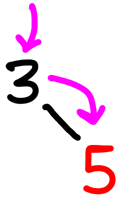
3



Task: sequence of operations on n keys (one at a time)

t_1 : insert(3)

t_2 : insert(5) Must access 3 first



t_3 : insert(1)

t_4 : insert(n)

t_5 : insert(4)

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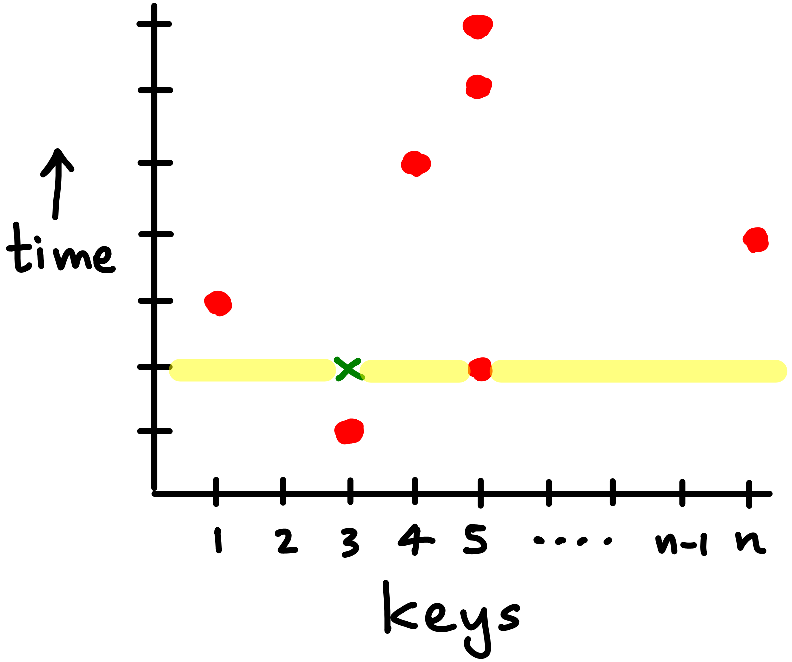

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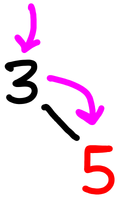

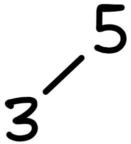

could rotate

t_3 : insert(1)

t_4 : insert(n)

t_5 : insert(4)

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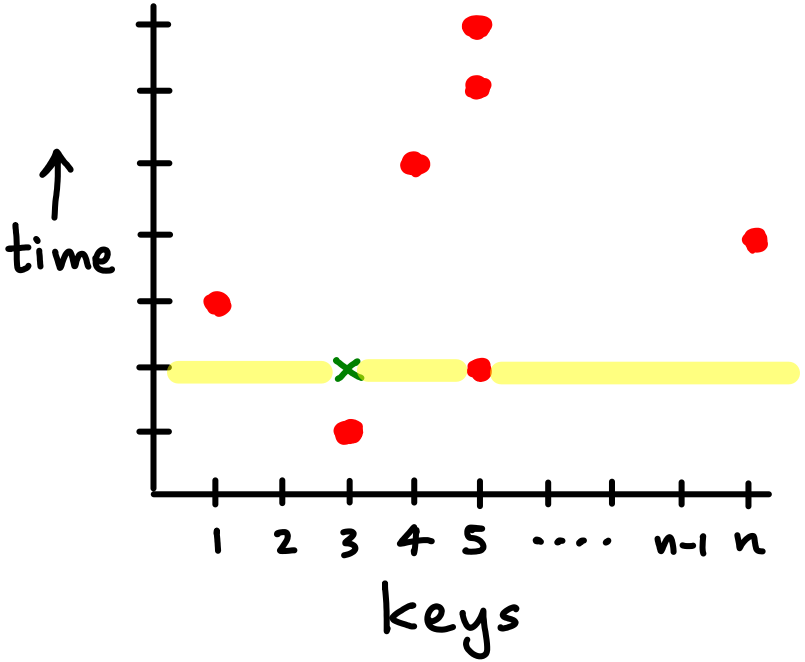
t_7 : delete(5)



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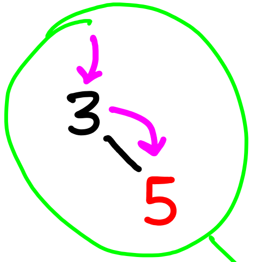
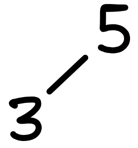
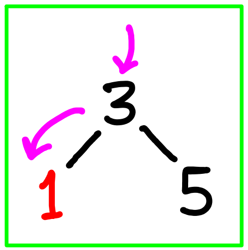
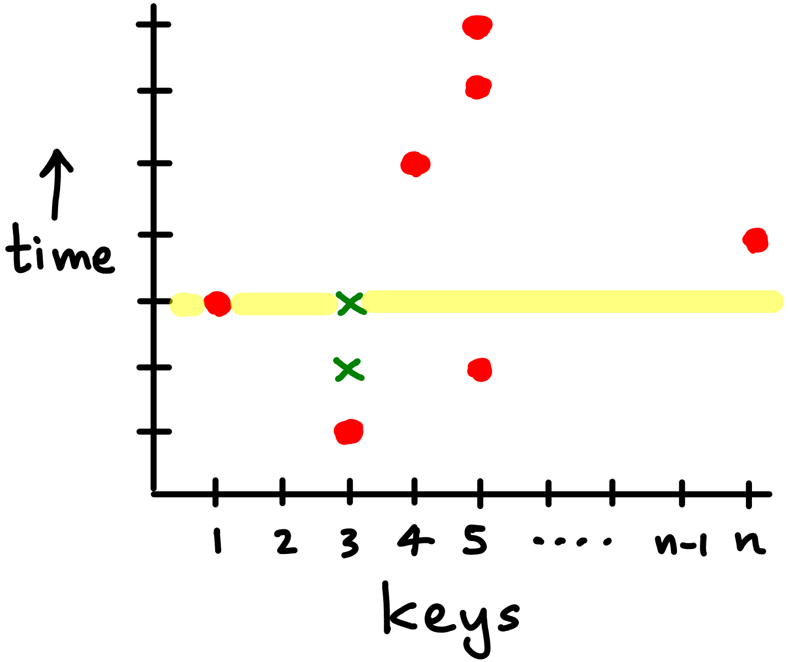




could rotate





Task: sequence of operations on n keys (one at a time)

t_1 : insert(3)

t_2 : insert(5) Must access 3 first

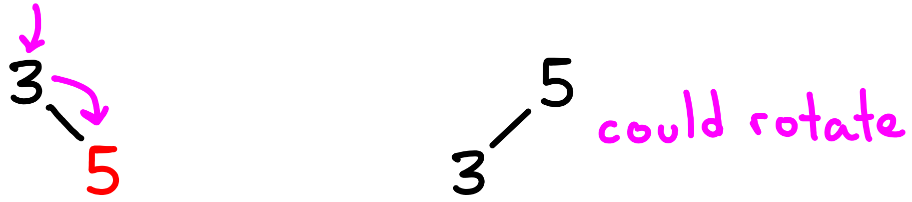
t_3 : insert(1) Must access 3 or 5 first (actually, if 5 then also 3)

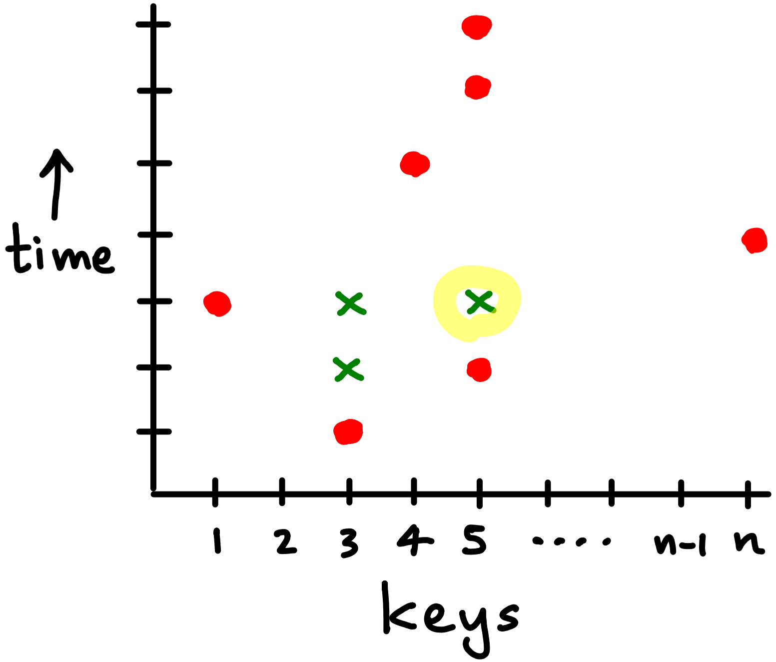
t_4 : insert(n)

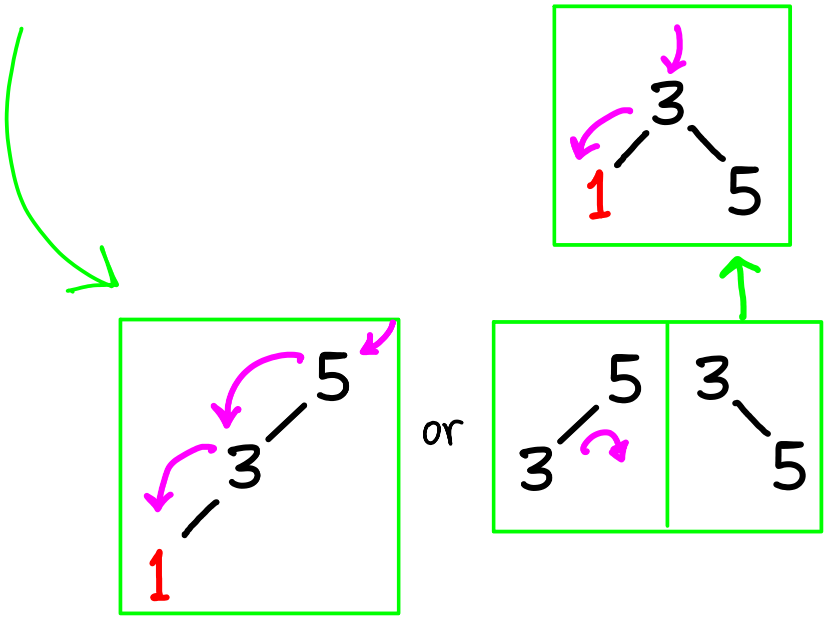
t_5 : insert(4)

t_6 : search(5)

t_7 : delete(5)



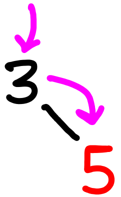


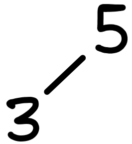


Task: sequence of operations on n keys (one at a time)

t_1 : insert(3)

t_2 : insert(5) Must access 3 first





could rotate

t_3 : insert(1) Must access 3 or 5 first (actually, if 5 then also 3)

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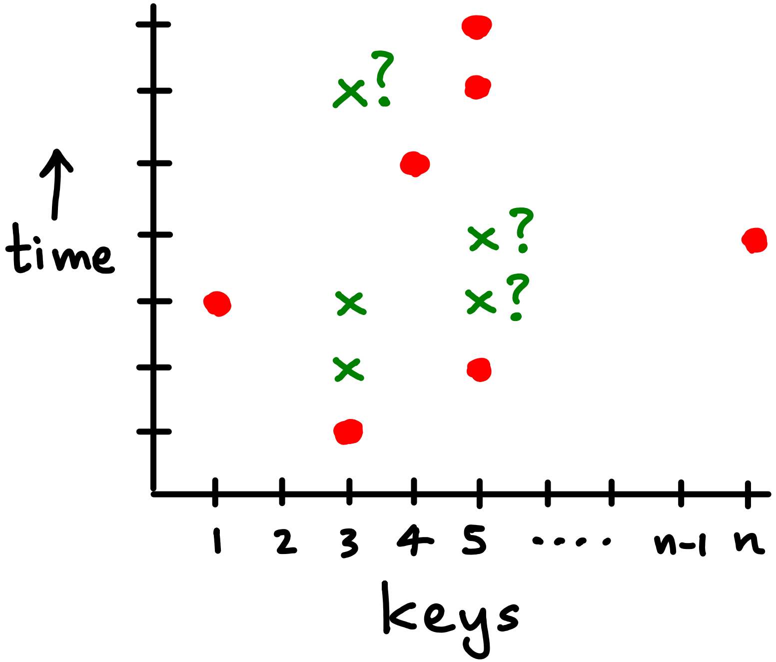
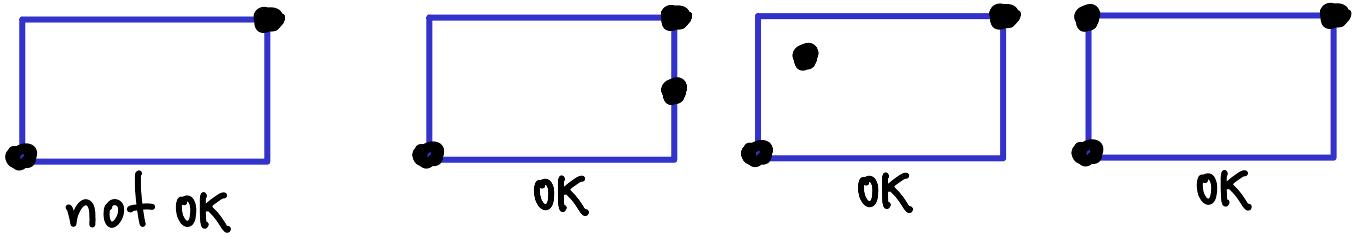


Besides the required ops there are other keys that we may need to go through

Theorem: the set of key accesses (over all ops) corresponds to a diagram where no two points form opposite corners of a closed rectangular region that is empty of other points



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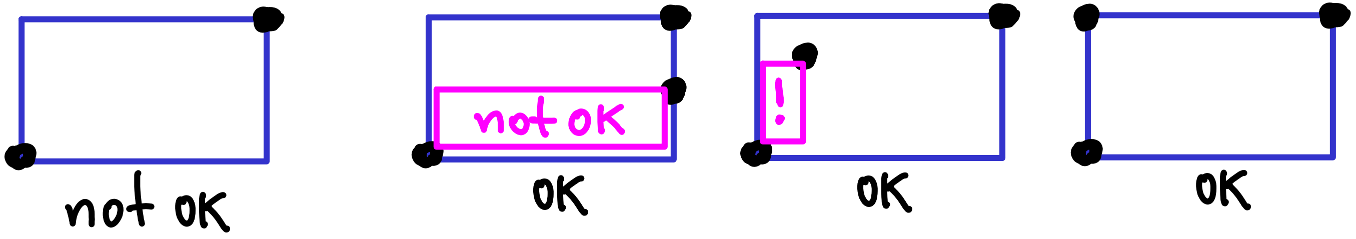

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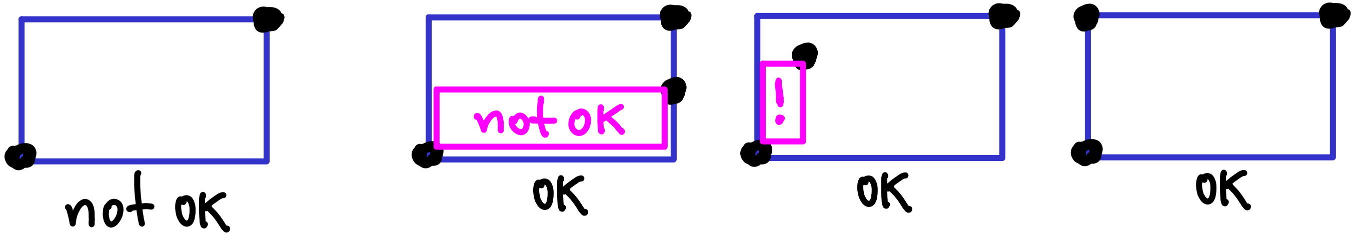

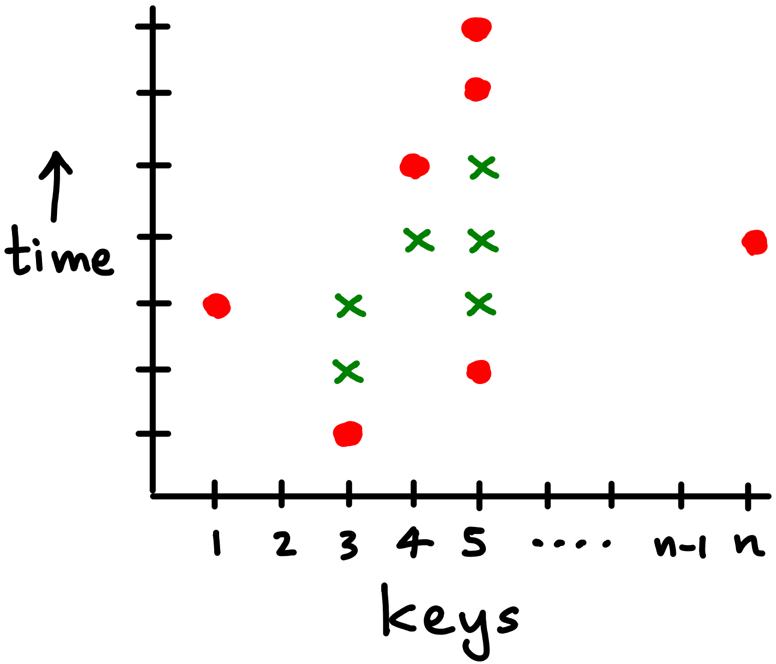

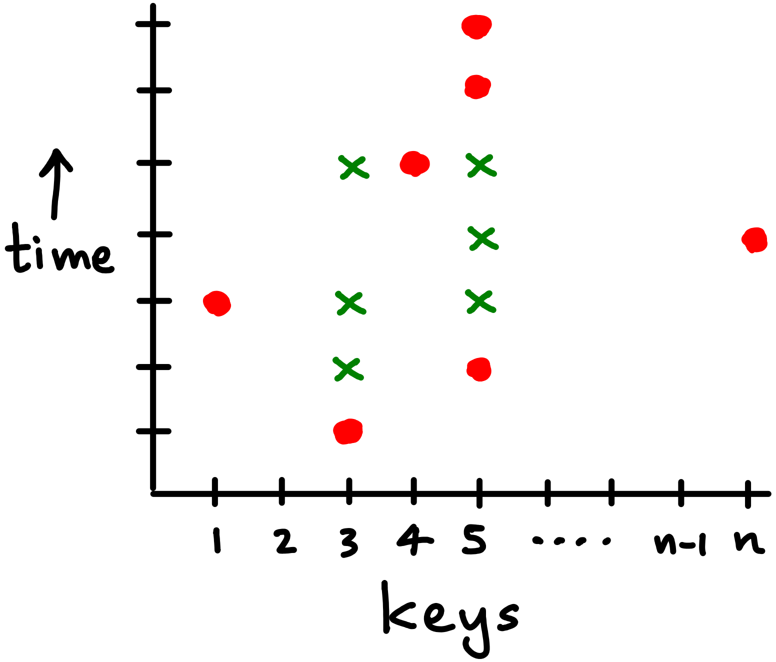

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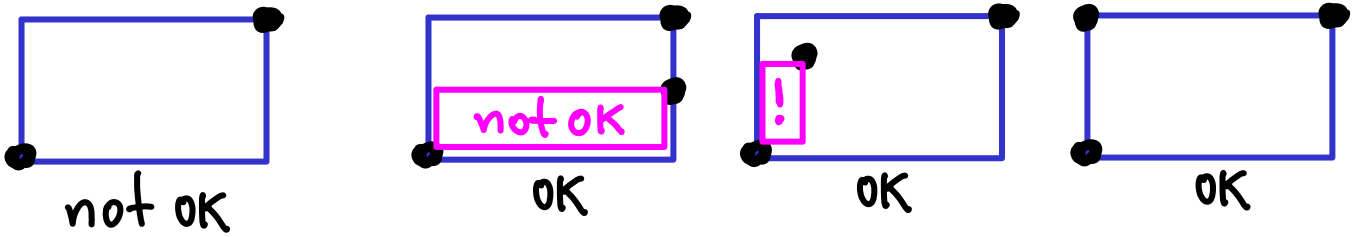
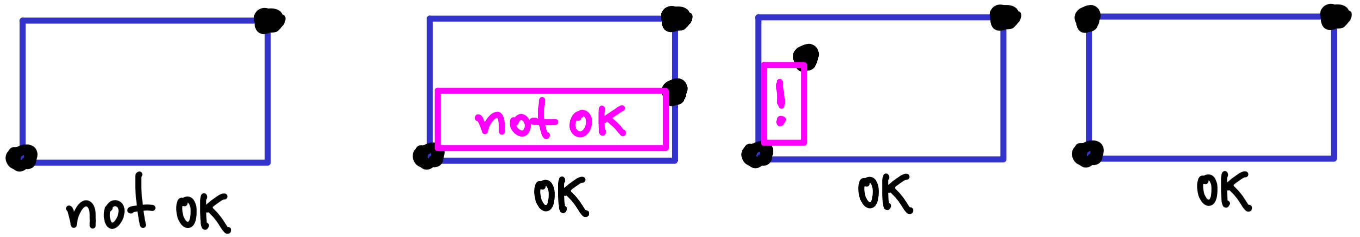
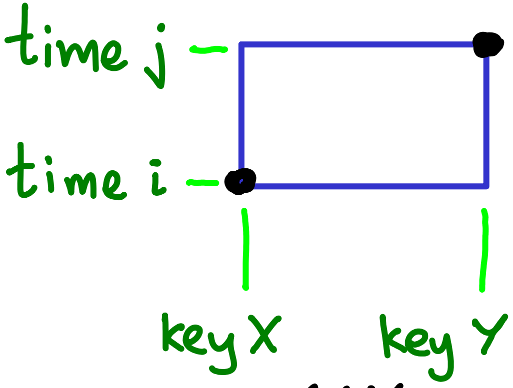


PROOF

(suppose all ops = search)

Theorem: the set of key accesses (over all ops) corresponds to a diagram where no two points form opposite corners of a closed rectangular region that is empty of other points





X<Y
(w.l.o.g.)

can this happen?

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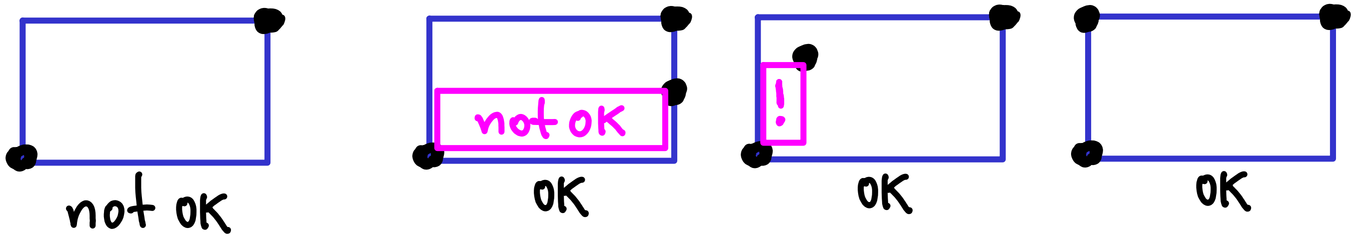
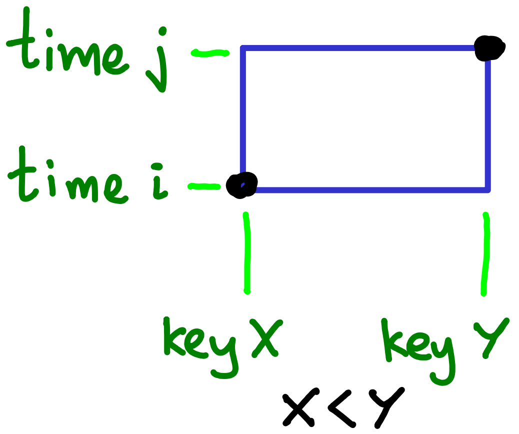
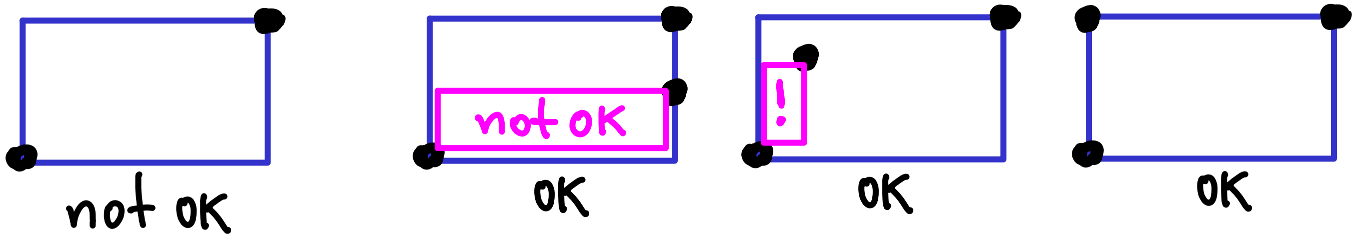
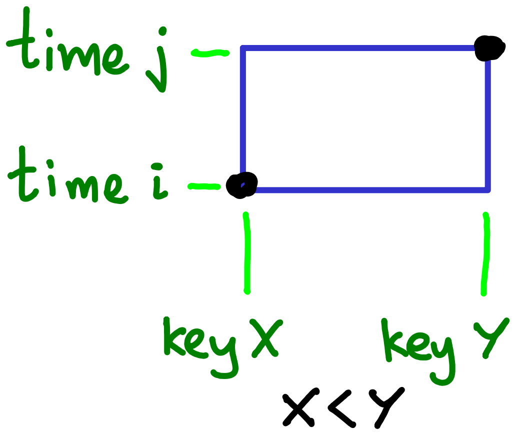




$LCA_i(X,Y)$ = lowest common ancestor at time i

Theorem: the set of key accesses (over all ops) corresponds to a diagram where no two points form opposite corners of a closed rectangular region that is empty of other points

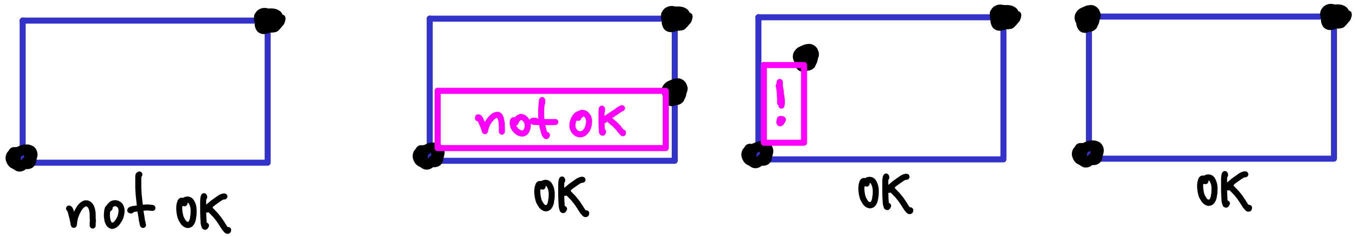


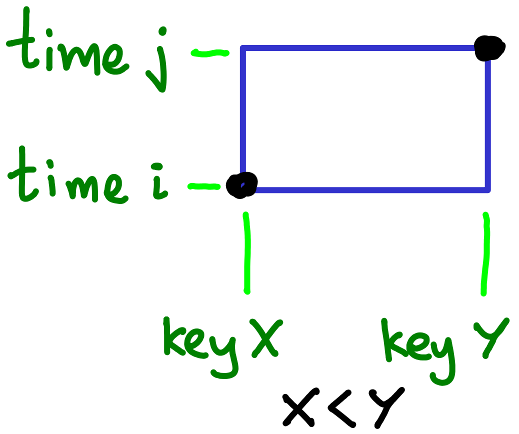


$LCA_i(X,Y)$ = lowest common ancestor at time i

if $LCA_i(X,Y) \neq X$ then ?

Theorem: the set of key accesses (over all ops) corresponds to a diagram where no two points form opposite corners of a closed rectangular region that is empty of other points





$LCA_i(X,Y)$ = lowest common ancestor at time i

if $LCA_i(X,Y) \neq X$ then: at time i, to access X...

?

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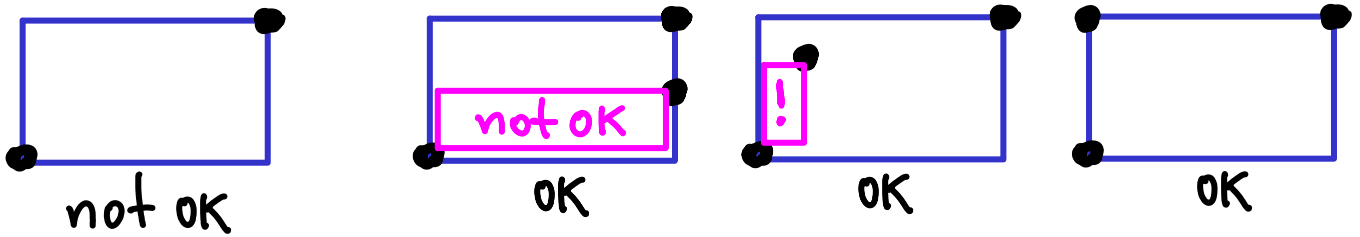
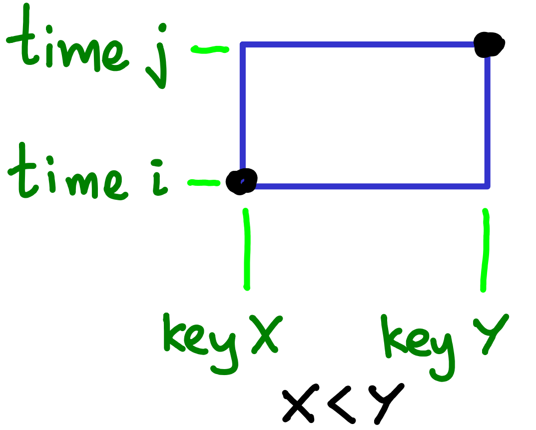


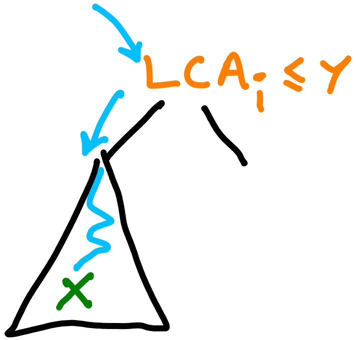


$\mathrm{LCA}_i(X,Y)$ = lowest common ancestor at time i

if $\mathrm{LCA}_i(X,Y) \neq X$ then: at time i, to access X...

...we must go through $\mathrm{LCA}_i(X,Y)$

So? ←



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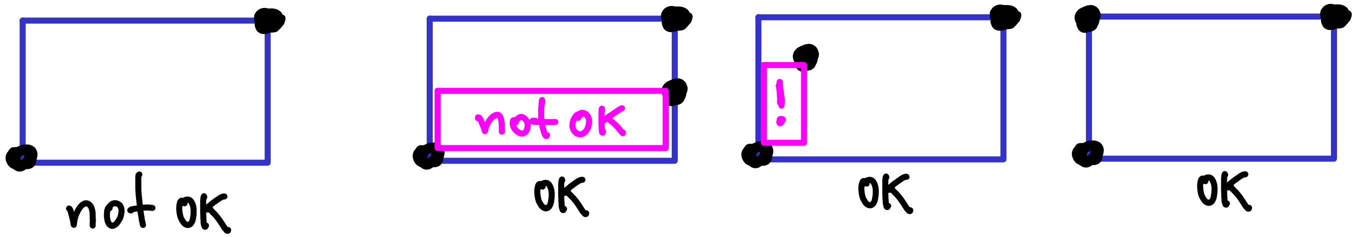

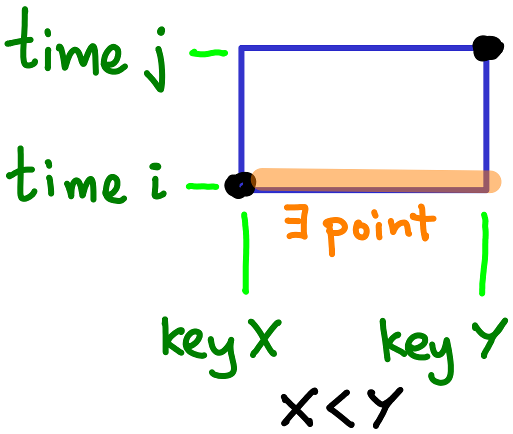

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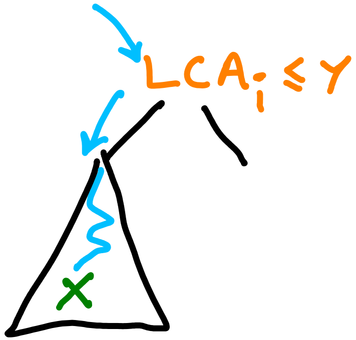

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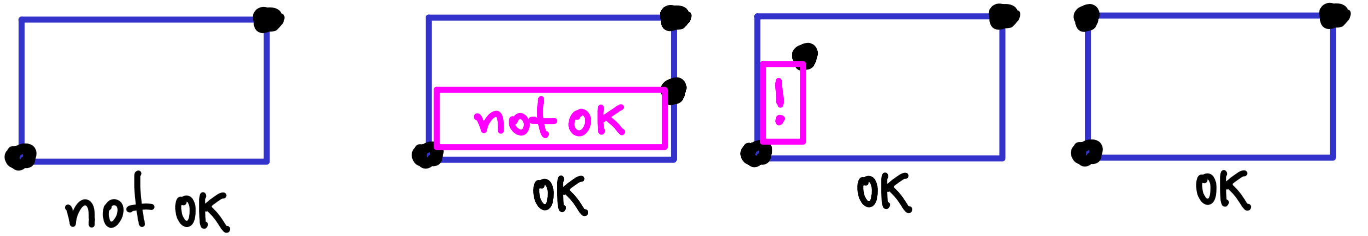
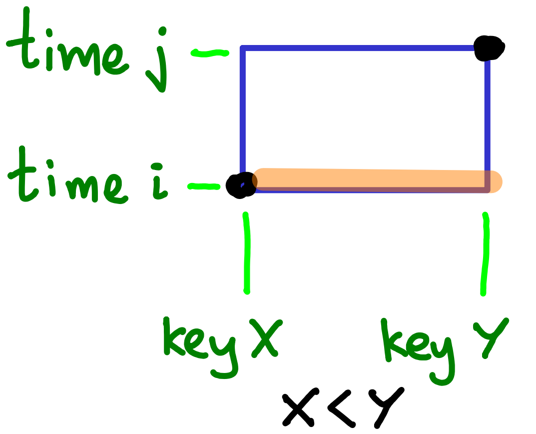
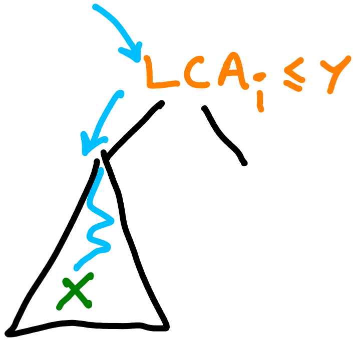




$LCA_i(X,Y)$ = lowest common ancestor at time i

if $LCA_i(X,Y) \neq X$ then: at time i, to access X...

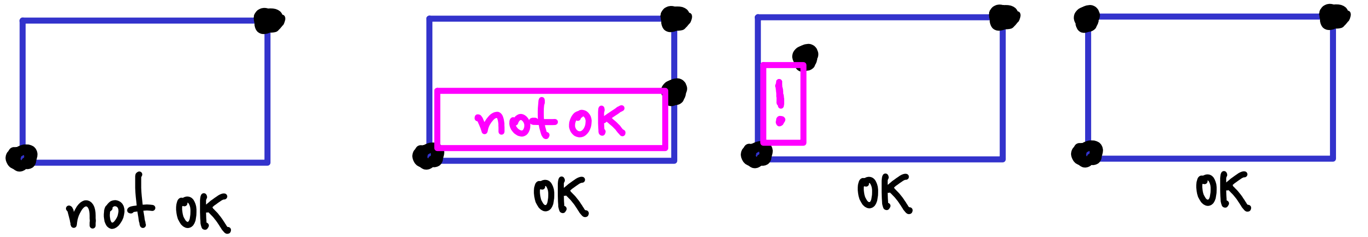
...we must go through $LCA_i(X,Y)$

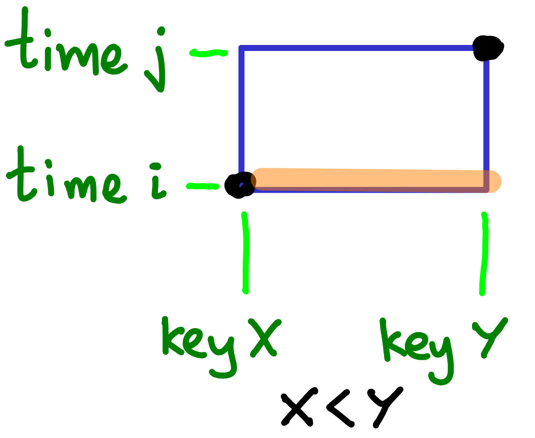


else

$X = LCA_i$

Theorem: the set of key accesses (over all ops) corresponds to a diagram where no two points form opposite corners of a closed rectangular region that is empty of other points

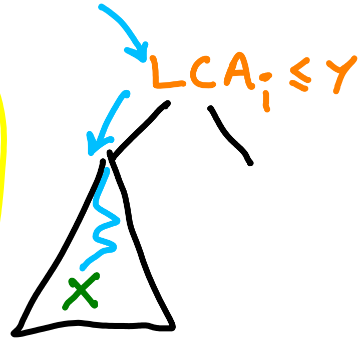




$LCA_i(X,Y)$ = lowest common ancestor at time i

if $LCA_i(X,Y) \neq X$ then: at time i, to access X...

...we must go through $LCA_i(X,Y)$



else if $LCA_j(X,Y) \neq Y$

?

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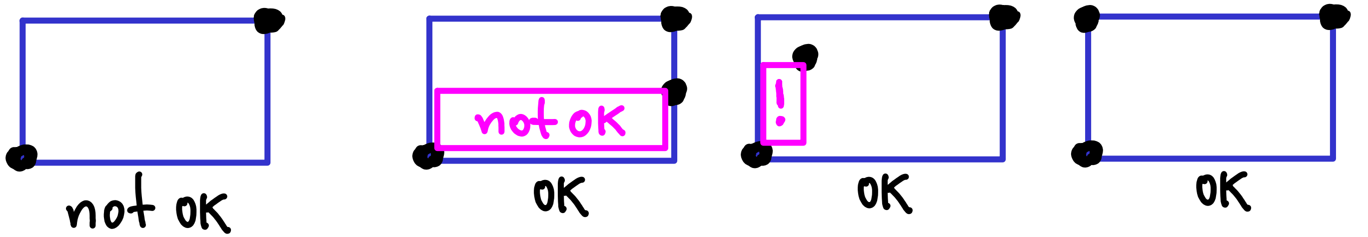

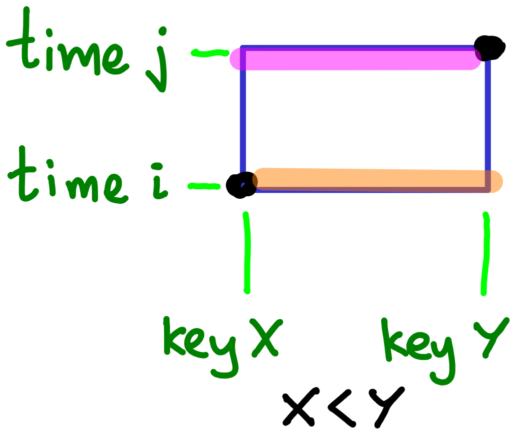

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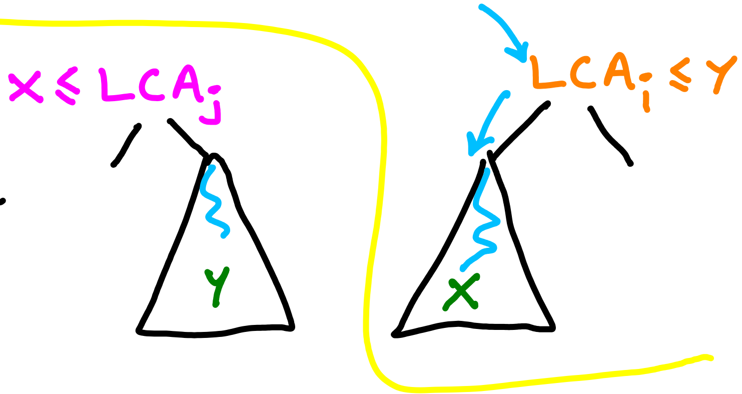
else if $LCA_j(X,Y) \neq Y$ then we must go through $LCA_j(X,Y)$



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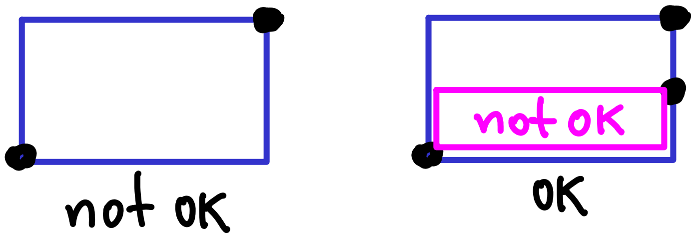

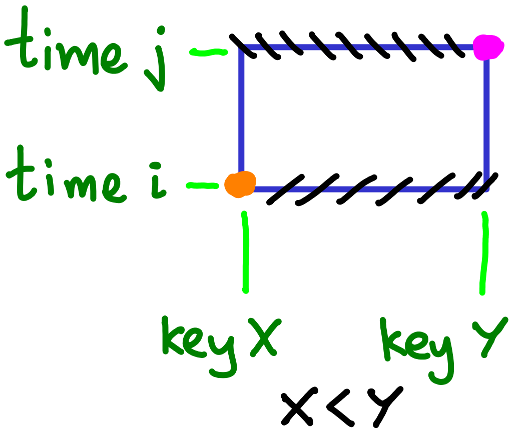

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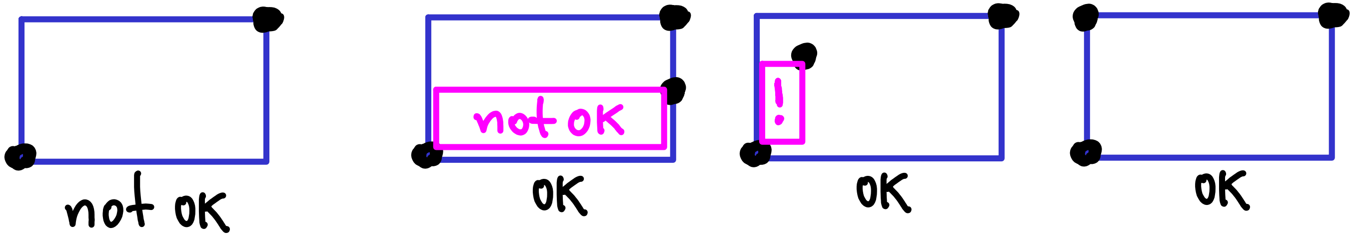
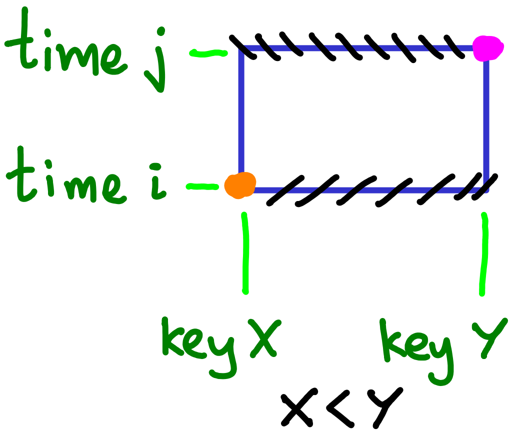
So far the only hope of contradicting the theorem is

Theorem: the set of key accesses (over all ops) corresponds to a diagram where no two points form opposite corners of a closed rectangular region that is empty of other points





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So far the only hope of contradicting the theorem is

but then some time t $(i < t \leq j)$ we must rotate X

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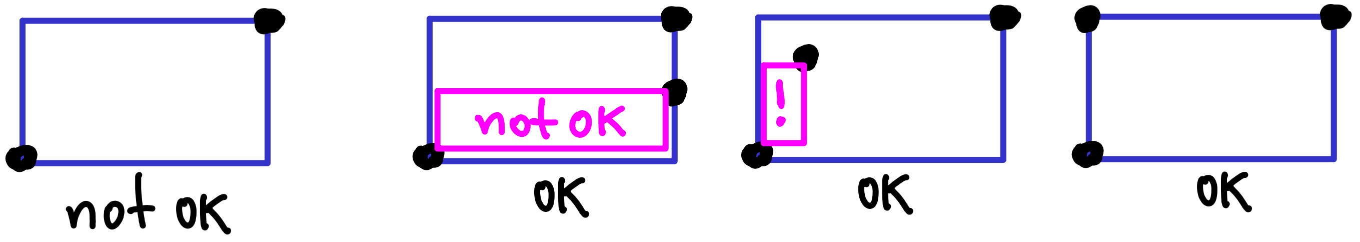

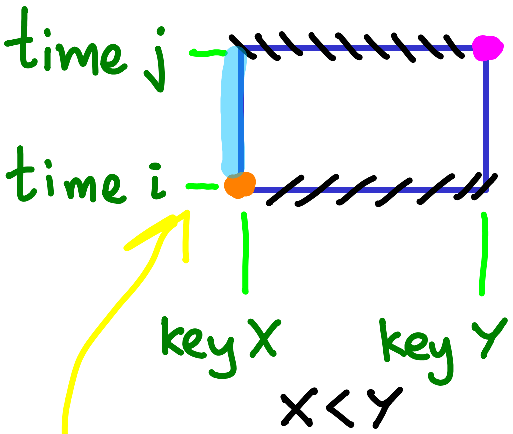

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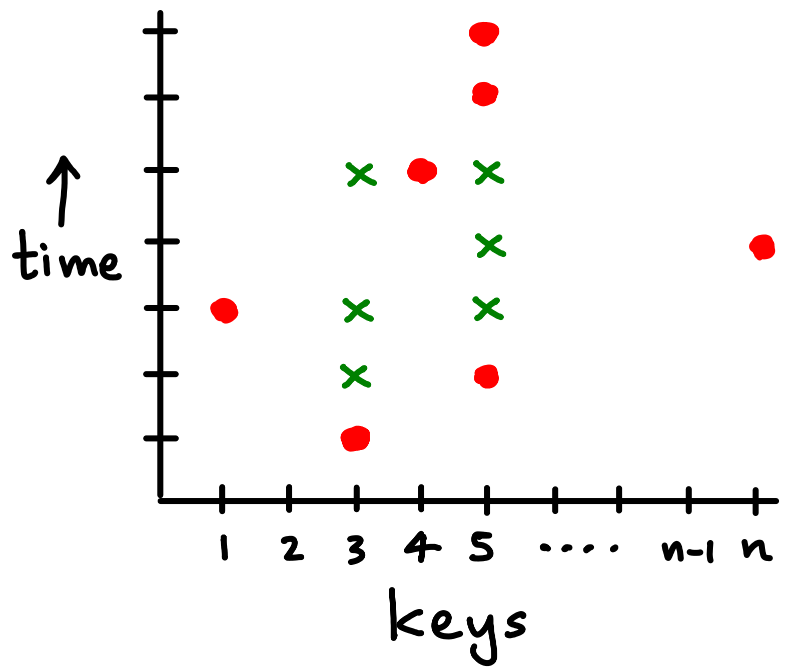
□

Theorem: any diagram where no two points form opposite corners of a closed rectangular region that is empty of other points corresponds to the set of key accesses for some BST algorithm

(reverse)



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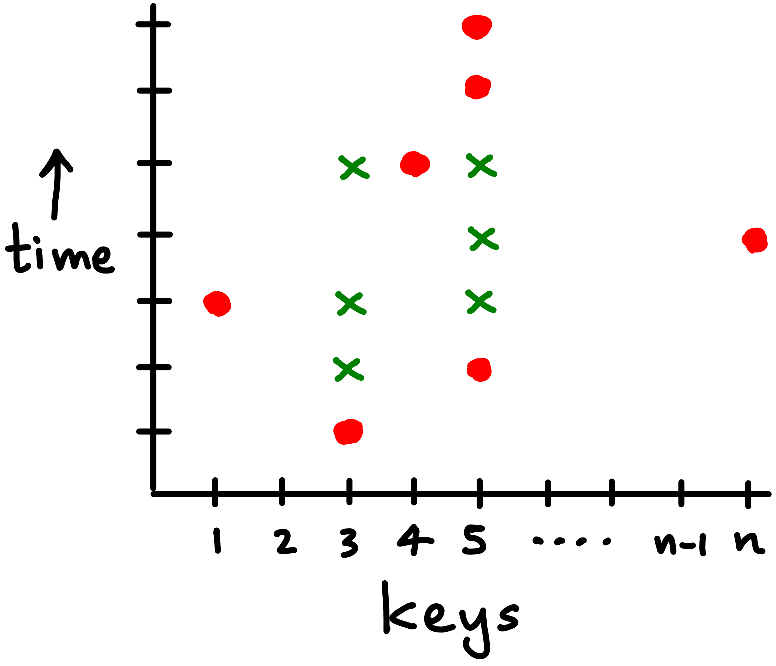

$\boxed{3}$

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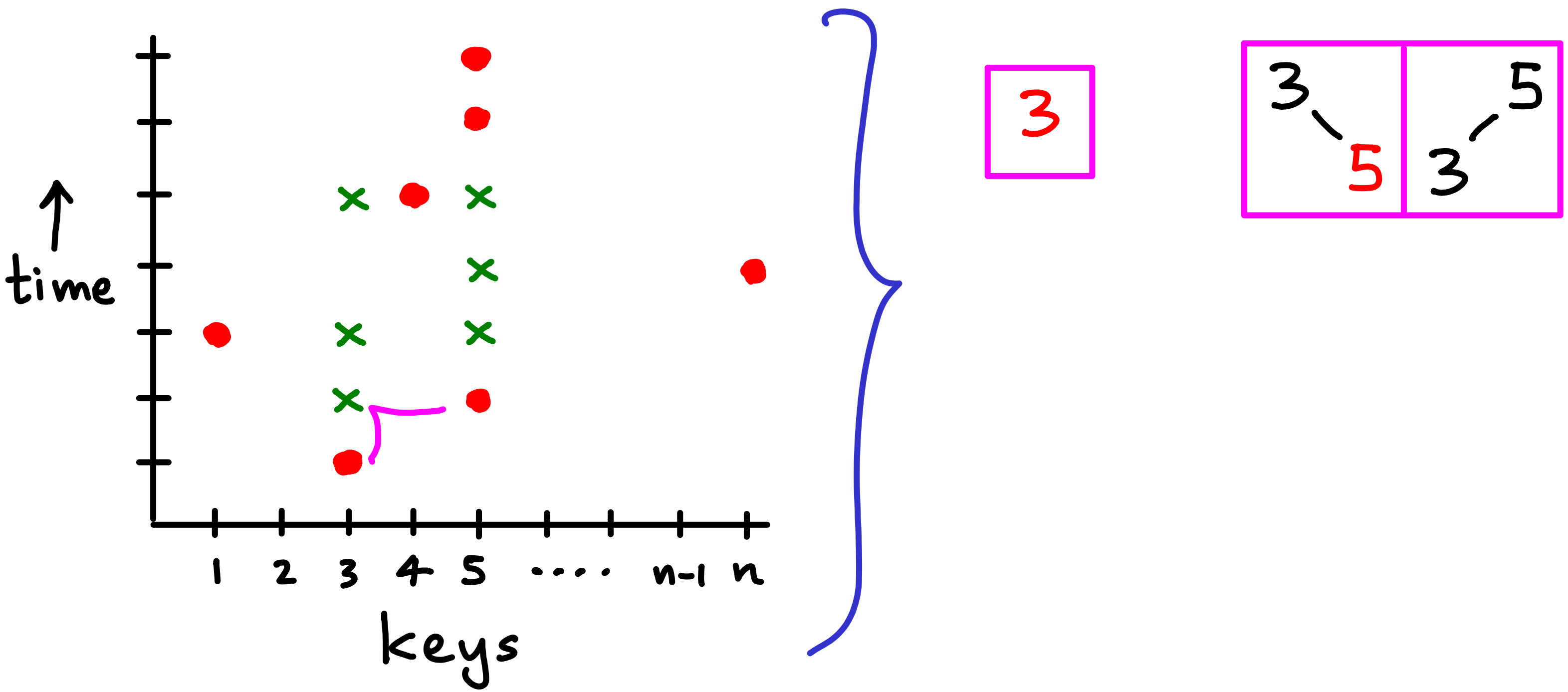

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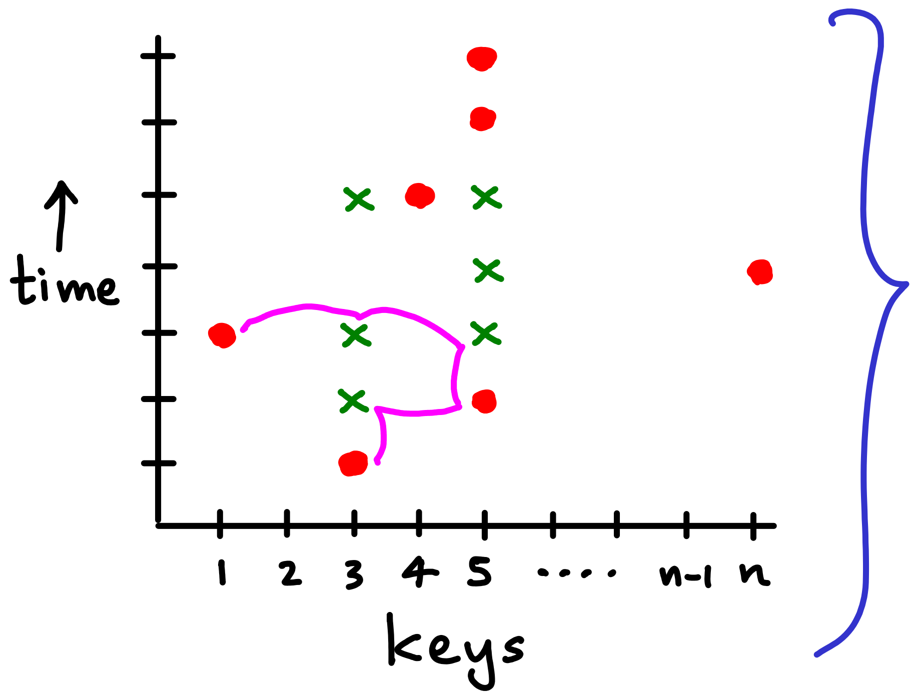

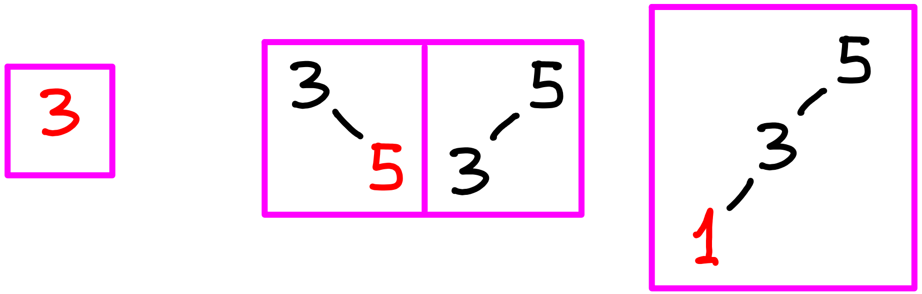

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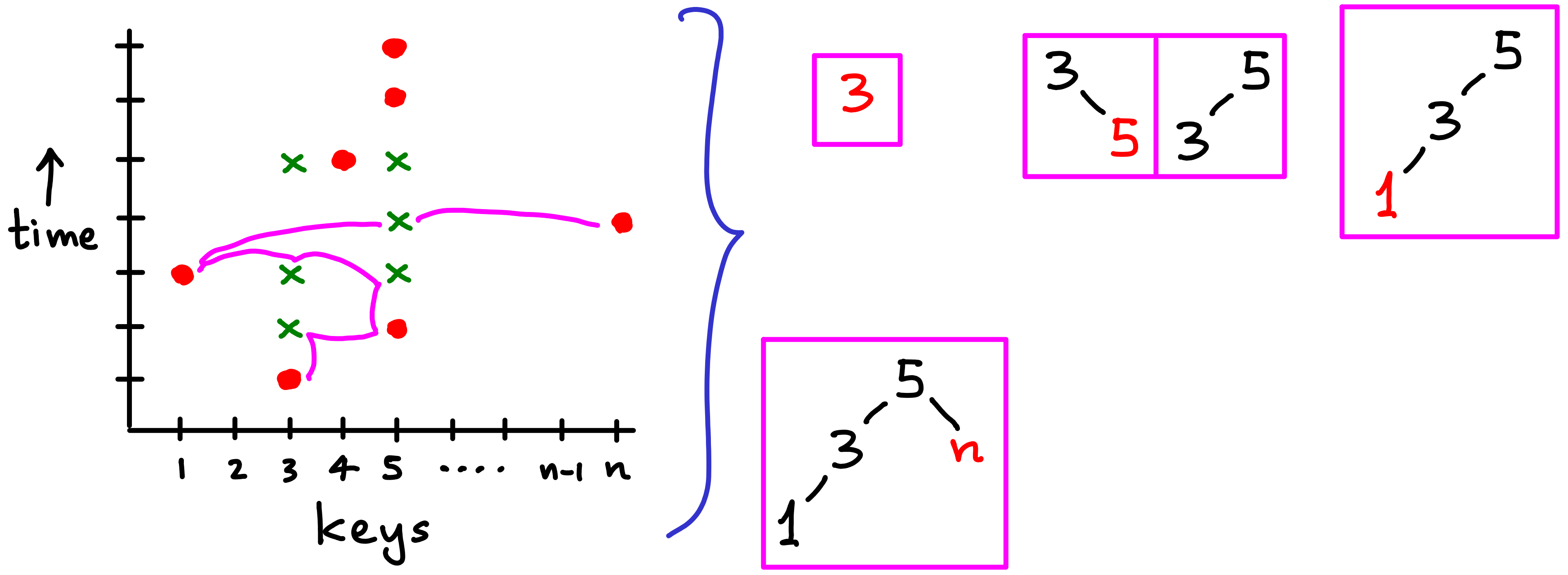

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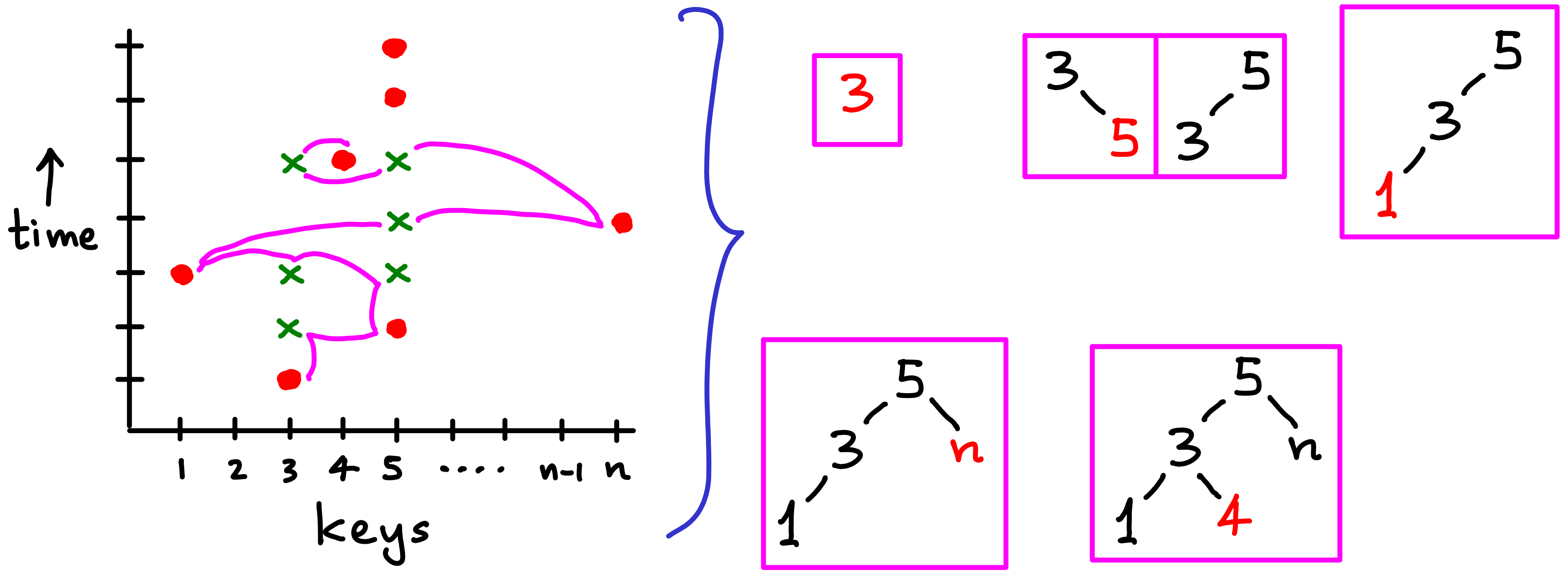

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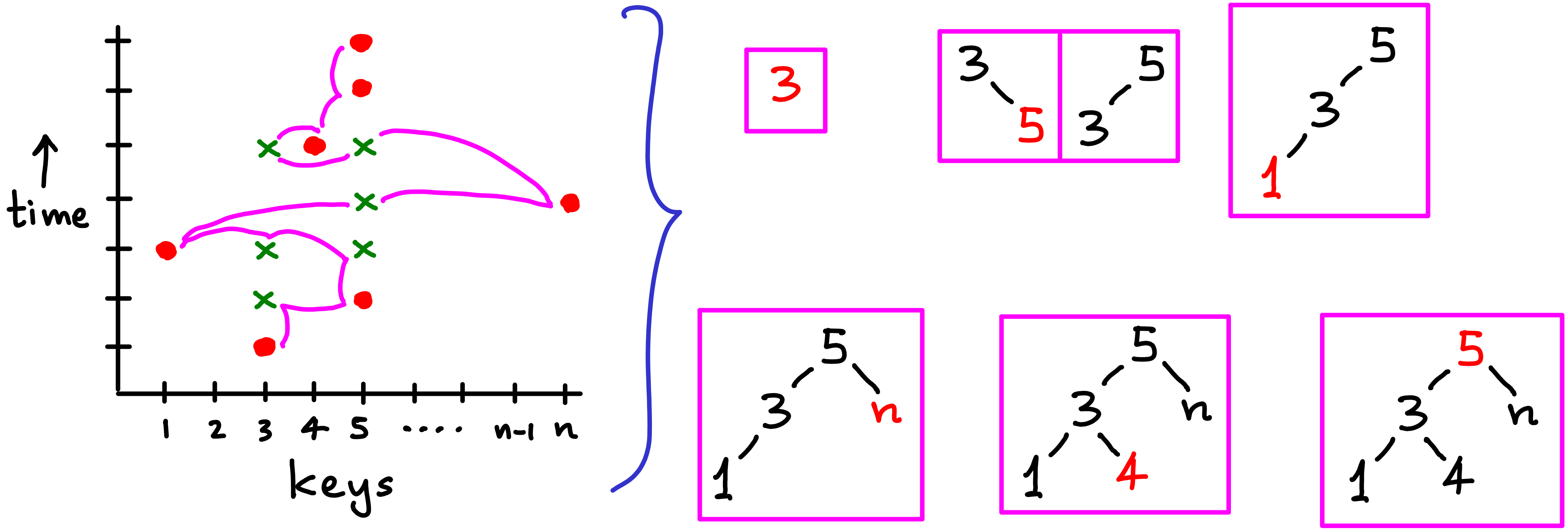
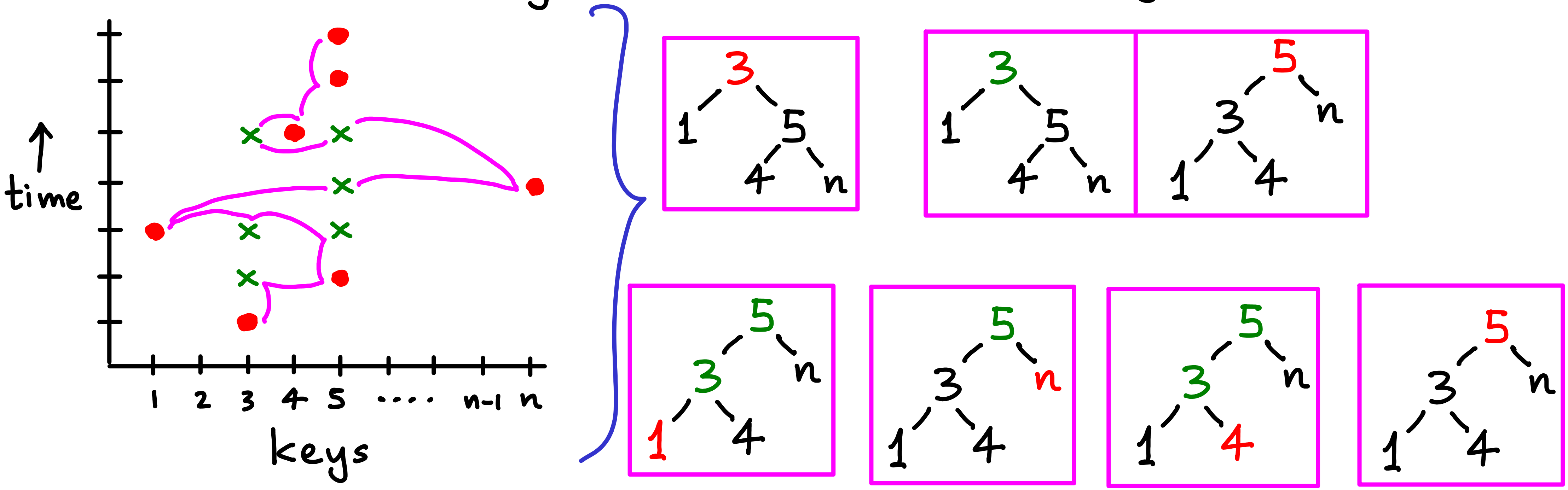


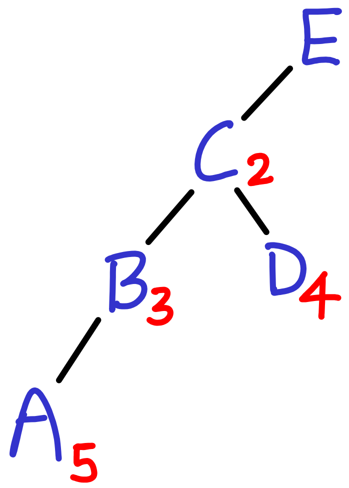
Theorem: // version for static set of keys (no insert-delete)

(reverse) any diagram where no two points form opposite corners of a closed rectangular region that is empty of other points corresponds to the set of key accesses for some BST algorithm



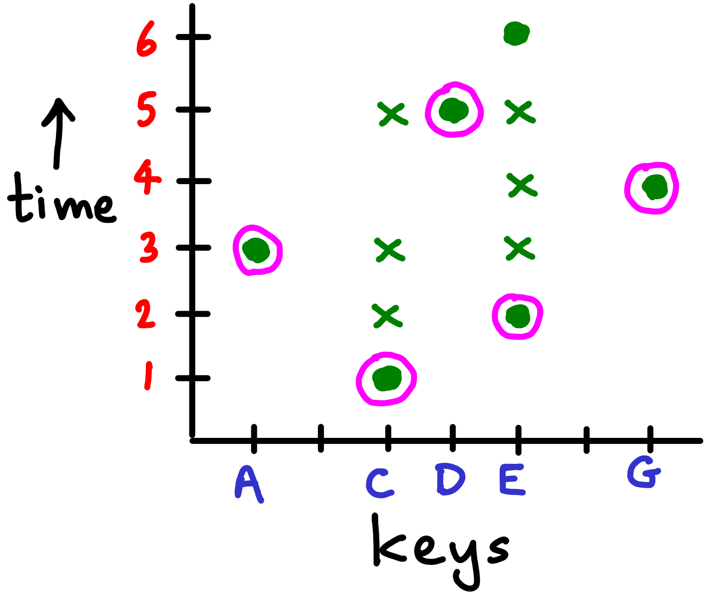
We will prove the theorem constructively, using TREAPS

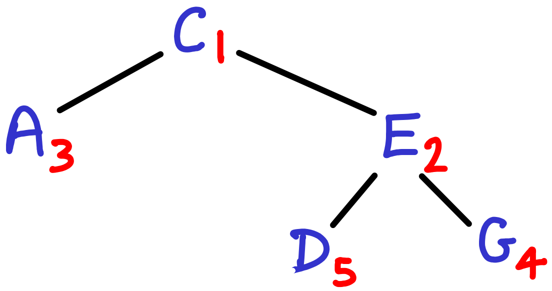
BST + HEAP



If heap values (and keys) are unique then shape is too.

Given a diagram, let every key have treap priority = lowest access time





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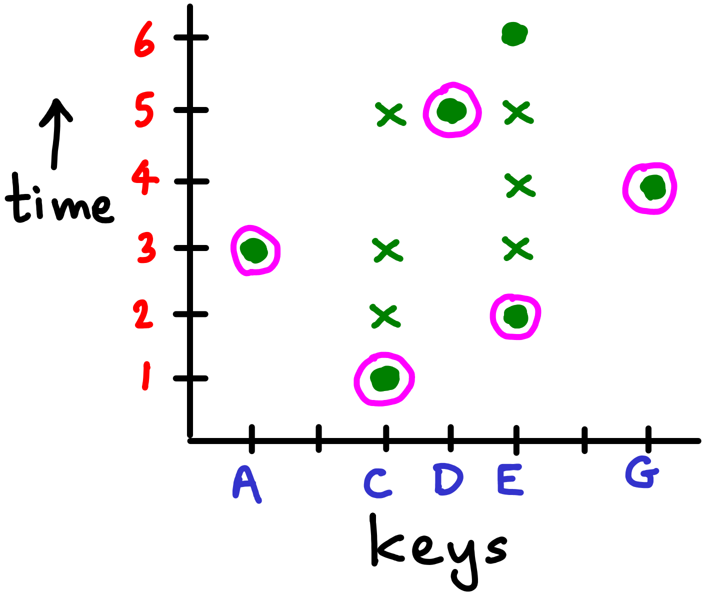
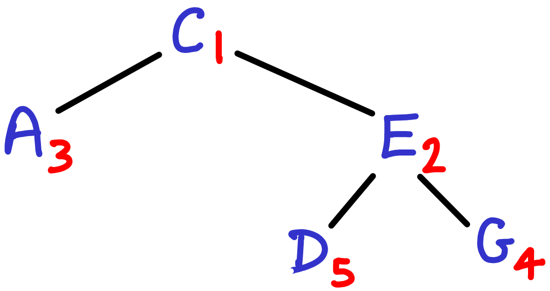
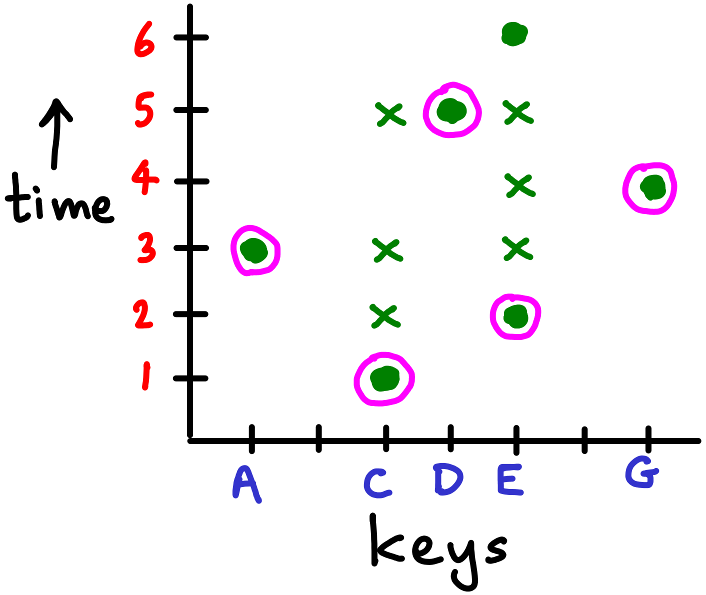
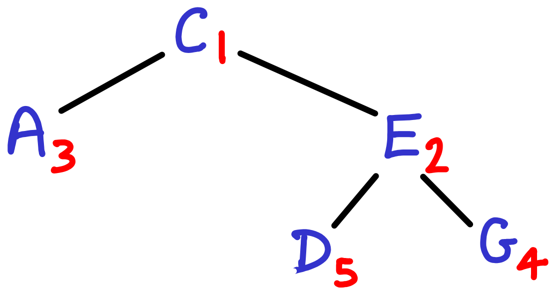




Treap priorities will increase over time

Given a diagram, let every key have treap priority = lowest access time

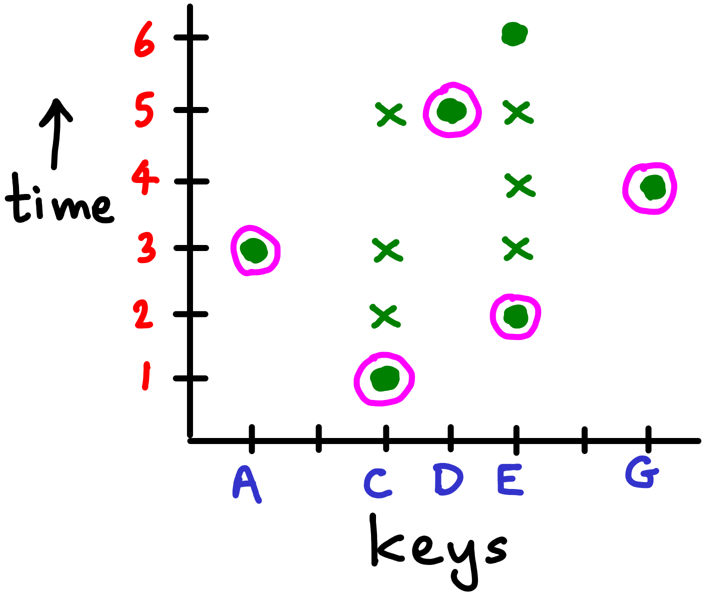


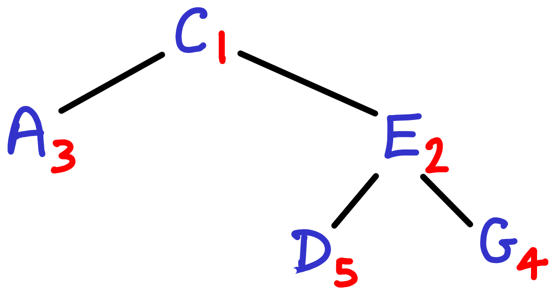


Treap priorities will increase over time

top of treap:
all nodes with
priority = root

Given a diagram, let every key have treap priority = lowest access time





Treap priorities will increase over time

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At time = min priority, we must access all of top and nothing below in treap

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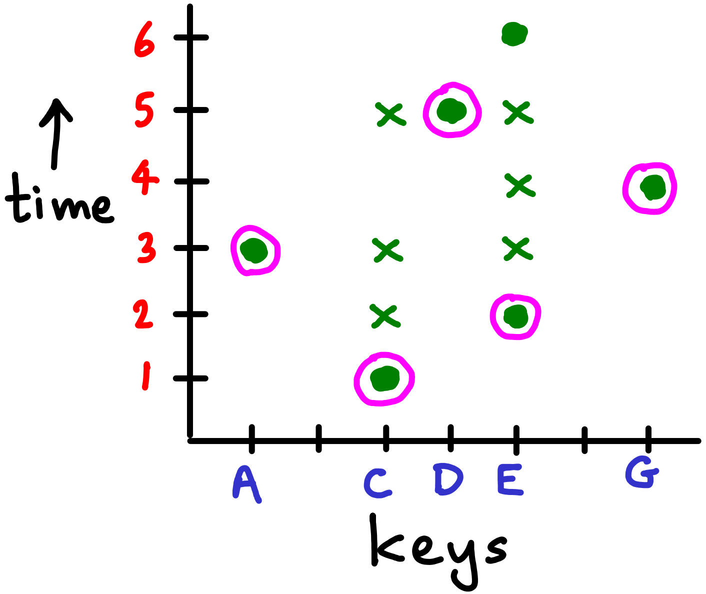
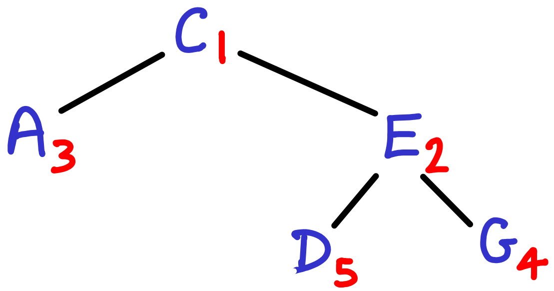


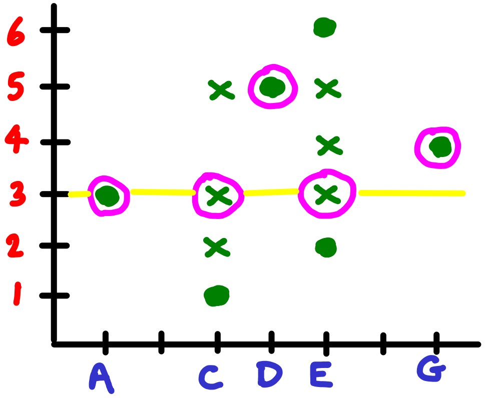


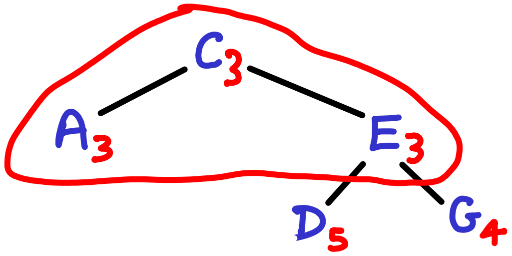
Treap priorities will increase over time

top of treap:
all nodes with
priority = root

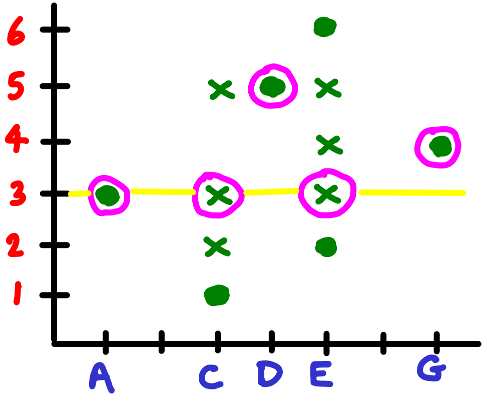
At time = min priority, we must access all of top
and nothing below in treap

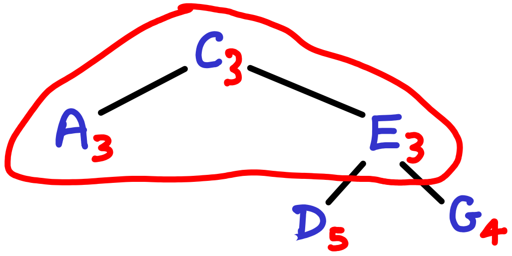
top may change shape via rotations,
and all priorities within will increase in value
(to next required access time)


6
5
4
3
2
1
A
C
D
E
G


C3
A3
E3
D5
G4

How should priorities in top change?





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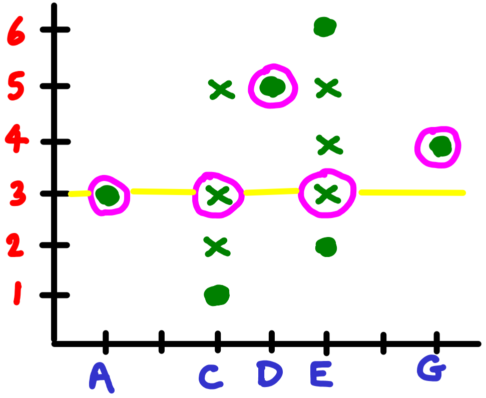

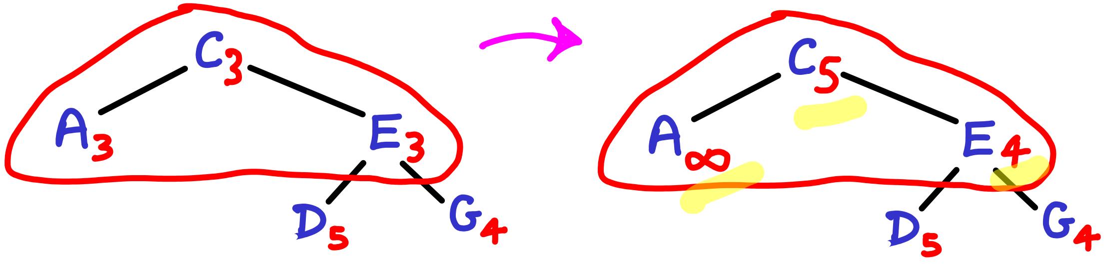

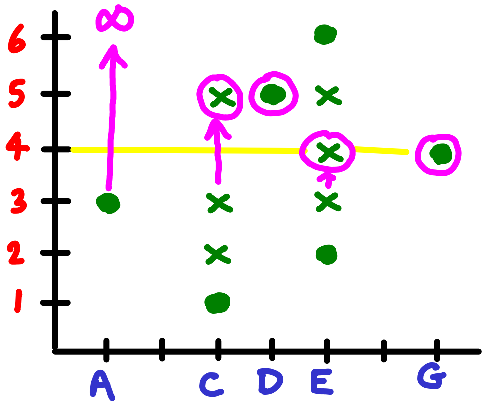

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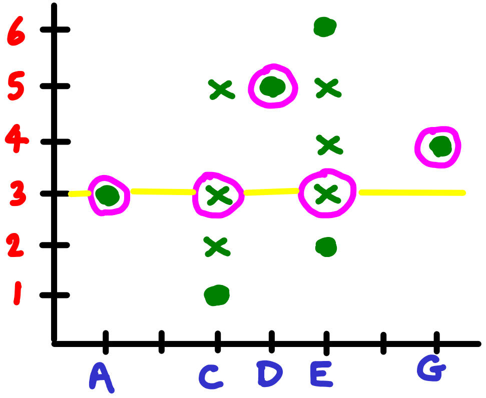

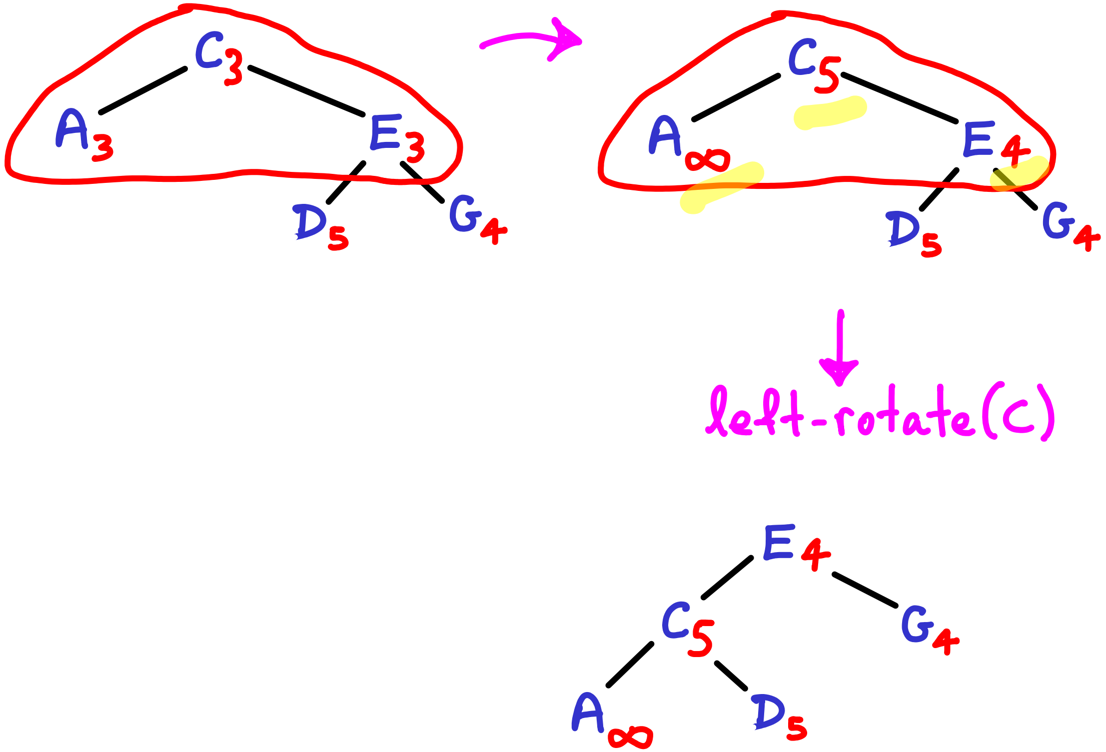

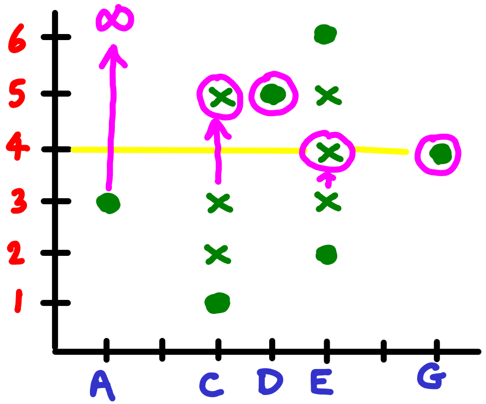

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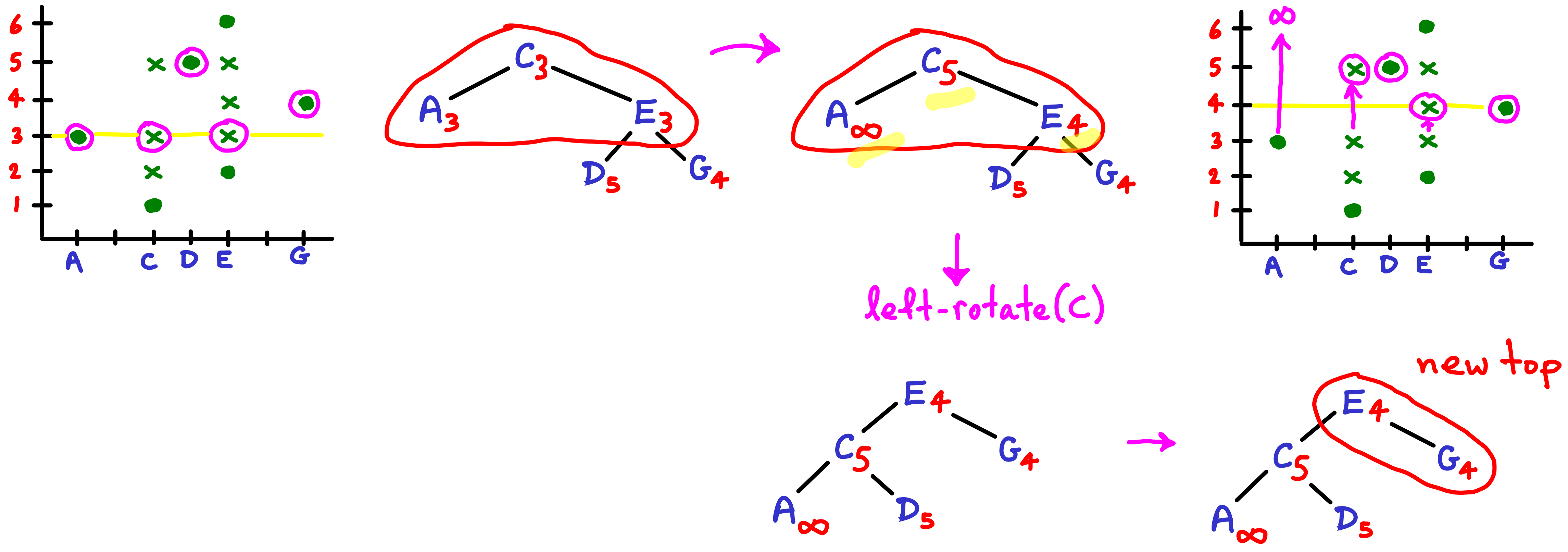

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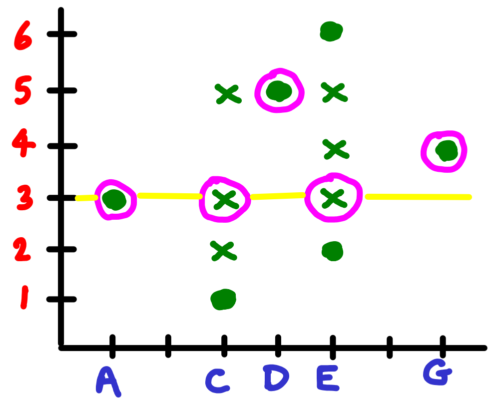
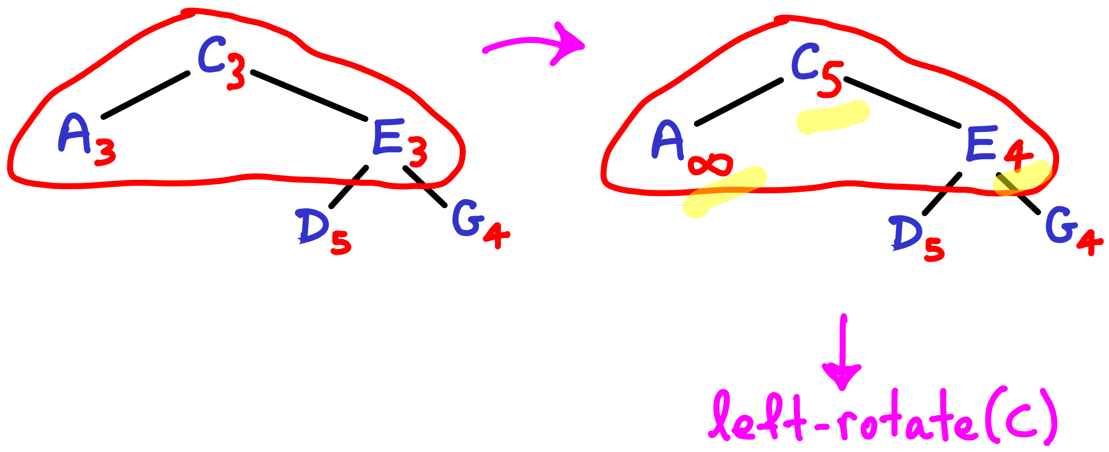
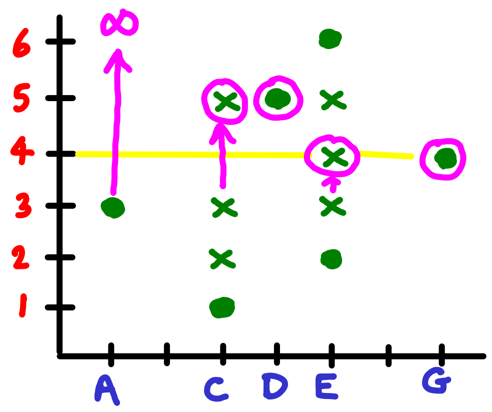
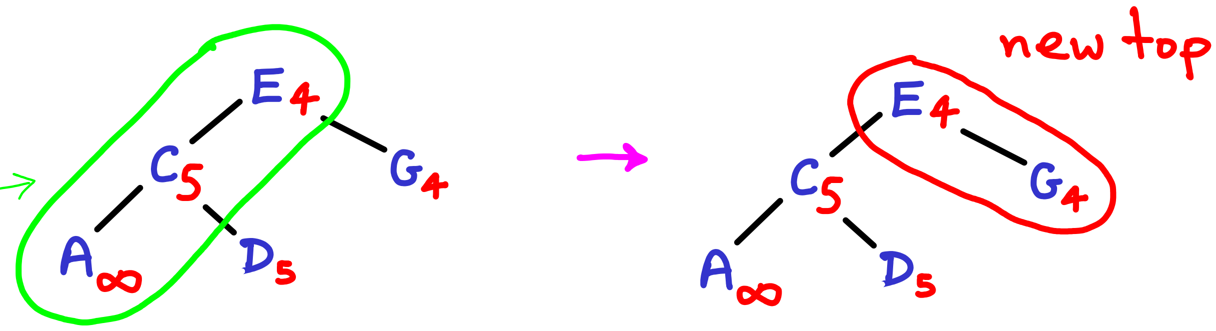






Just convert given shape of top
to whatever shape we need
s.t. heap property is restored



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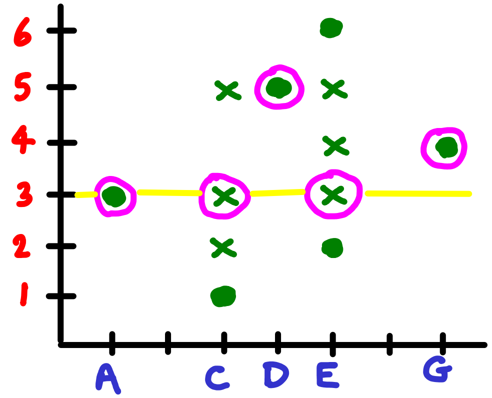

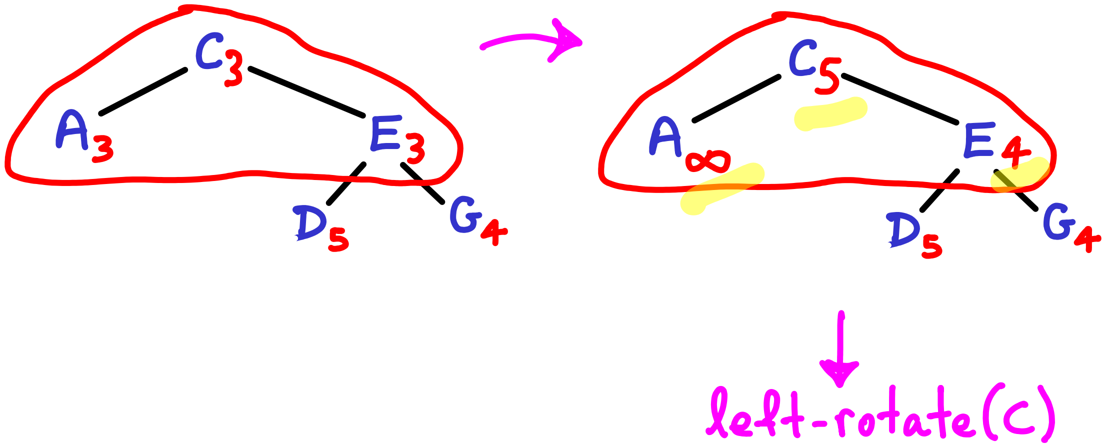

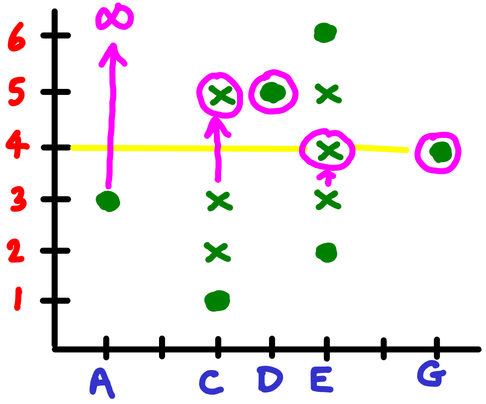

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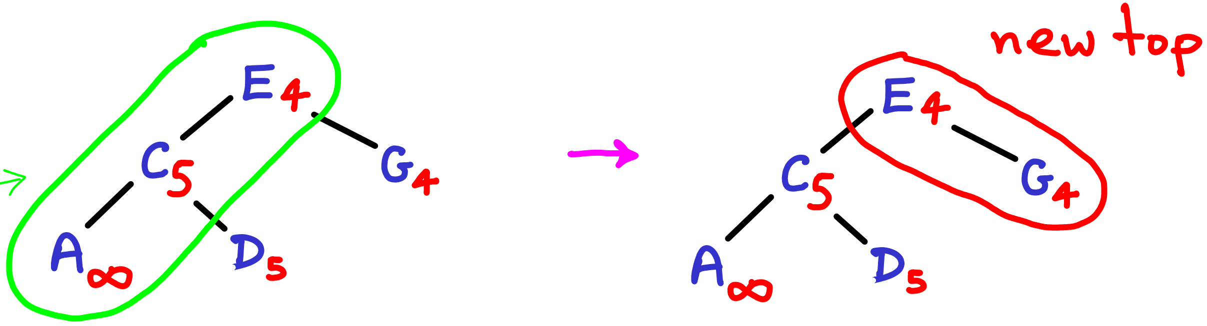
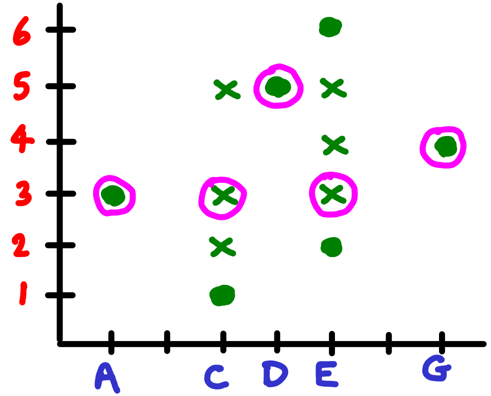
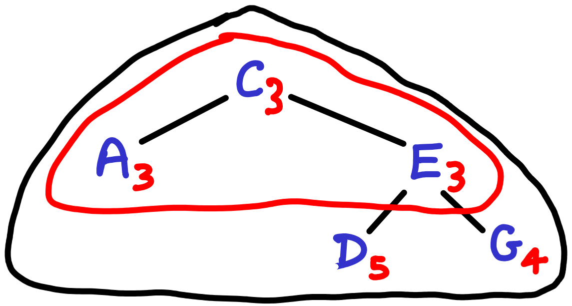
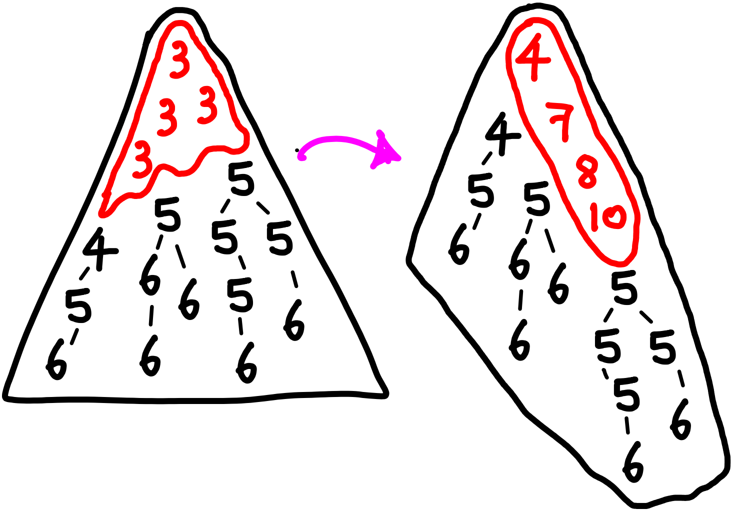


From COMP-160 we know this can always be done with <2n rotations



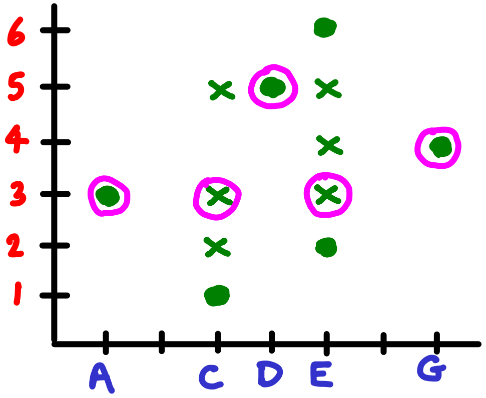


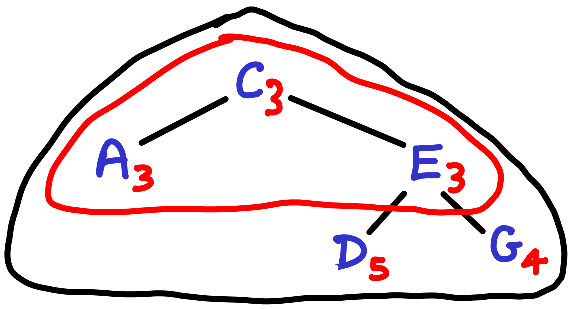
Method so far: at every time step, we will access (and possibly rotate) precisely the nodes in top, and update their priorities.



This restores the top as a treap.

All other nodes passively follow.

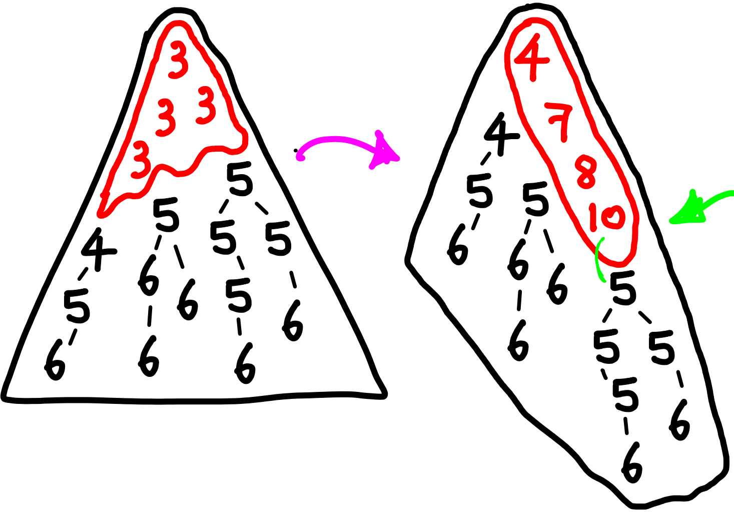




Method so far: at every time step, we will access (and possibly rotate) precisely the nodes in top, and update their priorities.

This restores the top as a treap.

All other nodes passively follow.

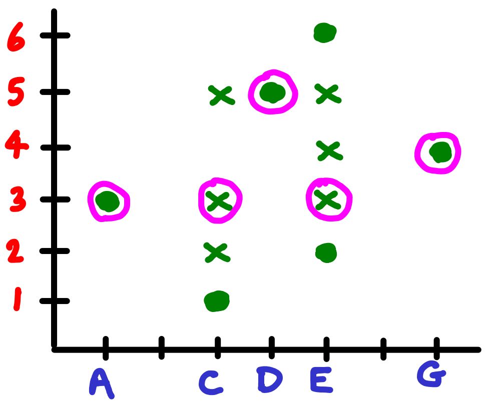


Claim:

new priorities won't violate heap property globally.

(this won't happen)

...so we always have a treap



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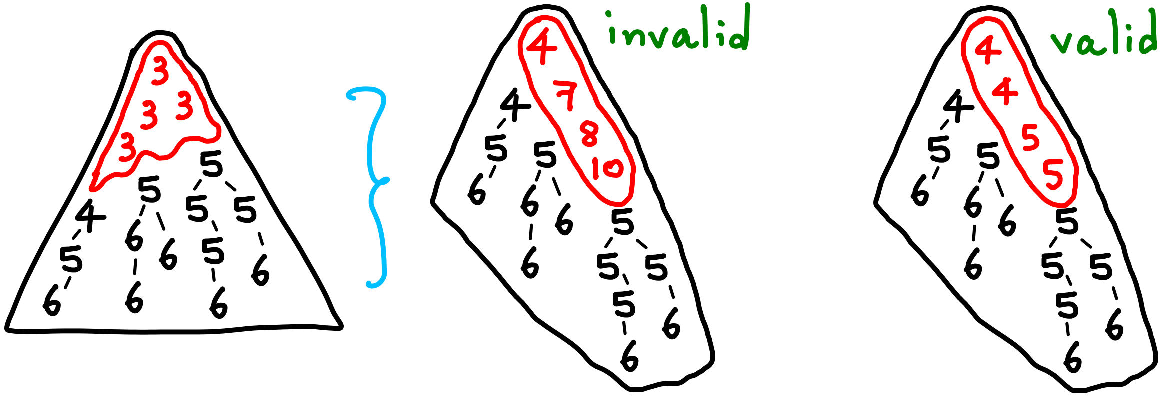
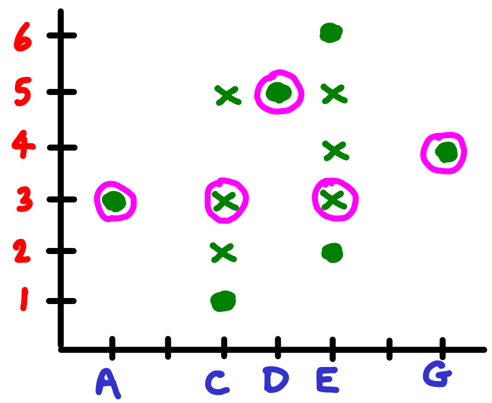


PROOF...



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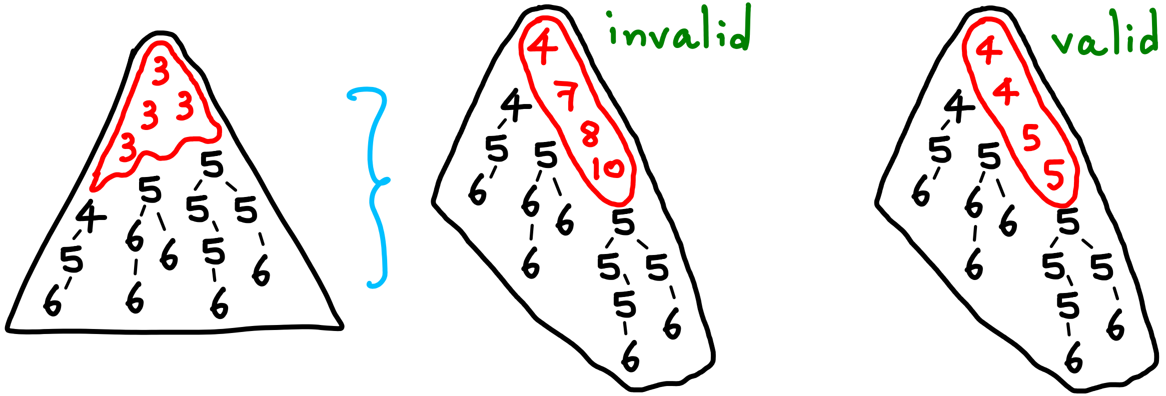
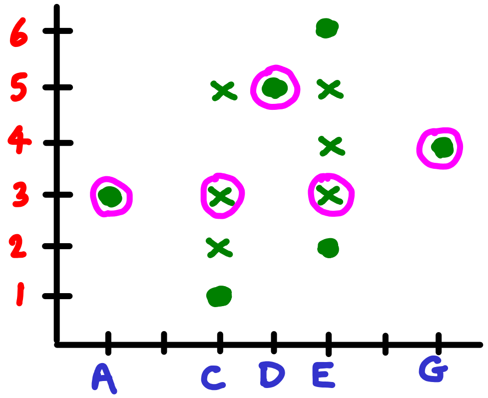


Proof by contradiction:



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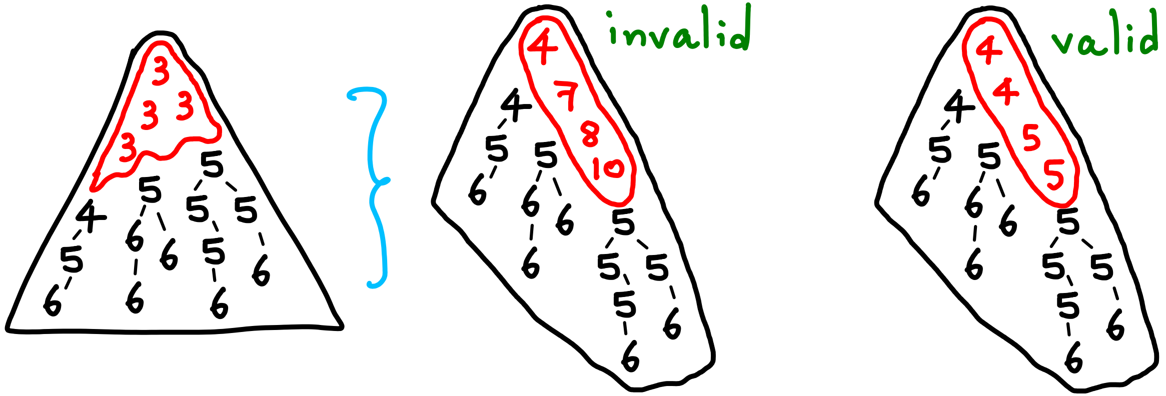
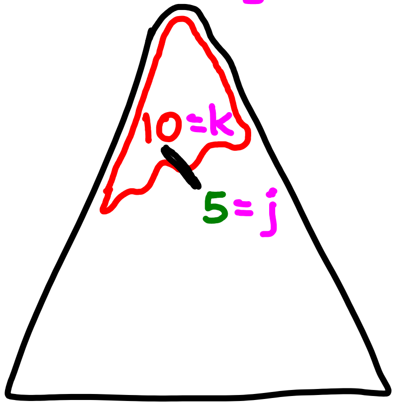
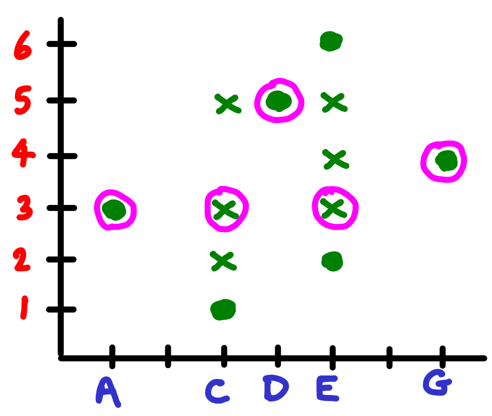


Proof by contradiction: suppose ∃ edge X — Y s.t. $\underbrace{\text{new priority}(X)}_{k} > \underbrace{\text{priority}(Y)}_{j}$

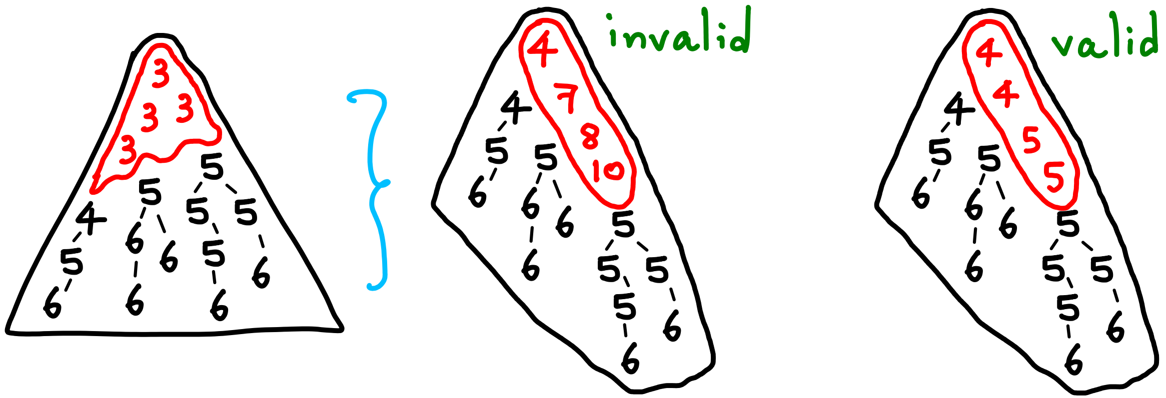


keys:

wlog X<Y



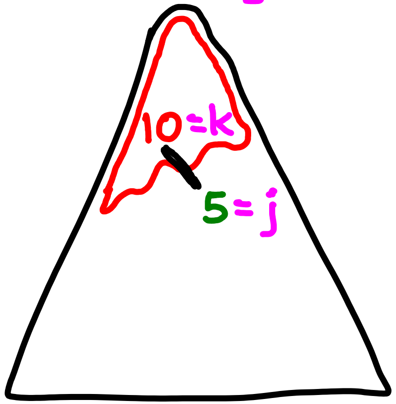
Claim:
new priorities won't violate heap property

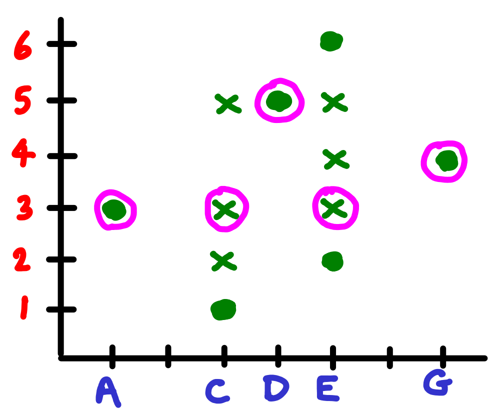


Proof by contradiction: suppose $\exists$ edge X–Y s.t. $\underbrace{\text{new priority}(X)}_{k} > \underbrace{\text{priority}(Y)}_{j}$

We know $\overbrace{\text{priority}(Y)}^{j} > \overbrace{\text{old priority}(X)}^{i}$

WHY?





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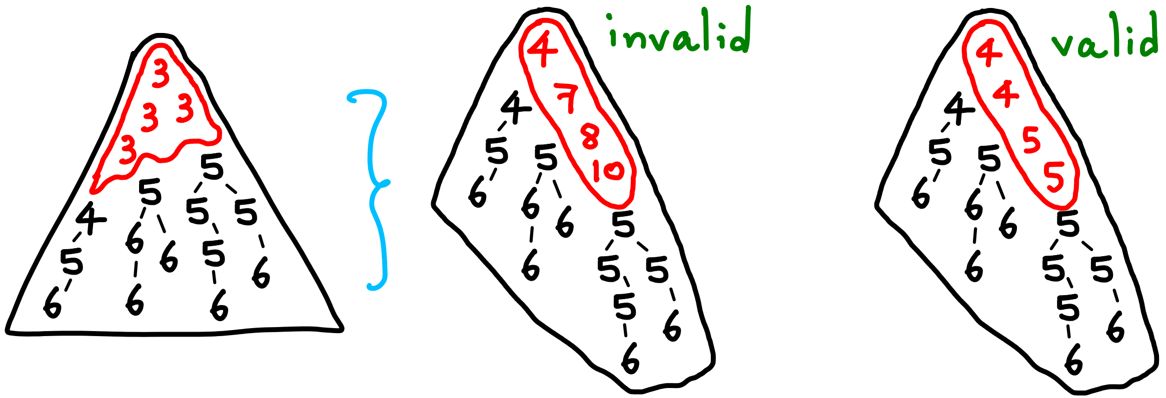
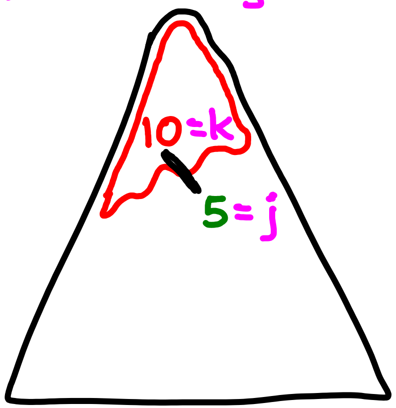
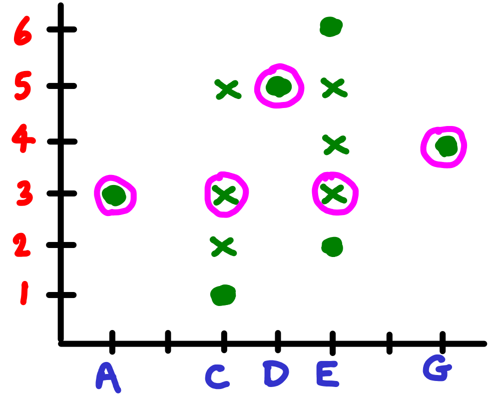


Proof by contradiction: suppose $\exists$ edge X–Y s.t. $\underbrace{\text{new priority}(X)}_{k} > \underbrace{\text{priority}(Y)}_{j}$

We know $\overbrace{\underbrace{\text{priority}(Y)}_{\text{wasn't in top}}}^{j} > \overbrace{\text{old priority}(X)}^{i}$





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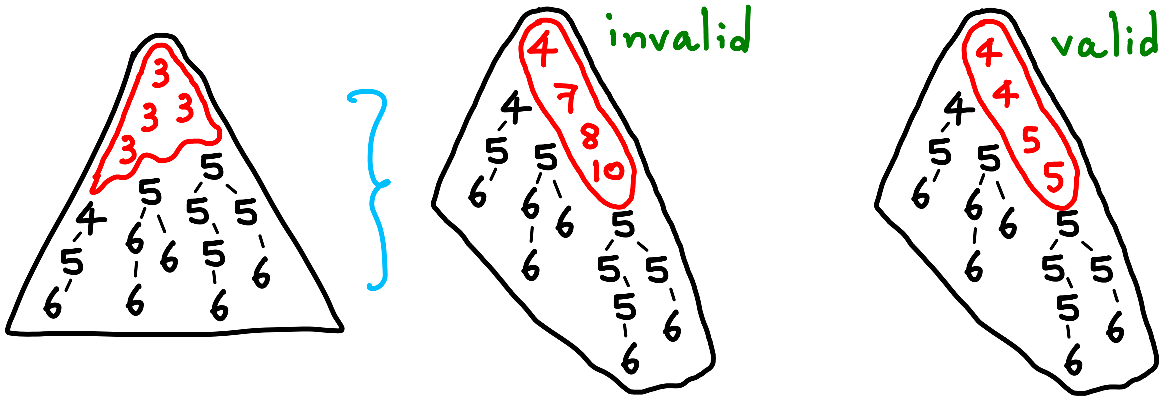

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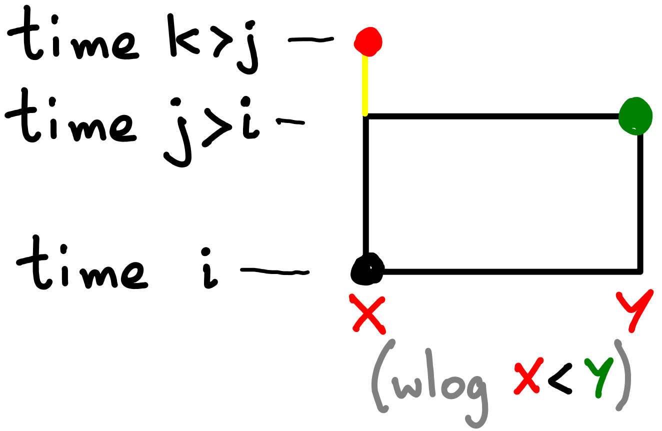
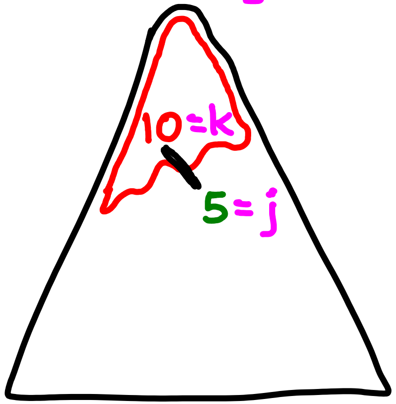
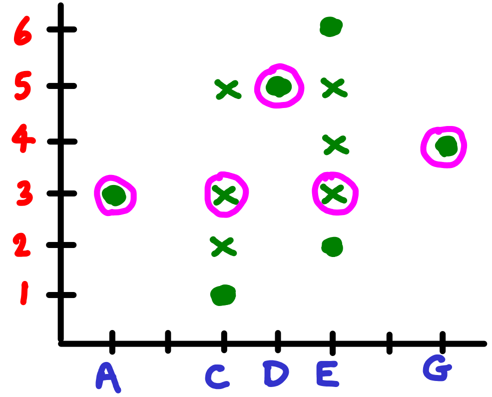


We know $\overbrace{\text{priority}(Y)}^{j} \overset{>}{>} \overbrace{\text{old priority}(X)}^{i}$

...?





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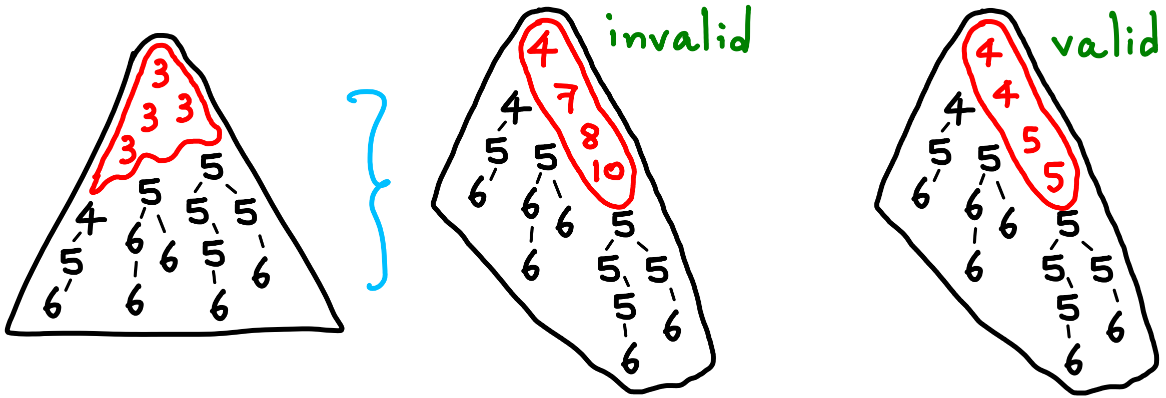

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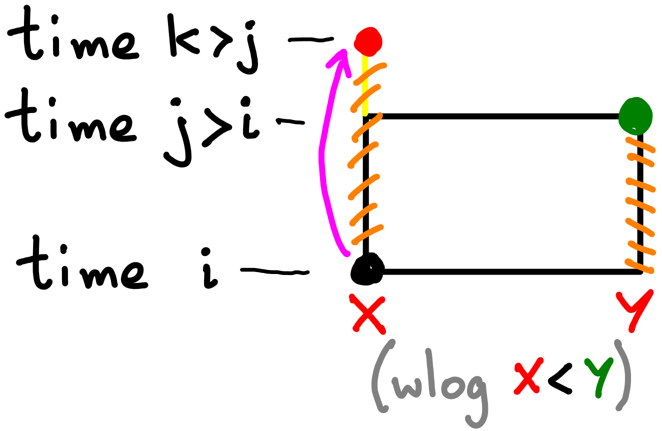
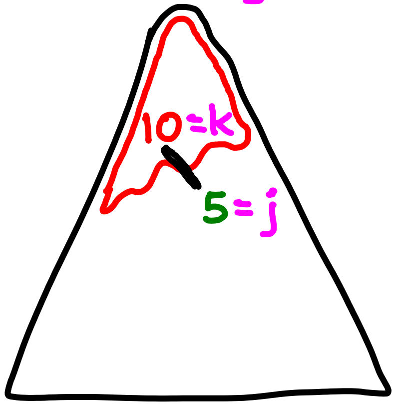
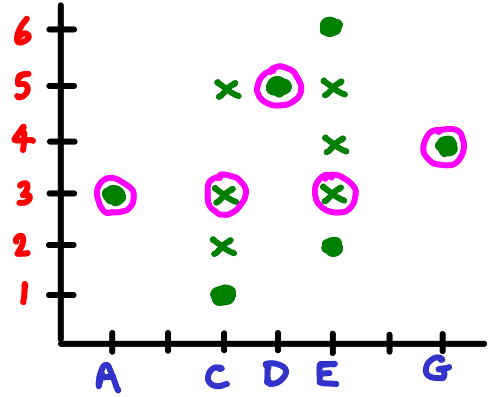


We know $\overbrace{\text{priority}(Y)}^{j} > \overbrace{\text{old priority}(X)}^{i}$

↳ so left & right sides of box are empty





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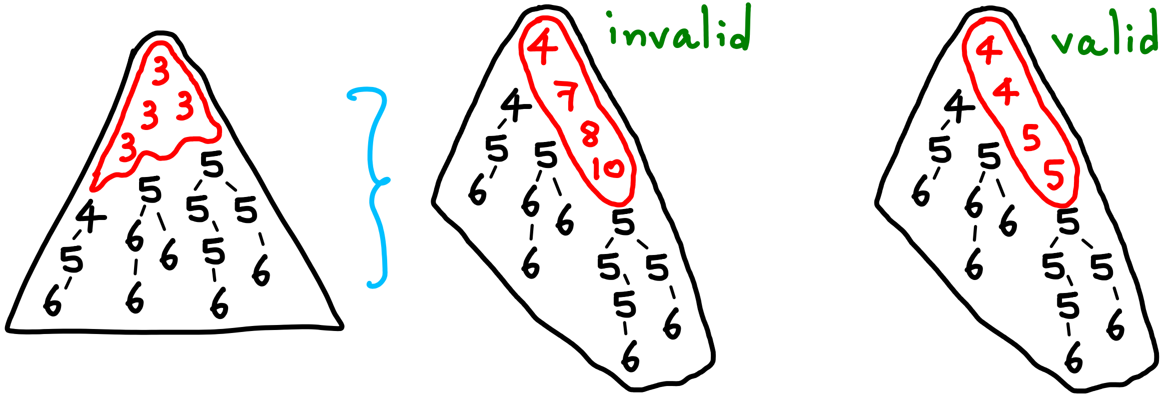

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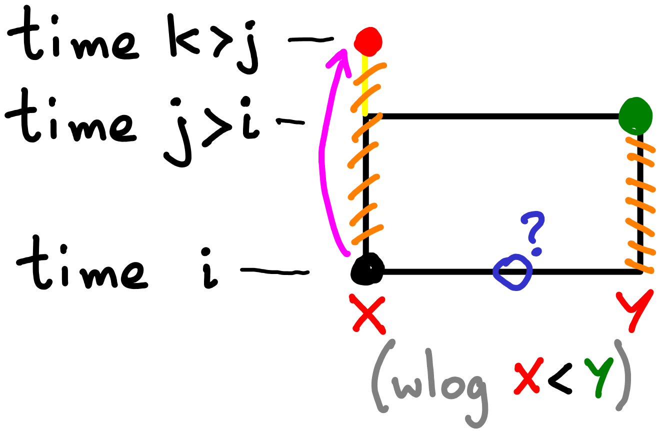
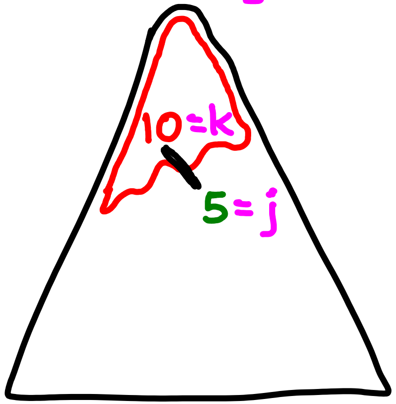
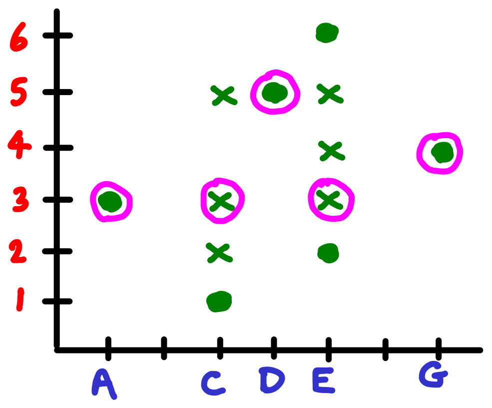


We know $\overbrace{\text{priority}(Y)}^{j} > \overbrace{\text{old priority}(X)}^{i}$

↪ so left & right sides of box are empty

What about bottom side?





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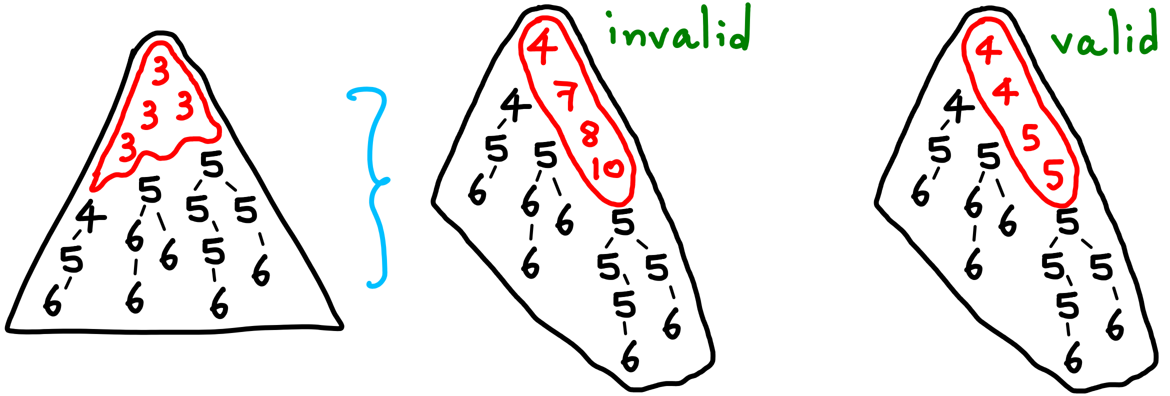

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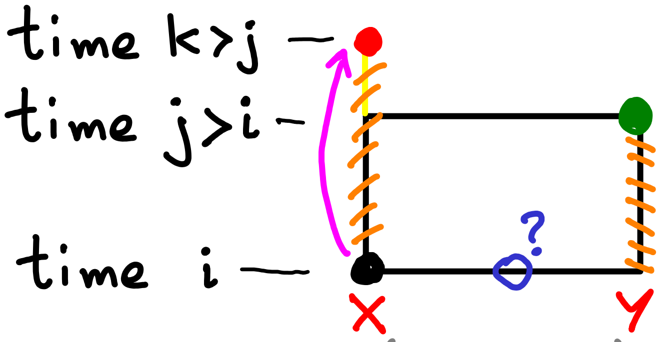
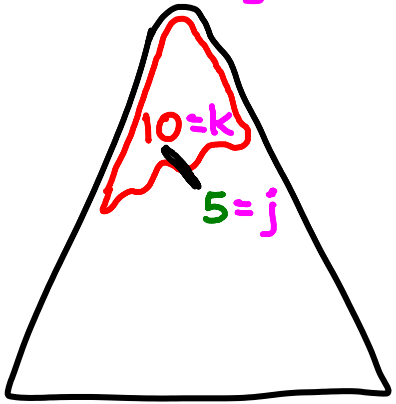


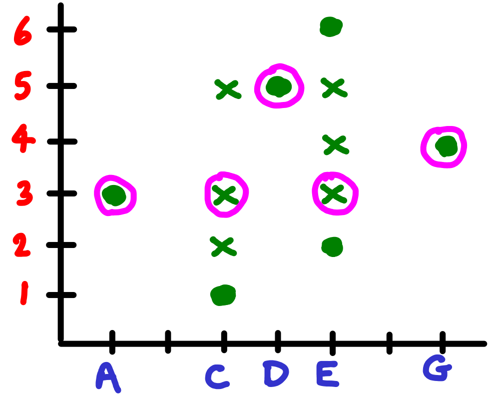
We know $\overbrace{\text{priority}(Y)}^{j} > \overbrace{\text{old priority}(X)}^{i}$

↳ so left & right sides of box are empty

What about bottom side?

Would need X<key<Y but then...





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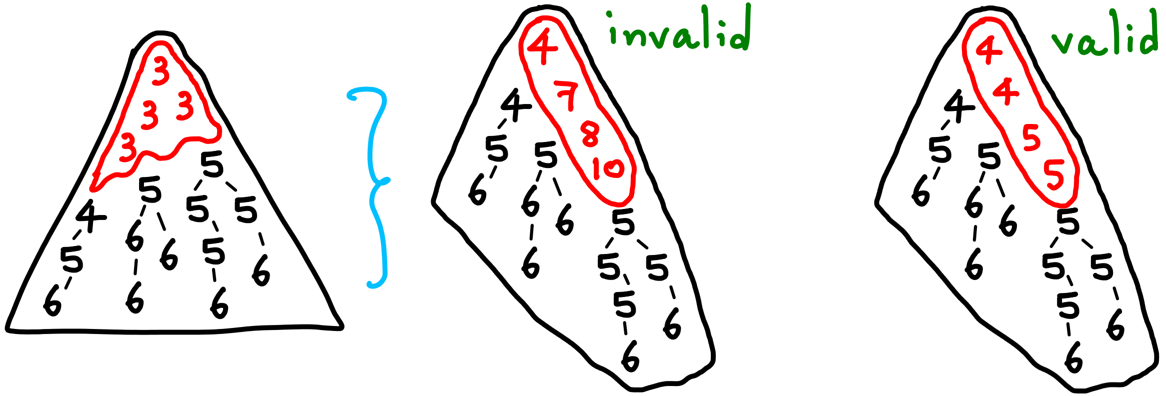

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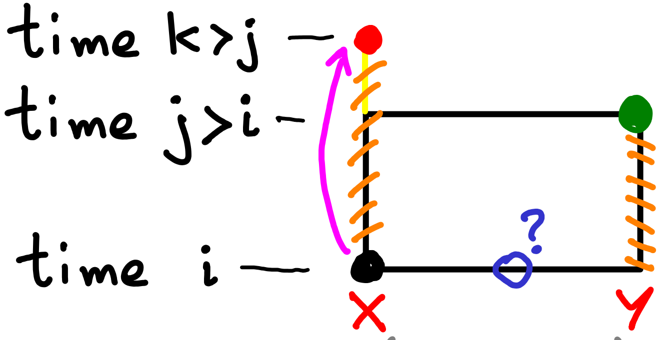


(wlog X<Y)

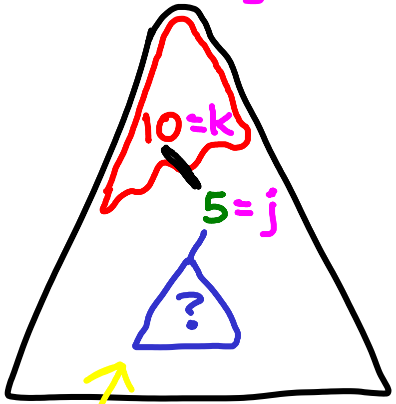
We know $\overbrace{\text{priority}(Y)}^{j} > \overbrace{\text{old priority}(X)}^{i}$

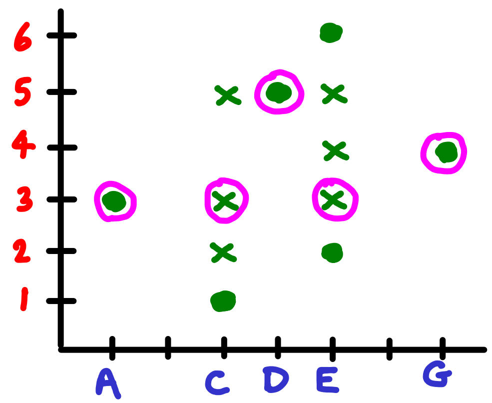
↳ so left & right sides of box are empty

What about bottom side?

Would need X<key<Y but then...

...key is in left subtree of Y (at time i)





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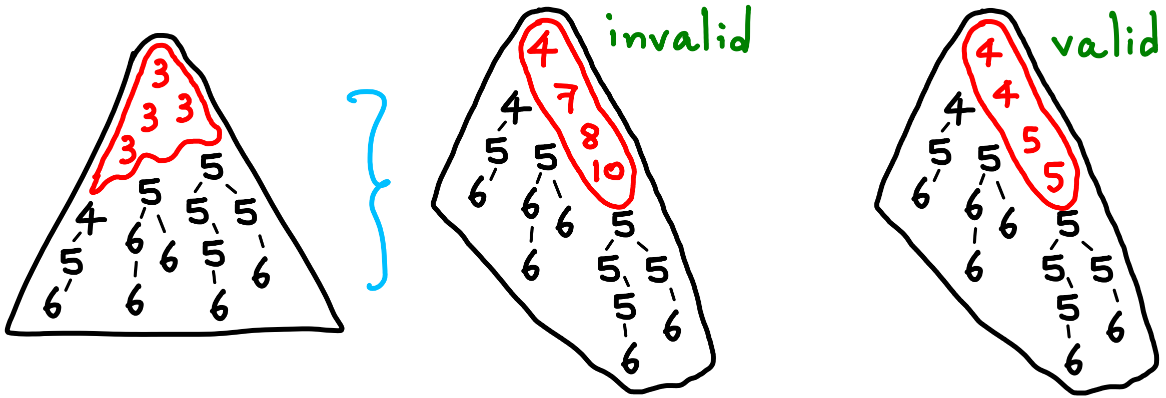

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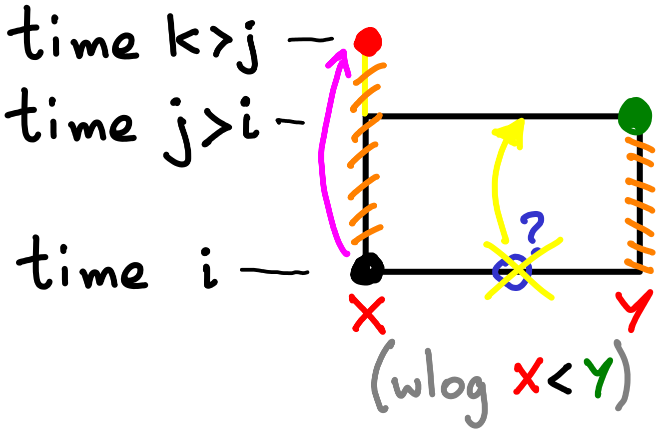


We know $\overbrace{\text{priority}(Y)}^{j} > \overbrace{\text{old priority}(X)}^{i}$

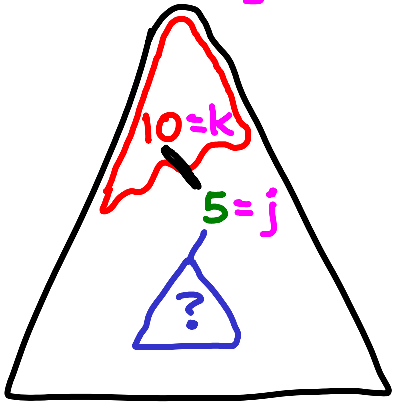
↳ so left & right sides of box are empty

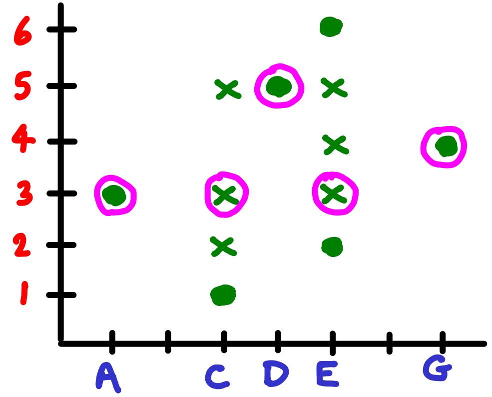
What about bottom side?

Would need X<key<Y but then...

...key is in left subtree of Y, so (unchanged) priority ≥ j (> i)

CONTRADICTION





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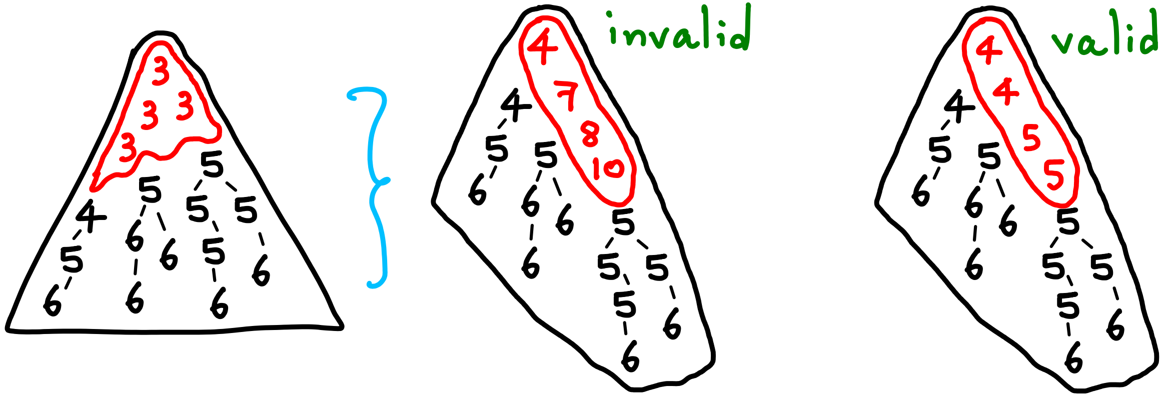

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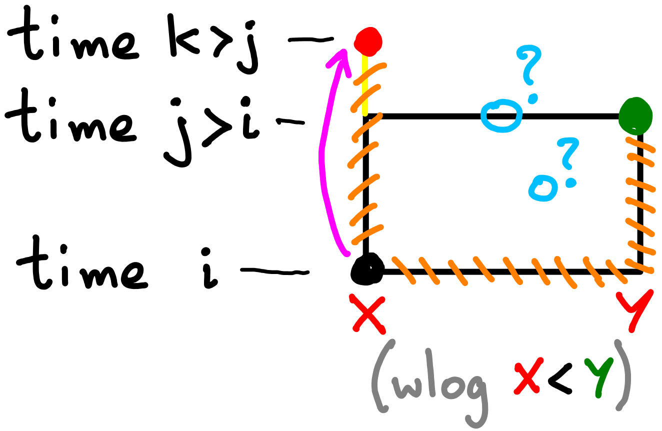


We know $\overbrace{\text{priority}(Y)}^{j} > \overbrace{\text{old priority}(X)}^{i}$

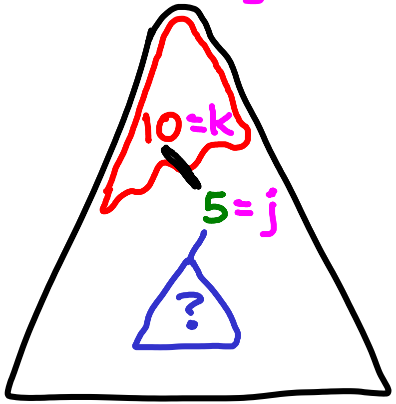
↳ so left & right sides of box are empty

What about bottom side?

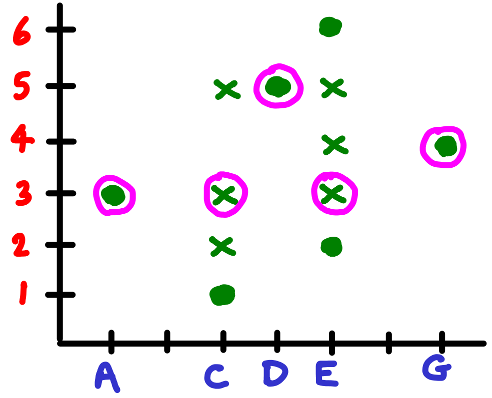
Would need X<key<Y but then...

...key is in left subtree of Y, so (unchanged) priority $\geq j$ $(> i)$

↳ so bottom side is empty.



Top side? Inside?



Claim:
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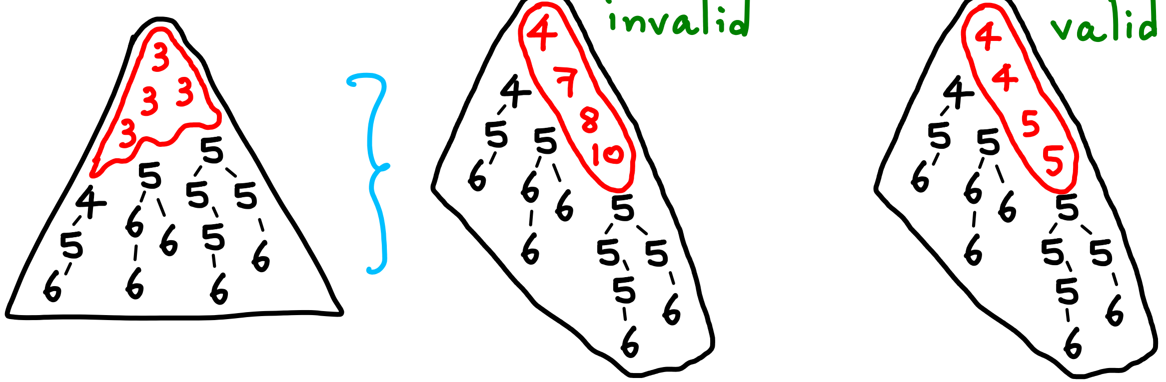

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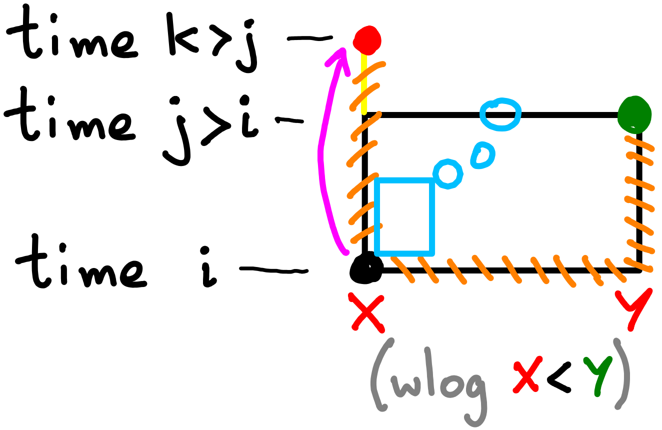


We know $\overbrace{\text{priority}(Y)}^{j} > \overbrace{\text{old priority}(X)}^{i}$

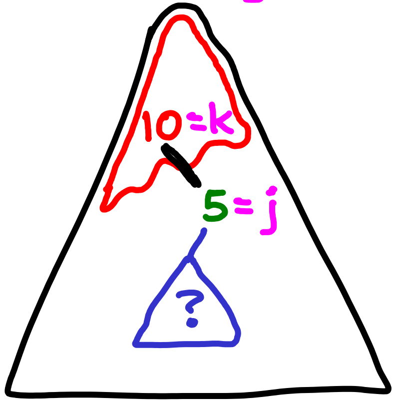
↳ so left & right sides of box are empty

What about bottom side?

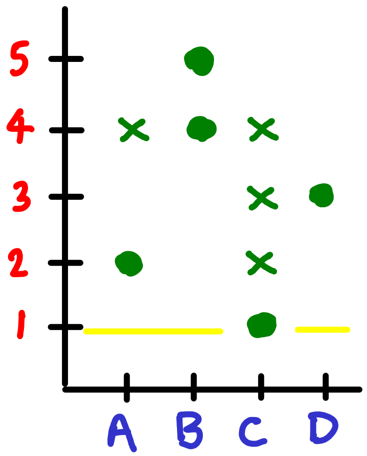
Would need X<key<Y but then...

...key is in left subtree of Y, so (unchanged) priority ≥j (> i)

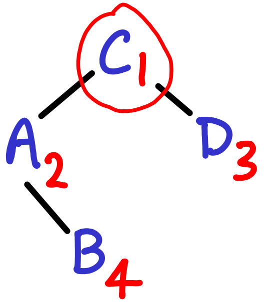
↳ so bottom side is empty.



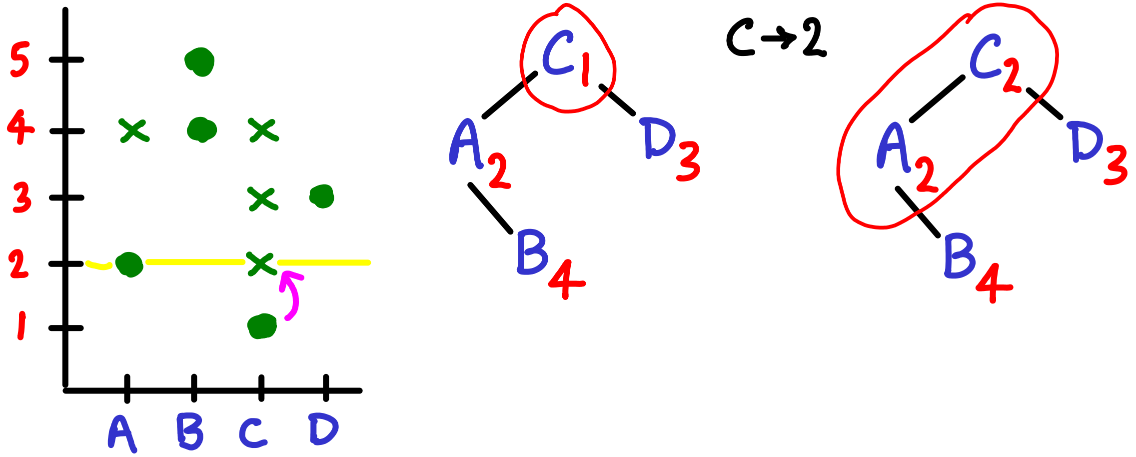
Now, if we place a point on top side, or inside, we will get an empty rectangle □



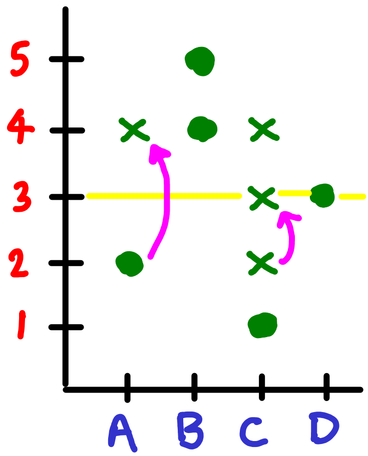
5
4
3
2
1
A
B
C
D

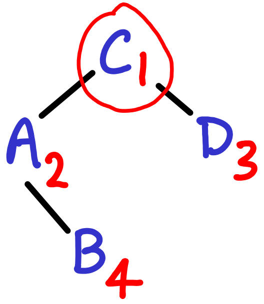


C1
A2
D3
B4

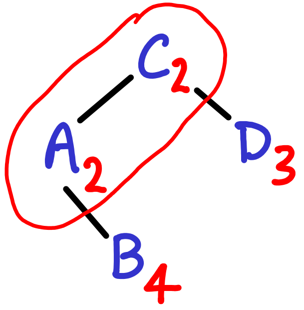


5
4
3
2
1
A B C D
C1
A2
D3
B4
C→2
C2
A2
D3
B4

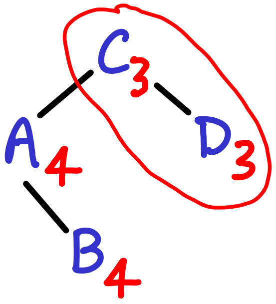


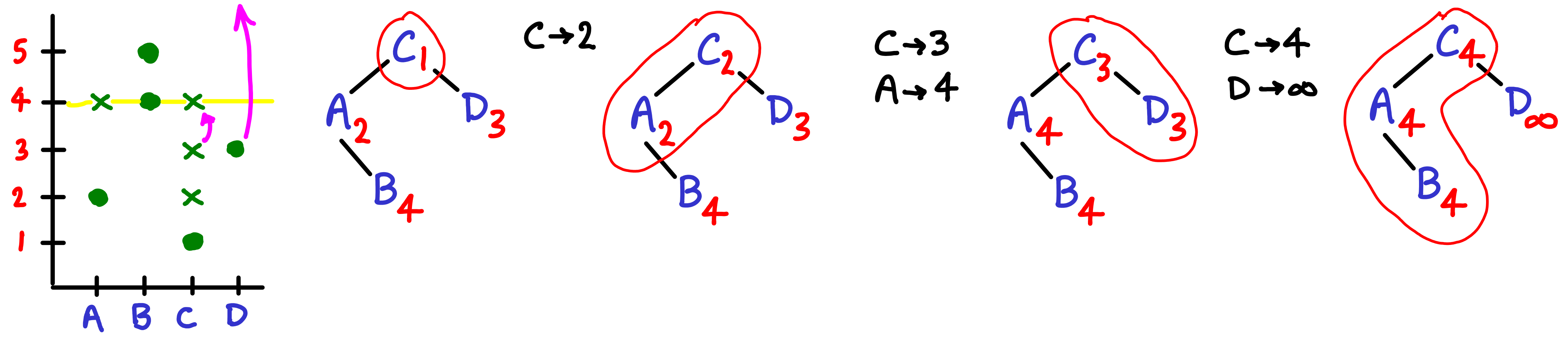


$C \to 2$

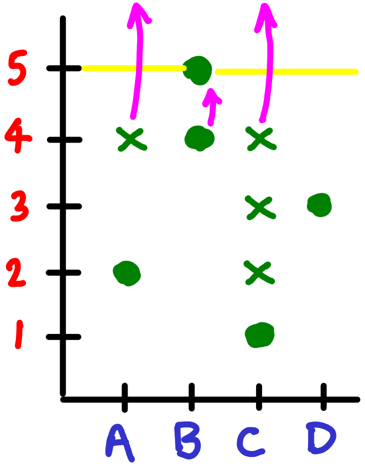


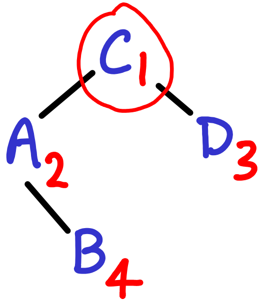
$C \to 3$
$A \to 4$



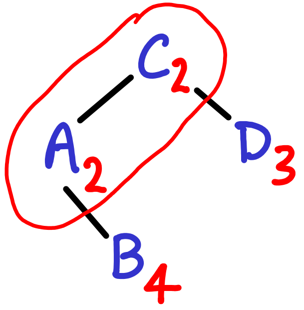


5
4
3
2
1
A B C D
C1
A2
D3
B4
C→2
C2
A2
D3
B4
C→3
A→4
C3
A4
D3
B4
C→4
D→∞
C4
A4
D∞
B4

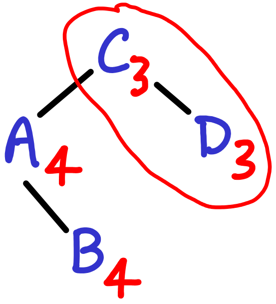




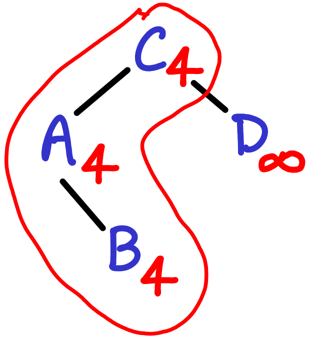
$C \to 2$



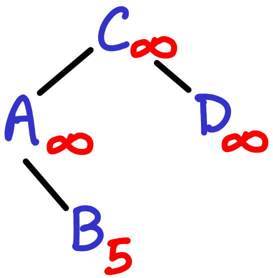
$C \to 3$
$A \to 4$

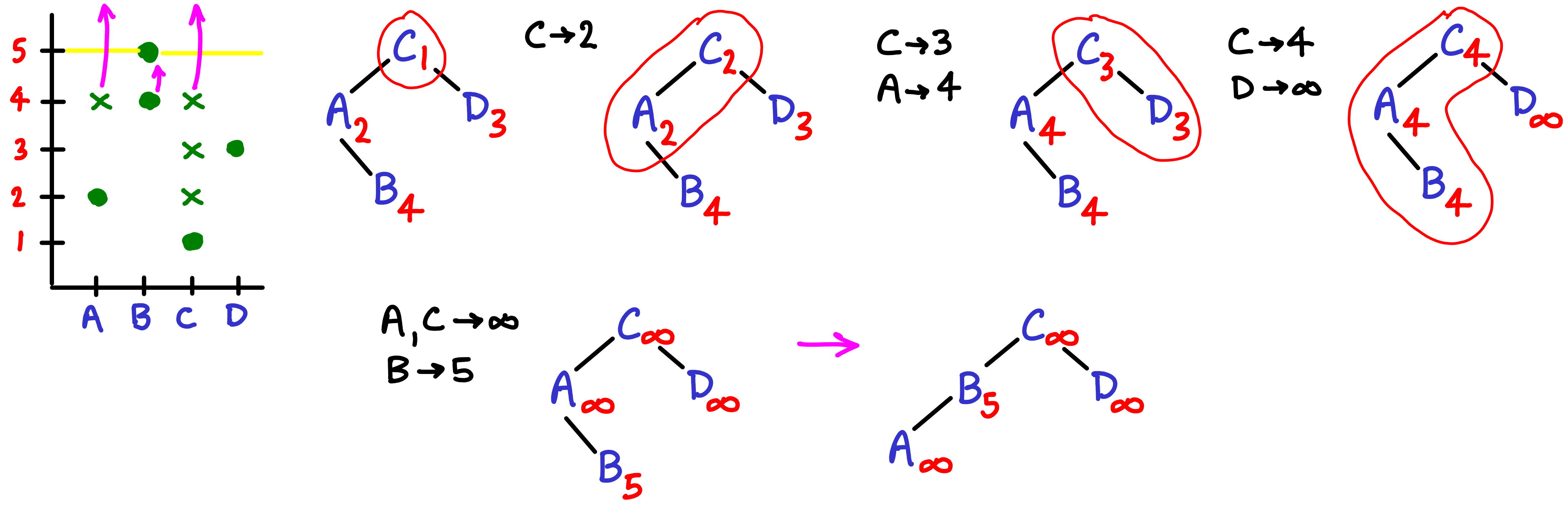


$C \to 4$
$D \to \infty$

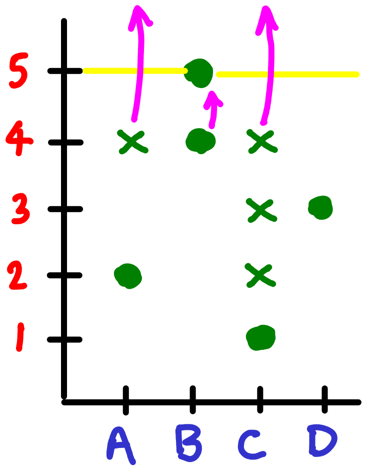


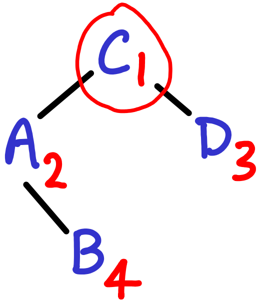
$A, C \to \infty$
$B \to 5$



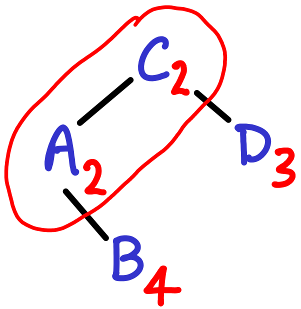


5
4
3
2
1
A B C D
C→2
C→3
A→4
C→4
D→∞
A, C→∞
B→5

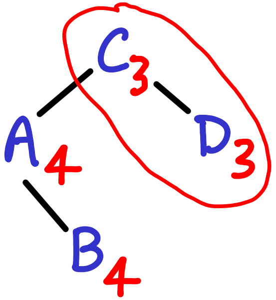




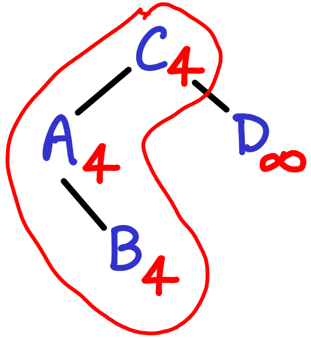
$C \to 2$



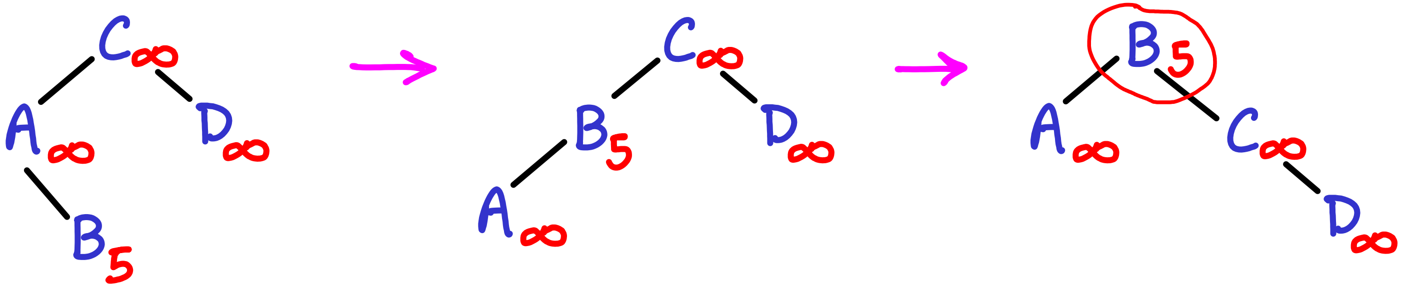
$C \to 3$
$A \to 4$

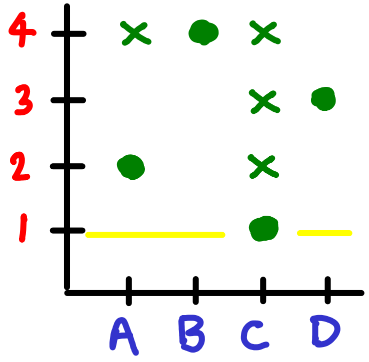


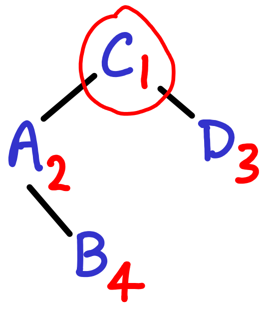
$C \to 4$
$D \to \infty$



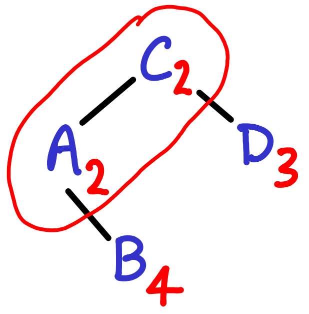
$A, C \to \infty$
$B \to 5$



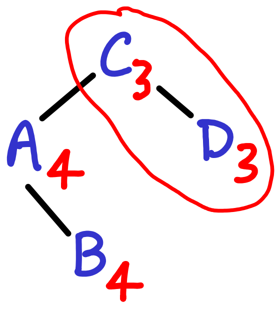




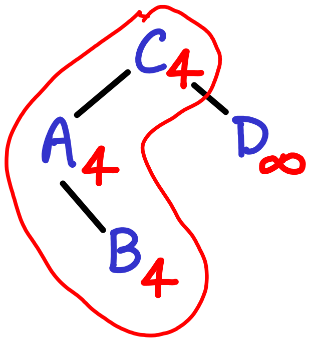
$C \to 2$

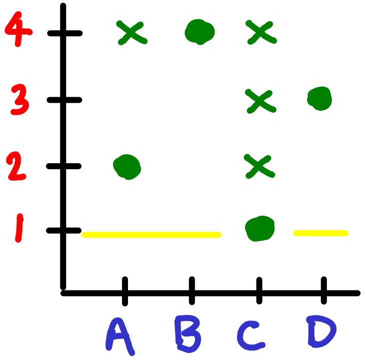


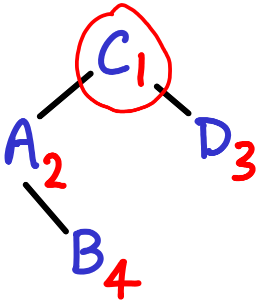
$C \to 3$
$A \to 4$



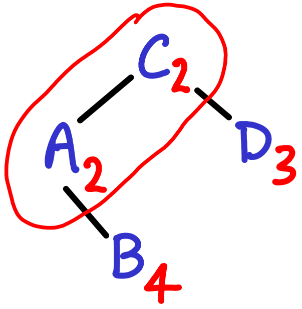
$C \to 4$
$D \to \infty$



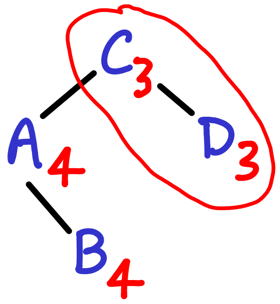




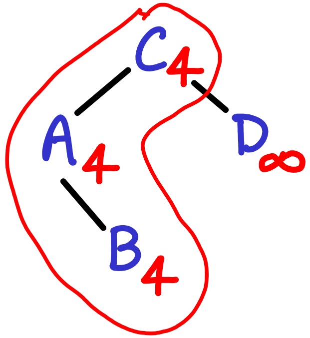
$C \to 2$

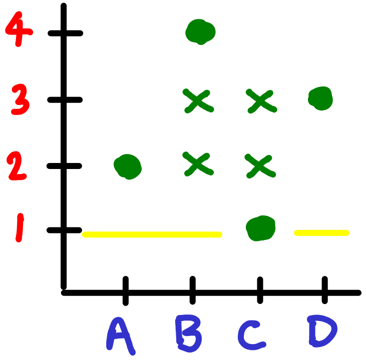


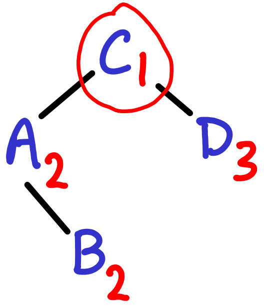
$C \to 3$
$A \to 4$

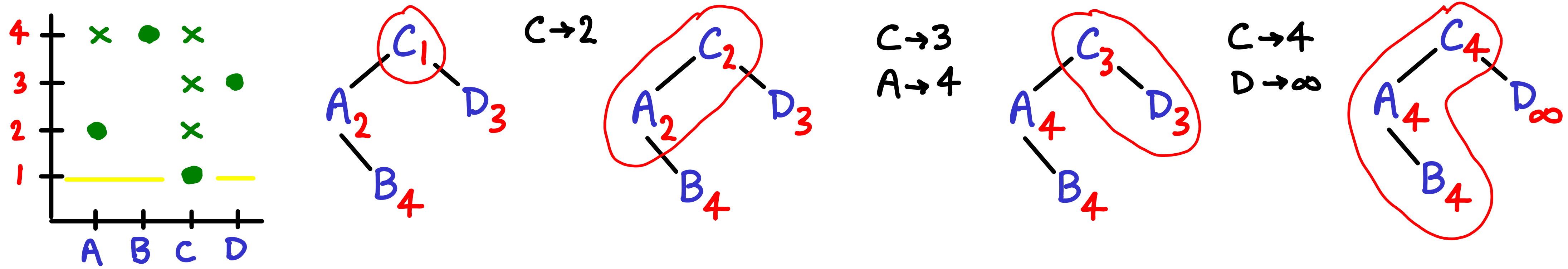


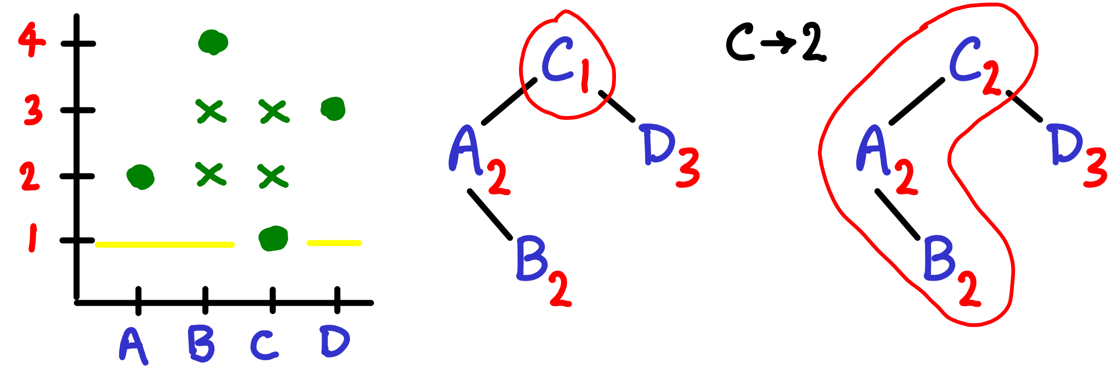
$C \to 4$
$D \to \infty$

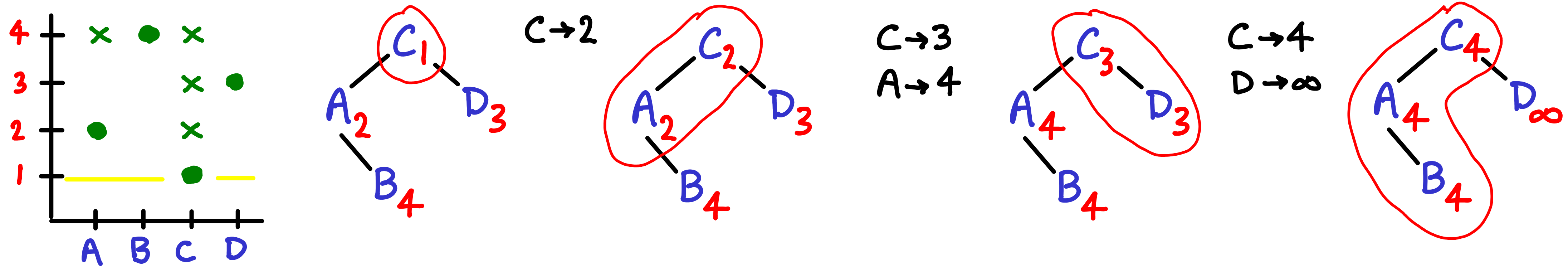







4
3
2
1
A B C D
C1
A2
D3
B4
C→2
C2
A2
D3
B4
C→3
A→4
C3
A4
D3
B4
C→4
D→∞
C4
A4
D∞
B4


4
3
2
1
A B C D
C1
A2
D3
B2
C→2
C2
A2
D3
B2


4
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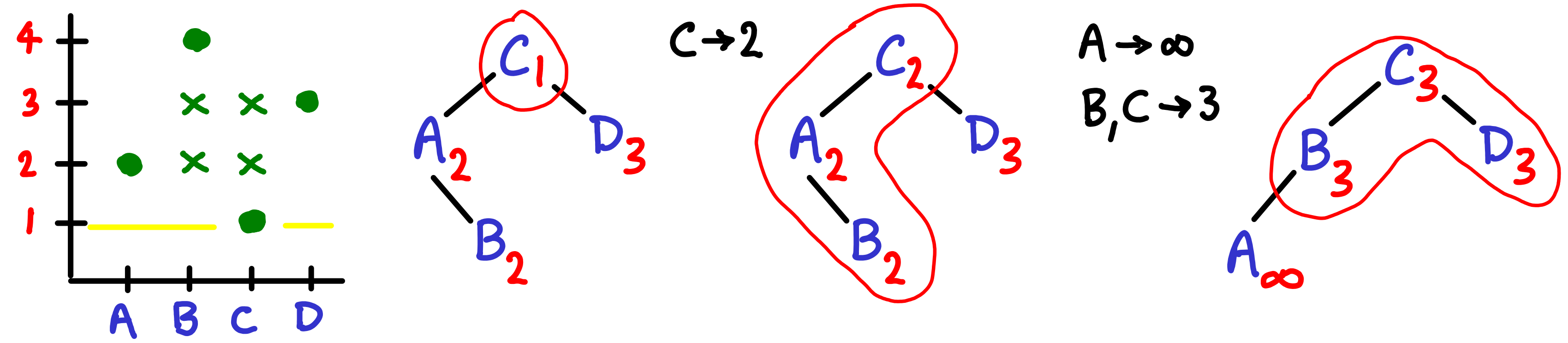
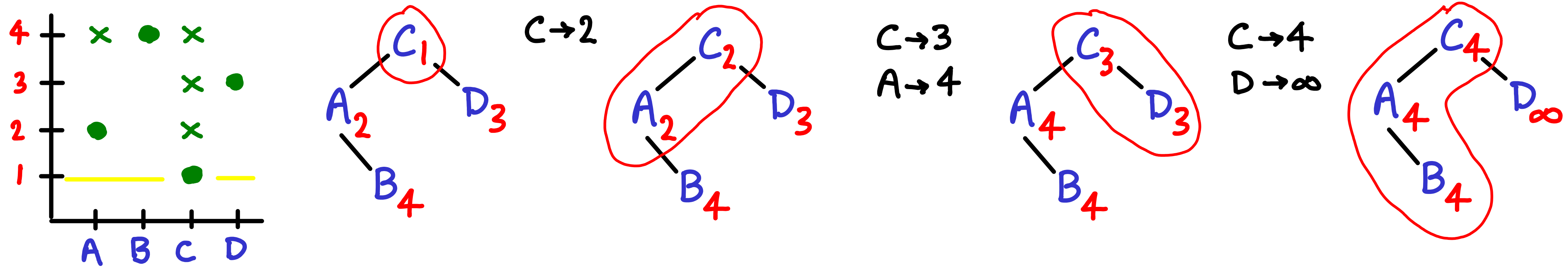
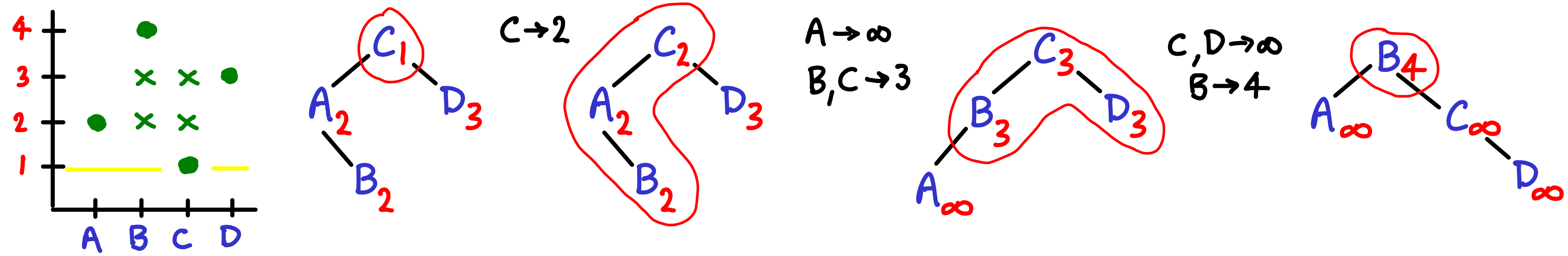
A B C D
C1
A2
D3
B4
C→2
C2
A2
D3
B4
C→3
A→4
C3
A4
D3
B4
C→4
D→∞
C4
A4
D∞
B4

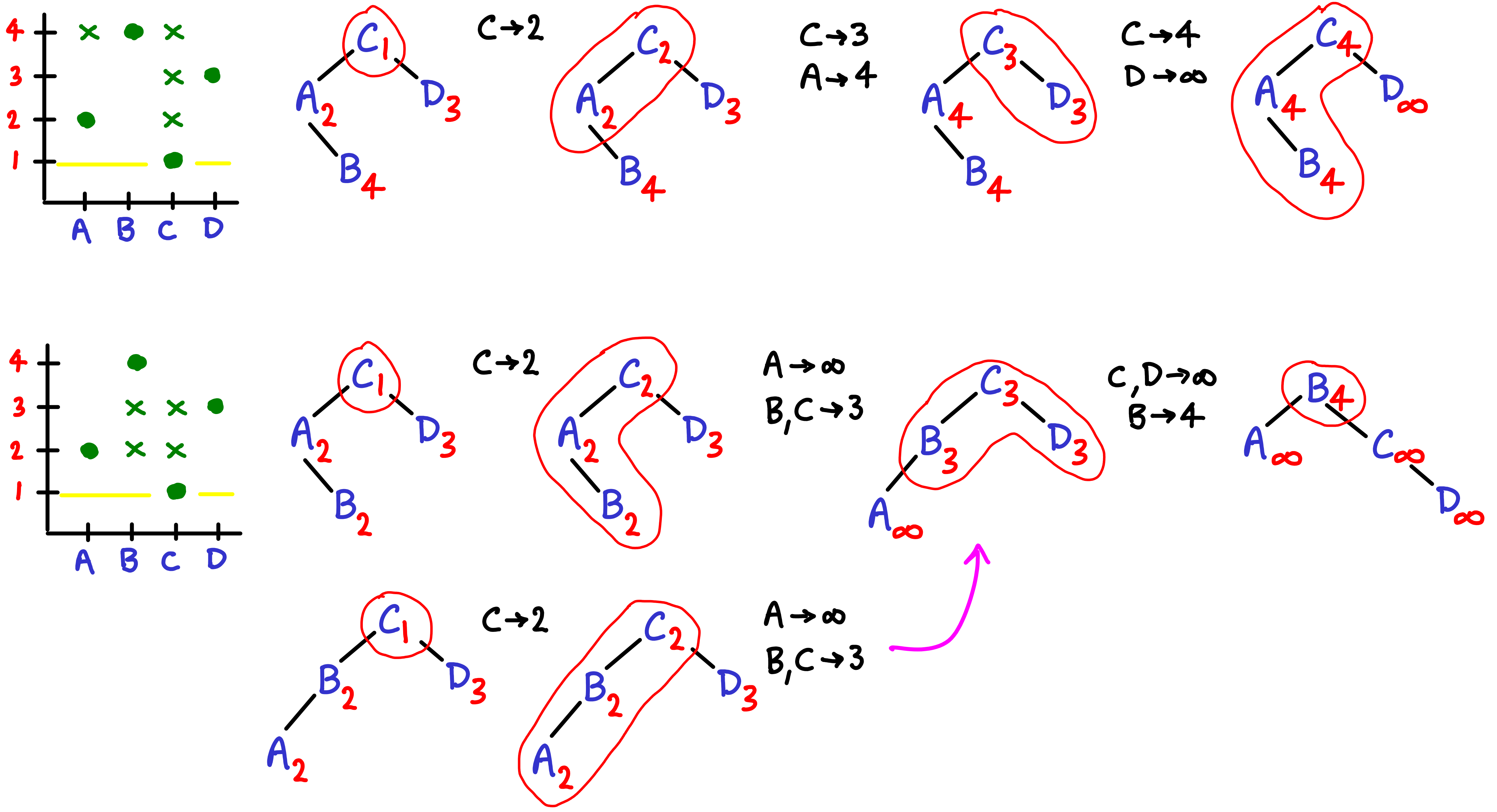

4
3
2
1
A B C D
C1
A2
D3
B2
C→2
C2
A2
D3
B2
A→∞
B,C→3
C3
B3
D3
A∞



4
3
2
1
A B C D
C1
A2
D3
B4
C→2
C2
A2
D3
B4
C→3
A→4
C3
A4
D3
B4
C→4
D→∞
C4
A4
D∞
B4



4
3
2
1
A B C D
C1
A2
D3
B2
C→2
C2
A2
D3
B2
A→∞
B,C→3
C3
B3
D3
A∞
C,D→∞
B→4
B4
A∞
C∞
D∞



4
3
2
1
A B C D
C→2
C→3
A→4
C→4
D→∞
4
3
2
1
A B C D
C→2
A→∞
B,C→3
C,D→∞
B→4
C→2
A→∞
B,C→3

Conclusion so far:

Every BST op sequence can be drawn as a special grid pattern and every valid grid pattern can be translated into a sequence of BST ops

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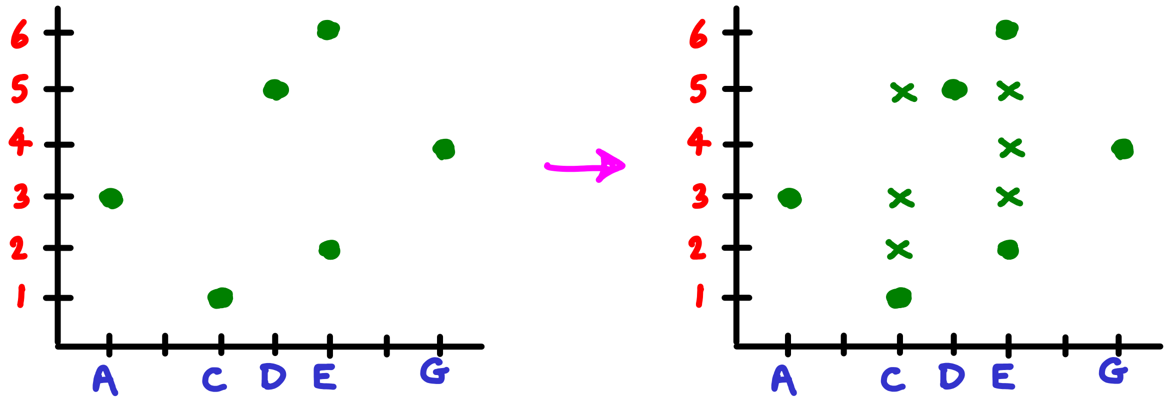
BST optimality (when we can modify the tree)

is related to

minimizing #added points to make a given grid pattern valid



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$\updownarrow$

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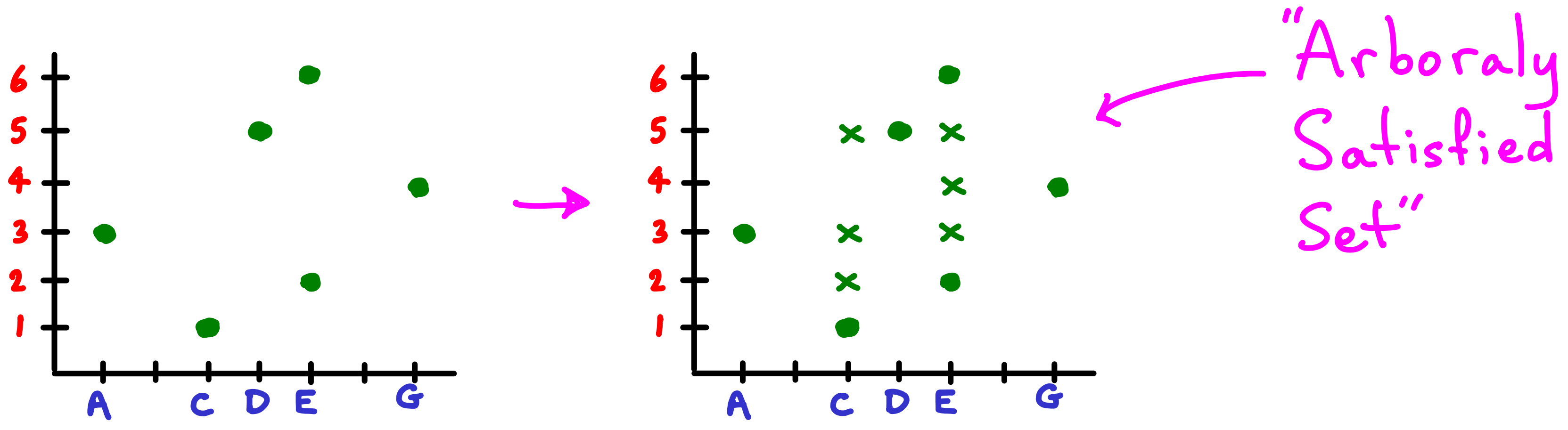
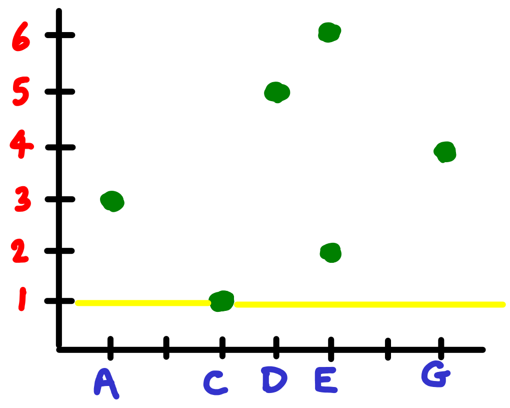


Greedy algorithm (online: doesn't even need to know sequence)

Sweep up, add points only when necessary



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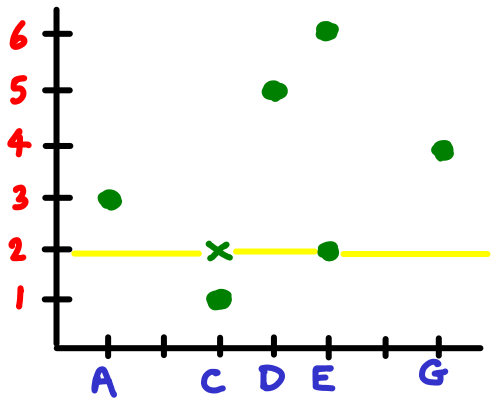

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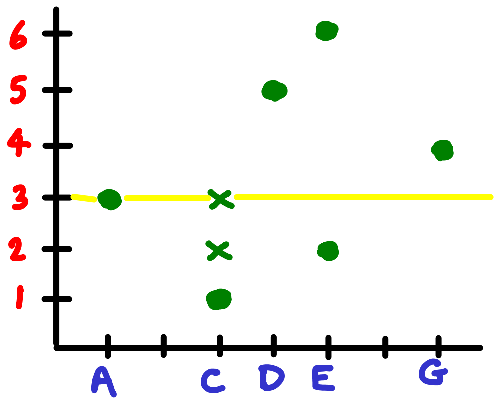

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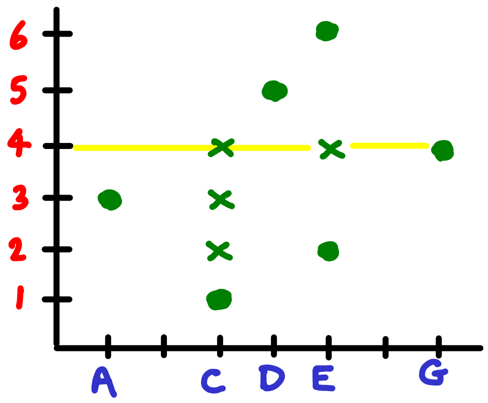

Greedy algorithm (online: doesn't even need to know sequence)

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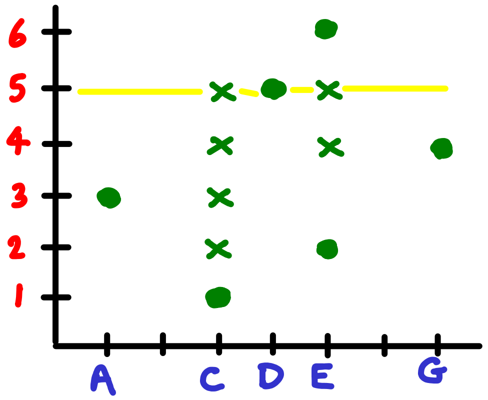

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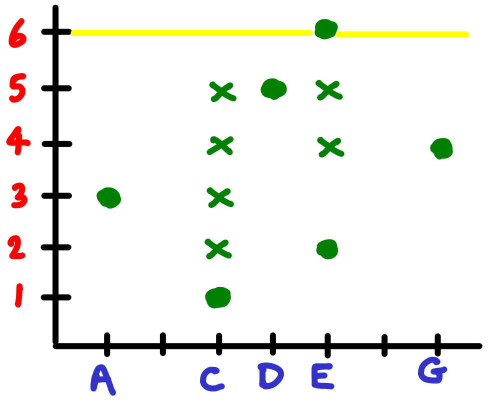

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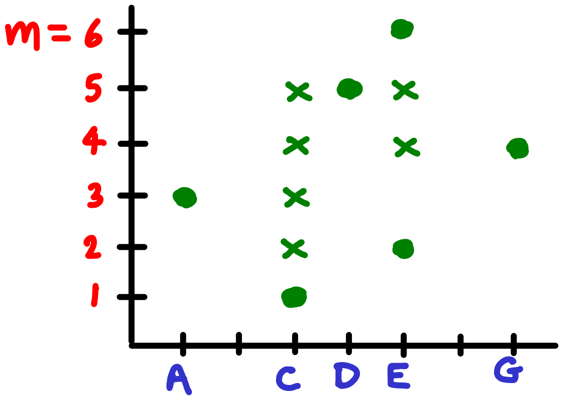

Conjectured $O(OPT)$ or $OPT + O(m)$

OPT : min # steps possible, with knowledge of sequence