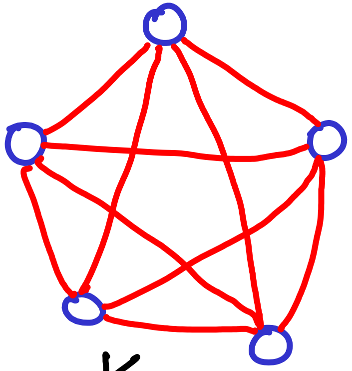
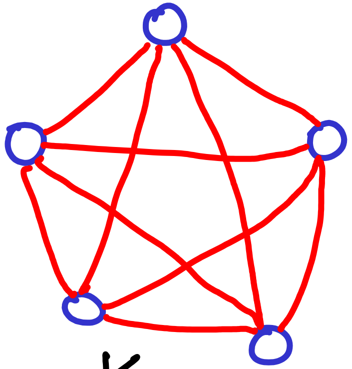
Non-planarity



K_5



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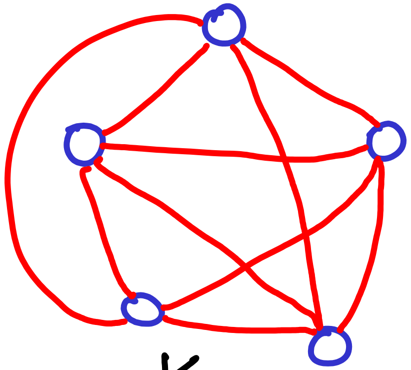

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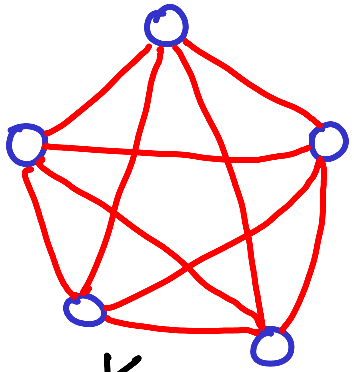

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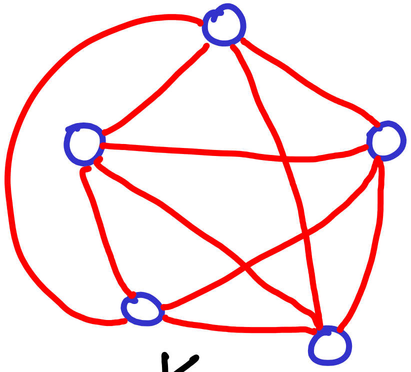

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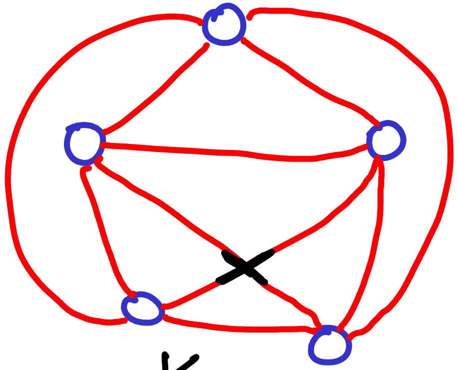

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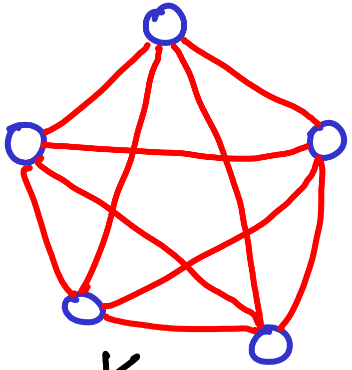

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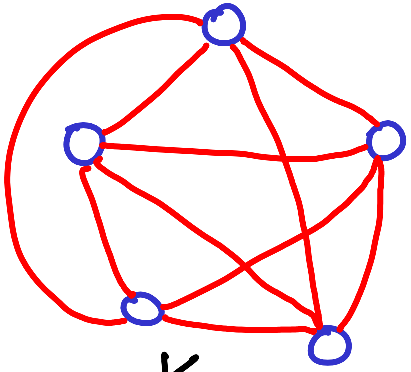

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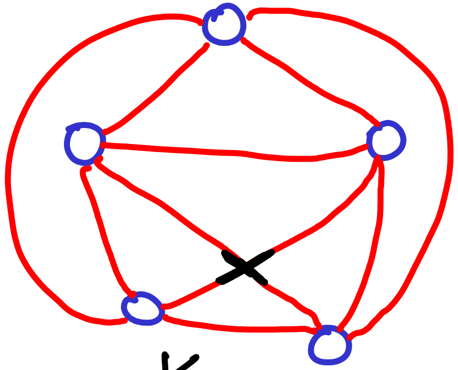

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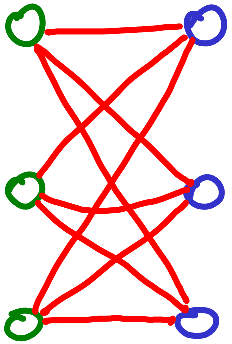
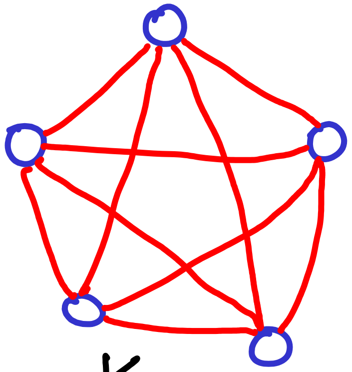


$K_{3,3}$



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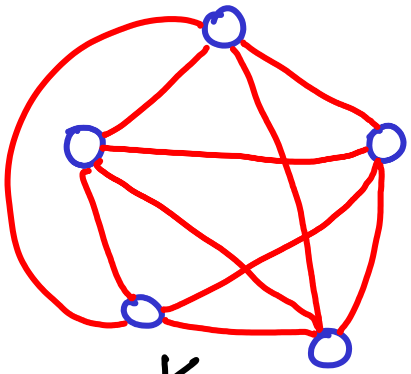

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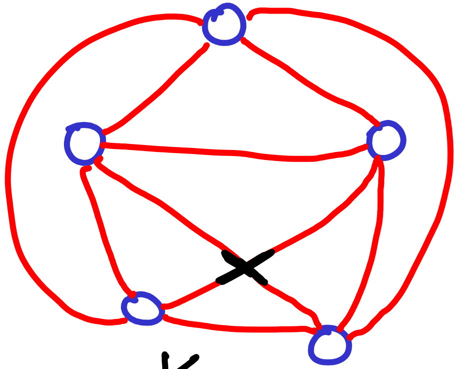

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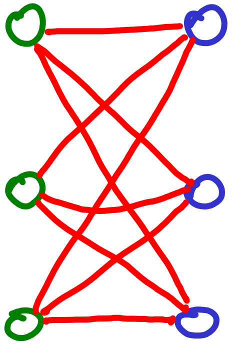
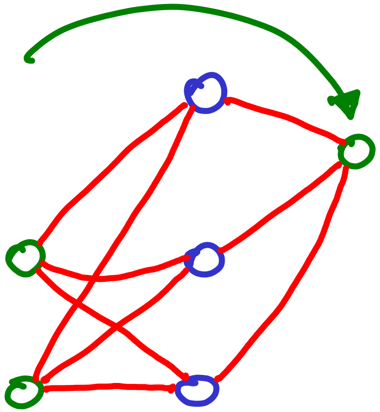


$K_{3,3}$



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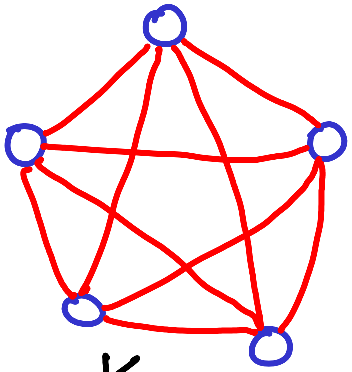

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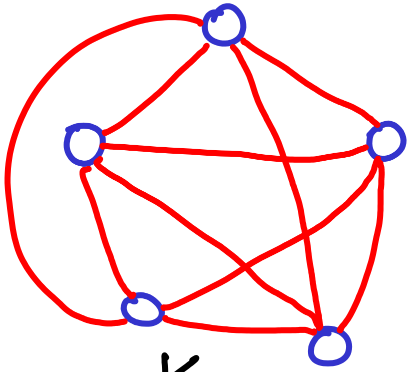

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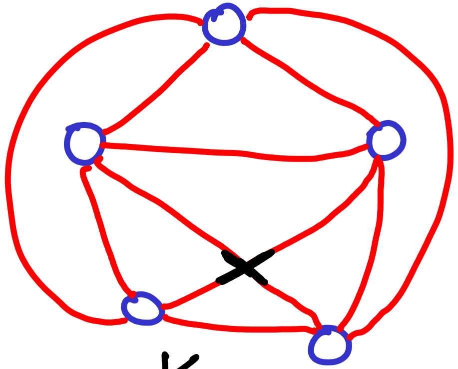

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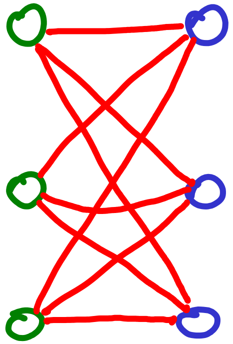
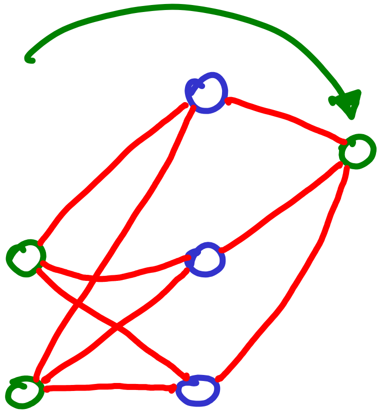
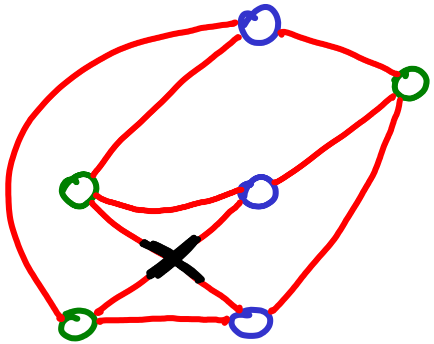


$K_{3,3}$



$K_{3,3}$

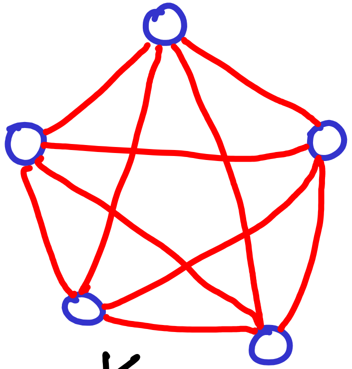


$K_{3,3}$

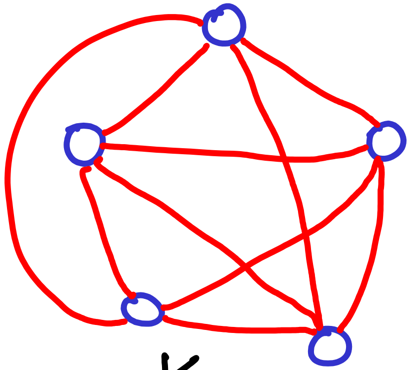
Graph Drawing & untangling

↓

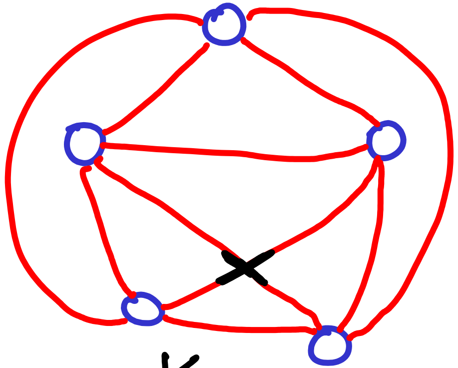
projects



K_5



K_5



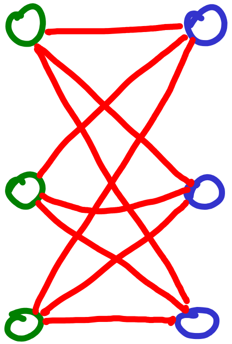
K_5

CROSSING NUMBER

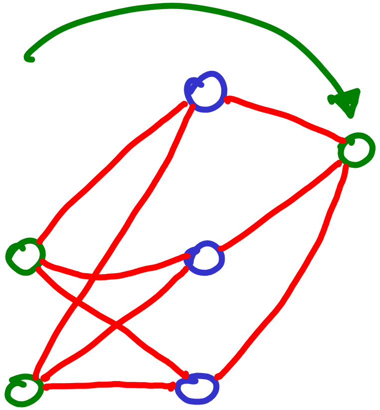
$cr(G)$

$\downarrow$

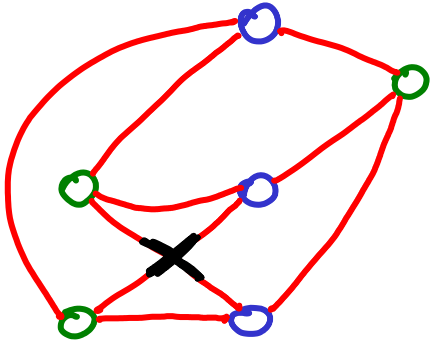
min # crossings possible for any embedding (drawing) of G



$K_{3,3}$



$K_{3,3}$



$K_{3,3}$

Given graph G & integer k, decide if $cr(G) \leq k$. $\rightarrow$ NPC

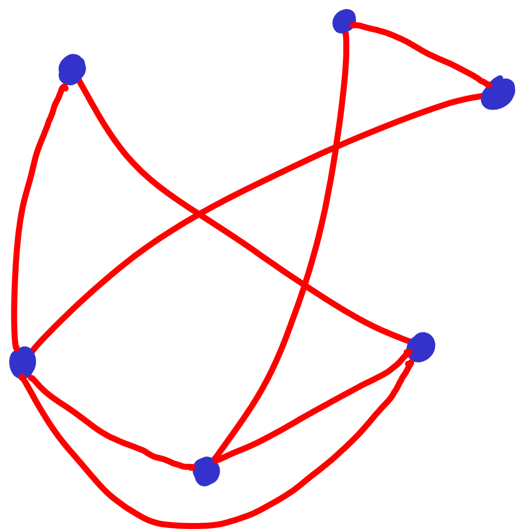
(unless k = const)

Look into algorithms
for $k = 0$ & $k = O(1)$

Given graph G & integer k, decide if $cr(G) \leq k$. → NPC

(unless k=const)

Here is a lower bound on $cr(G)$:

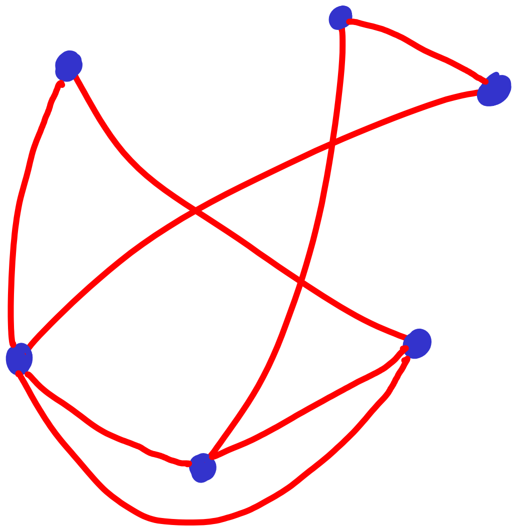


$G = (V, E)$

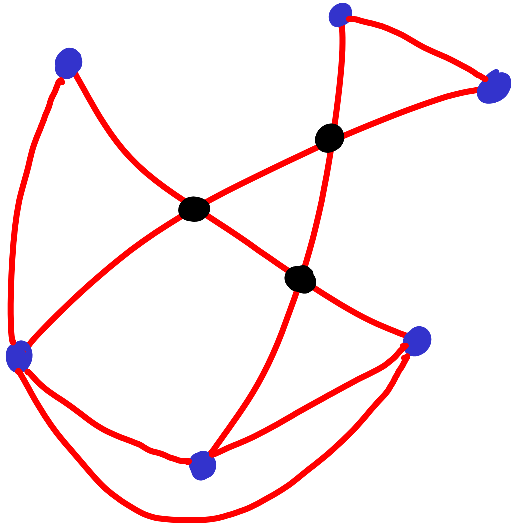
Given graph G & integer k, decide if $cr(G) \le k$. → NPC

(unless k=const)

Here is a lower bound on $cr(G)$:



$G = (V, E)$



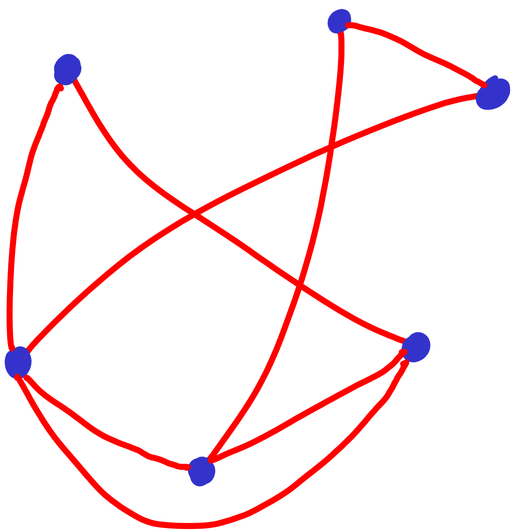
make plane

$G' = (V', E')$

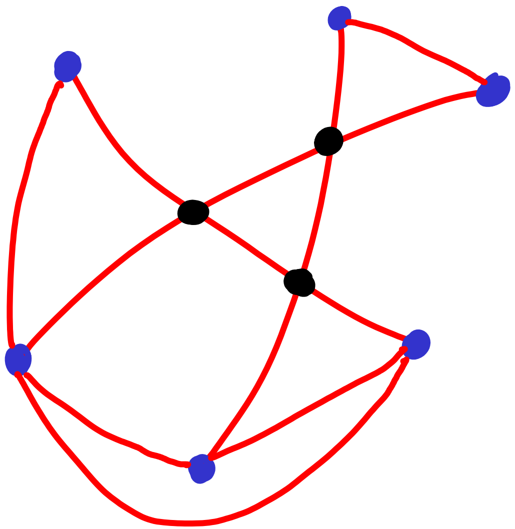
Given graph G & integer k, decide if $cr(G) \leq k$. → NPC

(unless k=const)

Here is a lower bound on $cr(G)$:



$G = (V, E)$



make plane

$G' = (V', E')$

$V' = V + \underline{\underline{cr(G)}}$

↓

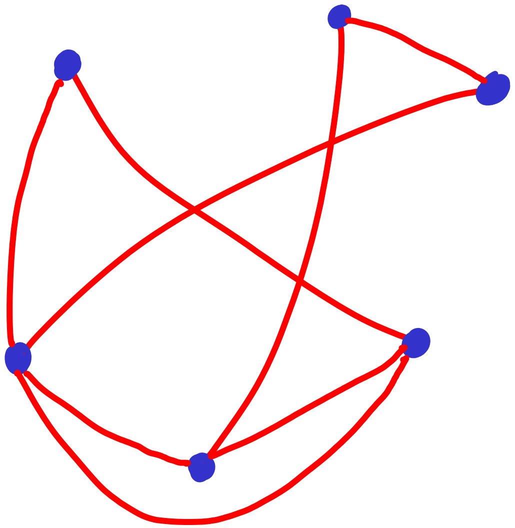
Note: you must start with the <u>best</u> drawing of G, unlike my example.

Here $V' > V + cr(G)$

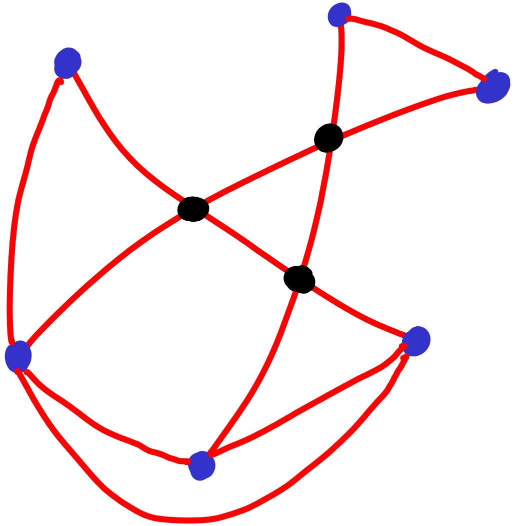
Given graph G & integer k, decide if $cr(G) \leq k$. $\rightarrow$ NPC

(unless k=const)

Here is a lower bound on $cr(G)$:



$G = (V, E)$



make plane

$G' = (V', E')$

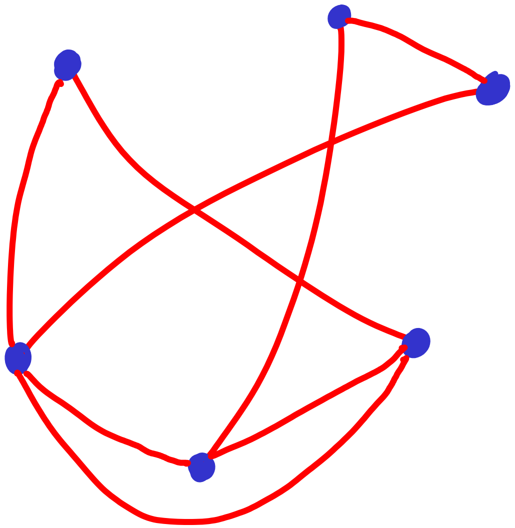
$V' = V + cr(G)$

$E' = ?$

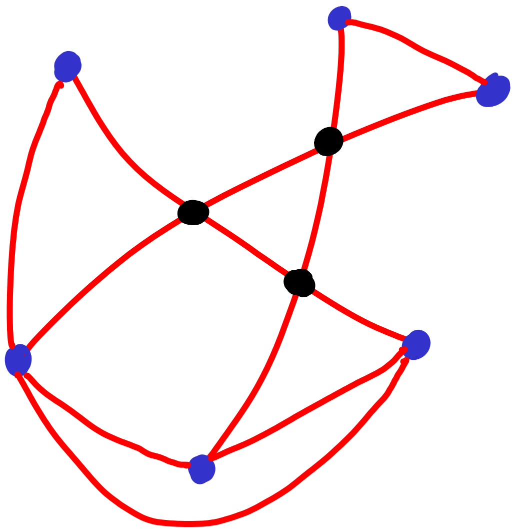
Given graph G & integer k, decide if $cr(G) \leq k$. → NPC

(unless k=const)

Here is a lower bound on $cr(G)$:



$G = (V, E)$



make plane

$G' = (V', E')$

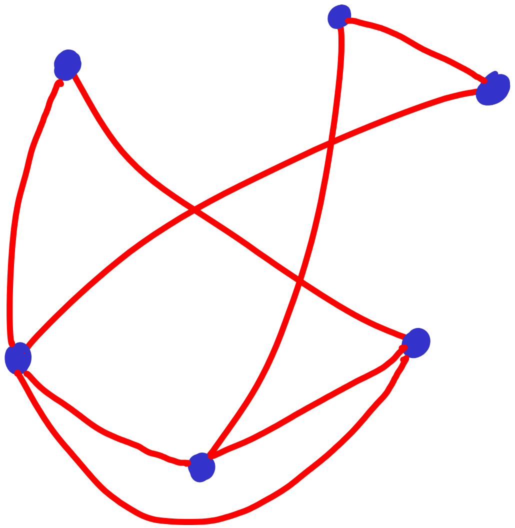
$V' = V + cr(G)$

$E' = ?$

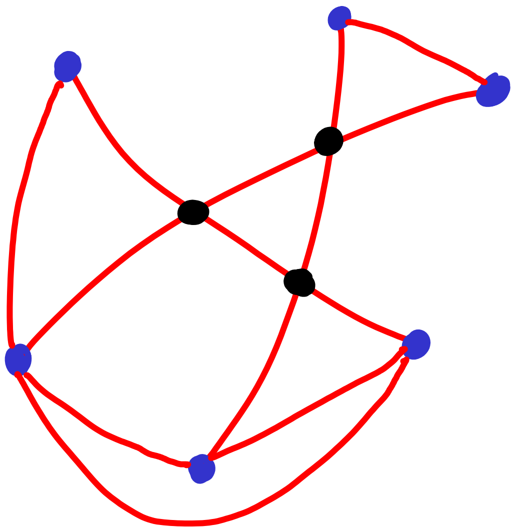
For every crossing (•) we get 2 new edges

Given graph G & integer k, decide if $cr(G) \leq k$. → NPC (unless k=const)

Here is a lower bound on $cr(G)$:



$G = (V, E)$



make plane

$G' = (V', E')$

$$V' = V + cr(G)$$

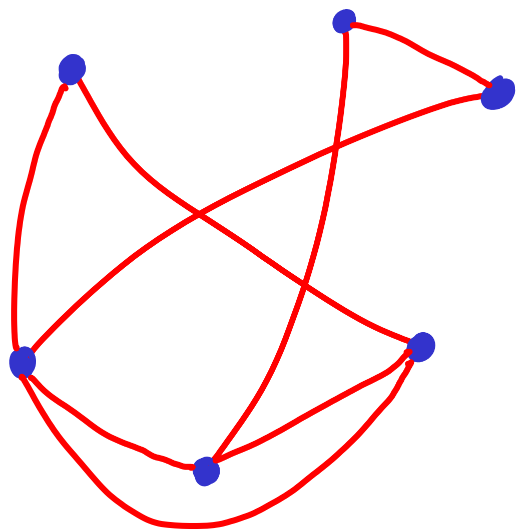
For every crossing (•) we get 2 new edges

$$E' = E + 2 \cdot cr(G)$$

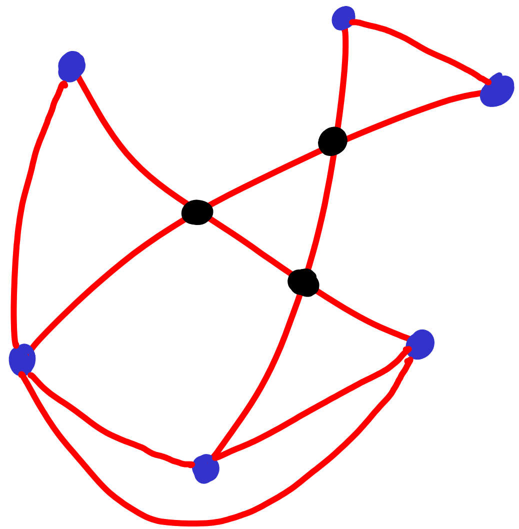
Given graph G & integer k, decide if $cr(G) \leq k$. → NPC

(unless k=const)

Here is a lower bound on $cr(G)$:



$G = (V, E)$



make plane

$G' = (V', E')$

$V' = V + cr(G)$

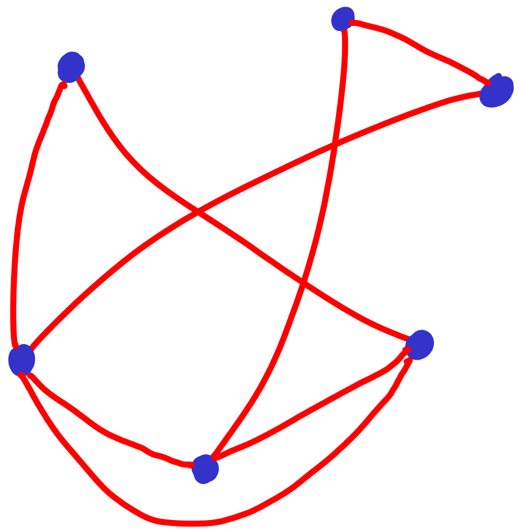
For every crossing (•) we get 2 new edges

$E' = E + 2 \cdot cr(G)$

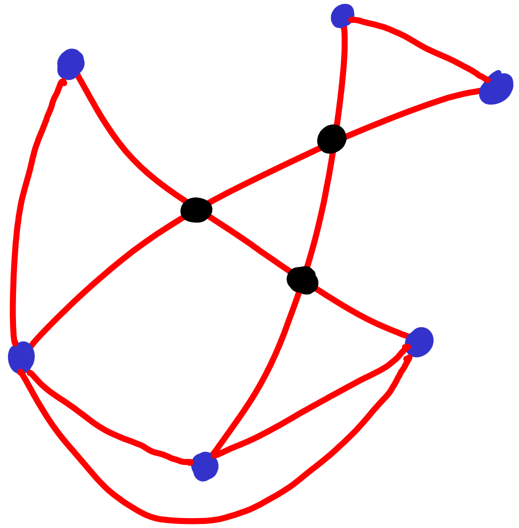
$E' \leq 3V' - 6$ (Euler)

Given graph G & integer k, decide if $cr(G) \le k$. → NPC (unless k=const)

Here is a lower bound on $cr(G)$:



$G = (V, E)$



make plane

$G' = (V', E')$

$V' = V + cr(G)$

For every crossing (•) we get 2 new edges

$E' = E + 2 \cdot cr(G)$

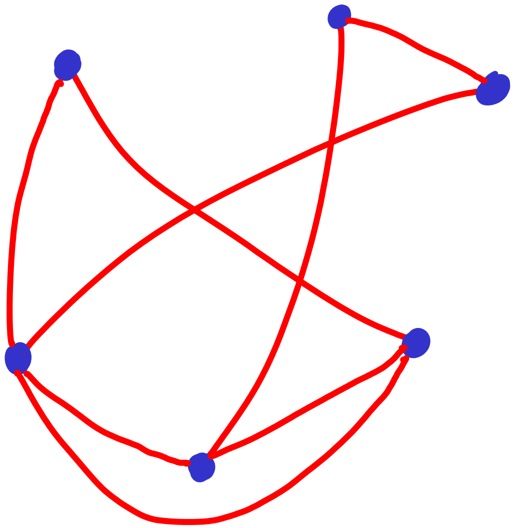
$E' \le 3V' - 6$

$E + 2cr(G) \le 3V + 3cr(G) - 6$

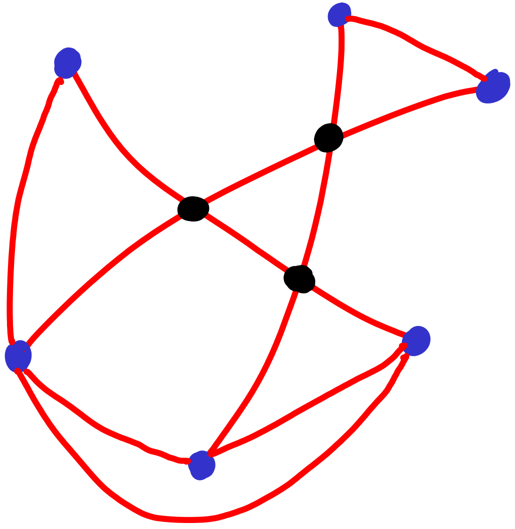
Given graph G & integer k, decide if $cr(G) \leq k$. → NPC

(unless k=const)

Here is a lower bound on $cr(G)$:



$G = (V, E)$



make plane

$G' = (V', E')$

$$V' = V + cr(G)$$

For every crossing (•) we get 2 new edges

$$E' = E + 2 \cdot cr(G)$$

$$E' \leq 3V' - 6$$

$$E + 2cr(G) \leq 3V + 3cr(G) - 6$$

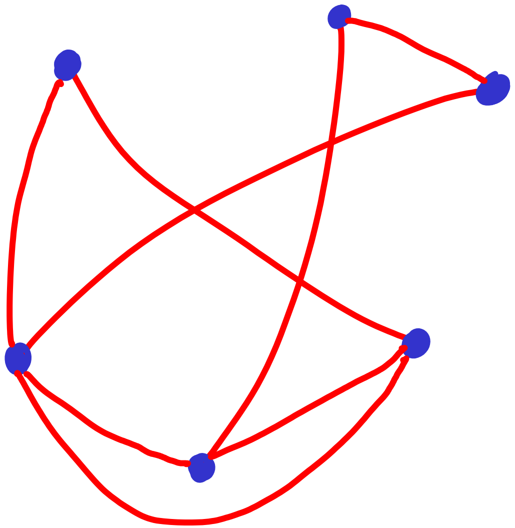
$$cr(G) \geq E - 3V + 6$$

(for $V > 3$)

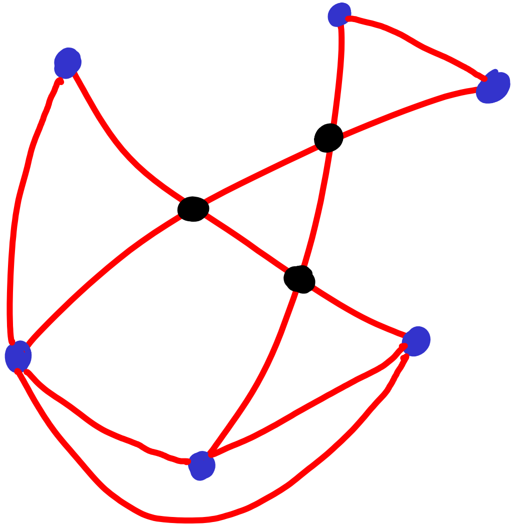
Given graph G & integer k, decide if $cr(G) \leq k$. → NPC

(unless k=const)

Here is a lower bound on $cr(G)$:



$G = (V, E)$



make plane

$G' = (V', E')$

$V' = V + cr(G)$

For every crossing (•) we get 2 new edges

$E' = E + 2 \cdot cr(G)$

$E' \leq 3V' - 6$

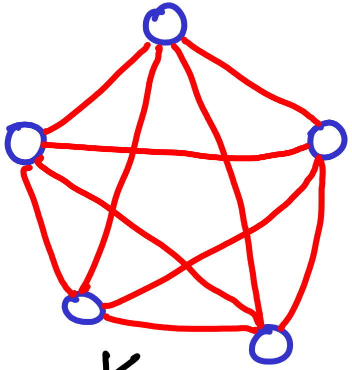
$E + 2cr(G) \leq 3V + 3cr(G) - 6$

$cr(G) \geq E - 3V + 6$

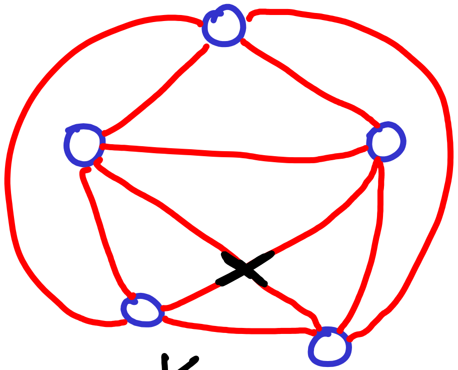
(for $V > 3$)

$\geq 8 - 18 + 6$

≥ -4 : no info (it's not an upper bound)

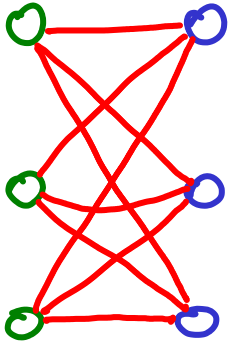


K_5

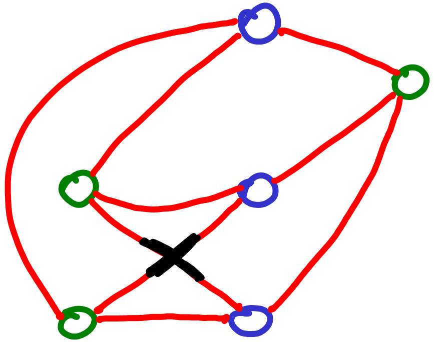


K_5

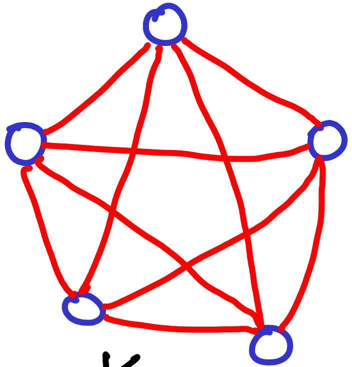
$$\boxed{cr(G) \geq E - 3V + 6}$$



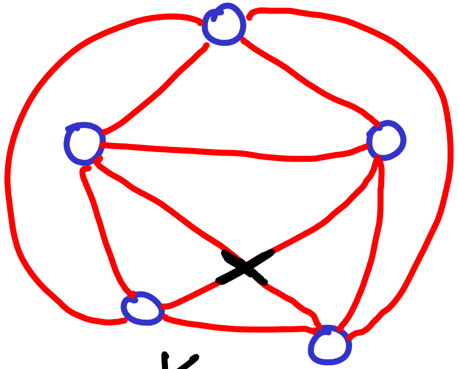
$K_{3,3}$



$K_{3,3}$



K_5

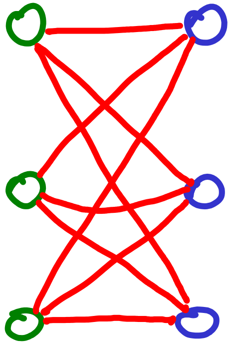


K_5

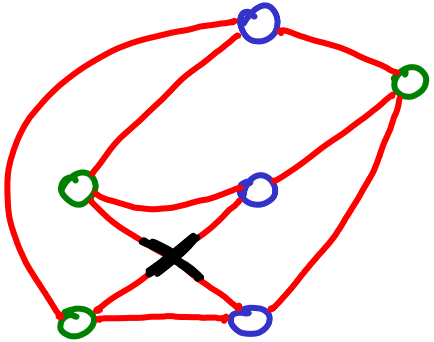
$$\boxed{cr(G) \geq E - 3V + 6}$$

$$\geq 10 - 15 + 6$$

$$\geq 1$$

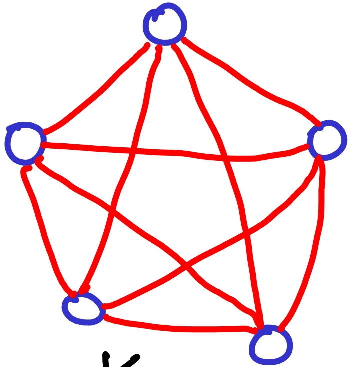


$K_{3,3}$

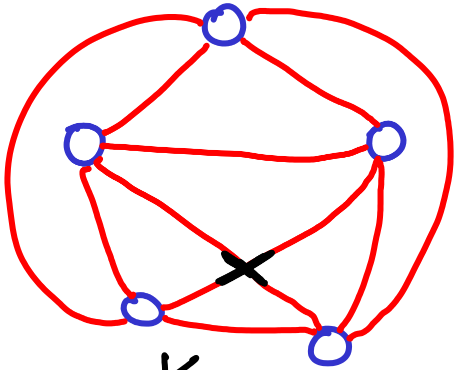


$K_{3,3}$

$$\boxed{cr(G) \geqslant E - 3V + 6}$$



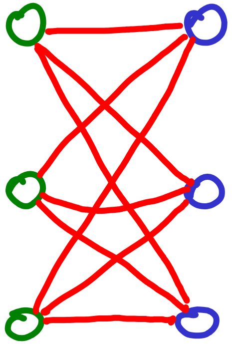
K_5



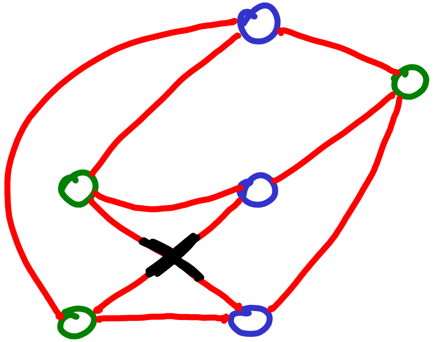
K_5

$\geqslant 10 - 15 + 6$

$\underline{\underline{\geqslant 1}}$



$K_{3,3}$



$K_{3,3}$

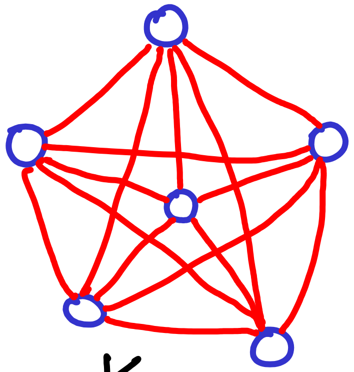
$\geqslant 9 - 18 + 6$

$\geqslant -3$

inconclusive

$cr(G) \geqslant E - 3V + 6$

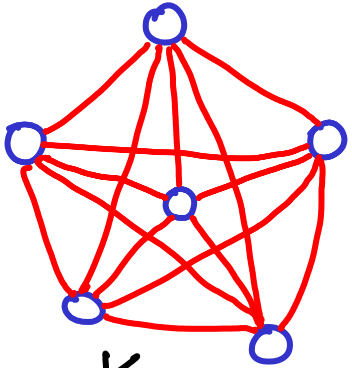
For K_n: $cr(K_n) \sim ?$



K_6

$cr(G) \geq E - 3V + 6$

For K_n: $cr(K_n) \geq \binom{V}{2} - 3V + 6$



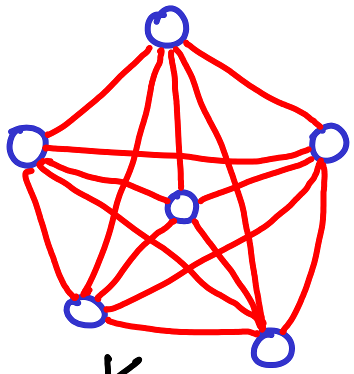
K_6

$cr(G) \geq E - 3V + 6$

For K_n : $cr(K_n) \geq \binom{V}{2} - 3V + 6$

$= \frac{1}{2}V(V-1) - 3V + 6$

$= \frac{1}{2}V^2 - 3.5V + 6$



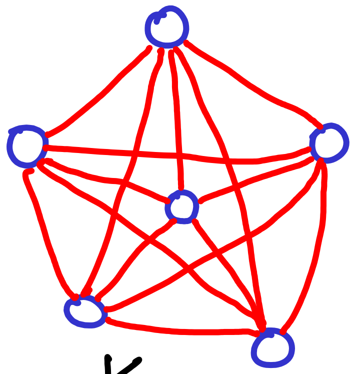
K_6

$cr(G) \geq E - 3V + 6$

For K_n: $cr(K_n) \geq \binom{V}{2} - 3V + 6$

$= \frac{1}{2}V(V-1) - 3V + 6$

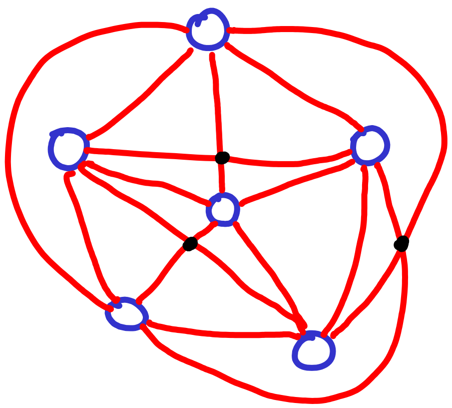
$= \frac{1}{2}V^2 - 3.5V + 6$



K_6

$cr(K_6) \geq \frac{1}{2}36 - 21 + 6$

$= 3$

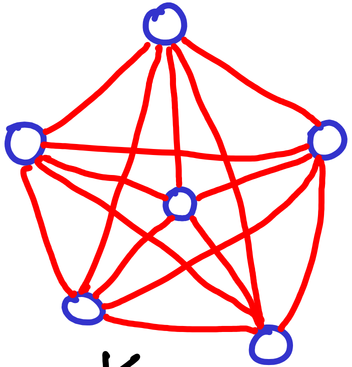


$cr(G) \geq E - 3V + 6$

For K_n: $cr(K_n) \geq \binom{V}{2} - 3V + 6$

$$= \frac{1}{2}V(V-1) - 3V + 6$$

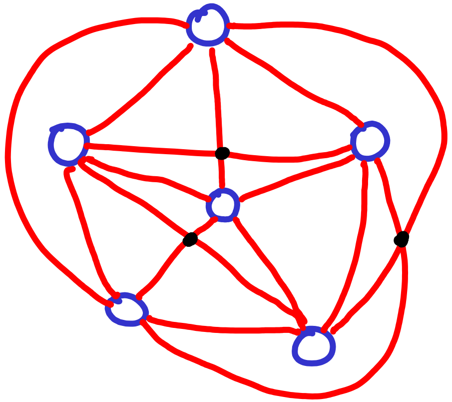
$$= \frac{1}{2}V^2 - 3.5V + 6$$



K_6

$cr(K_6) \geq \frac{1}{2}36 - 21 + 6$

$= 3$



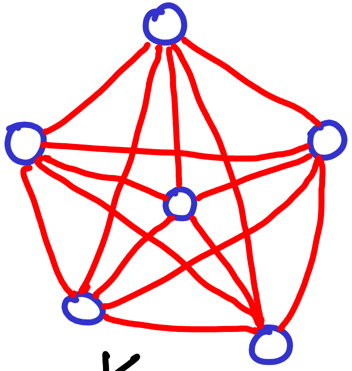
Also, trivially, $cr(K_n) \leq \binom{\binom{V}{2}}{2} = O(V^4)$

$cr(G) \geq E - 3V + 6$

For K_n: $cr(K_n) \geq \binom{V}{2} - 3V + 6$

$$= \frac{1}{2}V(V-1) - 3V + 6$$

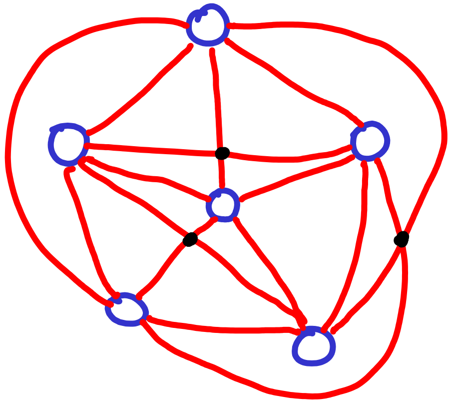
$$= \frac{1}{2}V^2 - 3.5V + 6$$



K_6

$cr(K_6) \geq \frac{1}{2}36 - 21 + 6$

$= 3$



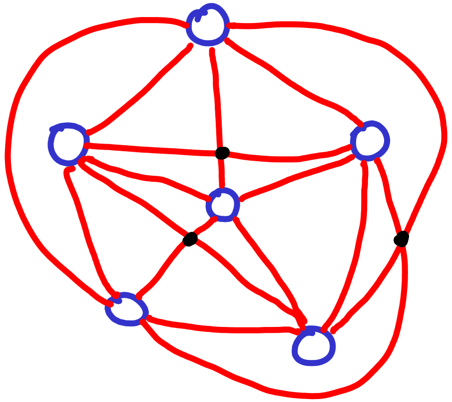
Also, trivially, $cr(K_n) \leq \binom{\binom{V}{2}}{2} = O(V^4)$

so $cr(G) = \Omega(V^2)$

$= O(V^4)$

A better bound

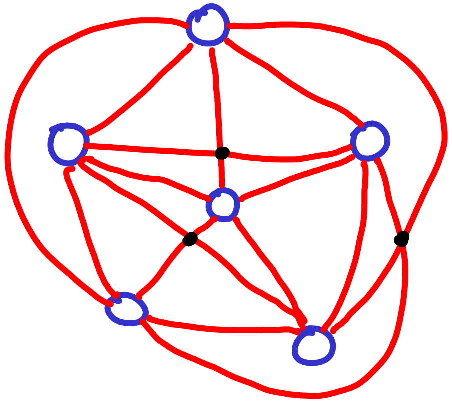
[Leighton 1983]



Given a graph G, $\exists$ a drawing w/ $cr(G)$ crossings

A better bound

[Leighton 1983]



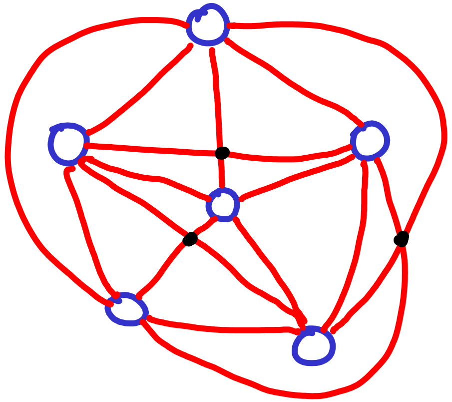
Given a graph G, $\exists$ a drawing w/ $cr(G)$ crossings

Suppose you also have a parameter $0 < p \leq 1$

G_p : subgraph of G :

A better bound

[Leighton 1983]



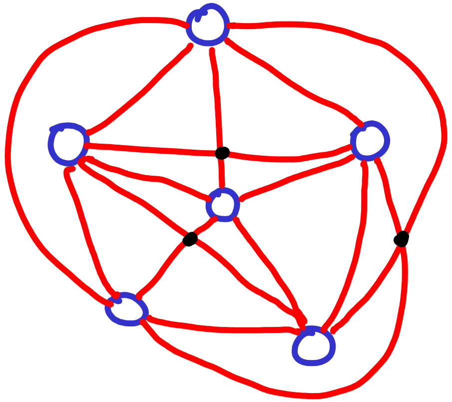
Given a graph G, $\exists$ a drawing w/ $cr(G)$ crossings

Suppose you also have a parameter $0 < p \leq 1$

G_p : subgraph of G : - for each $v \in V$, keep v with probability p

A better bound

[Leighton 1983]



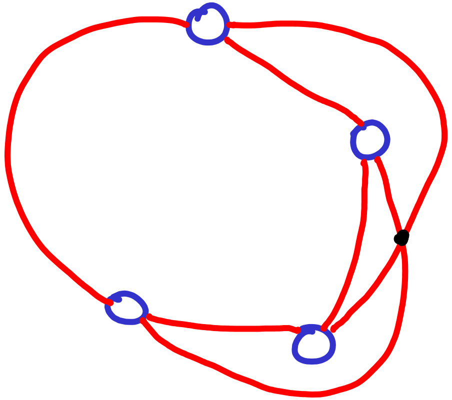
K_6

Given a graph G, $\exists$ a drawing w/ $cr(G)$ crossings

Suppose you also have a parameter $0 < p \leq 1$

G_p : subgraph of G :

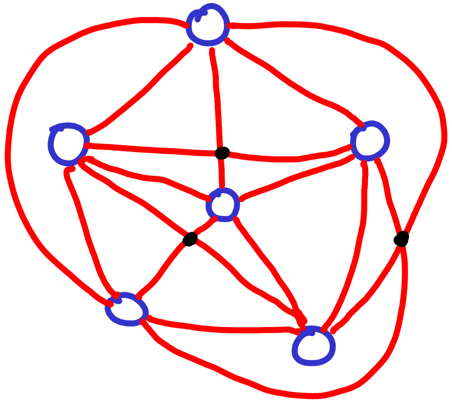
- for each $v \in V$, keep v with probability p
- for each $e \in E$ keep e iff both endpoints survive



$v_p = 4$

$e_p = 5$

A better bound [Leighton 1983]



Given a graph G, $\exists$ a drawing w/ $cr(G)$ crossings

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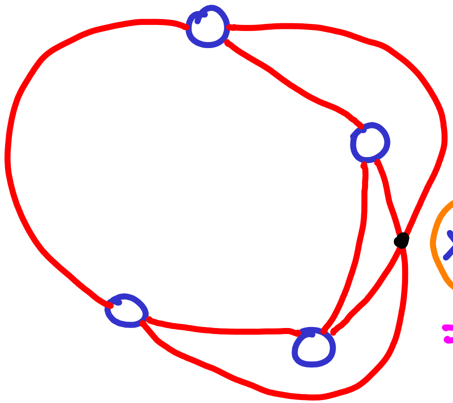


$x_p \geq cr(G_p)$

$=1$

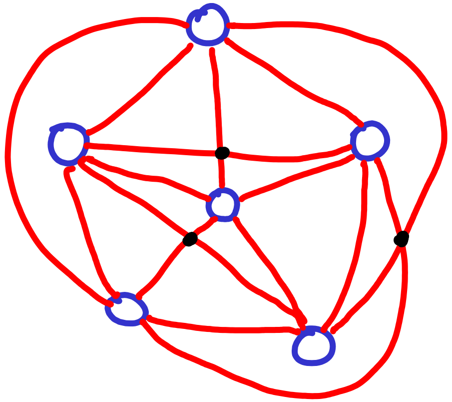
actual # crossings in G_p

$v_p = 4$

$e_p = 5$

A better bound

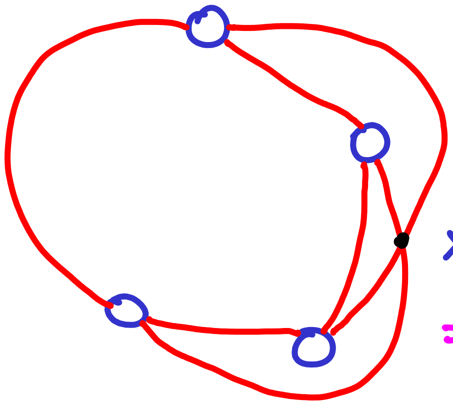
[Leighton 1983]



Given a graph G, $\exists$ a drawing w/ $cr(G)$ crossings

Suppose you also have a parameter $0 < p \leq 1$

G_p : subgraph of G :
- for each $v \in V$, keep v with probability p
- for each $e \in E$ keep e iff both endpoints survive



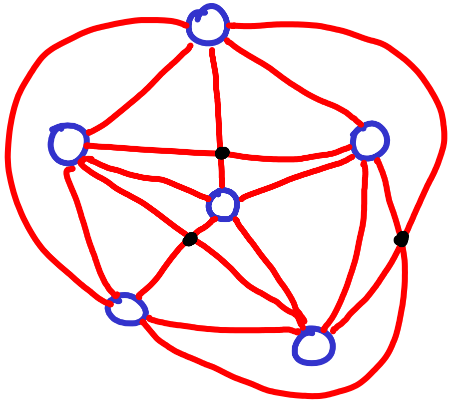
$x_p \geq cr(G_p)$
$= 1$

$v_p = 4$
$e_p = 5$

We have proved $cr(G) \geq E - 3V + 6$
(for $V > 3$)
(for any G)

A better bound

[Leighton 1983]

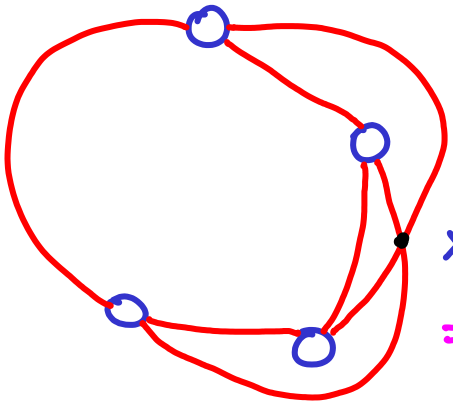


K_6

Given a graph G, $\exists$ a drawing w/ $cr(G)$ crossings

Suppose you also have a parameter $0 < p \leq 1$

G_p : subgraph of G :
- for each $v \in V$, keep v with probability p
- for each $e \in E$ keep e iff both endpoints survive



$x_p \geq cr(G_p)$
$= 1$

$v_p = 4$
$e_p = 5$

We have proved $cr(G) \geq E - 3V + 6$ (for $V > 3$)

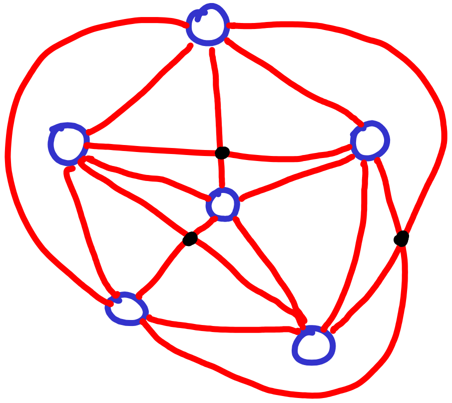
we can relax this:

$cr(G_p) - e_p + 3v_p \geq 0$ (for $V > 0$)

A better bound

[Leighton 1983]

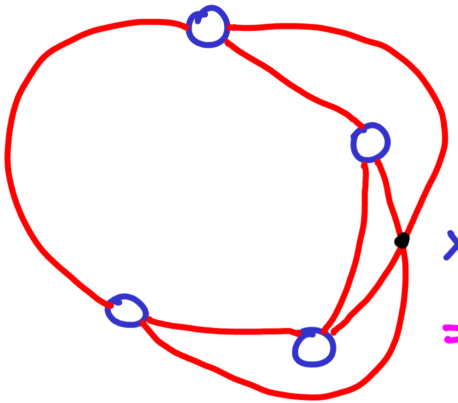
K_6



Given a graph G, $\exists$ a drawing w/ $cr(G)$ crossings

Suppose you also have a parameter $0 < p \leq 1$

G_p : subgraph of G :
- for each $v \in V$, keep v with probability p
- for each $e \in E$ keep e iff both endpoints survive



$x_p \geq cr(G_p)$
$= 1$

$v_p = 4$
$e_p = 5$

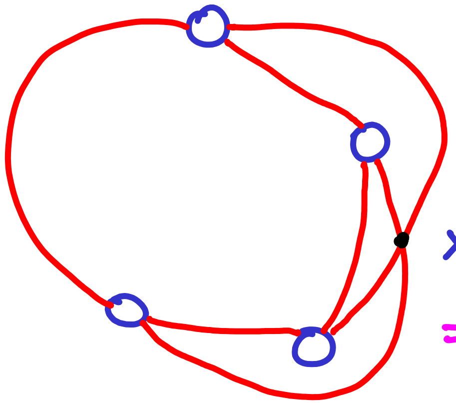
We have proved $cr(G) \geq E - 3V + 6$ (for $V > 3$)

we can relax this:

$cr(G_p) - e_p + 3v_p \geq 0$ (for $V > 0$)

$x_p - e_p + 3v_p \geq 0$

$x_p - e_p + 3v_p \geqslant 0$



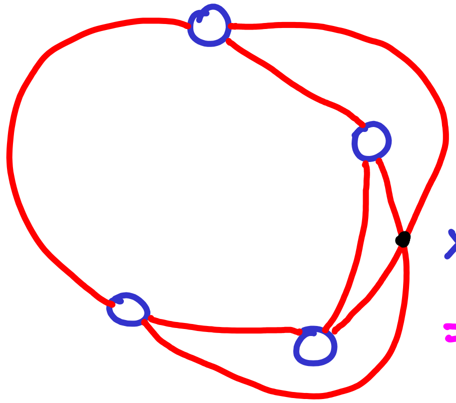
$v_p = 4$

$e_p = 5$

$x_p \geqslant cr(G_p)$

$= 1$

$x_p - e_p + 3v_p \geqslant 0$ These are random variables



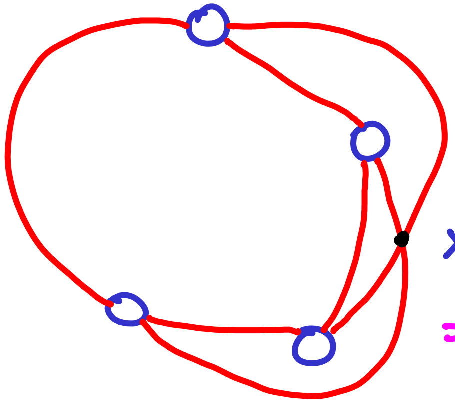
$v_p = 4$

$e_p = 5$

$x_p \geqslant cr(G_p)$
$= 1$

$x_p - e_p + 3v_p \geqslant 0$ These are random variables

$E[x_p - e_p + 3v_p] \geqslant 0$



$v_p = 4$

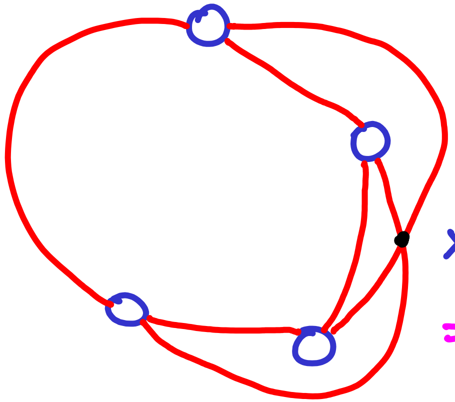
$e_p = 5$

$x_p \geqslant cr(G_p)$
$= 1$

$x_p - e_p + 3v_p \geqslant 0$ These are random variables

$E[x_p - e_p + 3v_p] \geqslant 0$

$E[x_p] - E[e_p] + E[3v_p] \geqslant 0$ (lin. of exp.)



$v_p = 4$

$e_p = 5$

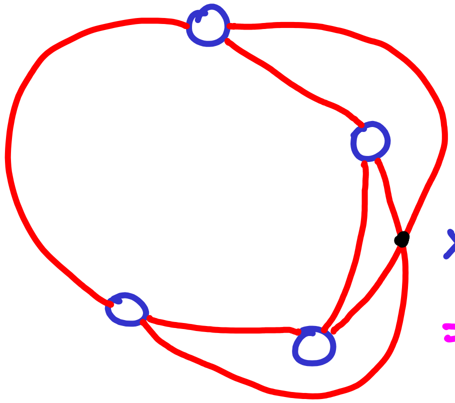
$x_p \geqslant cr(G_p) = 1$

$x_p - e_p + 3v_p \geqslant 0$ These are random variables

$E[x_p - e_p + 3v_p] \geqslant 0$

$E[x_p] - E[e_p] + E[3v_p] \geqslant 0$

$E[v_p] = \ ?$

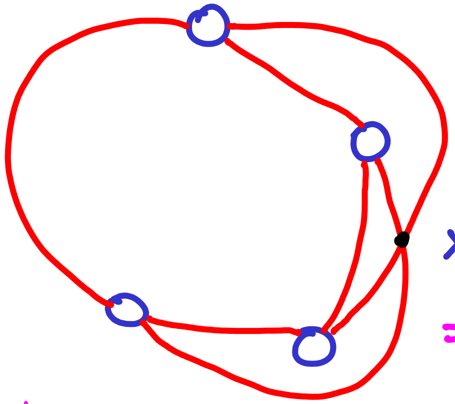


$v_p = 4$

$e_p = 5$

$x_p \geqslant cr(G_p) = 1$

$x_p - e_p + 3v_p \geq 0$ These are random variables



$v_p = 4$

$e_p = 5$

$x_p \geq cr(G_p)$

$= 1$

$E[x_p - e_p + 3v_p] \geq 0$

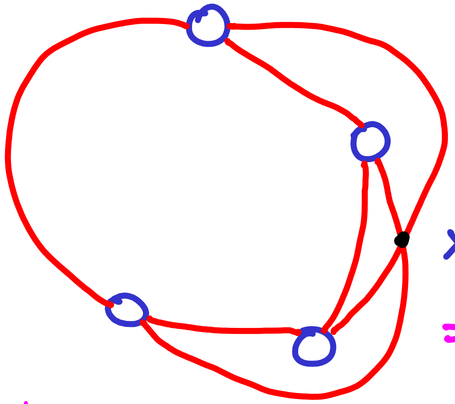
$E[x_p] - E[e_p] + E[3v_p] \geq 0$

$E[v_p] =$ // every vertex appears w/ prob p

$x_p - e_p + 3v_p \geq 0$ These are random variables

$E[x_p - e_p + 3v_p] \geq 0$

$E[x_p] - E[e_p] + E[3v_p] \geq 0$



$v_p = 4$

$e_p = 5$

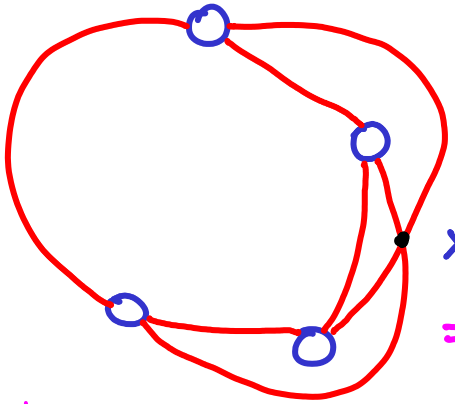
$x_p \geq cr(G_p)$

$= 1$

$E[v_p] = p \cdot V$ // every vertex appears w/ prob p

$E[e_p] =$

$x_p - e_p + 3v_p \geq 0$ These are random variables



$v_p = 4$

$e_p = 5$

$x_p \geq cr(G_p)$

$= 1$

$E[x_p - e_p + 3v_p] \geq 0$

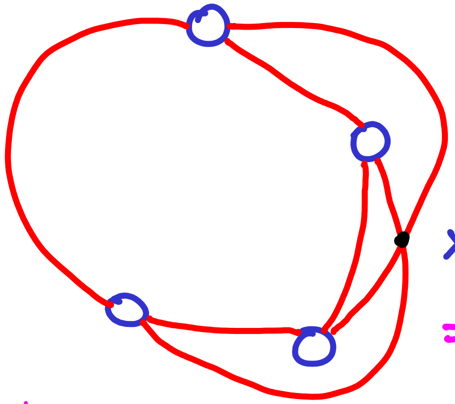
$E[x_p] - E[e_p] + E[3v_p] \geq 0$

$E[v_p] = p \cdot V$ // every vertex appears w/ prob p

$E[e_p] =$ // for every edge, both endpoints must survive

(note there are no self-loops)

$x_p - e_p + 3v_p \geq 0$ These are random variables



$E[x_p - e_p + 3v_p] \geq 0$

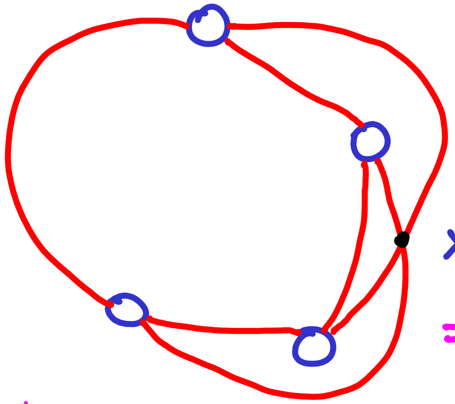
$E[x_p] - E[e_p] + E[3v_p] \geq 0$

$E[v_p] = p \cdot V$ // every vertex appears w/ prob p

$E[e_p] = p^2 \cdot E$ // for every edge, both endpoints must survive

$E[x_p] =$

$x_p - e_p + 3v_p \geq 0$ These are random variables



$E[x_p - e_p + 3v_p] \geq 0$

$E[x_p] - E[e_p] + E[3v_p] \geq 0$

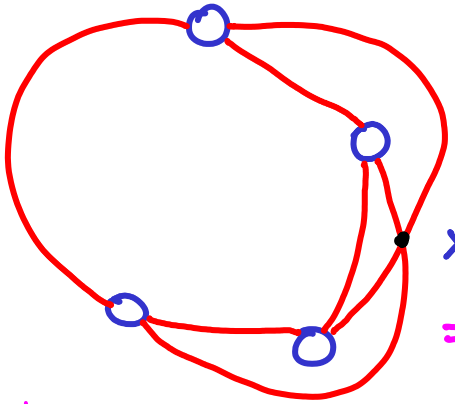
$E[v_p] = p \cdot V$ // every vertex appears w/ prob p

$E[e_p] = p^2 \cdot E$ // for every edge, both endpoints must survive

$E[x_p] =$ // any crossing in G will survive iff its 2 edges survive

= iff 4 endpoints survive

$x_p - e_p + 3v_p \geq 0$ These are random variables



$v_p = 4$

$e_p = 5$

$x_p \geq cr(G_p)$

$= 1$

$E[x_p - e_p + 3v_p] \geq 0$

$E[x_p] - E[e_p] + E[3v_p] \geq 0$

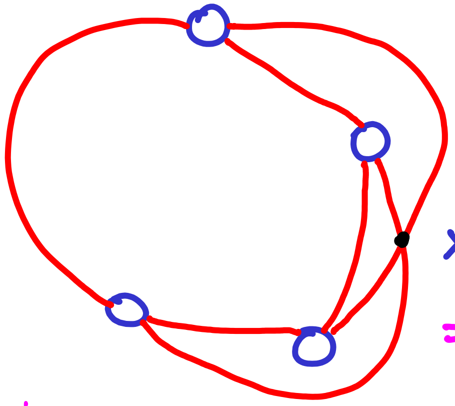
$E[v_p] = p \cdot V$ // every vertex appears w/ prob p

$E[e_p] = p^2 \cdot E$ // for every edge, both endpoints must survive

$E[x_p] = p^4 \cdot cr(G)$ // any crossing in G will survive iff its 2 edges survive

= iff 4 endpoints survive

$x_p - e_p + 3v_p \geq 0$ These are random variables



$E[x_p - e_p + 3v_p] \geq 0$

$E[x_p] - E[e_p] + E[3v_p] \geq 0$

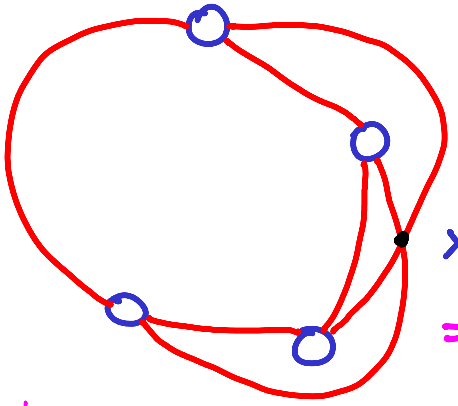
$E[v_p] = p \cdot V$ // every vertex appears w/ prob p

$E[e_p] = p^2 \cdot E$ // for every edge, both endpoints must survive

$E[x_p] = p^4 \cdot cr(G)$ // any crossing in G will survive iff its 2 edges survive = iff 4 endpoints survive

$p^4 \cdot cr(G) - p^2 \cdot E + 3p \cdot V \geq 0$

$x_p - e_p + 3v_p \geq 0$ These are random variables



$v_p = 4$

$e_p = 5$

$x_p \geq cr(G_p) = 1$

$E[x_p - e_p + 3v_p] \geq 0$

$E[x_p] - E[e_p] + E[3v_p] \geq 0$

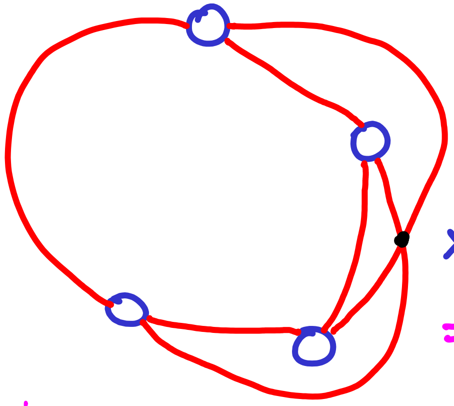
$E[v_p] = p \cdot V$ // every vertex appears w/ prob p

$E[e_p] = p^2 \cdot E$ // for every edge, both endpoints must survive

$E[x_p] = p^4 \cdot cr(G)$ // any crossing in G will survive iff its 2 edges survive = iff 4 endpoints survive

$p^4 \cdot cr(G) - p^2 \cdot E + 3p \cdot V \geq 0 \quad \Rightarrow cr(G) \geq \dfrac{p^2 E - 3pV}{p^4}$ (for any $0 < p \leq 1$)

$x_p - e_p + 3v_p \geq 0$ These are random variables



$E[x_p - e_p + 3v_p] \geq 0$

$E[x_p] - E[e_p] + E[3v_p] \geq 0$

$E[v_p] = p \cdot V$ // every vertex appears w/ prob p

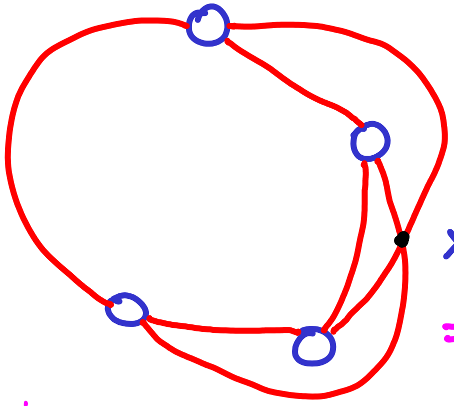
$E[e_p] = p^2 \cdot E$ // for every edge, both endpoints must survive

$E[x_p] = p^4 \cdot cr(G)$ // any crossing in G will survive iff its 2 edges survive = iff 4 endpoints survive

$p^4 \cdot cr(G) - p^2 \cdot E + 3p \cdot V \geq 0 \Rightarrow cr(G) \geq \dfrac{p^2 E - 3pV}{p^4}$ (for any $0 < p \leq 1$)

Choose $p = \dfrac{4V}{E}$ / Assumption $E \geq 4V$ /

$x_p - e_p + 3v_p \geq 0$ These are random variables



$v_p = 4$

$e_p = 5$

$x_p \geq cr(G_p) = 1$

$E[x_p - e_p + 3v_p] \geq 0$

$E[x_p] - E[e_p] + E[3v_p] \geq 0$

$E[v_p] = p \cdot V$ // every vertex appears w/ prob p

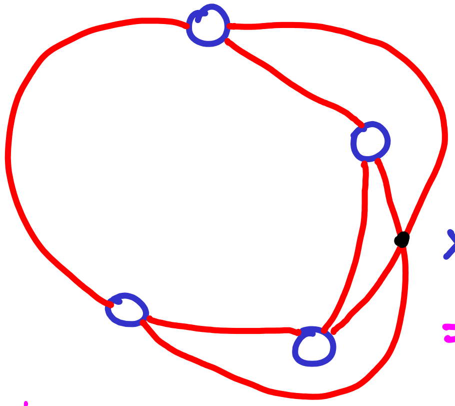
$E[e_p] = p^2 \cdot E$ // for every edge, both endpoints must survive

$E[x_p] = p^4 \cdot cr(G)$ // any crossing in G will survive iff its 2 edges survive = iff 4 endpoints survive

$p^4 \cdot cr(G) - p^2 \cdot E + 3p \cdot V \geq 0 \Rightarrow cr(G) \geq \dfrac{p^2 E - 3pV}{p^4}$ (for any $0 < p \leq 1$)

Choose $p = \dfrac{4V}{E}$ / Assumption $E \geq 4V$ / $\Rightarrow cr(G) \geq \dfrac{\frac{16V^2}{E^2}E - 3\frac{4V}{E}V}{256V^4/E^4}$

$x_p - e_p + 3v_p \geqslant 0$ These are random variables



$v_p = 4$
$e_p = 5$
$x_p \geqslant cr(G_p) = 1$

$E[x_p - e_p + 3v_p] \geqslant 0$

$E[x_p] - E[e_p] + E[3v_p] \geqslant 0$

$E[v_p] = p \cdot V$ // every vertex appears w/ prob p

$E[e_p] = p^2 \cdot E$ // for every edge, both endpoints must survive

$E[x_p] = p^4 \cdot cr(G)$ // any crossing in G will survive iff its 2 edges survive = iff 4 endpoints survive

$p^4 \cdot cr(G) - p^2 \cdot E + 3p \cdot V \geqslant 0 \Rightarrow cr(G) \geqslant \dfrac{p^2 E - 3pV}{p^4}$ (for any $0 < p \leq 1$)

Choose $p = \dfrac{4V}{E}$ / Assumption $E \geqslant 4V$ / $\Rightarrow cr(G) \geqslant \dfrac{\frac{16V^2}{E^2}E - 3\frac{4V}{E}V}{256V^4/E^4} \Rightarrow cr(G) \geqslant \dfrac{1}{64} \cdot \dfrac{E^3}{V^2}$

$$cr(G) \geq \frac{1}{64} \cdot \frac{E^3}{V^2}$$

$$cr(K_V) = \cancel{\Omega(V^2)} \geq \frac{1}{64} \frac{\binom{V}{2}^3}{V^2} \approx \frac{1}{512} \cdot V^4 = \Omega(V^4)$$

$$= O(V^4)$$

$$cr(G) \geqslant \frac{1}{64} \cdot \frac{E^3}{V^2}$$

$$cr(K_V) = \cancel{\Omega(V^2)} \geqslant \frac{1}{64} \frac{\binom{V}{2}^3}{V^2} \approx \frac{1}{512} \cdot V^4 = \Omega(V^4)$$

$$= O(V^4)$$

for K_V, if $p = \frac{4V}{E}$ then $p = \frac{4V}{\binom{V}{2}} = \frac{4V}{\frac{1}{2}V(V-1)} = \frac{8}{V-1}$

$cr(G) \geq \frac{1}{64} \cdot \frac{E^3}{V^2}$

$cr(K_V) = \cancel{\Omega(V^2)}$
$= O(V^4)$

$\geq \frac{1}{64} \frac{\binom{V}{2}^3}{V^2} \approx \frac{1}{512} \cdot V^4 = \Omega(V^4)$

for K_V, if $p = \frac{4V}{E}$ then $p = \frac{4V}{\binom{V}{2}} = \frac{4V}{\frac{1}{2}V(V-1)} = \frac{8}{V-1}$

we had:

$E[X_p] - E[e_p] + E[3V_p] \geq 0$

$$cr(G) \geq \frac{1}{64} \cdot \frac{E^3}{V^2}$$

$$cr(K_V) = \cancel{\Omega(V^2)} \geq \frac{1}{64} \frac{\binom{V}{2}^3}{V^2} \approx \frac{1}{512} \cdot V^4 = \Omega(V^4)$$

$$= O(V^4)$$

for K_V, if $p = \frac{4V}{E}$ then $p = \frac{4V}{\binom{V}{2}} = \frac{4V}{\frac{1}{2}V(V-1)} = \frac{8}{V-1}$

$$E[X_p] - E[e_p] + E[3V_p] \geq 0$$

for K_V $\left\{ \begin{array}{l} E[V_p] = \\ E[e_p] = \\ E[x_p] = \end{array} \right.$

$$cr(G) \geq \frac{1}{64} \cdot \frac{E^3}{V^2}$$

$$cr(K_V) = \cancel{\Omega(V^2)} \geq \frac{1}{64} \frac{\binom{V}{2}^3}{V^2} \approx \frac{1}{512} \cdot V^4 = \Omega(V^4)$$

$$= O(V^4)$$

for K_V, if $p = \frac{4V}{E}$ then $p = \frac{4V}{\binom{V}{2}} = \frac{4V}{\frac{1}{2}V(V-1)} = \frac{8}{V-1}$

$$E[X_p] - E[e_p] + E[3V_p] \geq 0$$

for K_V

$$E[V_p] = p \cdot V \approx 8$$

$$E[e_p] =$$

$$E[x_p] =$$

$$cr(G) \geq \frac{1}{64} \cdot \frac{E^3}{V^2}$$

$$cr(K_V) = \cancel{\Omega(V^2)} \geq \frac{1}{64} \frac{\binom{V}{2}^3}{V^2} \approx \frac{1}{512} \cdot V^4 = \Omega(V^4)$$

$$= O(V^4)$$

for K_V, if $p = \frac{4V}{E}$ then $p = \frac{4V}{\binom{V}{2}} = \frac{4V}{\frac{1}{2}V(V-1)} = \frac{8}{V-1}$

$$E[X_p] - E[e_p] + E[3V_p] \geq 0$$

for K_V

$$E[V_p] = p \cdot V \approx 8$$

$$E[e_p] = p^2 \cdot E \approx 32$$

$$E[x_p] =$$

$$cr(G) \geq \frac{1}{64} \cdot \frac{E^3}{V^2}$$

$$cr(K_V) = \cancel{\Omega(V^2)} \geq \frac{1}{64} \frac{\binom{V}{2}^3}{V^2} \approx \frac{1}{512} \cdot V^4 = \Omega(V^4)$$

$$= O(V^4)$$

for K_V, if $p = \frac{4V}{E}$ then $p = \frac{4V}{\binom{V}{2}} = \frac{4V}{\frac{1}{2}V(V-1)} = \frac{8}{V-1}$

$$E[X_p] - E[e_p] + E[3V_p] \geq 0$$

for K_V

$$E[V_p] = p \cdot V \approx 8$$

$$E[e_p] = p^2 \cdot E \approx 32$$

$$E[X_p] = p^4 \cdot cr(K_V)$$

$$cr(G) \geq \frac{1}{64} \cdot \frac{E^3}{V^2}$$

$$cr(K_V) = \cancel{\Omega(V^2)} = O(V^4)$$

$$\geq \frac{1}{64} \frac{\binom{V}{2}^3}{V^2} \approx \frac{1}{512} \cdot V^4 = \Omega(V^4)$$

for K_V, if $p = \frac{4V}{E}$ then $p = \frac{4V}{\binom{V}{2}} = \frac{4V}{\frac{1}{2}V(V-1)} = \frac{8}{V-1}$

$$E[X_p] - E[e_p] + E[3V_p] \geq 0$$

for K_V $\left\{ \begin{array}{l} E[V_p] = p \cdot V \approx 8 \\ E[e_p] = p^2 \cdot E \approx 32 \\ E[x_p] = p^4 \cdot cr(K_V) \geq 8 \approx \frac{4096}{V^4} \cdot cr(K_V) \end{array} \right.$

$$cr(G) \geq \frac{1}{64} \cdot \frac{E^3}{V^2}$$

$$cr(K_V) = \cancel{\Omega(V^2)} \geq \frac{1}{64} \frac{\binom{V}{2}^3}{V^2} \approx \frac{1}{512} \cdot V^4 = \Omega(V^4)$$

$$= O(V^4)$$

for K_V, if $p = \frac{4V}{E}$ then $p = \frac{4V}{\binom{V}{2}} = \frac{4V}{\frac{1}{2}V(V-1)} = \frac{8}{V-1}$

$$E[X_p] - E[e_p] + E[3V_p] \geq 0$$

for K_V

$$E[V_p] = p \cdot V \approx 8$$

$$E[e_p] = p^2 \cdot E \approx 32$$

$$E[X_p] = p^4 \cdot cr(K_V) \geq 8 \approx \frac{4096}{V^4} \cdot cr(K_V)$$

Intuition:

We discard almost all vertices, but still get ≥ 8 crossings, so there must have been many to begin with