worst case		AVL	RB
	height	1.4 log n	2 log n
insert	# rotations	O(1)	O(1)
delete	# rotations	O(log n)	O(1)

WAVL - Weak AVL trees

(Haeupler, Sen, Tarjan - 2009)

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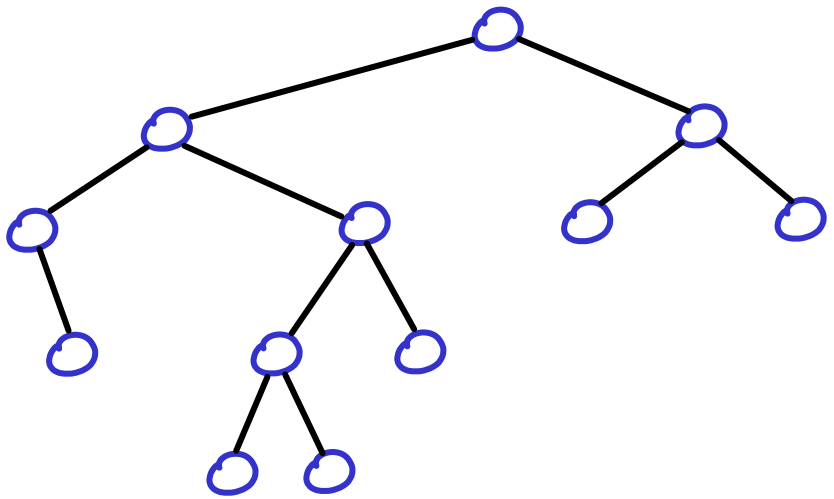

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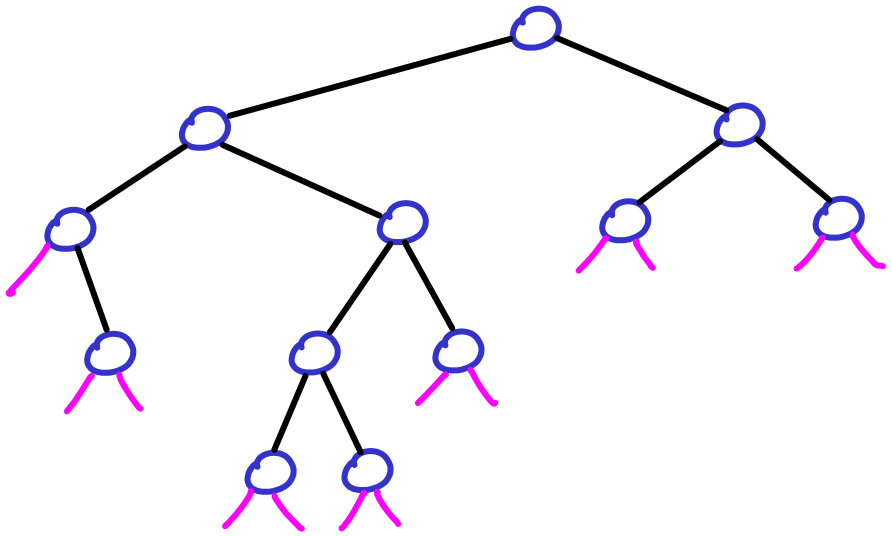

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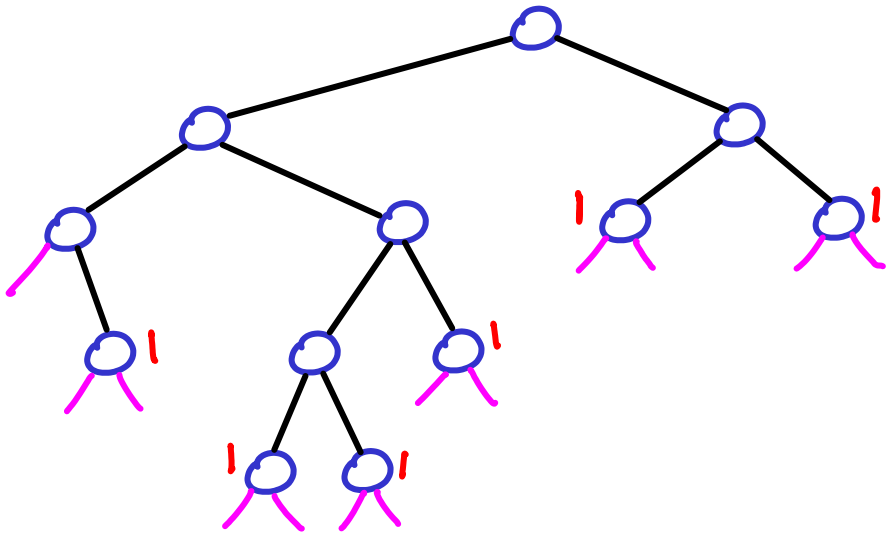

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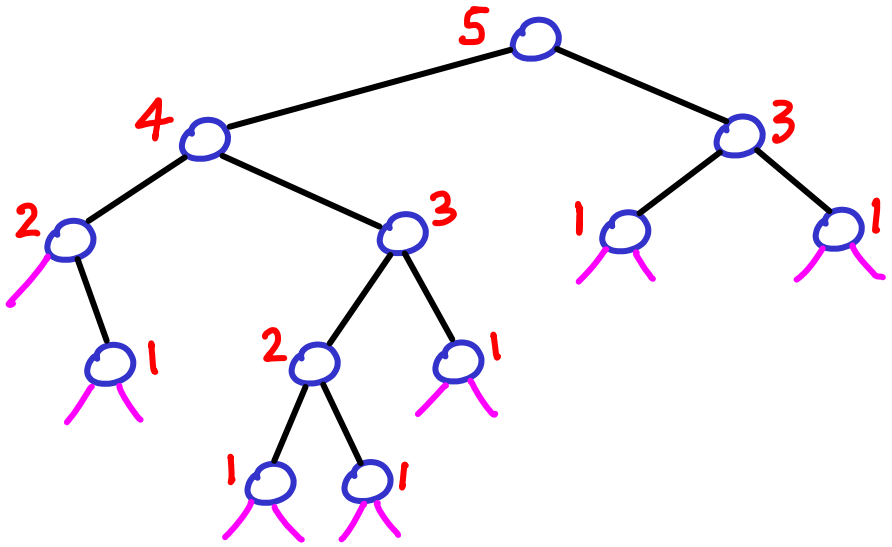

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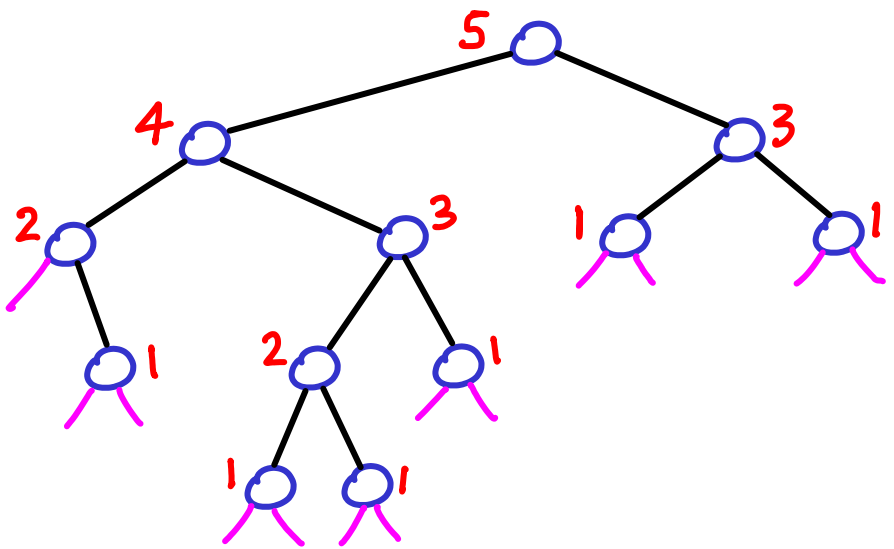
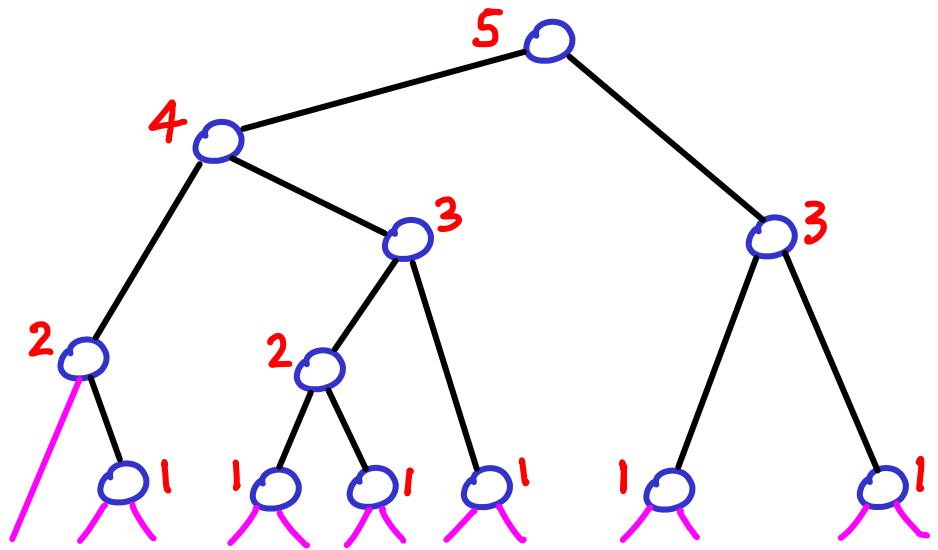
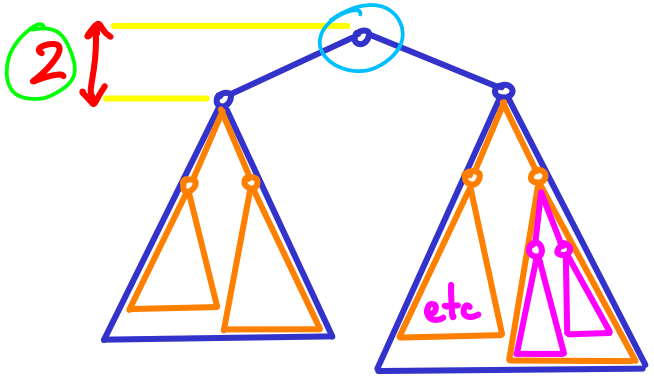


not AVL



$N(r)$ = min #nodes to achieve score r.



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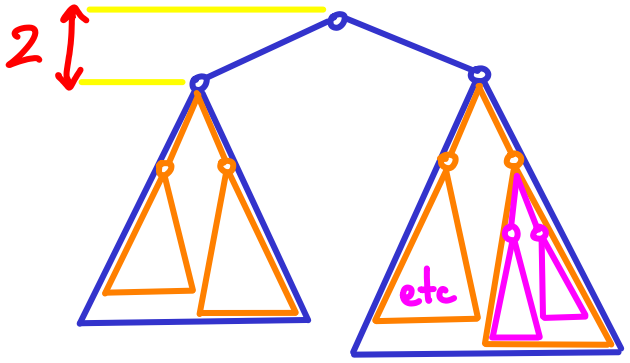

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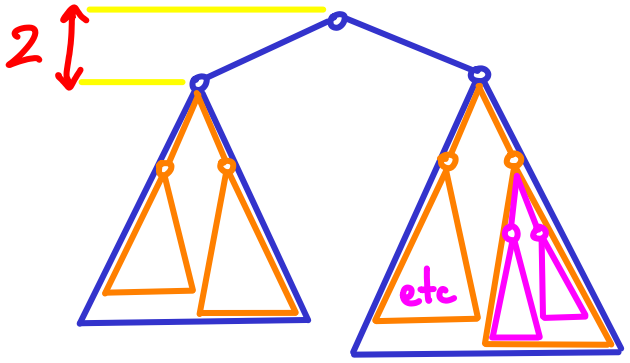

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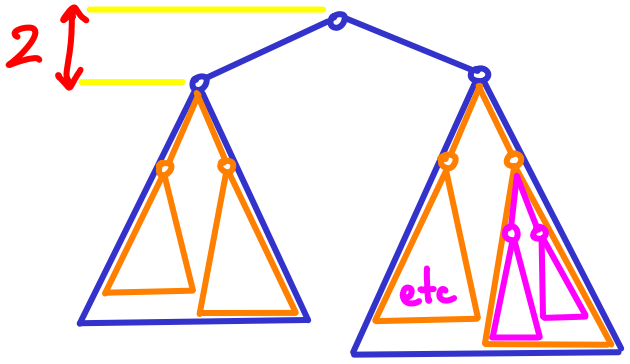

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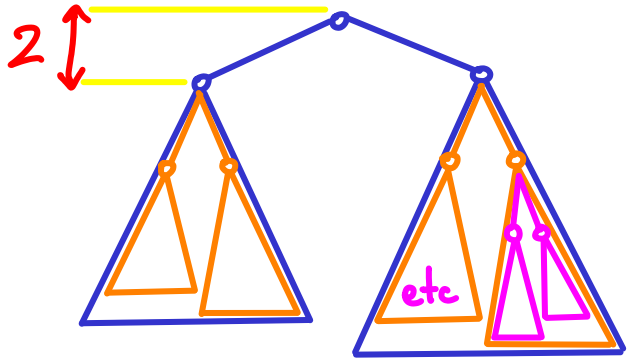

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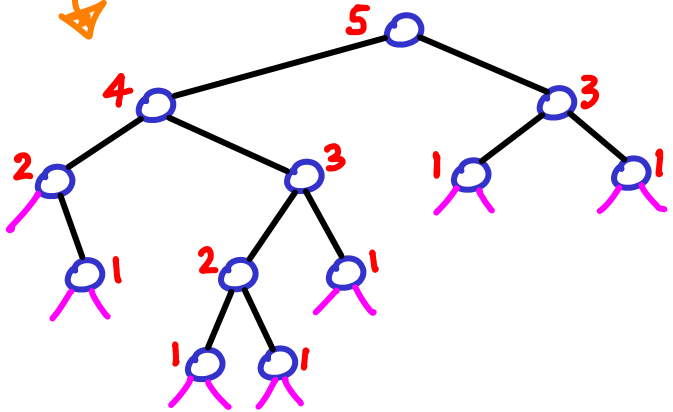

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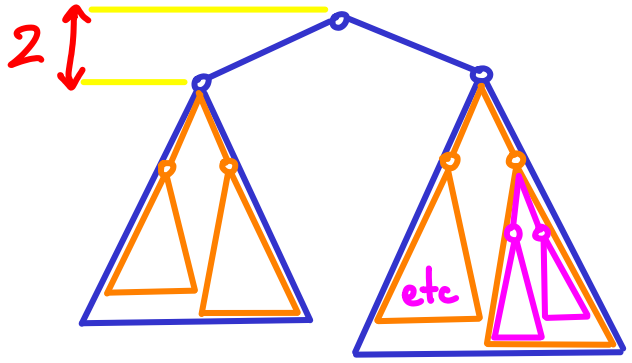

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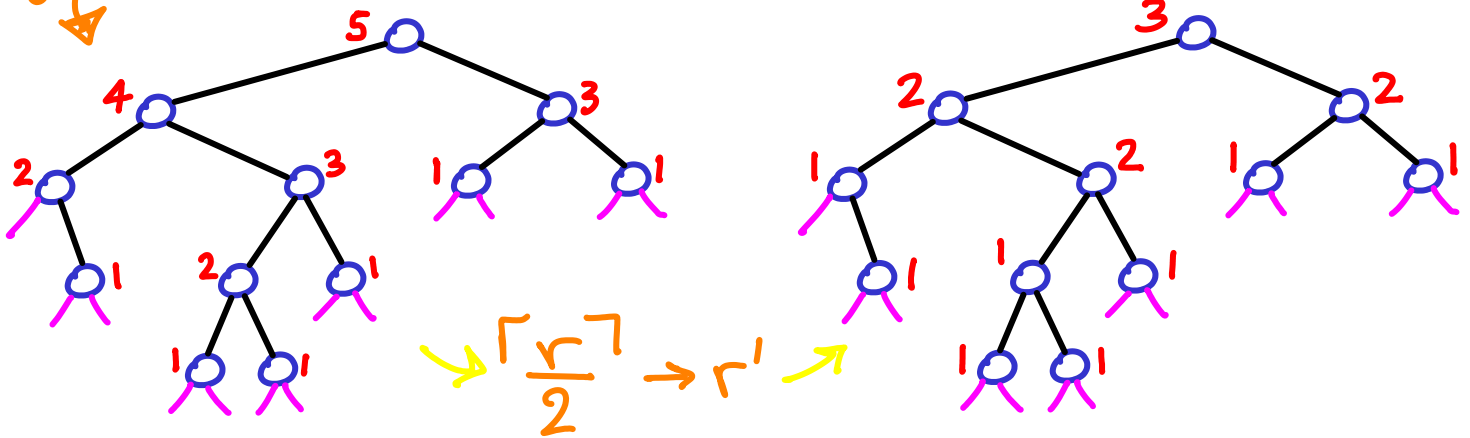

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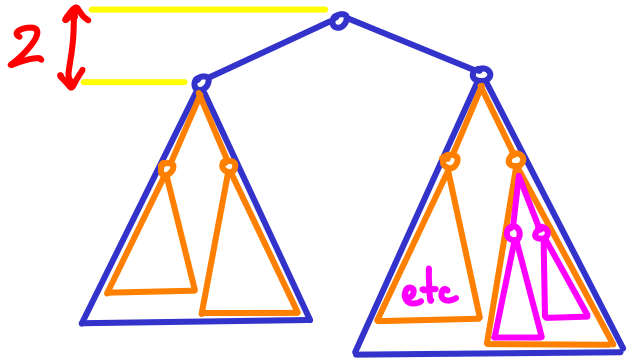

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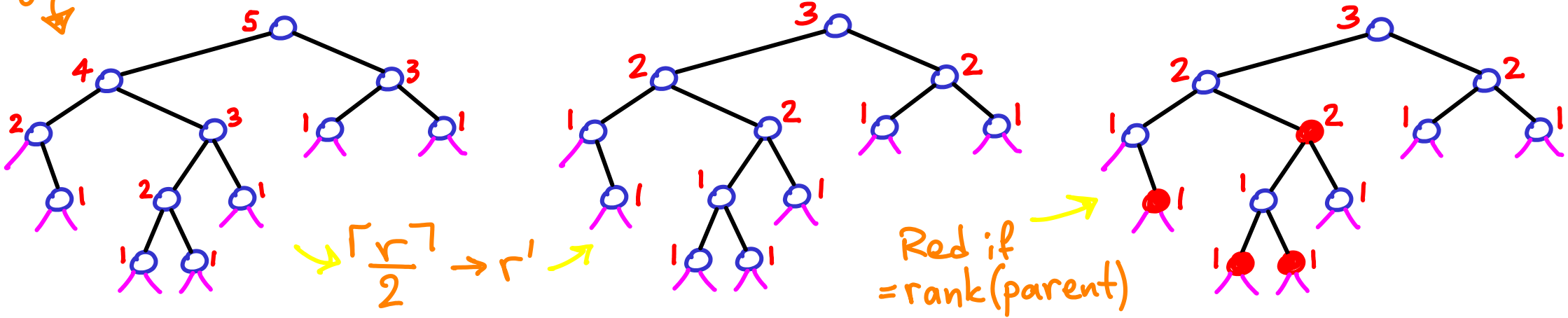

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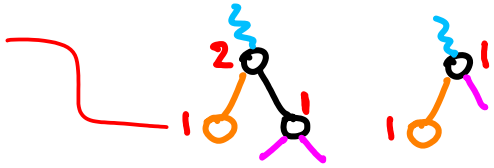

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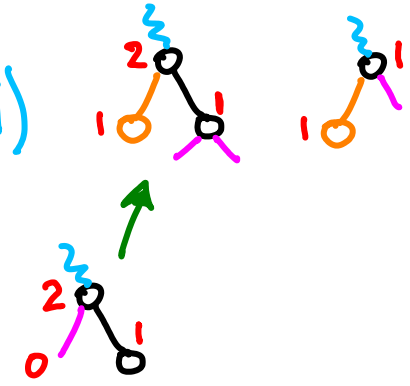

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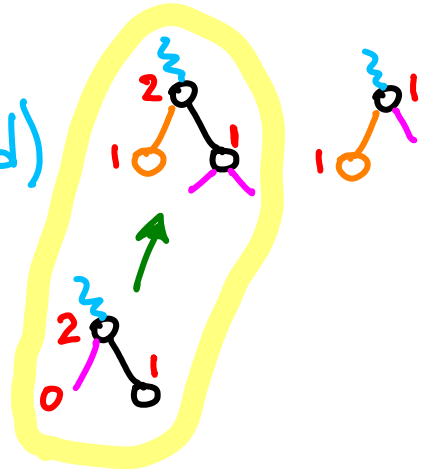

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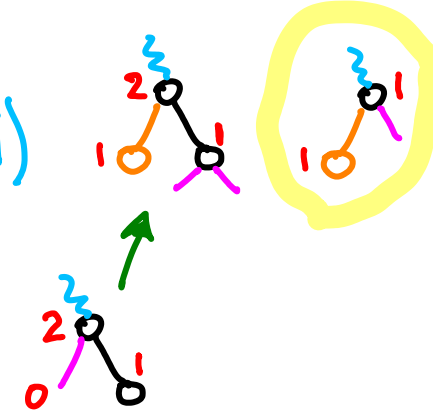

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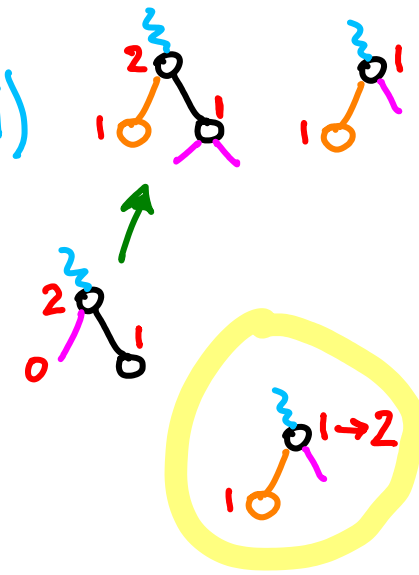

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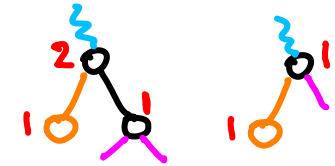

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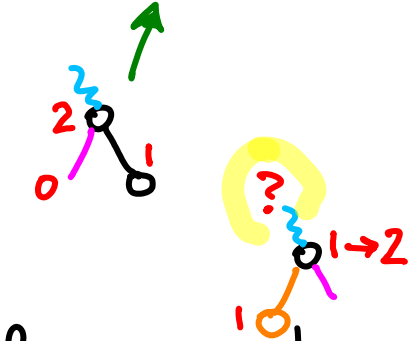

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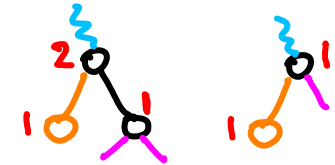

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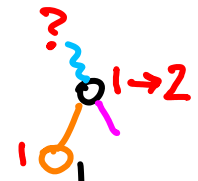

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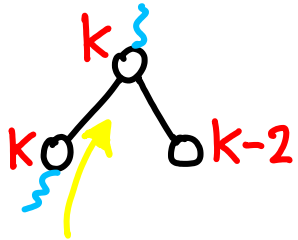

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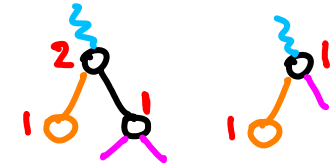

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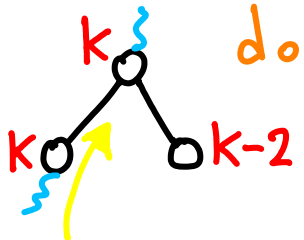

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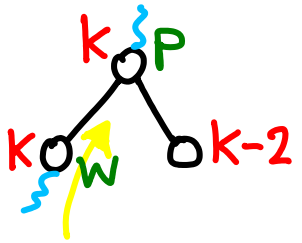

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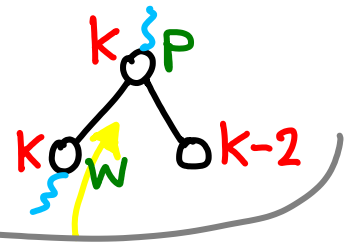

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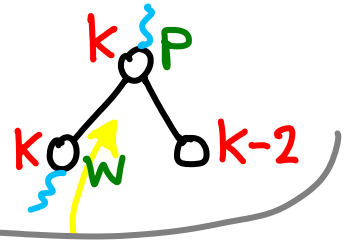

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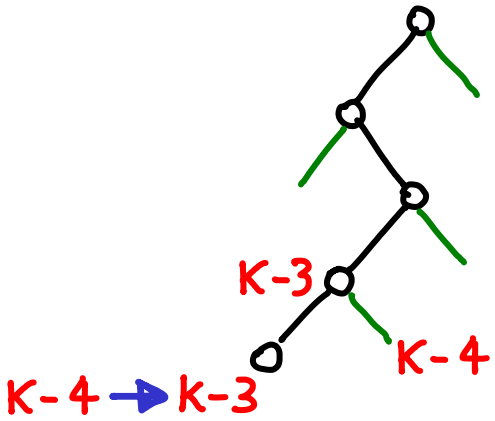

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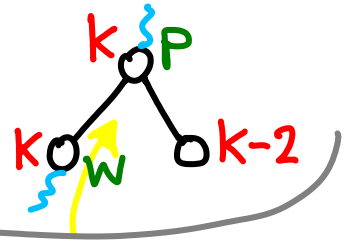

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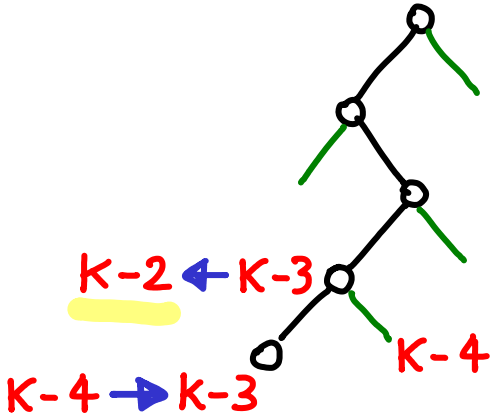

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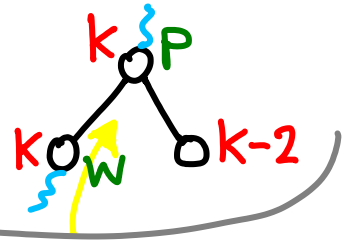

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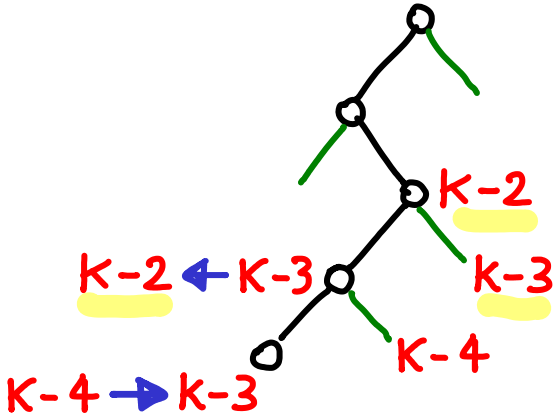

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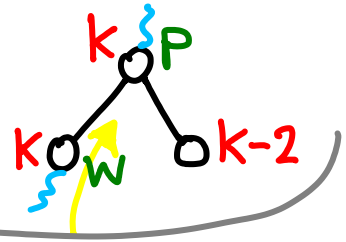

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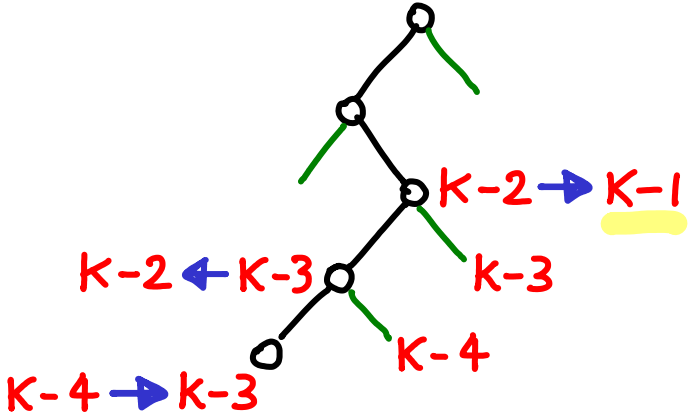

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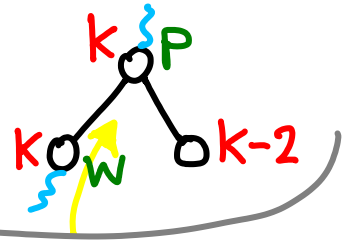

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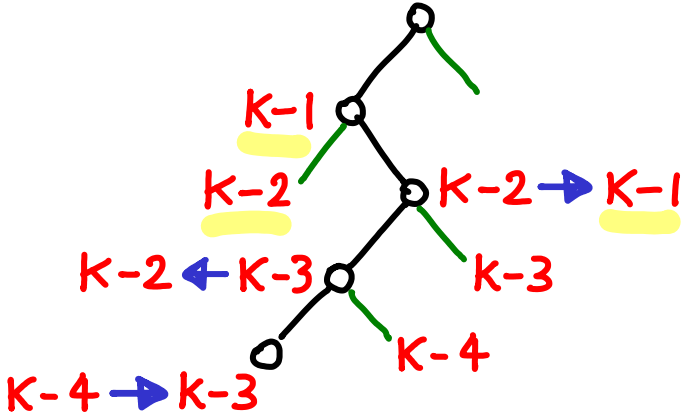

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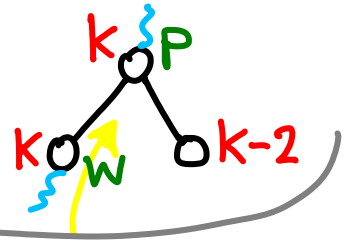

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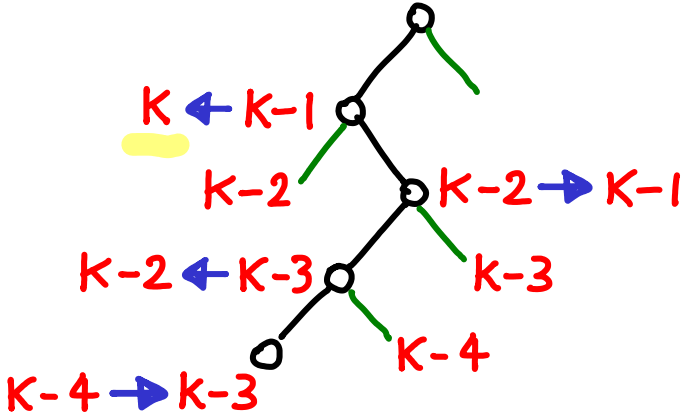

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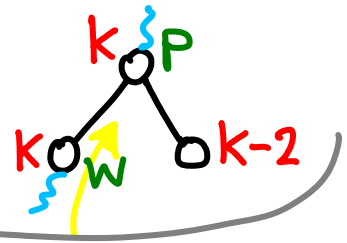

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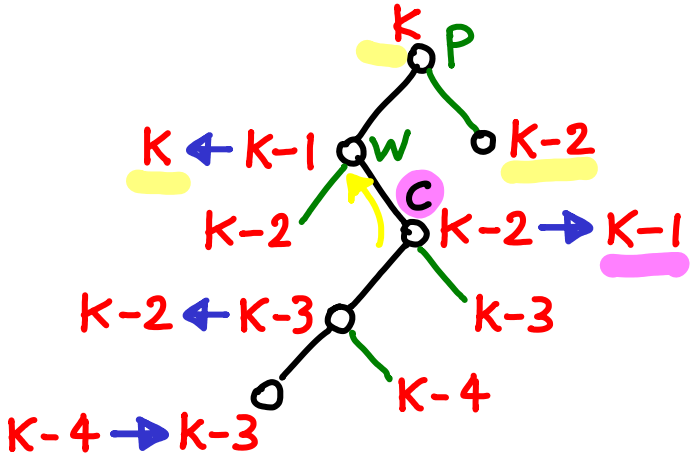

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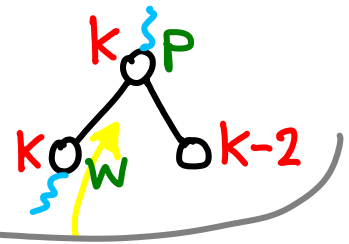

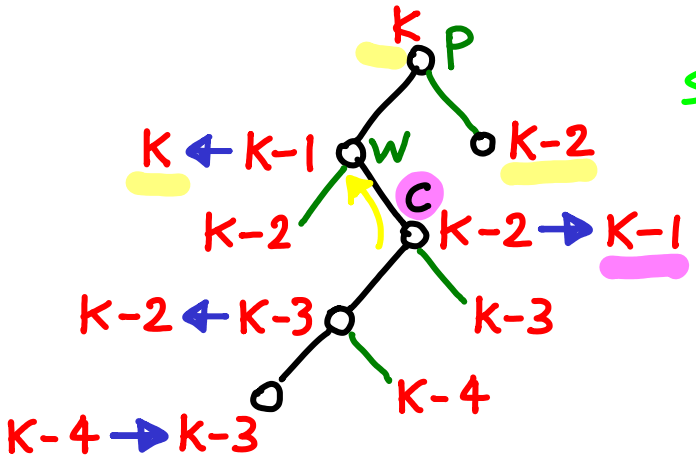

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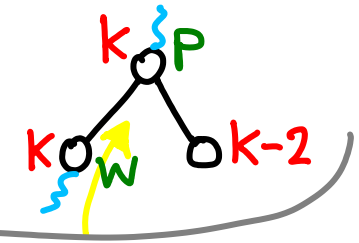

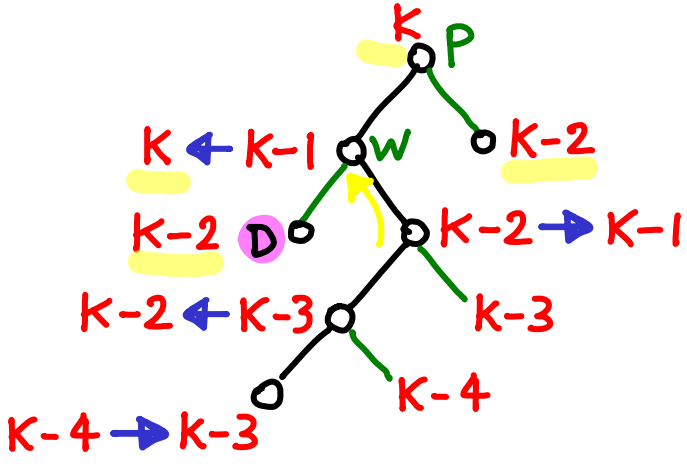

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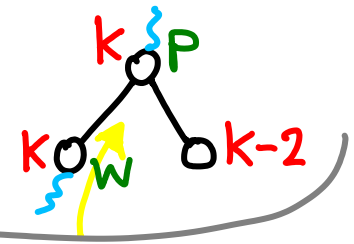

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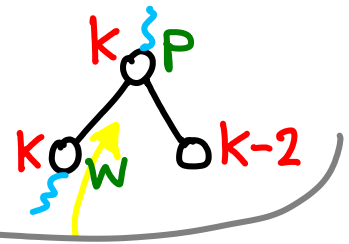

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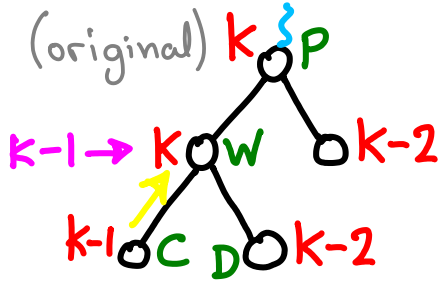

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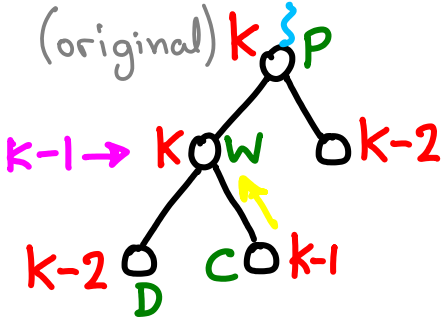

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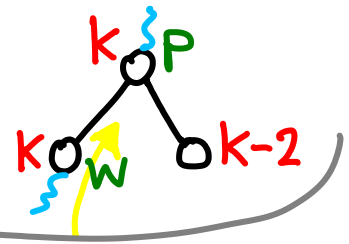

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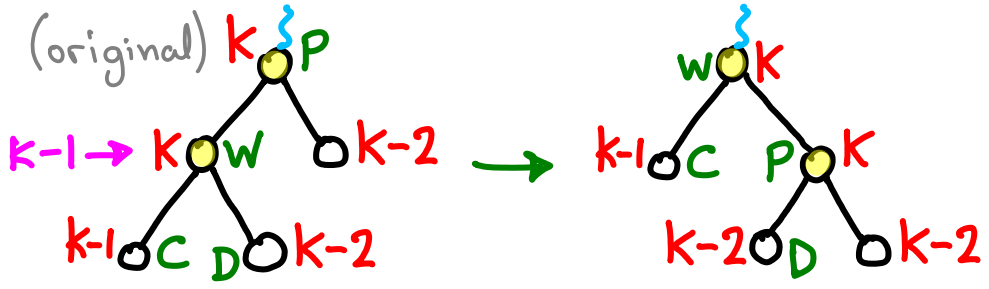

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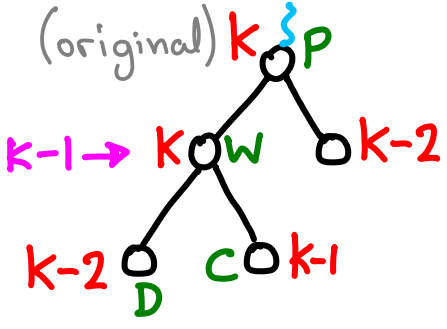

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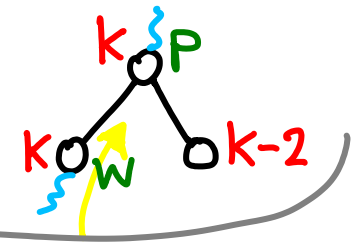

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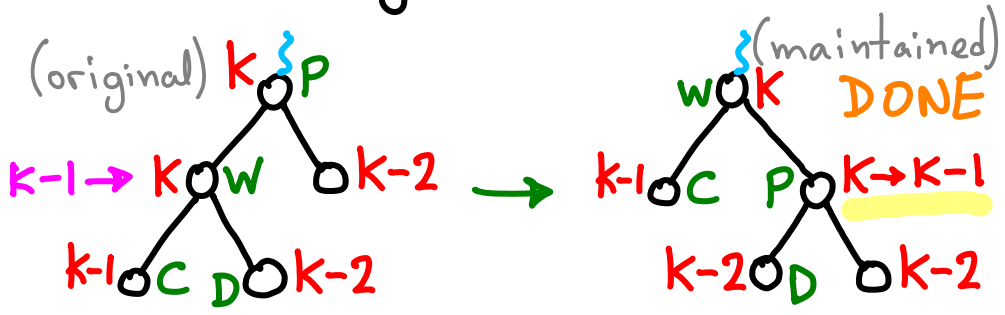

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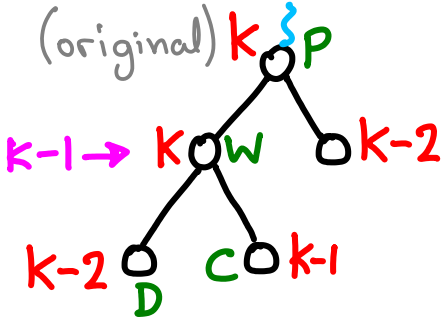

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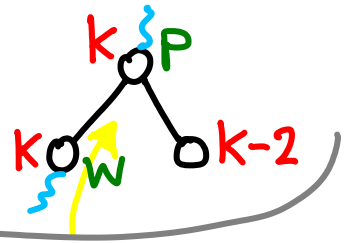

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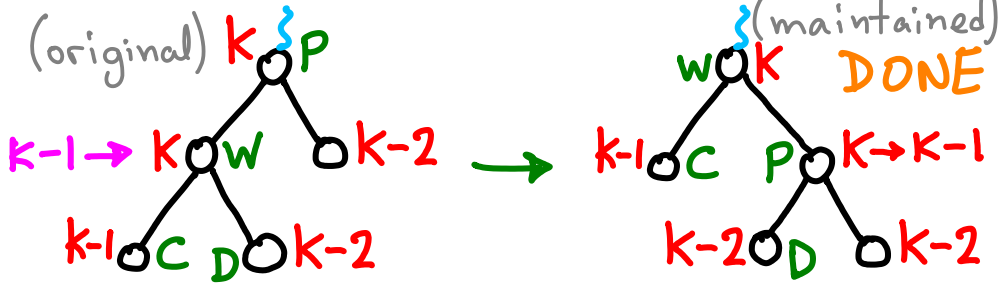

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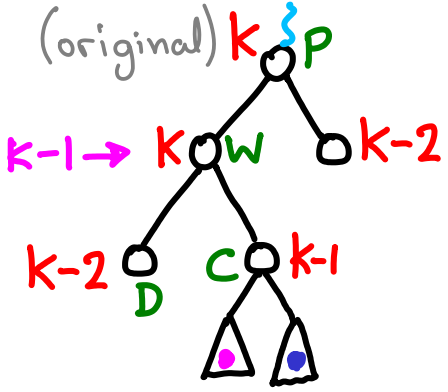

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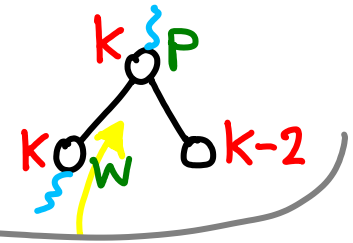

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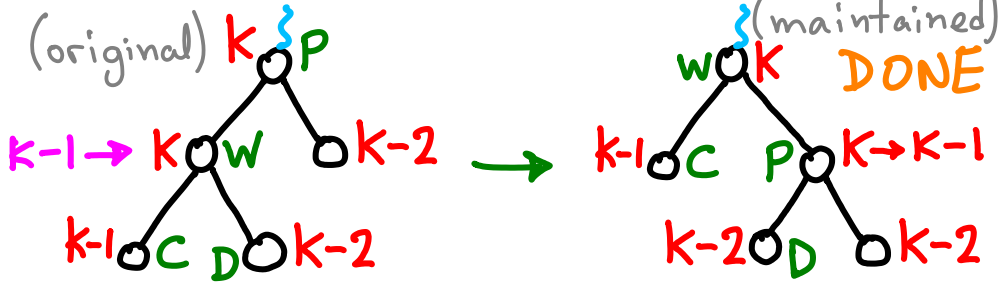

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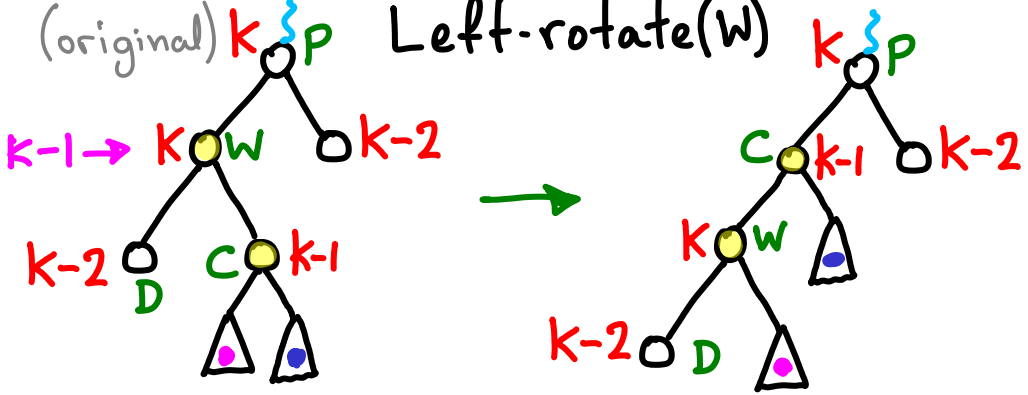
(original) k P Left-rotate(W)



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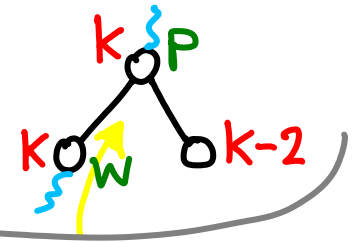

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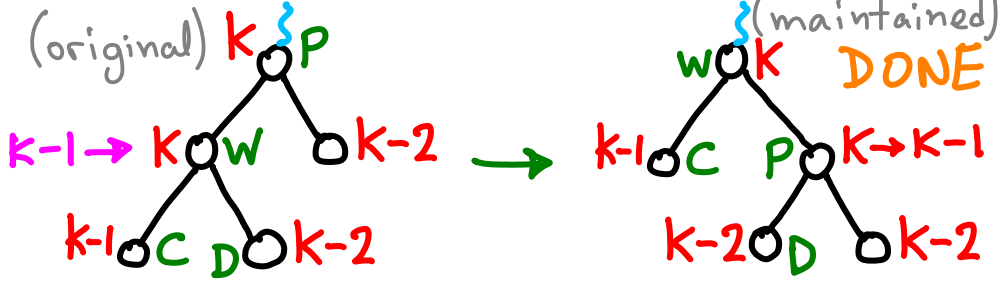

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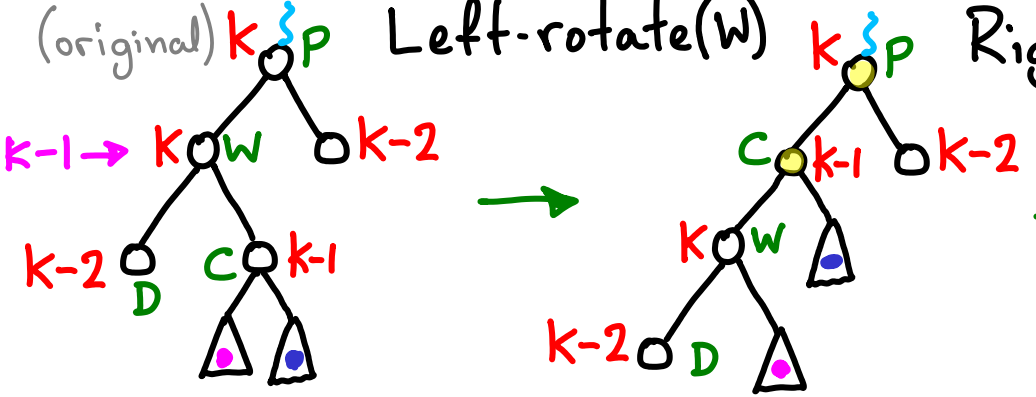

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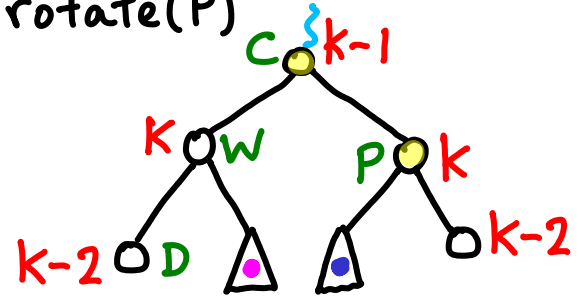

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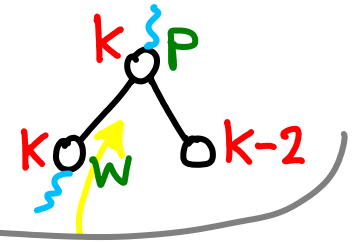

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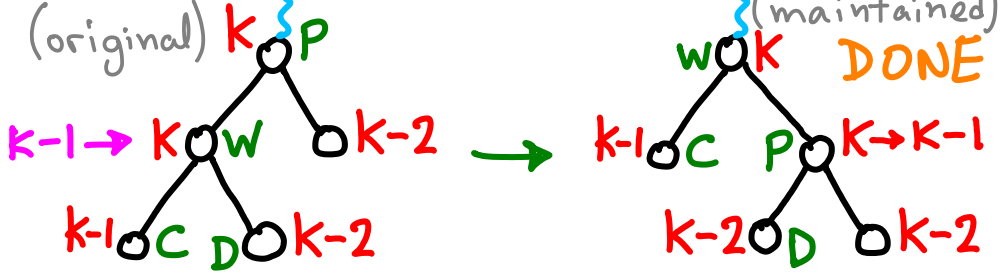

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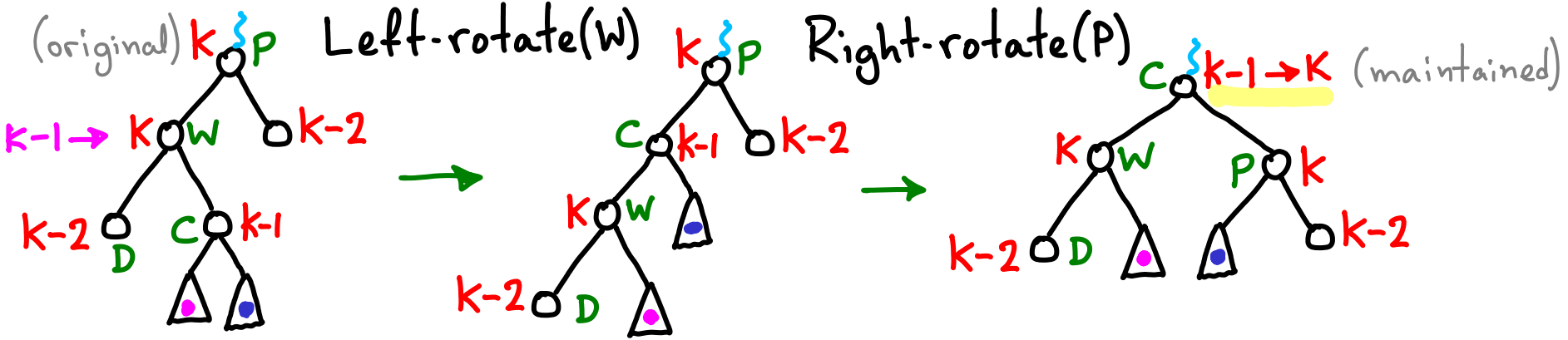

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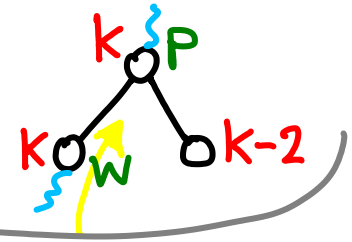

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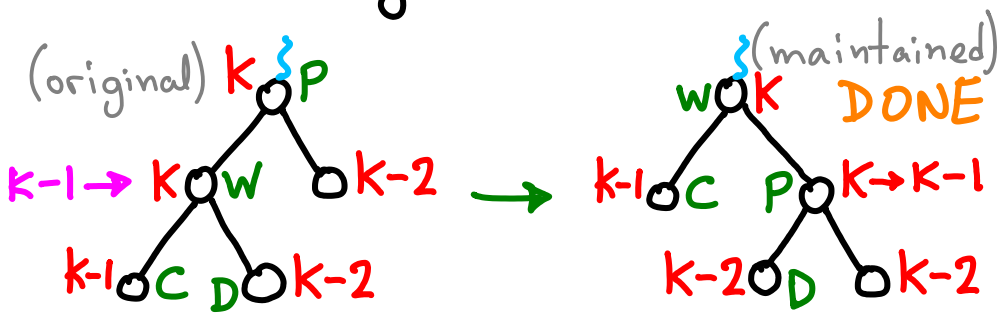

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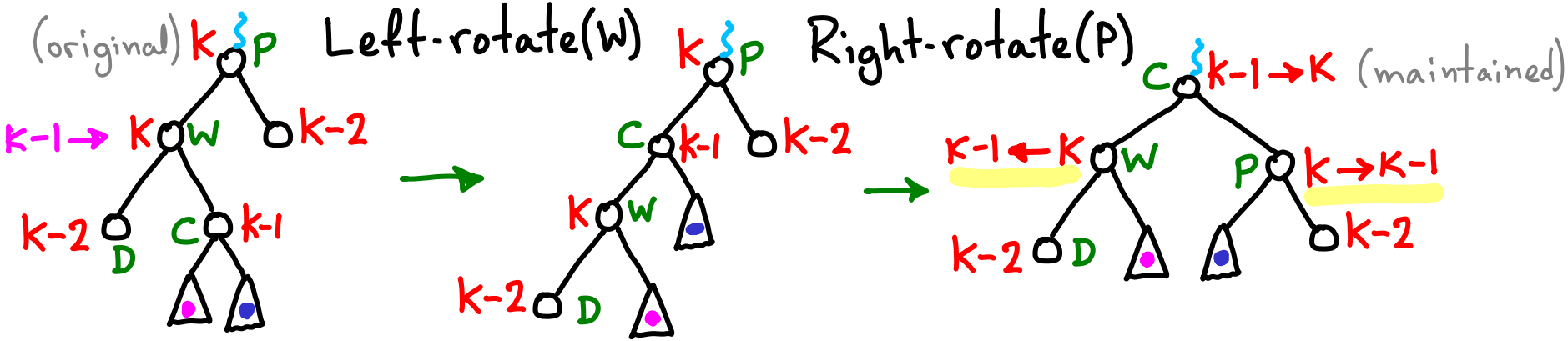

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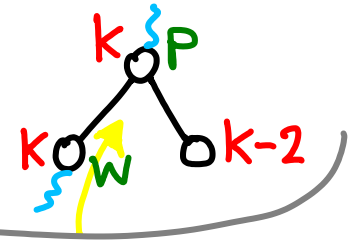

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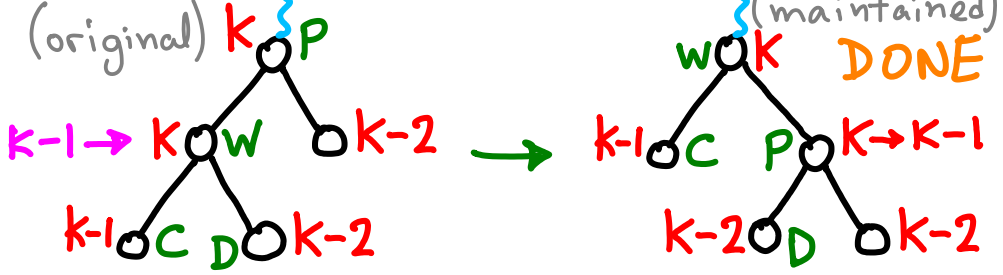

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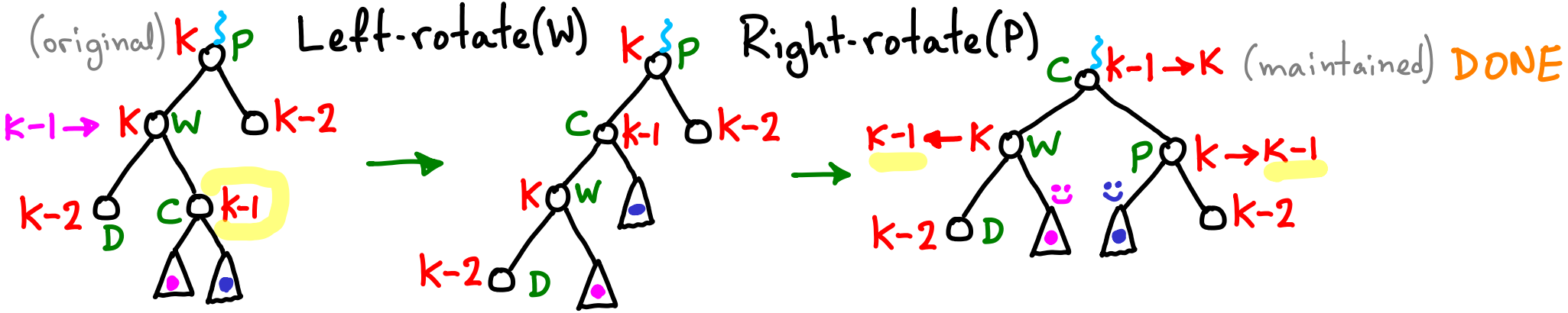

WAVL deletion is also straightforward

worst case	AVL	RB	WAVL
height	1.4 log n	2 log n	2 log n 1.4 log n insert only
insert # rotations	O(1)	O(1)	O(1) better than RB
delete # rotations	O(log n)	O(1)	O(1)