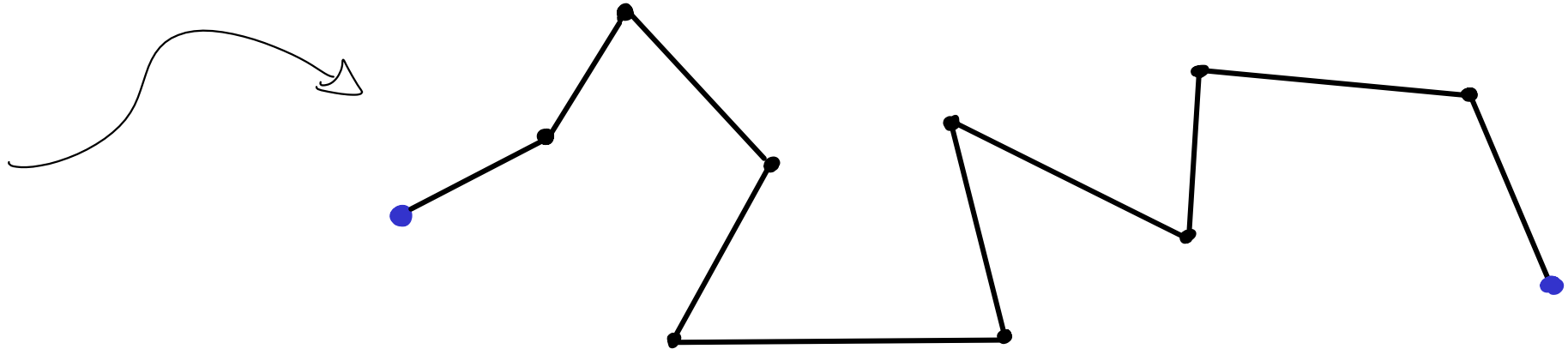
CHAIN SIMPLIFICATION

input:

a chain



CHAIN SIMPLIFICATION

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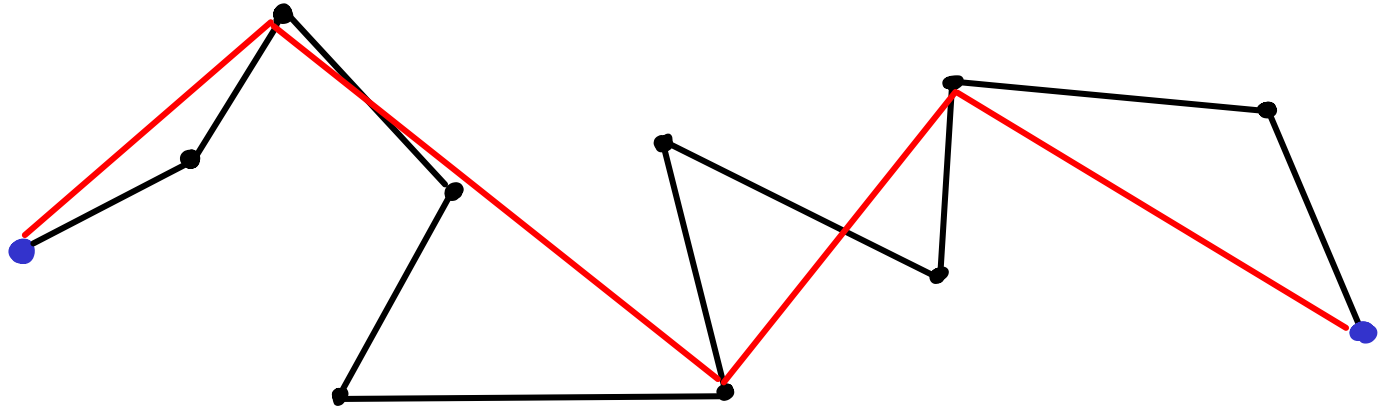
a chain

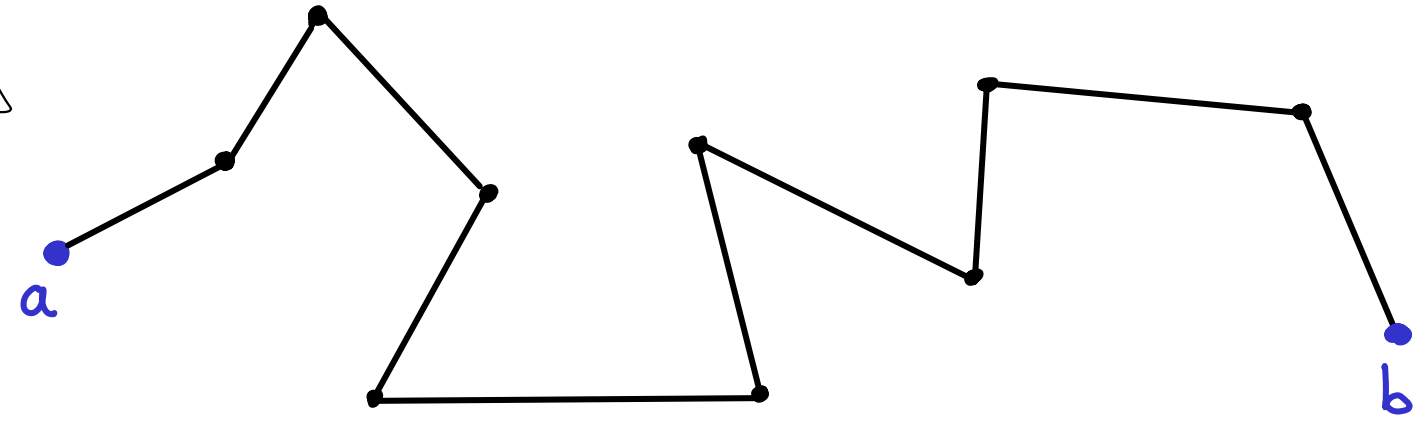


output : "something simpler"

CHAIN SIMPLIFICATION

input:

a chain



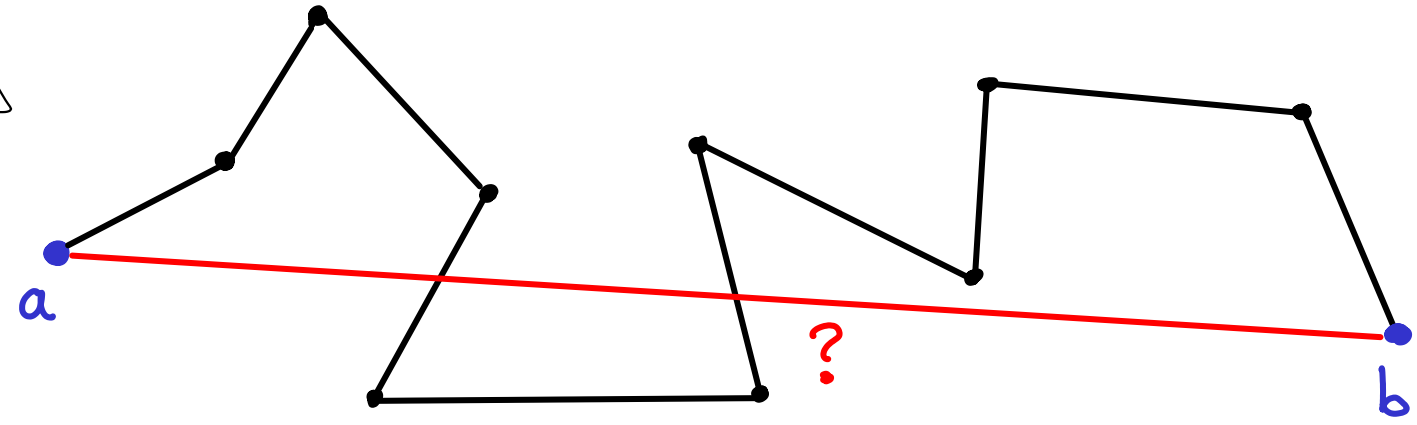
Algorithm:

```
simplify(a,b)   // chain from a to b
```

CHAIN SIMPLIFICATION : "ITERATIVE ENDPOINTS FIT"

input:

a chain



Algorithm:

simplify(a,b) // chain from a to b

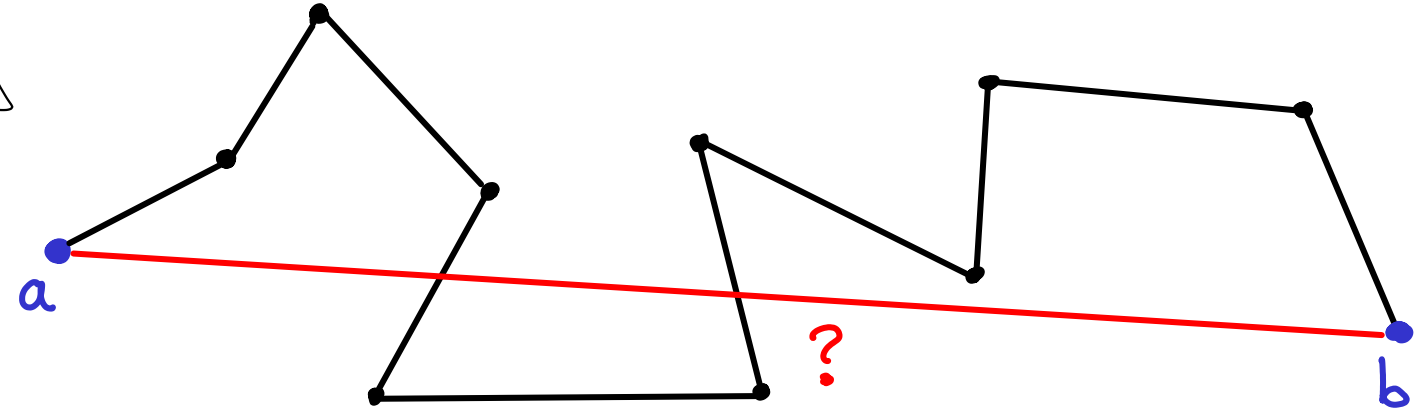
if $\overline{ab}$ doesn't approximate chain(a,b) "well" ?

CHAIN SIMPLIFICATION : "ITERATIVE ENDPOINTS FIT"

input:

① a chain

② an error tolerance



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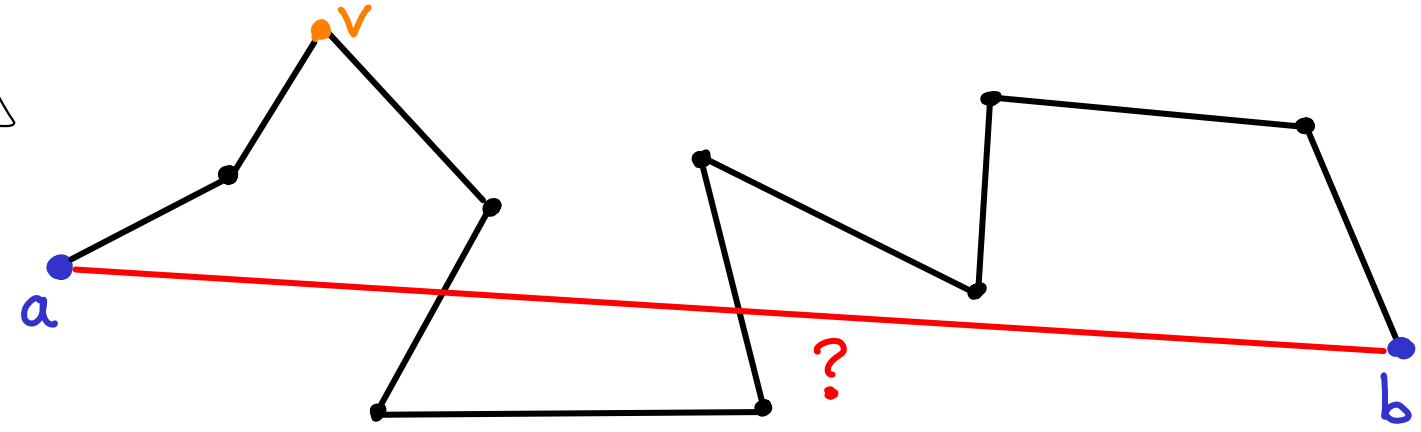

Algorithm:

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```

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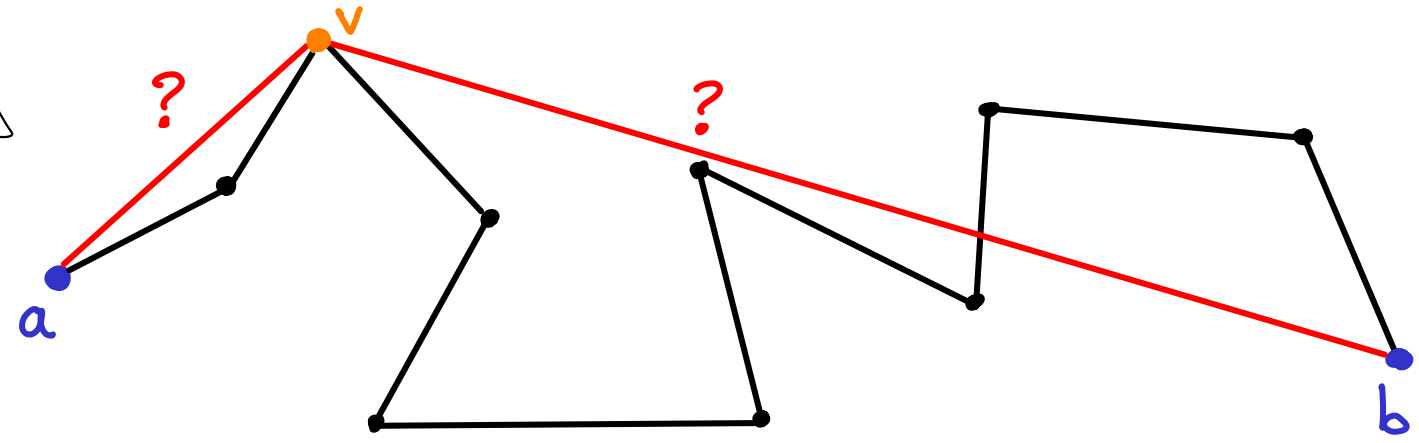

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```
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        simplify(a,v)
        simplify(v,b)
```

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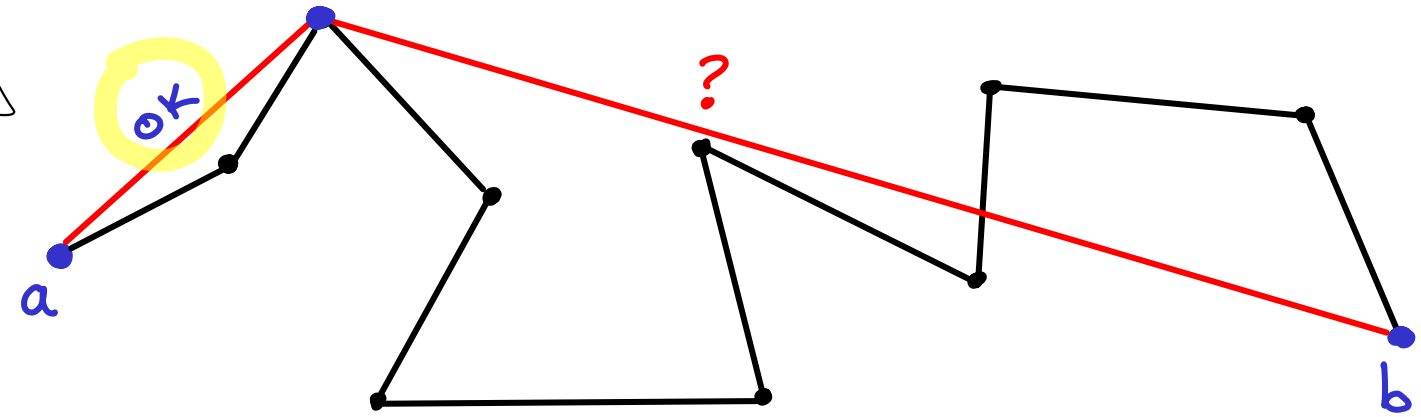
input:

① a chain

② an error tolerance



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    else output ab
```

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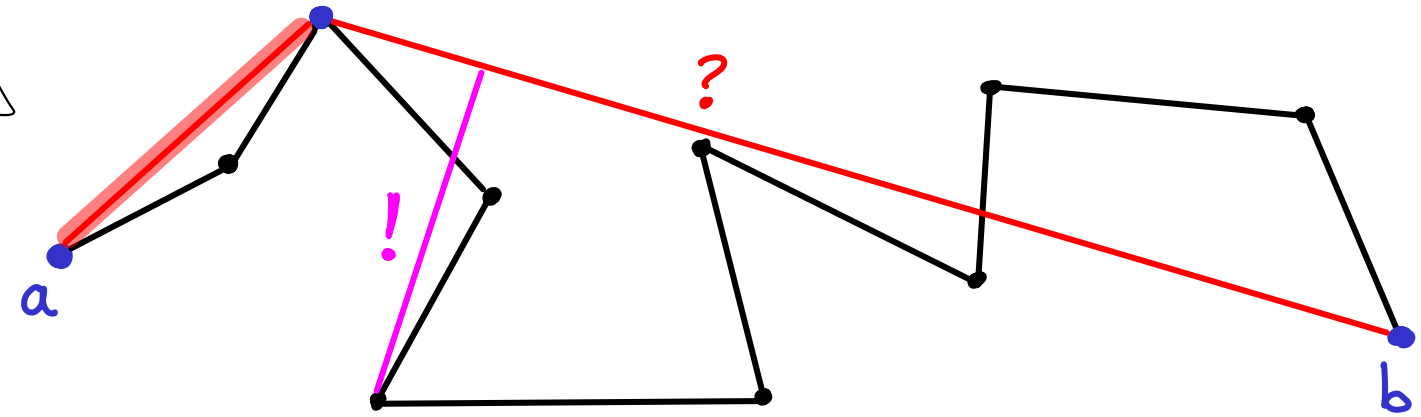

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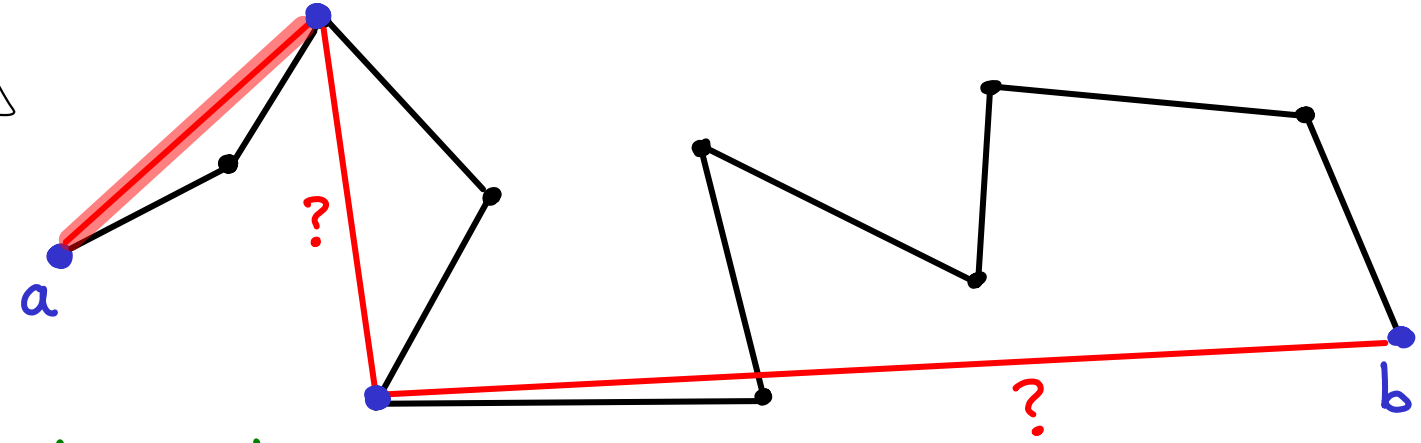

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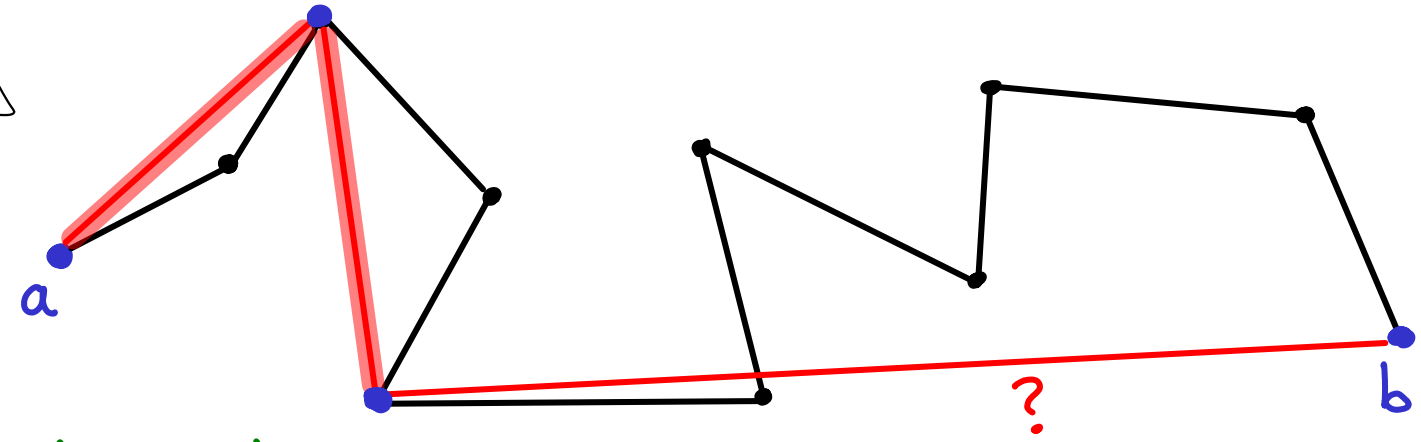

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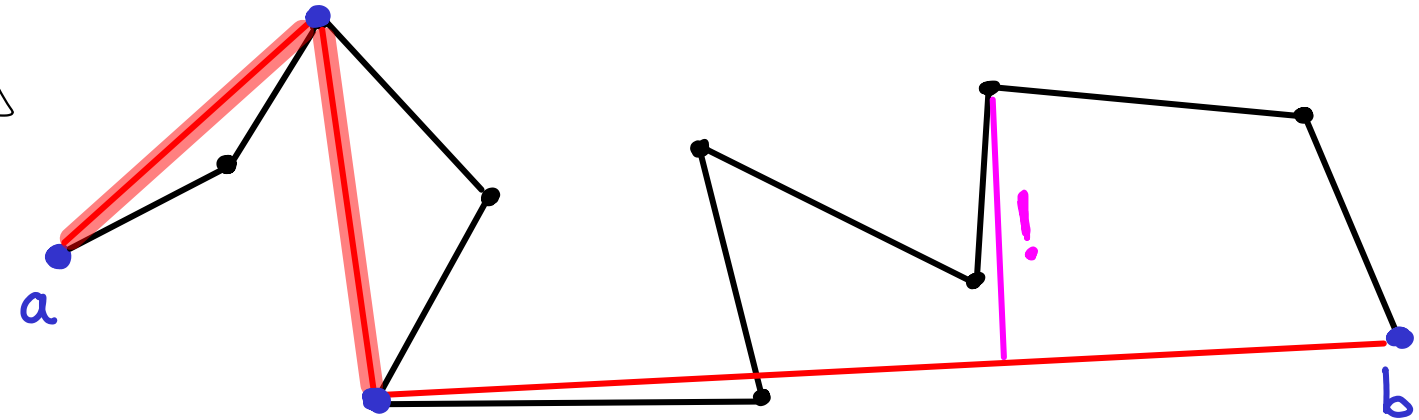

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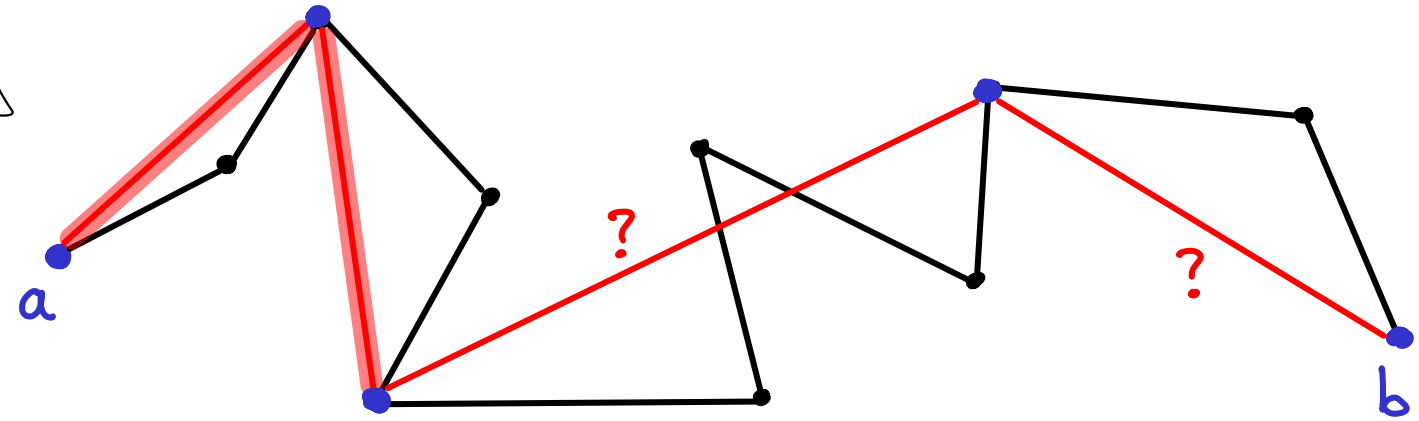

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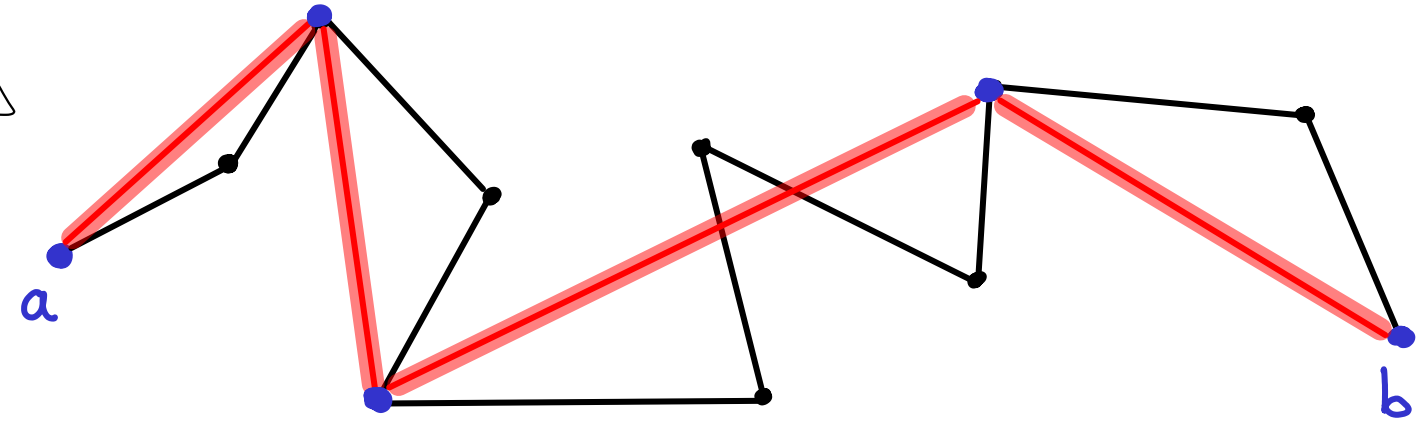
② an error tolerance



one detail missing

Algorithm:

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CHAIN SIMPLIFICATION : "ITERATIVE ENDPOINTS FIT"

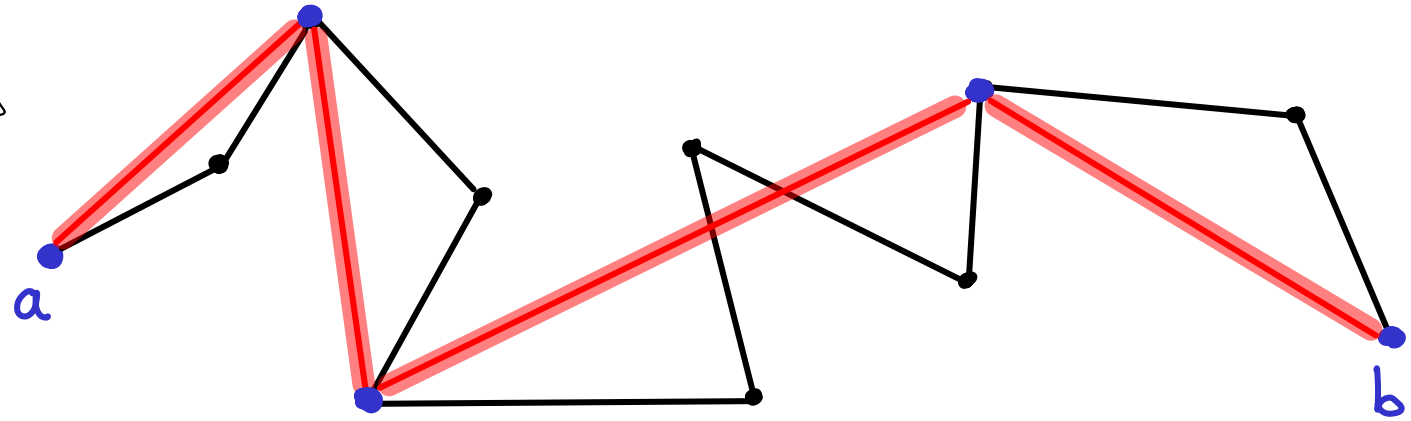
input:

① a chain

② an error tolerance

③ a distance definition

(between points & segments)



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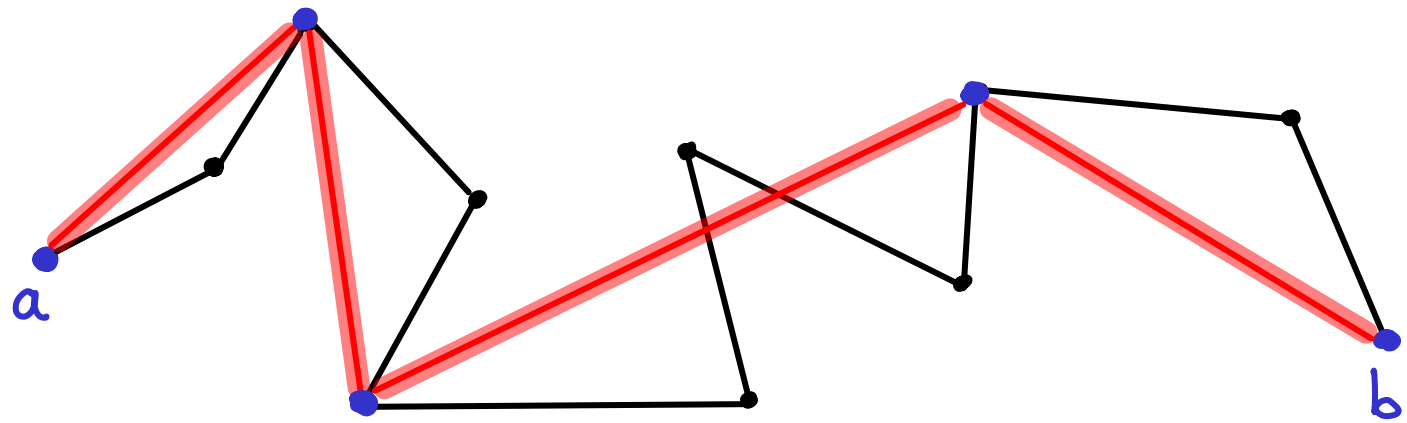
```

CHAIN SIMPLIFICATION : "ITERATIVE ENDPOINTS FIT"

Time (chain length: n)

?



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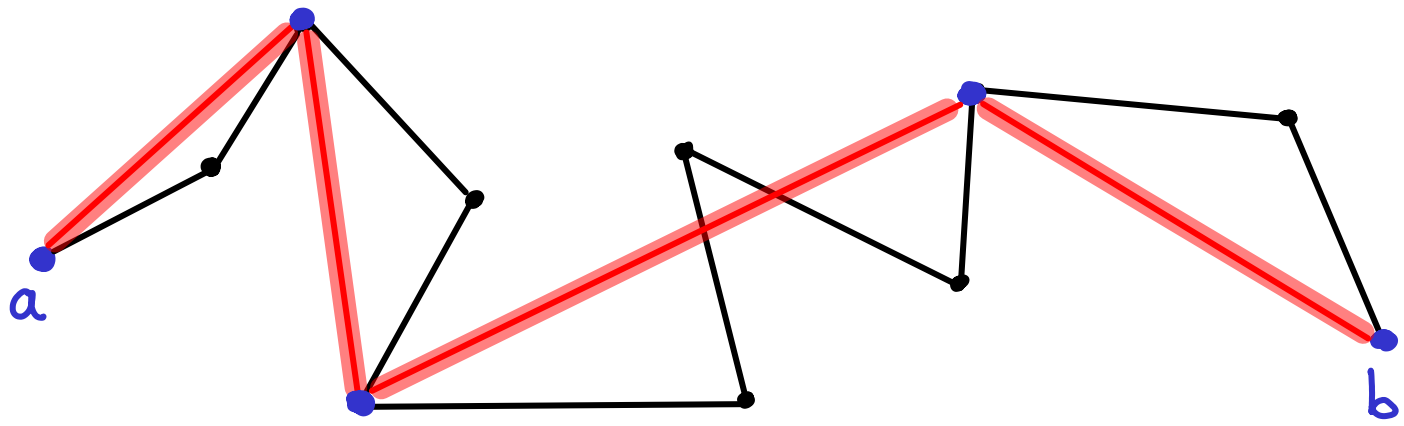
CHAIN SIMPLIFICATION : "ITERATIVE ENDPOINTS FIT"

Time (chain length: n)

- testing $\overline{ab}$: O(n)
  (for reasonable dist)



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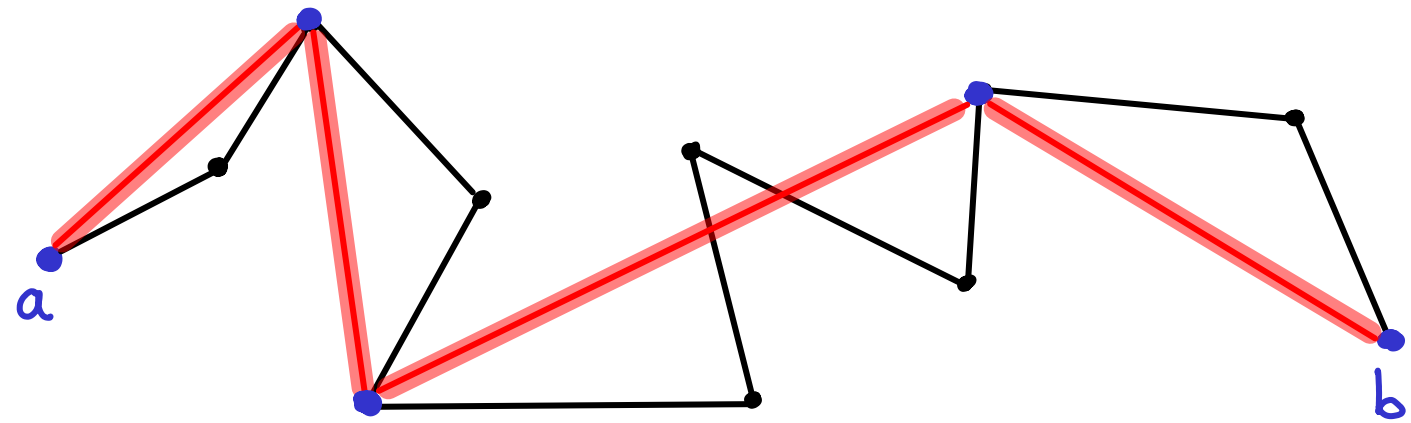
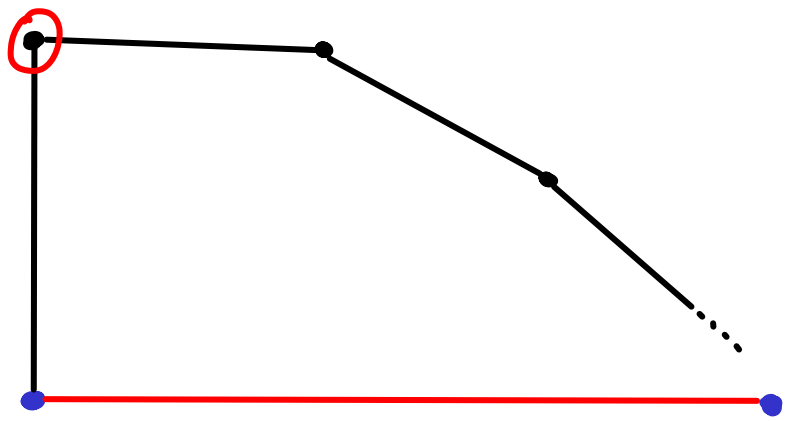
CHAIN SIMPLIFICATION : "ITERATIVE ENDPOINTS FIT"

Time (chain length: n)

- testing $\overline{ab}$: O(n)
  (for reasonable dist)

- worst case:
  unbalanced recursion





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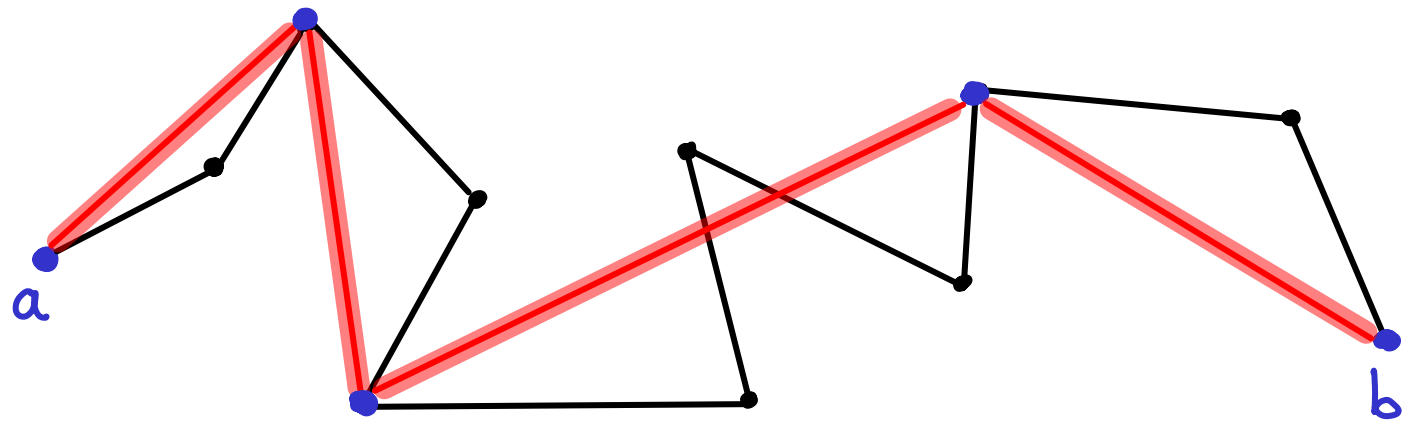

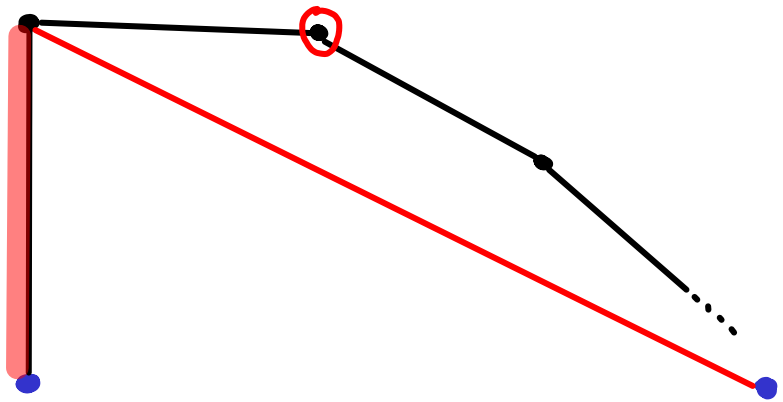

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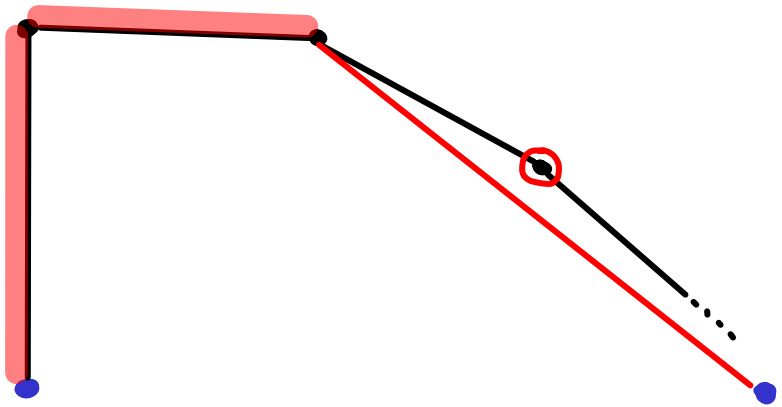
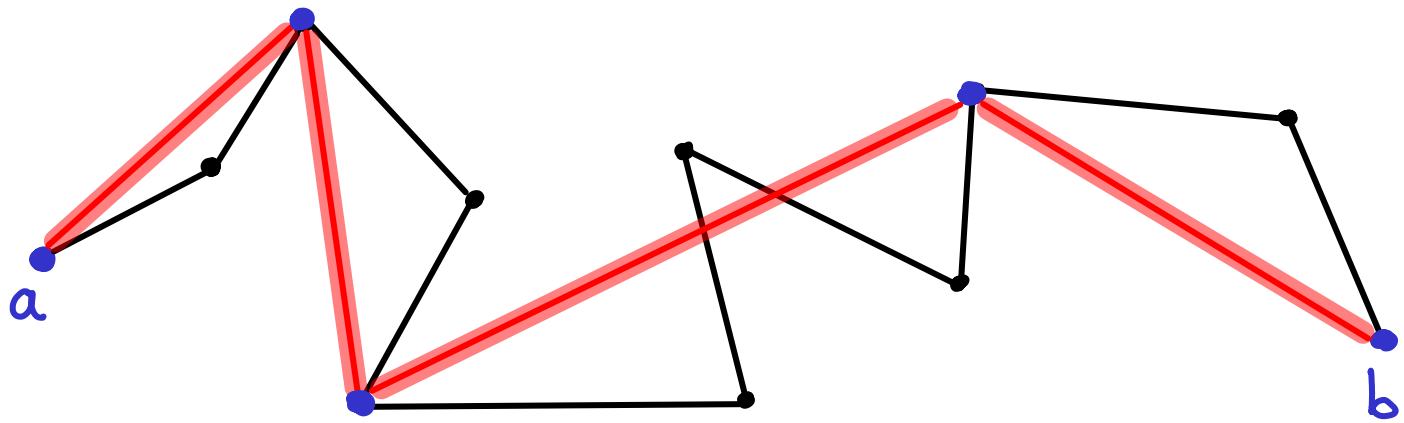
CHAIN SIMPLIFICATION : "ITERATIVE ENDPOINTS FIT"

Time (chain length: n)

- testing $\overline{ab}$: O(n)
  (for reasonable dist)

- worst case:
  unbalanced recursion
  ... $O(n^2)$





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do we always get a simple (non-crossing) chain?

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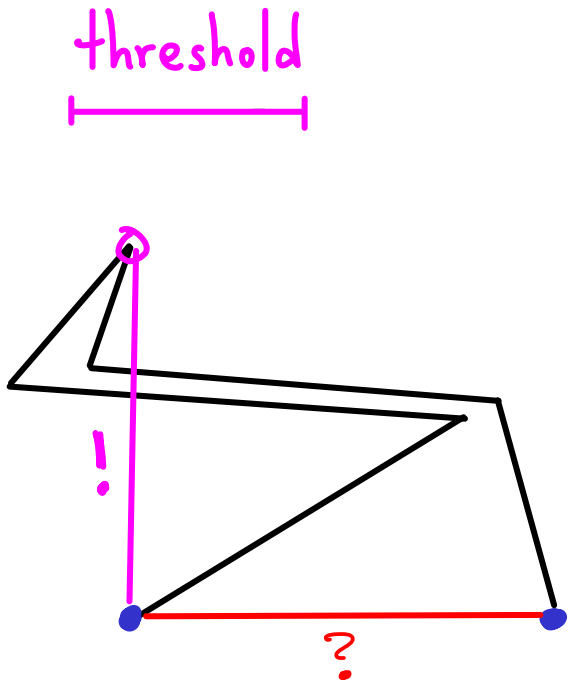

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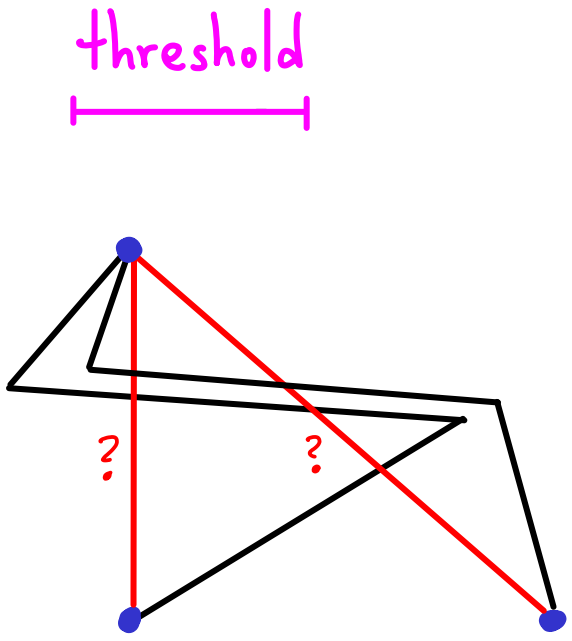

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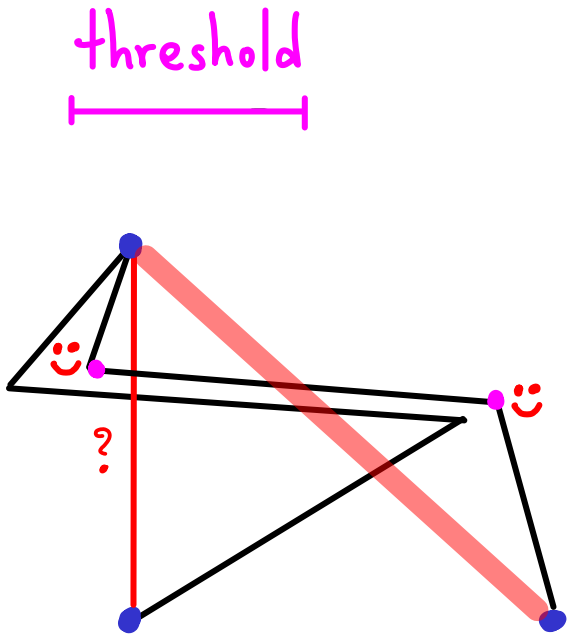

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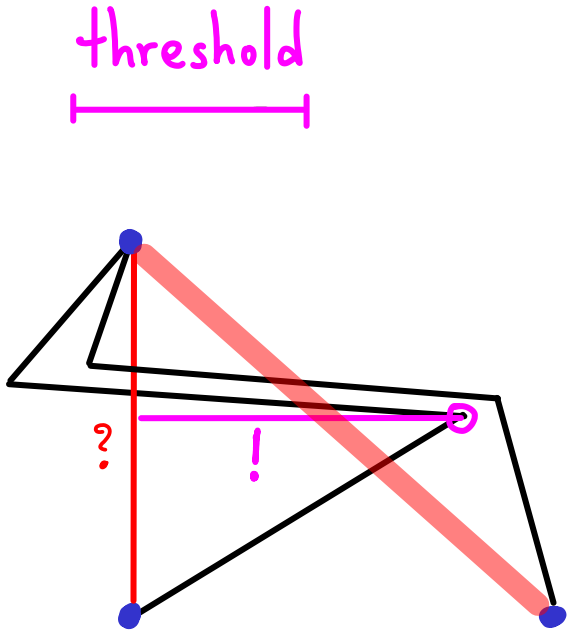
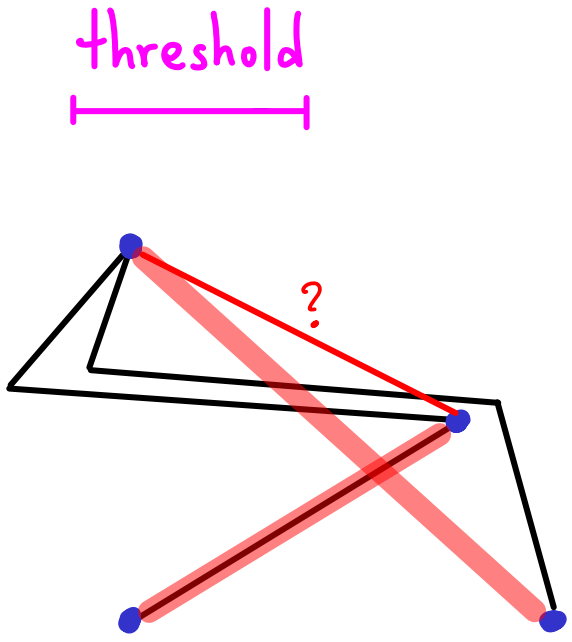
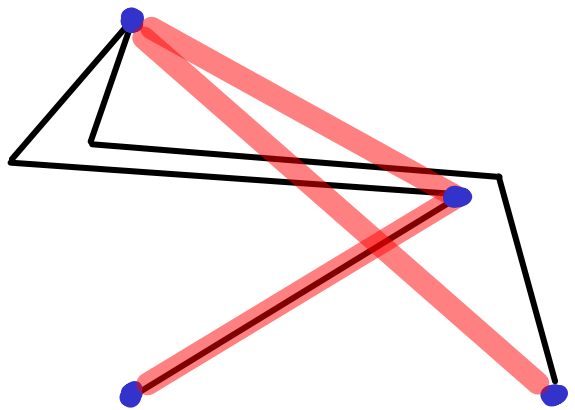


do we always get a simple (non-crossing) chain?



do we always get a simple (non-crossing) chain? NO

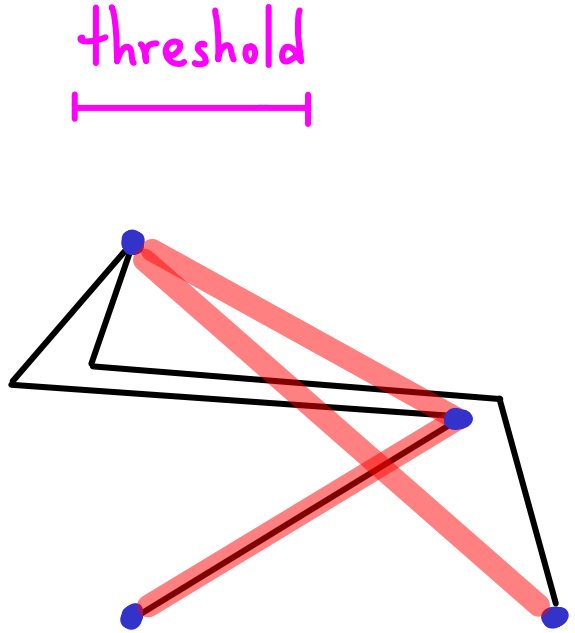
threshold



variants (possibly unexplored)

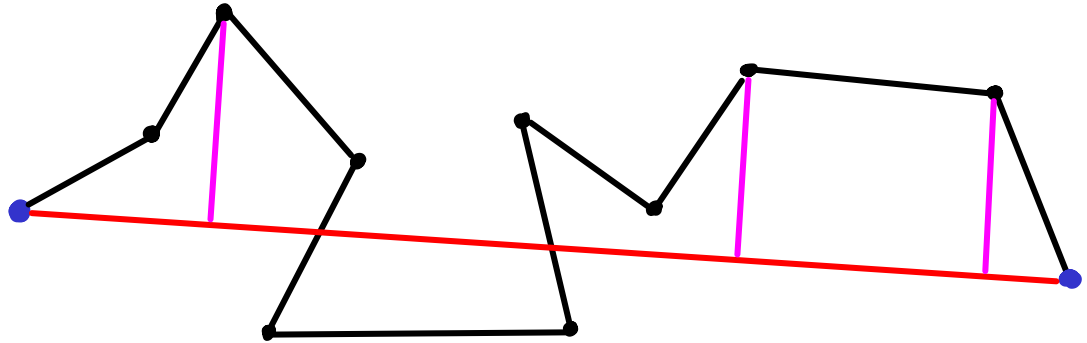
- require non-crossing output

do we always get a simple (non-crossing) chain? NO



variants (possibly unexplored)

- require non-crossing output
- deal with multiple error violations
  - ↳ use median "offender"?
  - ↳ ≥2 recursive calls?



Does the algorithm minimize the number of segments used ?

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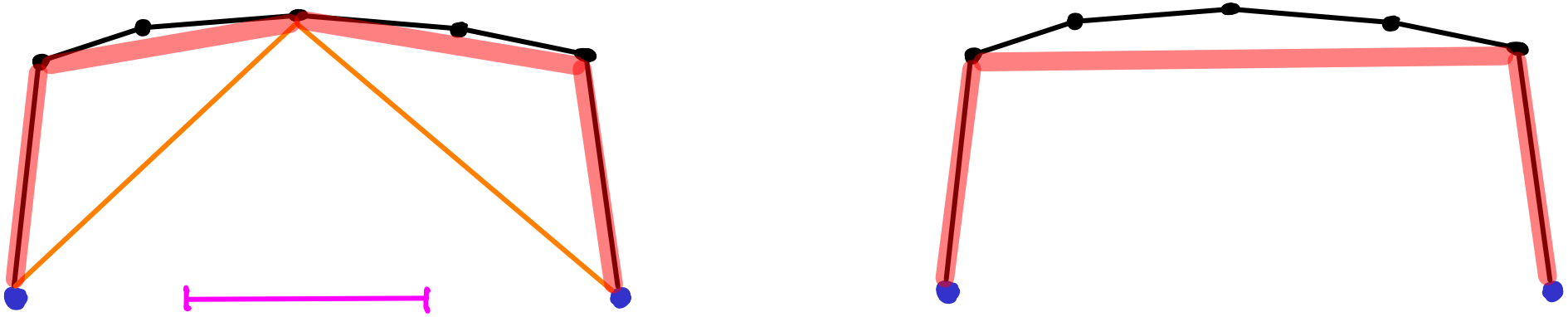

No. can be made arbitrarily worse

IRI-IMAI algorithm to minimize segments (given path & tolerance)

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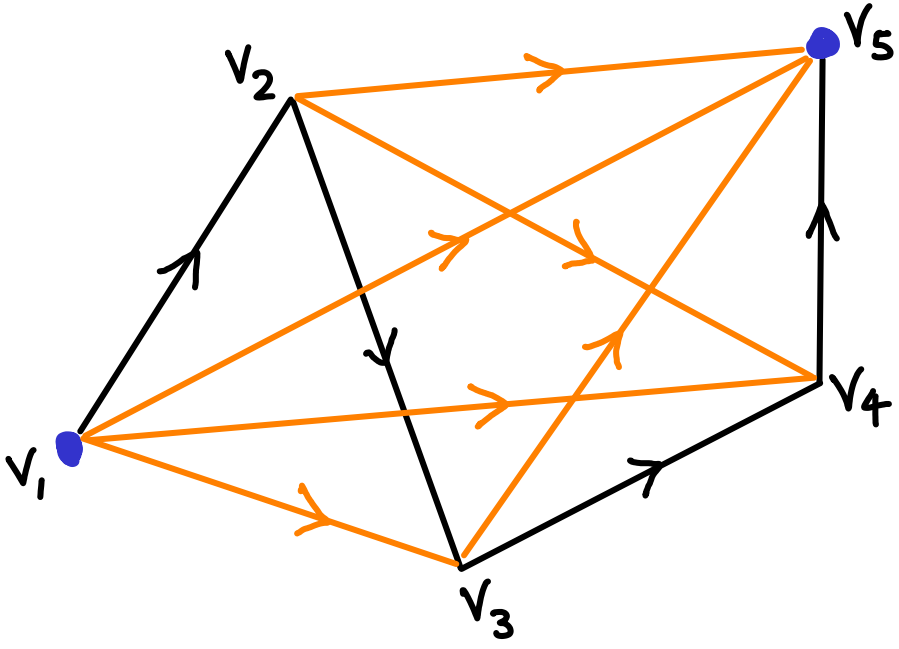
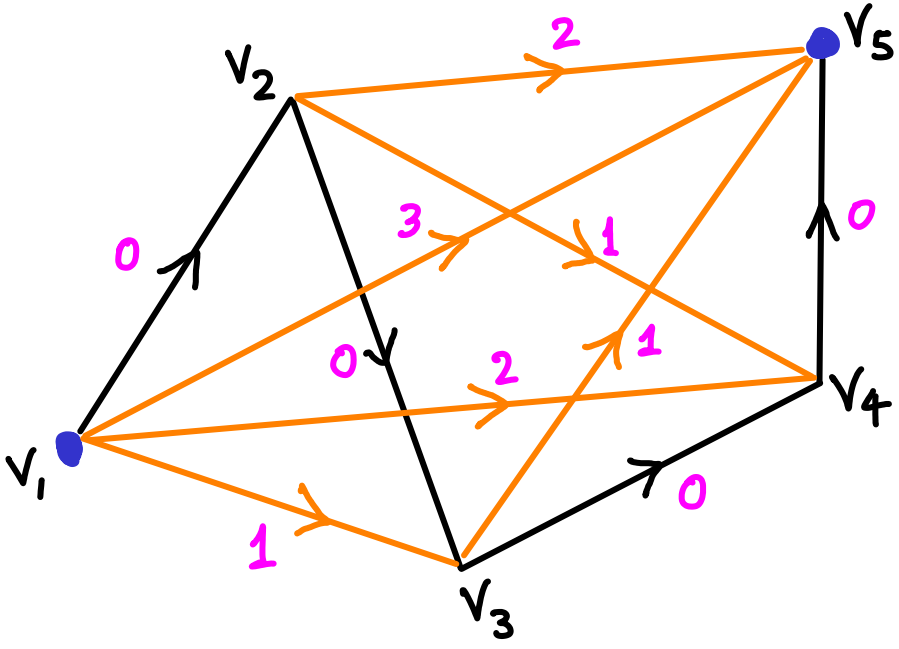


- form directed graph:
  - ↳ make $\overrightarrow{v_i v_j}$ iff $i < j$

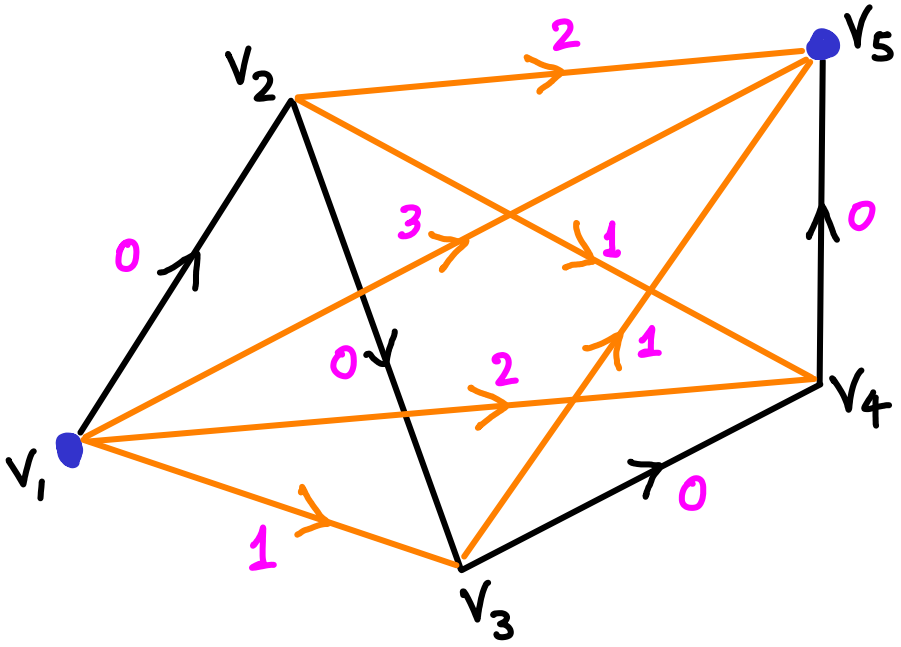
IRI-IMAI algorithm to minimize segments (given path & tolerance)



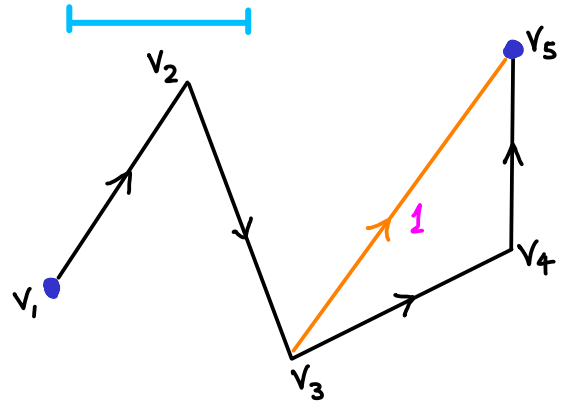
- form directed graph:
  - ↳ make $\overrightarrow{v_i v_j}$ iff $i<j$

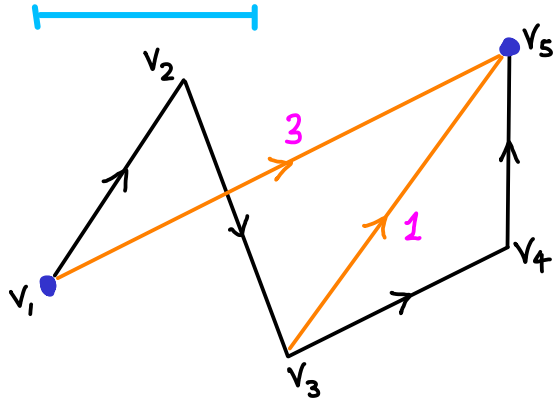
- give a score to each edge:
  - ↳ # path edges skipped
    - ↓
    - i.e. avoided

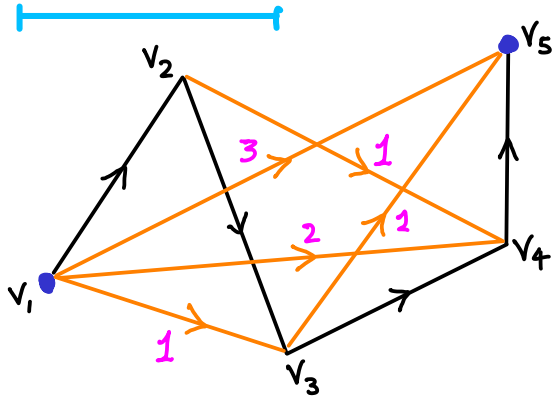
IRI-IMAI algorithm to minimize segments (given path & tolerance)



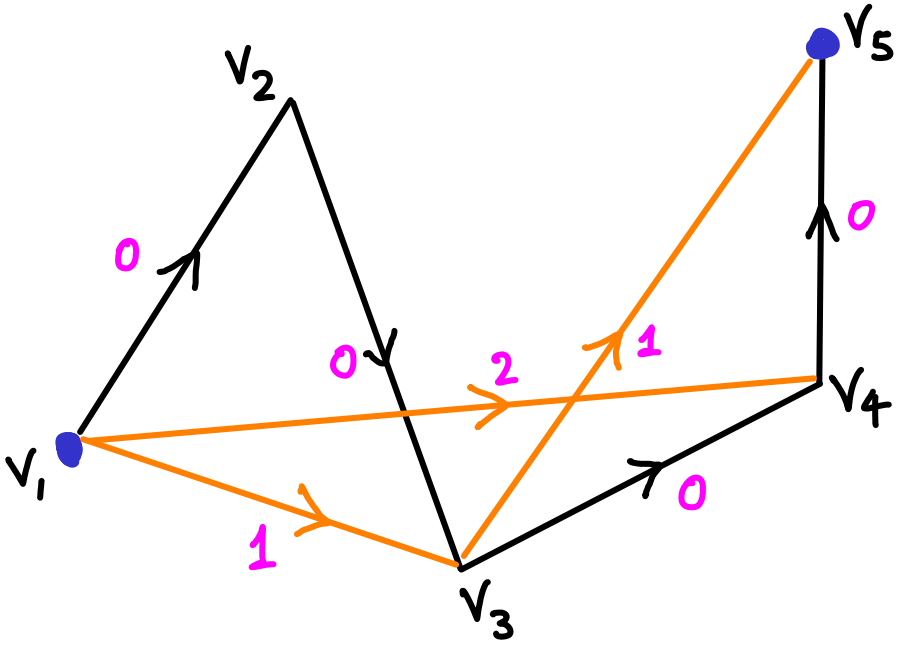
- form directed graph:
  - ↳ make $\overrightarrow{v_i v_j}$ iff $i<j$
- give a score to each edge:
  - ↳ # path edges skipped
- keep only edges satisfying tolerance







IRI-IMAI algorithm to minimize segments (given path & tolerance)



- form directed graph:
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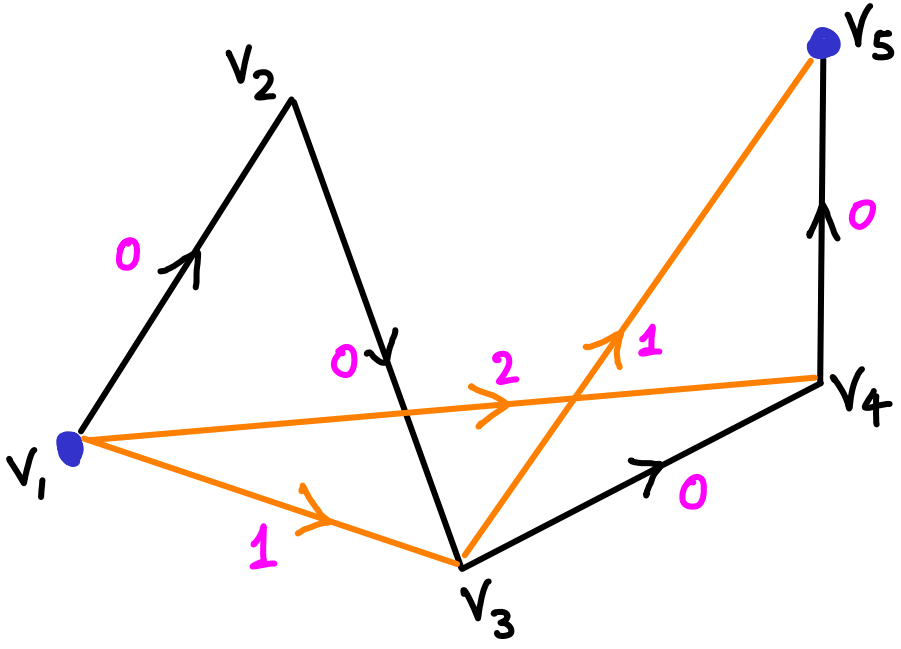
time so far?

IRI-IMAI algorithm to minimize segments (given path & tolerance)



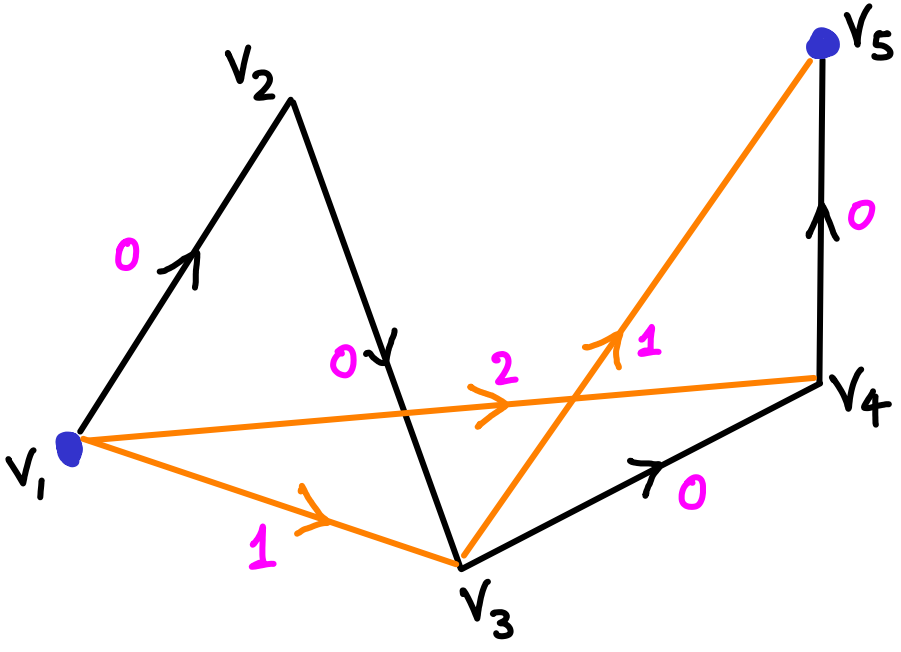
1 - form directed graph:
↳ make $\overrightarrow{v_i v_j}$ iff $i<j$

2 - give a score to each edge:
↳ # path edges skipped

3 - keep only edges satisfying tolerance

$\Theta(v^2)$ edges $\begin{cases} 1 & \Theta(v^2) \\ 2 & >> \\ 3 & \Theta(v^2)\cdot O(v) \end{cases}$

IRI-IMAI algorithm to minimize segments (given path & tolerance)



1 - form directed graph:
↳ make $\overrightarrow{v_i v_j}$ iff $i<j$

2 - give a score to each edge:
↳ # path edges skipped

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$\Theta(v^2)$ edges $\begin{cases} 1 \\ 2 \\ 3 \end{cases}$ $\Theta(v^2)$ >> $\Theta(v^2)\cdot O(v)$

Next?

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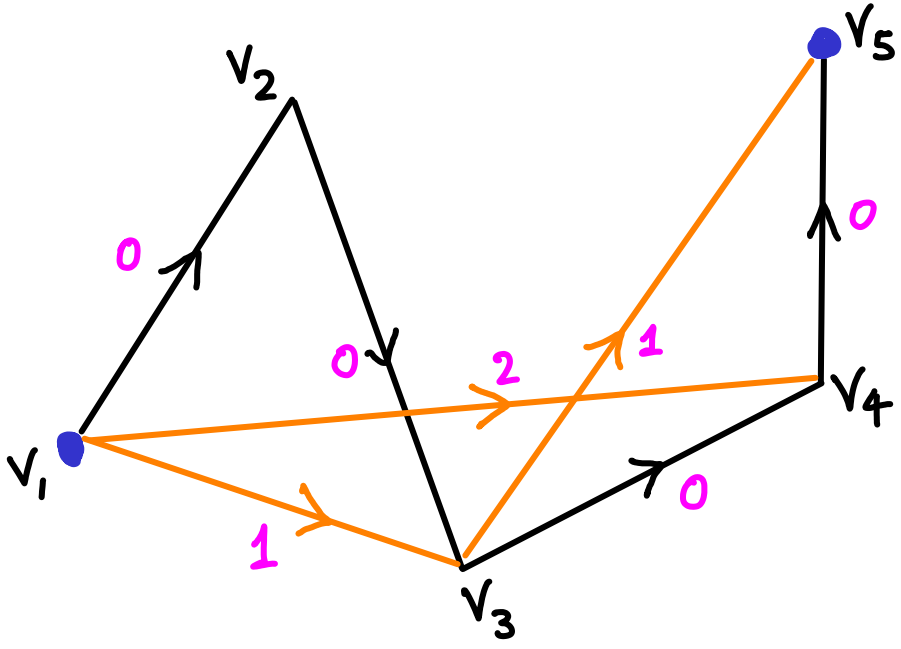

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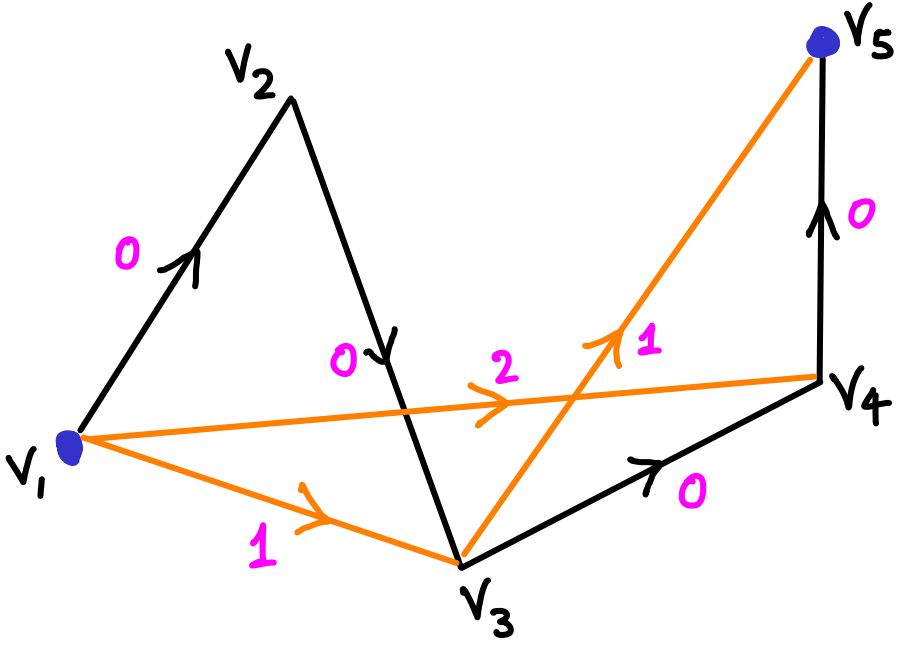

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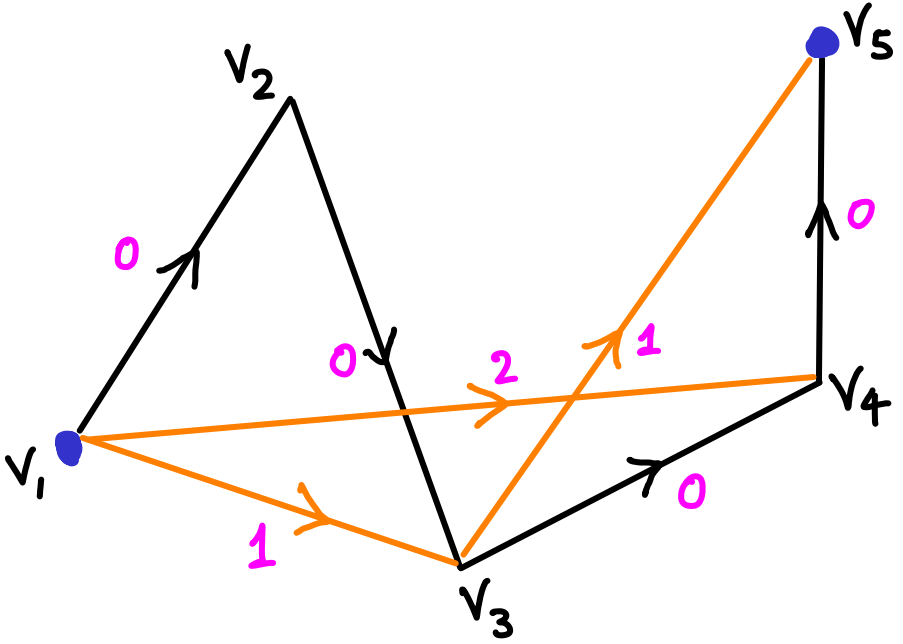

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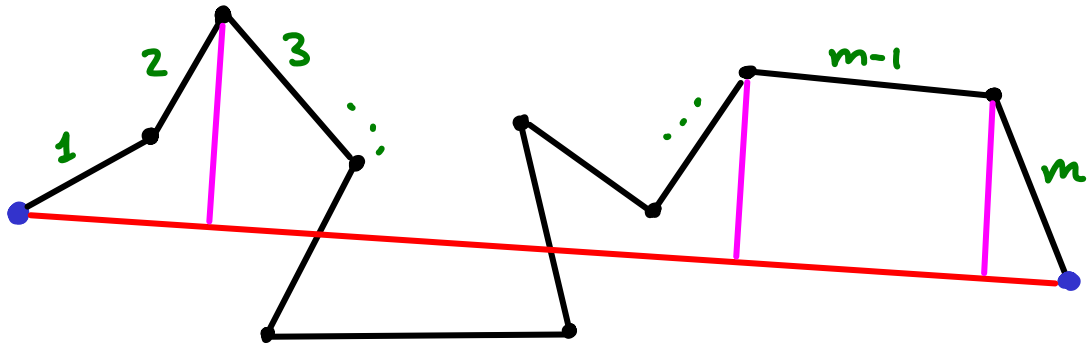

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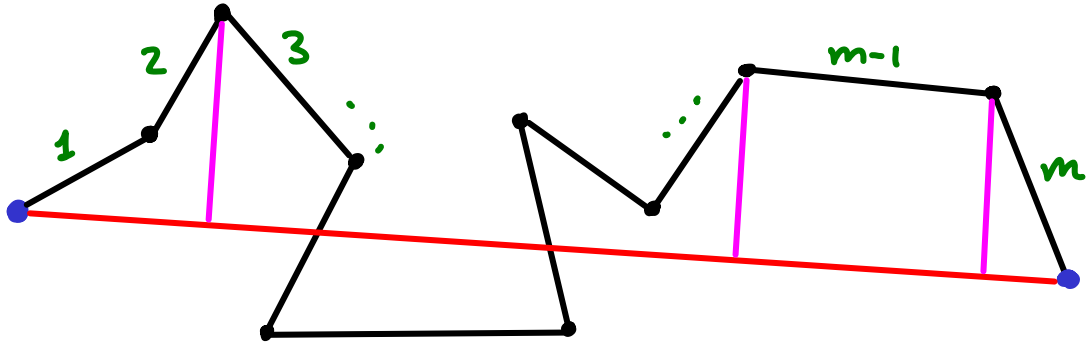


Testing an edge that connects endpoints of a path w/ m segments

↳ brute force : $\Theta(m)$ time

↳ under certain conditions (e.g. parallel strip distance)
& w/ preprocessing, we can do better

data structure?

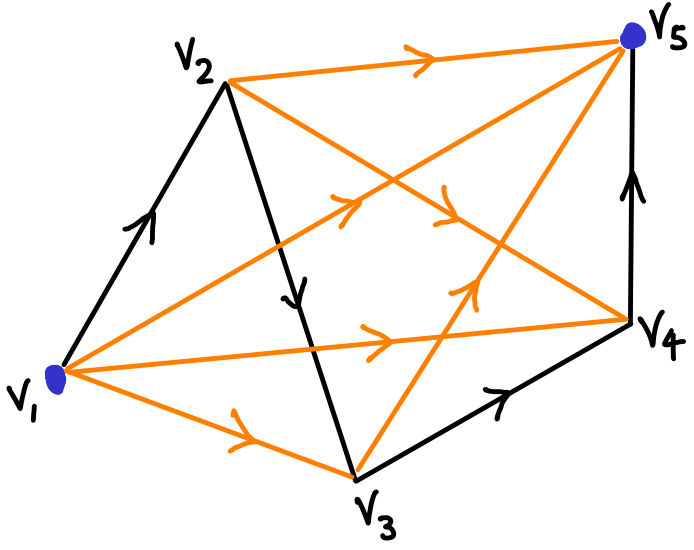
(and thus get $o(V^3)$ overall)

PROJECT

Suppose we are willing to use up to k edges.

↳ goal: minimize error

↘ & get a chain for output



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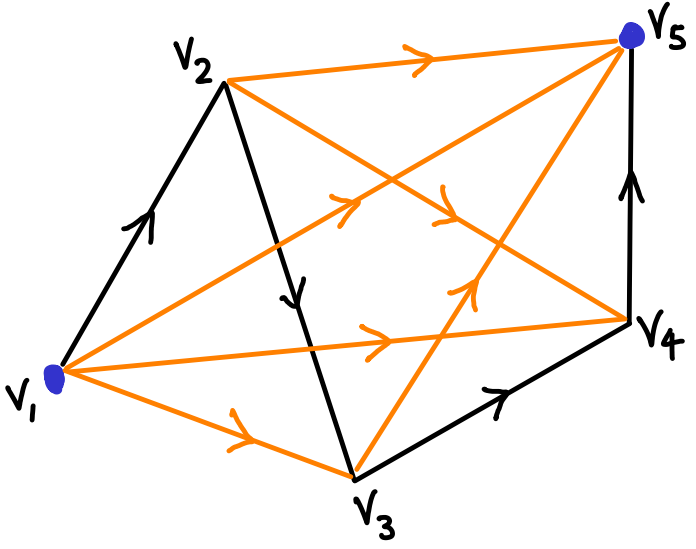

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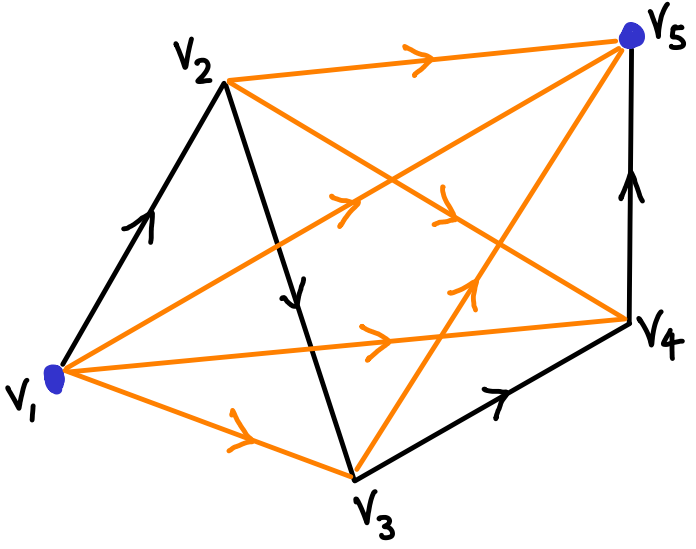

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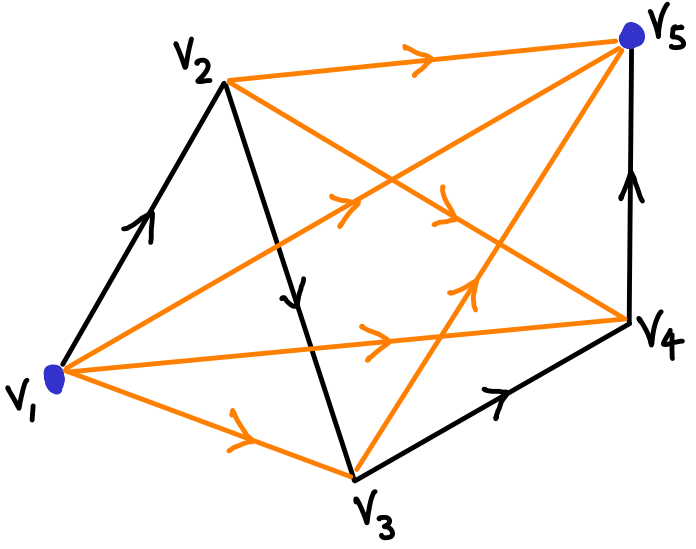

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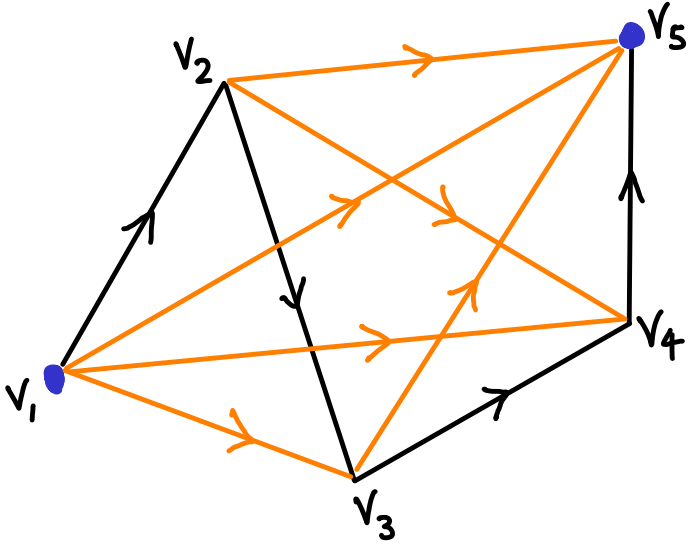

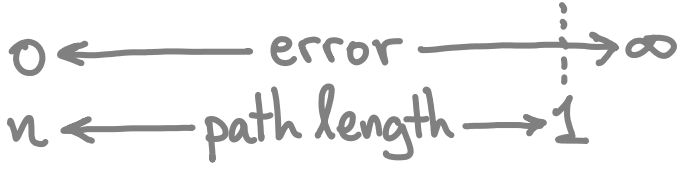

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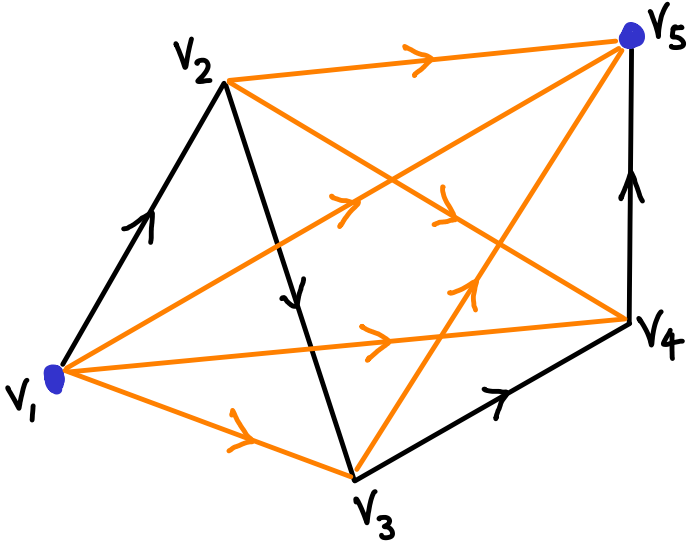

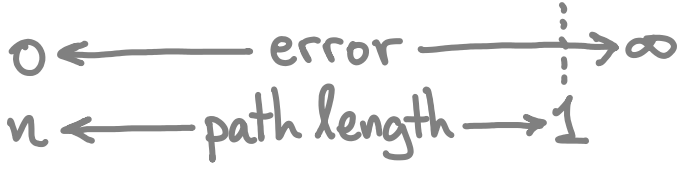

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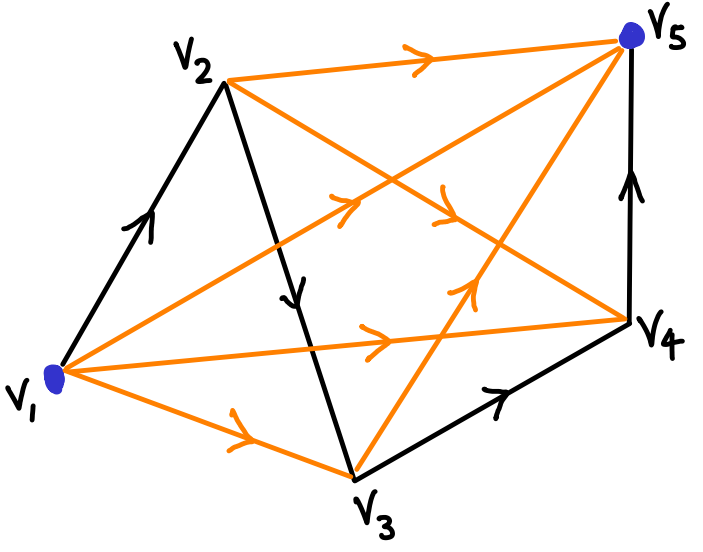

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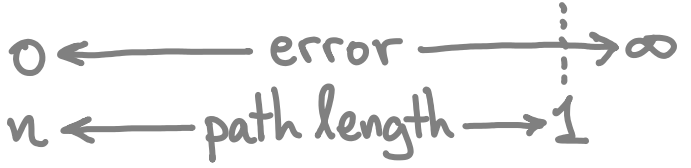

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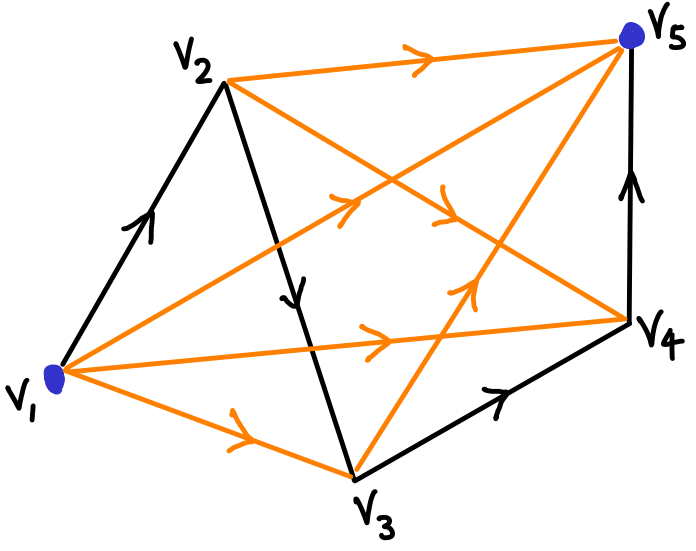

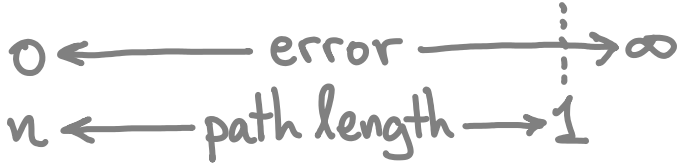

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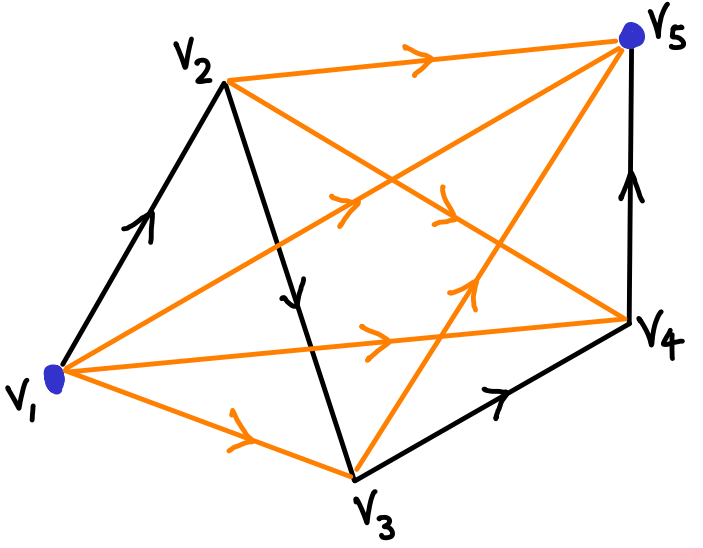

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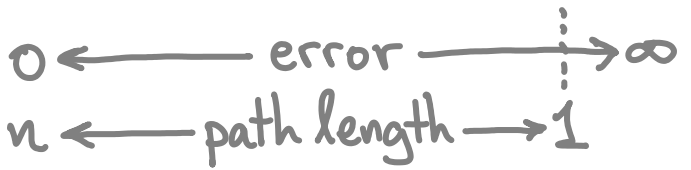

Dynamic programming improvement?

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Lots of project possibilities