Partial Decode-Forward Binning for Full-Duplex Causal Cognitive Interference Channels

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***Abstract*—The causal cognitive interference channel (CCIC) is a four-node channel, in which the second sender obtains information from the first sender causally and assists in the transmission of both. We propose a new coding scheme called Han-Kobayashi partial decode-forward binning (HK-PDF-binning), which combines the ideas of Han-Kobayashi coding, partial decode-forward relaying, conditional Gelfand-Pinsker binning and relaxed joint decoding. The second sender decodes a part of the message from the first sender, then uses Gelfand-Pinsker binning to bin against the decoded codeword. When applied to the Gaussian channel, this HK-PDF-binning essentializes to a correlation between the transmit signal and the state, which encompasses the traditional dirty-paper-coding binning as a special case when this correlation factor is zero. The proposed scheme encompasses the Han-Kobayashi rate region and achieves both partial decode-forward relaying rate for the first user and interference-free rate for the second user.**

I. Introduction

The causal Cognitive Interference Channel (CCIC) is a practically-oriented cognitive channel, in which the second (cognitive) sender obtains information from the first (primary) sender causally, then uses that to assist the transmissions of the first sender and its own message. This is different from the traditional cognitive channel in which the cognitive user knows the primary user's message non-causally.

Coding for the traditional (non-causal) CIC is mainly based on combining Gelfand-Pinsker binning technique [1] with Han-Kobayashi coding [2], [3] for the interference channel. Since the cognitive user has the primary user's message non-causally, the fact that it relays this message is implicit. In the causal-CIC, however, the cognitive user has to relay explicitly. Specifically in this paper, we apply partial decode-forward relaying [4], in which the cognitive user first decodes the primary user's message causally, then transmits the decoded message and its own message cognitively.

The CCIC can also be considered as a special case of the interference channel with source cooperation (IC-SC) in which both senders exchange information. Several coding schemes have been proposed for the IC-SC by applying different ways of rate splitting, block Markov encoding and/or Gelfand-Pinsker binning [5]–[8]. These existing schemes may encompass the Han-Kobayashi rate region or the partial decode-forward (PDF) relaying rate, but none achieve both.

In this paper, we propose a new coding scheme for the CCIC based on block Markov encoding, partial decode-forward relaying, Gelfand-Pinsker binning and Han-Kobayashi coding by splitting the first user's message into 3 parts and the second user's into 2 parts. The proposed scheme achieves both the Han-Kobayashi region and the PDF rate for the primary user.

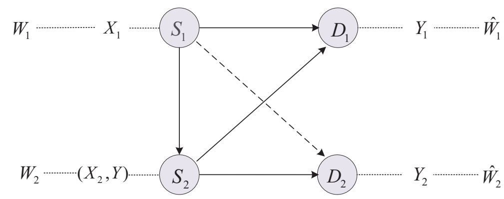

Fig. 1. The full-duplex causal cognitive interference channel.

This scheme therefore brings a new way of coding and can be combined with existing schemes for the IC-SC to improve rates further. We then apply our scheme to the Gaussian channel and show that introducing a correlation between the state and the transmit signal can enlarge the rate region by allowing both state nullifying and forwarding at the cognitive user.

II. CCIC Channel Models

A. Full-duplex DM-CCIC model

The full-duplex causal cognitive interference channel consists of two input alphabets $\mathcal{X}_1, \mathcal{X}_2$, and three output alphabets $\mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}$. The channel is characterized by a channel transition probability $p(y_1, y_2, y|x_1, x_2)$, where x_1 and x_2 are the transmit signals of S_1 and S_2, y_1, y_2 and y are the received signals of D_1, D_2 and S_2. Figure 1 illustrates the channel model, where W_1 and W_2 are the messages of S_1 and S_2.

A (2^{nR_1}, 2^{nR_2}, n) code consists of the following:

- Two message sets $\mathcal{W}_1 \times \mathcal{W}_2 = [1, 2^{nR_1}] \times [1, 2^{nR_2}]$ with uniform, independent messages W_1, W_2, respectively.
- Two encoders: one maps message w_1 into codeword $x_1^n(w_1) \in \mathcal{X}_1^n$, and one maps w_2 and each received sequence y^{k-1} into a symbol $x_{2k}(w_2, y^{k-1}) \in \mathcal{X}_2$.
- Two decoders: one maps y_1^n into $\hat{w}_1 \in \mathcal{W}_1$; one maps y_2^n into $\hat{w}_2 \in \mathcal{W}_2$.

The definitions for the error probability, achievable rates and capacity region follow the standard ones in [9].

B. Full-duplex Gaussian CCIC model

The standard full-duplex Gaussian causal cognitive interference channel is shown in Figure 2 as

$$\begin{aligned} Y_1 &= X_1 + bX_2 + Z_1, \\ Y_2 &= aX_1 + X_2 + Z_2, \\ Y &= cX_1 + Z, \end{aligned} \tag{1}$$

where Z_1, Z_2, $Z \sim \mathcal{N}(0, 1)$ are independent Gaussian noises, and a, b and c are cross-channel gains. If the original channel

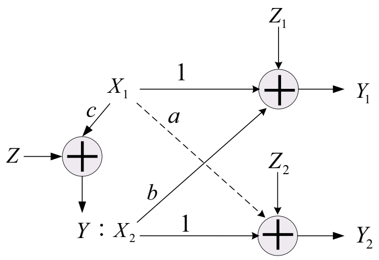

Fig. 2. Standard full-duplex Gaussian causal cognitive interference channel.

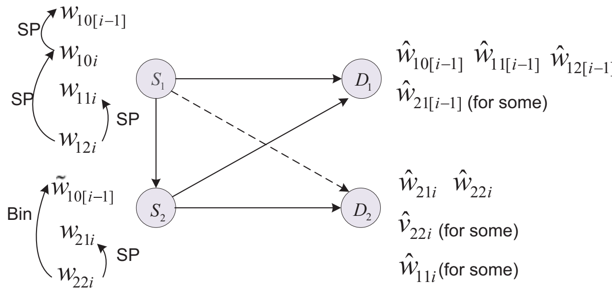

Fig. 3. Coding structure for the full-duplex Han-Kobayashi PDF-binning scheme at block i. (SP = superposition, Bin = binning)

is not in this standard form, we can always transform it into the standard form using a procedure similar to the interference channel [10]. The transmit signals X_1 and X_2 are subject to power constraints P_1 and P_2, respectively.

III. PARTIAL DECODE-FORWARD BINNING SCHEME

A. Han-Kobayashi PDF-binning scheme

Figure 3 illustrates the idea of the full-duplex Han-Kobayashi PDF-binning scheme. Message w_1 is split into three parts: w_{10}, w_{11}, w_{12}, corresponding to the common (forwarding), public and private parts, and message w_2 is split into two parts: w_{21}, w_{22}, corresponding to the public and private parts. Take the transmission in block i as an example. At S_1, the current common message w_{10i} is superimposed on the previous commons message $w_{10[i-1]}$; message w_{11i} is encoded independently of both $w_{10[i-1]}$ and w_{10i}; message w_{12i} is then superimposed on all three messages $w_{10[i-1]}$, w_{10i} and w_{10i}. S_2 decodes $\tilde{w}_{10[i-1]}$ of the previous block and uses conditional binning to bin the codeword for its private part $w_{22[i]}$ against $\tilde{w}_{10[i-1]}$, conditionally on knowing the public part $w_{21[i]}$. At the end of block i, D_1 uses joint decoding over two consecutive blocks to decode a unique tuple $(\hat{w}_{10[i-1]}, \hat{w}_{11[i-1]}, \hat{w}_{12[i-1]})$ for some $\hat{w}_{21[i-1]}$ without requiring this message part to be correct. D_2 treats the codeword for $w_{10[i-1]}$ as the state and searches for a unique pair (w_{21i}, w_{22i}) for some w_{11i}.

Theorem 1. *The convex hull of the following rate region is achievable for the CCIC using HK-PDF-binning:*

$$\bigcup_{P_1} \left\{ \begin{array}{ll} R_1 & \leq \min\{I_2 + I_5, I_6\} \\ R_2 & \leq I_{12} \\ R_1 + R_2 & \leq \min\{I_2 + I_7, I_8\} + I_{13} \\ R_1 + R_2 & \leq \min\{I_2 + I_3, I_4\} + I_{14} \\ R_1 + R_2 & \leq \min\{I_2 + I_9, I_{10}\} + I_{11} \\ 2R_1 + R_2 & \leq \min\{I_2 + I_3, I_4\} \\ & \quad + \min\{I_2 + I_9, I_{10}\} + I_{13} \\ R_1 + 2R_2 & \leq \min\{I_2 + I_7, I_8\} + I_{11} + I_{14} \end{array} \right. \tag{2}$$

where

$$\begin{aligned} P_1 = & p(t_{10})p(u_{10}|t_{10})p(u_{11})p(x_1|t_{10}, u_{10}, u_{11})p(u_{21}) \\ & p(u_{22}|u_{21}, t_{10})p(x_2|t_{10}, u_{21}, u_{22})p(y_1, y_2, y|x_1, x_2), \end{aligned} \tag{3}$$

and I_2 — I_{14} are defined as

$$\begin{aligned} I_2 &= I(U_{10}; Y|T_{10}) \\ I_3 &= I(X_1; Y_1|T_{10}, U_{10}, U_{11}, U_{21}) \\ I_4 &= I(U_{10}, X_1; Y_1|T_{10}, U_{11}, U_{21}) + I(T_{10}; Y_1) \\ I_5 &= I(U_{11}, X_1; Y_1|T_{10}, U_{10}, U_{21}) \\ I_6 &= I(U_{10}, U_{11}, X_1; Y_1|T_{10}, U_{21}) + I(T_{10}; Y_1) \\ I_7 &= I(X_1, U_{21}; Y_1|T_{10}, U_{10}, U_{11}) \\ I_8 &= I(U_{10}, X_1, U_{21}; Y_1|T_{10}, U_{11}) + I(T_{10}; Y_1) \\ I_9 &= I(U_{11}, X_1, U_{21}; Y_1|T_{10}, U_{10}) \\ I_{10} &= I(T_{10}, U_{10}, U_{11}, X_1, U_{21}; Y_1) \\ I_{11} &= I(U_{22}; Y_2|U_{21}, U_{11}) - I(U_{22}; T_{10}|U_{21}) \\ I_{12} &= I(U_{21}, U_{22}; Y_2|U_{11}) - I(U_{22}; T_{10}|U_{21}) \\ I_{13} &= I(U_{11}, U_{22}; Y_2|U_{21}) - I(U_{22}; T_{10}|U_{21}) \\ I_{14} &= I(U_{11}, U_{21}, U_{22}; Y_2) - I(U_{22}; T_{10}|U_{21}). \end{aligned} \tag{4}$$

Proof: Fix a joint probability distribution as in (3).

1) Codebook generation: For each block i:

- Independently generate $2^{nR_{10}}$ sequences $t_{10}^n(w'_{10}) \sim \prod_{k=1}^n p(t_{10k})$, $w'_{10} \in [1, 2^{nR_{10}}]$.
- For each $t_{10}^n(w'_{10})$, independently generate $2^{nR_{10}}$ sequences $u_{10}^n(w_{10}|w'_{10}) \sim \prod_{k=1}^n p(u_{10k}|t_{10k})$, $w_{10} \in [1, 2^{nR_{10}}]$. w'_{10} and w_{10} are the common (forwarding) messages of the previous and current blocks, respectively.
- Independently generate $2^{nR_{11}}$ sequences $u_{11}^n(w_{11}) \sim \prod_{k=1}^n p(u_{11k})$, $w_{11} \in [1, 2^{nR_{11}}]$.
- For each $t_{10}^n(w'_{10})$, $u_{10}^n(w_{10}|w'_{10})$ and $u_{11}^n(w_{11})$, independently generate $2^{nR_{12}}$ sequences $x_1^n(w_{12}|w_{11}, w_{10}, w'_{10}) \sim \prod_{k=1}^n p(x_{1k}|t_{10k}, u_{10k}, u_{11k})$, $w_{12} \in [1, 2^{nR_{12}}]$.
- Independently generate $2^{nR_{21}}$ sequences $u_{21}^n(w_{21}) \sim \prod_{k=1}^n p(u_{21k})$, $w_{21} \in [1, 2^{nR_{21}}]$.
- For each $u_{21}^n(w_{21})$, independently generate $2^{n(R_{22}+R'_{22})}$ sequences $u_{22}^n(w_{22}, v_{22}|w_{21}) \sim \prod_{k=1}^n p(u_{22k}|u_{21k})$, $w_{22} \in [1, 2^{nR_{22}}]$ and $v_{22} \in [1, 2^{nR'_{22}}]$.
- For each $t_{10}(w'_{10})$, $u_{21}^n(w_{21})$ and $u_{22}^n(w_{22}, v_{22}|w_{21})$, generate one sequence $x_2^n(w'_{10}, w_{21}, w_{22}, v_{22}) \sim \prod_{k=1}^n p(x_{2k}|t_{10k}, u_{21i}, u_{22i})$.

2) Encoding: Let $(w_{10i}, w_{11i}, w_{12i}, w_{21i}, w_{22i})$ be the new messages to be sent in block i, and $(w_{10[i-1]}, w_{11[i-1]}, w_{12[i-1]}, w_{21[i-1]}, w_{22[i-1]})$ be the messages sent in block $i-1$. At the beginning of block i:

- S_1 transmits $x_1^n(w_{12i}|w_{11i}, w_{10i}, w_{10[i-1]})$.
- S_2 searches for a v_{22i} such that

$$(t_{10}^n(w_{10[i-1]}), u_{21}^n(w_{21i}), u_{22}^n(w_{22i}, v_{22i}|w_{21i})) \in A_\epsilon^{(n)}(P_{T_{10}U_{22}|U_{21}}). \quad (5)$$

  S_2 then transmits $x_2^n(w_{10[i-1]}, w_{21i}, w_{22i}, v_{22i})$.

3) Decoding: At the end of block i:

- S_2 knows $w_{10[i-1]}$ and declares message $\hat{w}_{10i}$ was sent if it is the unique message such that

$$(t_{10}^n(w_{10[i-1]}), u_{10}^n(\hat{w}_{10i}|w_{10[i-1]}), y^n(i)) \in A_\epsilon^{(n)}(P_{T_{10}U_{10}Y}).$$

- D_1 knows $w_{10[i-2]}$ and searches for a unique tuple $(\hat{w}_{10[i-1]}, \hat{w}_{11[i-1]}, \hat{w}_{12[i-1]})$ for some $\hat{w}_{21[i-1]}$ such that

$$\begin{aligned}&(t_{10}^n(w_{10[i-2]}), u_{10}^n(\hat{w}_{10[i-1]}|w_{10[i-2]}), u_{11}^n(\hat{w}_{11[i-1]}),\\ &x_1^n(\hat{w}_{12[i-1]}|\hat{w}_{11[i-1]}, \hat{w}_{10[i-1]}, w_{10[i-2]}),\\ &u_{21}^n(\hat{w}_{21[i-1]}), y_1^n(i-1)) \in A_\epsilon^{(n)}(P_{T_{10}U_{10}U_{11}X_1U_{21}Y_1})\\ &\text{and} \quad (t_{10}^n(\hat{w}_{10[i-1]}), y_1^n(i)) \in A_\epsilon^{(n)}(P_{T_{10}Y_1}). \quad (6)\end{aligned}$$

- D_2 treats $T_{10}^n(w'_{10[i-1]})$ as the state and searches for a unique $(\hat{w}_{21i}, \hat{w}_{22i})$ for some $(\hat{w}_{11i}, \hat{v}_{22i})$ such that

$$(u_{11}^n(\hat{w}_{11i}), u_{21}^n(\hat{w}_{21i}), u_{22}^n(\hat{w}_{22i}, \hat{v}_{22i}|\hat{w}_{21i}), y_2^n(i)) \in A_\epsilon^{(n)}(P_{U_{11}U_{21}U_{22}Y_2}). \quad (7)$$

Applying standard error analysis and Fourier Motzkin elimination, we obtain rate region (2). For details see [11]. ■

Remark 1. Even though at S_2 we use standard Gelfand-Pinsker binning technique, but depending on the joint distribution between the binning auxiliary random variable (U_{22}) and the state (T_{10}), S_2 can also forward a part of the state (i.e. message w'_{10} of the previous block) to D_1.

Remark 2. In the binning step (5) at S_2, we use conditional binning instead of unconditional binning. The binning is only between the codeword for Han-Kobayashi private message part $U_{22}(w_{22})$ and the state $T_{10}(w'_{10})$, conditionally on knowing the Han-Kobayashi public messsage part w_{21}. This conditional binning is possible since w_{21} is decoded at both destinations.

Remark 3. In the decoding step (7) at D_2, we use joint decoding of both the Gelfand-Pinsker auxiliary random variable (u_{22}) and the Han-Kobayashi public message parts (w_{11} and w_{21}), instead of decoding Gelfand-Pinsker and Han-Kobayashi codewords separately. This joint decoding is possible since the codewords for w_{11} and w_{21} (i.e. U_{11}^n and U_{21}^n) are independent of the state in Gelfand-Pinsker coding (i.e. T_{10}^n). Joint decoding at both D_1 (6) and D_2 (7) help achieve the largest rate region for this coding structure.

Remark 4. Inclusion of Han-Kobayashi rate region and the maximum rate for each user

- The HK-PDF-binning scheme becomes the Han-Kobayashi scheme if $T_{10} = U_{10} = \emptyset$ and $X_2 = U_{22}$.
- S_1 achieves the partial decode-forward relaying rate if we set $U_{11} = U_{21} = U_{22} = \emptyset$, and $X_2 = T_{10}$.

$$R_1^{\max} = \max_{p(u_{10},x_2)p(x_1|u_{10},x_2)} \min \{I(U_{10}; Y|X_2) + I(X_1; Y_1|U_{10}, X_2), I(X_1, X_2; Y_1)\} \quad (8)$$

  Here, there is no binning but only forwarding at S_2.
- S_2 achieves the maximum rate as in Gelfand-Pinsker coding if we set $U_{11} = U_{21} = \emptyset$, $T_{10} = U_{10} = X_1$.

$$R_2^{\max} = \max_{\substack{p(x_1,u_{22})\\ p(x_2|x_1,u_{22})}} \{I(U_{22}; Y_2) - I(U_{22}; X_1)\} \quad (9)$$

  In this case, there is no forwarding of the state at S_2.

B. Comparison with existing schemes for the IC-SC

The interference channel with source cooperation (IC-SC) is a 4-node channel in which both S_1 and S_2 can receive signal from each other and use that cooperatively in sending messages to D_1 and D_2. This channel therefore includes the CCIC as a special case (when S_2 sends no information to S_1).

1) Host-Madsen's scheme [5]: This scheme for Gaussian IC-SC is based on dirty paper coding and block Markov encoding; it includes the rate for decode-forward relaying but not the Han-Kobayashi region. However, since both senders must decode the cooperative message from the other sender in order to apply dirty paper coding, the scheme cannot be applied to the CCIC which has only uni-directional cooperation.

2) Prabhakaran-Viswanath's scheme [6]: This scheme is based on 4-part rate splitting and block Markov superposition coding. It may not contain the Han-Kobayashi region or the PDF rate, depending on channel parameters. The ideas in this scheme, however, can be combined with our scheme to further improve the rate region.

3) Cao-Chen's scheme [7]: This scheme is quite close to our proposed scheme. It is also based on 3-part message splitting, block Markov encoding, Gelfand-Pinkser binning and random binning, and achieves the Han-Kobayashi region. But this scheme cannot achieve the decode-forward relaying rate because of no block Markovity between the current and the previous cooperative messages, hence no coherent transmission between source and relay.

4) Yang-Tuninetti's scheme [8]: This scheme combines ideas in previous schemes by doing 4-part message splitting, block Markov superposition coding, Marton double binning and Gelfand-Pinsker binning. It achieves the Han-Kobayashi region but not the partial decode-forward relaying rate as in (8). In this scheme, destination 2 decodes the cooperative-common part of user 1, thus limits rate R_1 to be below the decode-forward relaying rate. In our proposed scheme, the forwarding part of user 1 is not decoded at destination 2. (In [8], it is claimed to achieve the partial decode-forward relaying rate but only by setting $Y_1 = Y_2$, which is not necessary in our scheme for the CCIC.)

Furthermore, in both [7] and [8], joint decoding of both the state and the binning auxiliary variables is used at the destinations, but this joint decoding is invalid as it results in a rate region larger than is possible. In our proposed scheme, all message parts that are jointly decoded with the binning auxiliary variable are encoded independently of the state.

Hence none of the existing schemes for the IC-SC include both the HK rate region and the decode-forward relaying rate as our proposed scheme (see Remark 4). More detailed analysis can be found in [11].

IV. Gaussian CCIC rate regions

A. Signaling and rates for Han-Kobayashi PDF-binning

In the Gaussian channel, input signals for the HK-PDF-binning scheme in Section III-A can be represented as

$$
\begin{aligned}
T_{10} &= \alpha S'_{10}(w'_{10}), \qquad (10)\\
U_{10} &= \alpha S'_{10}(w'_{10}) + \beta S_{10}(w_{10}),\\
U_{11} &= \gamma S_{11}(w_{11}),\\
X_1 &= \alpha S'_{10}(w'_{10}) + \beta S_{10}(w_{10}) + \gamma S_{11}(w_{11}) + \delta S_{12}(w_{12}),\\
U_{21} &= \theta S_{21}(w_{21}),\\
X_2 &= \theta S_{21}(w_{21}) + \mu\left(\rho S'_{10}(w'_{10}) + \sqrt{1-\rho^2}S_{22}\right),\\
U_{22} &= X_2 + \lambda S'_{10} = (\mu\rho+\lambda)S'_{10} + \theta S_{21}(w_{21}) + \mu\sqrt{1-\rho^2}S_{22},
\end{aligned}
$$

where S'_{10}, S_{10}, S_{11}, S_{12}, S_{21}, S_{22} are independent $\mathcal{N}(0,1)$ random variables to encode w'_{10}, w_{10}, w_{11}, w_{12}, w_{21}, w_{22}, respectively. U_{22} is the auxiliary random variable for binning that encodes w_{22}. X_1 and X_2 are the transmit signals of S_1 and S_2. The parameters α, β, γ, δ, θ and μ are power allocation factors satisfying power constraints

$$
\begin{aligned}
\alpha^2+\beta^2+\gamma^2+\delta^2 &\le P_1\\
\theta^2+\mu^2 &\le P_2. \qquad (11)
\end{aligned}
$$

An important feature of the signaling design in (10) is ρ ($-1 \le \rho \le 1$), the correlation factor between the transmit signal (X_2) and the state (S'_{10}) at S_2. In traditional dirty paper coding, the transmit signal and the state are independent. Here, we introduce correlation between them, which includes dirty paper coding as a special case when $\rho = 0$. This correlation allows both signal forwarding and traditional binning at the same time. λ is the partial decode-forward binning parameter which will be optimized later.

Corollary 1. *The achievable rate region for the full-duplex Gaussian-CCIC using the Han-Kobayashi PDF-binning scheme is the convex hull of all rate pairs (R_1, R_2) satisfying* (2) *with*

$$
\begin{aligned}
I_2 &= C\left(\frac{c^2\beta^2}{c^2\gamma^2+c^2\delta^2+1}\right); \quad I_3 = C\left(\frac{\delta^2}{b^2\mu^2(1-\rho^2)+1}\right)\\
I_4 &= C\left(\frac{\beta^2+\delta^2}{b^2\mu^2(1-\rho^2)+1}\right)\\
&\quad + C\left(\frac{(\alpha+b\mu\rho)^2}{\beta^2+\gamma^2+\delta^2+b^2\theta^2+b^2\mu^2(1-\rho^2)+1}\right)\\
I_5 &= C\left(\frac{\gamma^2+\delta^2}{b^2\mu^2(1-\rho^2)+1}\right)\\
I_6 &= C\left(\frac{\beta^2+\gamma^2+\delta^2}{b^2\mu^2(1-\rho^2)+1}\right)\\
&\quad + C\left(\frac{(\alpha+b\mu\rho)^2}{\beta^2+\gamma^2+\delta^2+b^2\theta^2+b^2\mu^2(1-\rho^2)+1}\right)\\
I_7 &= C\left(\frac{\delta^2+b^2\theta^2}{b^2\mu^2(1-\rho^2)+1}\right)\\
I_9 &= C\left(\frac{\gamma^2+\delta^2+b^2\theta^2}{b^2\mu^2(1-\rho^2)+1}\right); \quad I_{11} = C\left(\frac{\mu^2(1-\rho^2)}{a^2\beta^2+a^2\delta^2+1}\right)
\end{aligned}
$$

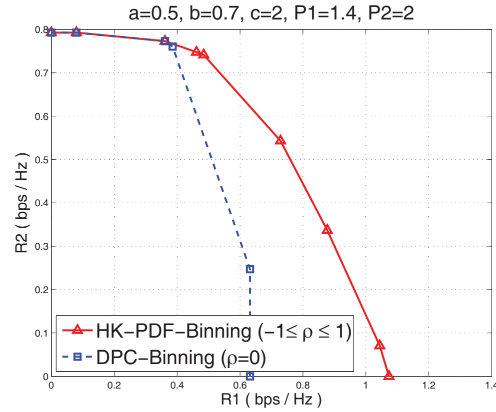

Fig. 4. Effect of the binning correlation factor ρ.

$$
\begin{aligned}
I_8 &= C\left(\frac{\beta^2+\delta^2+b^2\theta^2}{b^2\mu^2(1-\rho^2)+1}\right)\\
&\quad + C\left(\frac{(\alpha+b\mu\rho)^2}{\beta^2+\gamma^2+\delta^2+b^2\theta^2+b^2\mu^2(1-\rho^2)+1}\right)\\
I_{10} &= C\left(\frac{(\alpha+b\mu\rho)^2+\beta^2+\gamma^2+\delta^2+b^2\theta^2}{b^2\mu^2(1-\rho^2)+1}\right)\\
I_{12} &= C\left(\frac{\mu^2(1-\rho^2)}{a^2\beta^2+a^2\delta^2+1}\right)\\
&\quad + C\left(\frac{\theta^2}{(a\alpha+\mu\rho)^2+a^2\beta^2+a^2\delta^2+\mu^2(1-\rho^2)+1}\right)\\
I_{13} &= C\left(\frac{\mu^2(1-\rho^2)}{a^2\beta^2+a^2\delta^2+1}\right)\\
&\quad + C\left(\frac{a^2\gamma^2}{(a\alpha+\mu\rho)^2+a^2\beta^2+a^2\delta^2+\mu^2(1-\rho^2)+1}\right)\\
I_{14} &= C\left(\frac{\mu^2(1-\rho^2)}{a^2\beta^2+a^2\delta^2+1}\right) \qquad (12)\\
&\quad + C\left(\frac{a^2\gamma^2+\theta^2}{(a\alpha+\mu\rho)^2+a^2\beta^2+a^2\delta^2+\mu^2(1-\rho^2)+1}\right)
\end{aligned}
$$

with $C(x) = \frac{1}{2}\log(1+x)$, α, β, γ, δ, θ *and* μ *satisfy the power constraints* (11) *and* $-1 \le \rho \le 1$.

Proof: Applying Theorem 1 with the signaling in (10), we obtain the rate region in Corollary 1. ■

Remark 5. Maximum rates for each sender

- Setting $\delta = 0$, $\rho = \pm 1$, $\mu = \rho\sqrt{P_2}$, we obtain the maximum R_1 as in partial decode-forward relaying:

$$
\begin{aligned}
R_1^{\max} = \max_{\alpha^2+\beta^2+\gamma^2 \le P_1} \min\Bigg\{ & C\left(\frac{c^2\beta^2}{c^2\gamma^2+1}\right) + C(\gamma^2),\\
& C\left(\left(\alpha + b\sqrt{P_2}\right)^2 + \beta^2+\gamma^2\right)\Bigg\}. \qquad (13)
\end{aligned}
$$

- Setting $\rho = 0$, $\beta = \gamma = \delta = 0$, $\theta = 0$ and $\mu = \sqrt{P_2}$, we obtain the maximum R_2 as in dirty paper coding:

$$
R_2^{\max} = C(P_2). \qquad (14)
$$

This is also the interference-free rate for user 2.

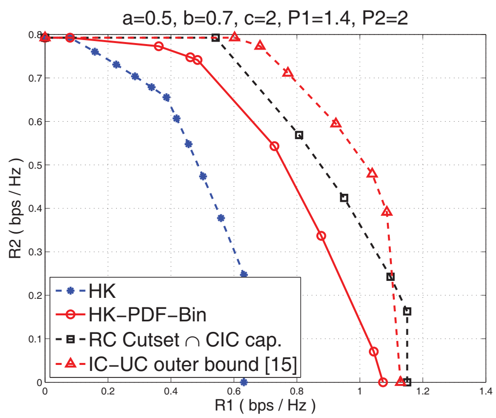

Fig. 5. Rate regions for full-duplex schemes in weak interference.

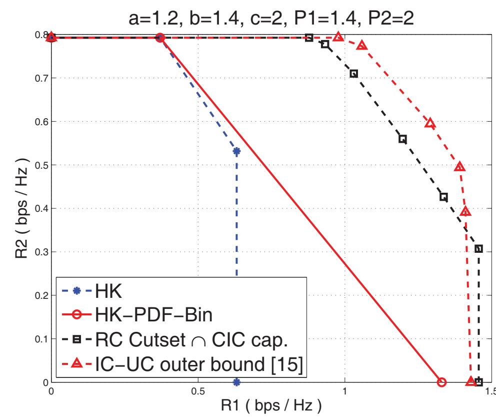

Fig. 6. Rate regions for full-duplex schemes in strong interference.

B. Optimal binning parameter for HK-PDF-binning

Corollary 2. *The optimal λ^* for the HK-PDF-binning is*

$$\lambda^* = \frac{a\alpha\mu^2(1-\rho^2) - \mu\rho(a^2\beta^2 + a^2\delta^2 + 1)}{a^2\beta^2 + a^2\delta^2 + \mu^2(1-\rho^2) + 1} \tag{15}$$

Proof: λ^* is obtained by maximizing I_{11} in (2). The detailed proof is omitted and can be seen in [11]. ∎

Remark 6. Effect of ρ:

- If $\rho = 0$, λ^* becomes the optimal λ for traditional dirty paper coding [12], which achieves $R_2^{\max}$ as in (14).
- If $\rho = \pm 1$, λ^* differs from the λ in traditional dirty paper coding and achieves $R_1^{\max}$ as in (13).
- The effect of ρ is shown in Figure 4. The dashed line represents the rate region for DPC-binning ($\rho = 0$), while the solid line represents the region for HK-PDF-binning when we adapt $\rho \in [-1, 1]$. Figure 4 illustrates that the correlation factor ρ can enlarge the rate region.

C. Numerical examples

In this section, we provide numerical comparison among the proposed HK-PDF-binning scheme, the original Han-Kobayashi scheme, and an outer bound combining the capacity for the (non-causal) CIC [13], [14] and the outer bound for the IC with user cooperation (IC-UC) [15]. Figure 5 shows an example for weak interference and Figure 6 for strong interference. We can see that the proposed HK-PDF-binning scheme contains the Han-Kobayashi region, partial decode-forward relaying rate for user 1 as in (13) and interference-free rate for user 2 as in (14). The outer bound is the intersection of the two bounds drawn and is loose as this bound is not achievable. However, we observe that as b decreases, the HK-PDF-binning rate region becomes closer to the outer bound.

V. Conclusion

In this paper, we have proposed a new coding scheme for the full-duplex causal cognitive interference channel based on partial decode-forward relaying, Gelfand-Pinsker binning and Han-Kobayashi coding. For the Gaussian channel, we introduce a correlation between the transmit signal and the state, which enlarges the rate region by allowing both state nullifying and forwarding. We also derive the optimal binning parameter for the coding scheme. Results show that the Han-Kobayashi PDF-binning scheme for the CCIC contains both the Han-Kobayashi region and partial decode-forward relaying rate. Thus cognitive communication is also beneficial even in causal setting.

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