A robust transmit CSI framework with applications in MIMO wireless precoding

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***Abstract*– Transmit channel side information (CSIT) can improve performance of MIMO wireless systems by means of precoding. More complete and reliable CSIT provides more precoding gain. Instantaneous channel measurements provide the most potential gain, but suffer from delay-induced error due to channel temporal variation. Statistical channel knowledge provides less gain, but is reliable. In this paper, we propose a framework combining a possibly outdated channel measurement with the channel statistics – the mean and the covariance – to create a dynamic CSIT form, as a function of a correlation factor between the measurement and the current channel. The CSIT consists of an estimate of the current channel and the associated estimation error covariance, which function effectively as the channel mean and covariance. We apply a precoder design exploiting these channel statistics to illustrate the achievable gain and the robustness of the new CSIT framework.**

1. INTRODUCTION

Exploiting transmit channel side information (CSIT) in MIMO wireless to improve performance has recently been an active area of research. CSIT helps increase the channel capacity and improve the system error performance. It has applications in areas such as precoding, power control, and link adaptation.

CSIT can be obtained based on the reciprocity principle or via feedback from the receiver. In both cases, there is usually a delay between when the information is obtained and when it is used at the transmitter. Such delay can affect the reliability of the CSIT, depending on the CSIT form. Several forms exist, including instantaneous channel measurements, channel statistics, and channel parameters (such as the K factor and the condition number). We focus on instantaneous channel measurements and channel statistics information. An instantaneous channel measurement possesses the most potential performance gain, but is sensitive to channel variation due to the delay. In contrast, the channel statistics, including the channel mean and the covariance (or antenna correlations), provide less gain, but are stable over much longer periods of time and can be obtained reliably.

A number of earlier precoding designs have focused on exploiting the channel statistics alone, such as the transmit antenna correlation [1], the channel mean [2], or both [3]. In addition, the scheme in [2] uses posterior channel statistics given an imperfect channel estimate, but without an explicit formulation of these statistics. Fundamentally, more complete and reliable information brings more performance gain, suggesting the combination of different forms of channel information.

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In this paper, we provide an explicit formulation of CSIT combining a potentially outdated channel measurement with the channel statistics to create a robust CSIT form. The formulation allows evaluating the CSIT based on a temporal correlation factor ρ, which is a function of the delay and the channel Doppler spread. When $\rho = 1$, the CSIT is perfect; when $\rho \to 0$, the CSIT approaches the actual channel statistics. Specifically, the new CSIT consists of an estimate of the channel at the transmit time and the associated error covariance, which function effectively as the channel mean and the covariance (for brevity, we will refer to these parameters as the functional channel mean and the functional covariance), respectively. This framework enables applications of precoding schemes exploiting both the channel mean and covariance, optimally utilizing the available transmit channel information.

The rest of the paper is organized as follows. In Section 2, we discuss the channel model and the effect of a delay on instantaneous CSIT and statistical CSIT. We construct the dynamic CSIT framework in Section 3. We outline a precoding design based on a pair-wise error probability criterion in Section 4. We then apply this design to the dynamic CSIT framework in Section 5 to illustrate the range of the precoding gain, the value of the initial channel measurement, and the robustness of the framework. Finally, we conclude in Section 6.

Notations: We use a boldface capital letter for a matrix, and the corresponding lowercase letter for its vectorized version (for example, $\mathbf{h} = \text{vec}(\mathbf{H})$), obtained by vertically concatenating the columns of the matrix. We use $(.)^*$ for conjugate transpose; and $E[.]$ for expectation.

2. CHANNEL MODEL AND CSIT

Consider a frequency flat, quasi-static block fading MIMO channel with N transmit and M receive antennas, modeled as a random matrix $\mathbf{H}$ of size $M \times N$. The channel is Gaussian distributed with mean $\bar{\mathbf{H}}$ and covariance $\mathbf{R}_0$, representing the Rician component and the correlation among all transmit and receive antennas, respectively. This model includes Rayleigh fading ($\bar{\mathbf{H}} = \mathbf{0}$) as a special case. The channel $\mathbf{H}$ can thus be decomposed as

$$\mathbf{H} = \bar{\mathbf{H}} + \tilde{\mathbf{H}} \;, \tag{1}$$

where $\tilde{\mathbf{H}}$ is the zero-mean Gaussian component, and the channel covariance $\mathbf{R}_0$ of size $MN \times MN$ is then defined as

$$\mathbf{R}_0 = E[\tilde{\mathbf{h}}\tilde{\mathbf{h}}^*], \tag{2}$$

where $\tilde{\mathbf{h}} = \text{vec}(\tilde{\mathbf{H}})$. The channel statistics, $\bar{\mathbf{H}}$ and $\mathbf{R}_0$, can be obtained by averaging instantaneous channel measurements over tens of channel coherence times; they remain valid for a period of tens to hundreds coherence time, during which, the channel can be considered as stationary [4].

Channel side information at the transmitter, or CSIT, can be obtained by using the reversed-channel information, relying on the reciprocity principle, or by feedback from the receiver. Reciprocity is usually applicable only in time-division-duplex systems, due to stringent matching constraints in all dimensions (time, frequency, and space); whereas feedback can be used in both time and frequency division-duplex systems. In either case, there exists a delay from when the channel information is obtained to when it is used by the transmitter, such as a scheduling or a feedback delay. This delay may affect the reliability of the CSIT obtained, depending on the CSIT form.

Consider the channel statistics and instantaneous channel measurements. Because of (short-term) stationarity, the channel statistics remain unchanged for a relatively long time compared to the transmission intervals. Therefore, these statistics are not affected by channel acquisition delay, and they become a reliable form of CSIT. Instantaneous channel measurements, on the other hand, are sensitive to the delay due to channel temporal variation, leading to a potential mismatch between the measurement and the channel at the time of use. In a well-designed system, this delay-induced mismatch is the main source of irreducible channel estimation errors. Thus, we assume that the initial measurements are accurate and only focus on the delay-induced effect on measurement reliability.

A more reliable and more complete form of CSIT provides more gain in system capacity and performance. This principle suggests combining both the channel statistics and instantaneous measurements to create a CSIT framework robust to channel variation, while optimally capturing the potential gain.

3. A TRANSMIT CSI FRAMEWORK

Assume that the transmitter has an initial channel measurement $\mathbf{H}_0$ at time 0, together with the channel statistics $\bar{\mathbf{H}}$ and $\mathbf{R}_0$. We aim to establish an estimate of the current channel $\mathbf{H}_s$ at the transmit time s. The channel measurement is correlated with the current channel, captured by the channel auto-covariance

$$\mathbf{R}_s = E[\tilde{\mathbf{h}}_0 \tilde{\mathbf{h}}_s^*] \ . \tag{3}$$

Because of stationarity, the auto-covariance $\mathbf{R}_s$ depends only on the time difference s, but not on the absolute time; at a zero delay, we have $\mathbf{R}_0$ in (2). Again, $\mathbf{R}_s$ can be obtained by averaging operations on channel measurements over several (tens of) channel coherence times.

Given the channel measurement $\mathbf{H}_0$ and the statistics $\bar{\mathbf{H}}$, $\mathbf{R}_0$, and $\mathbf{R}_s$, an estimate of the channel at time s follows from MMSE estimation theory [5] as

$$\begin{aligned} \hat{\mathbf{h}}_s &= E\big[\mathbf{h}_s|\mathbf{h}_0\big] = \bar{\mathbf{h}} + \mathbf{R}_s^* \mathbf{R}_0^{-1}\big[\mathbf{h}_0 - \bar{\mathbf{h}}\big] \\ \mathbf{R}_{e,s} &= \text{cov}\big[\mathbf{h}_s|\mathbf{h}_0\big] = \mathbf{R}_0 - \mathbf{R}_s^* \mathbf{R}_0^{-1} \mathbf{R}_s \ , \end{aligned}$$

where $\hat{\mathbf{h}}_s = \text{vec}(\hat{\mathbf{H}}_s)$ is the channel estimate, and $\mathbf{R}_{e,s}$ is the estimation error covariance at time s. These two quantities function effectively as the channel mean and the channel covariance, constituting the new CSIT. A similar model was proposed in [6] for estimating a scalar time-varying channel from a vector of outdated estimates. CSIT formulations conditioned on noisy channel estimates were also studied in [7, 2].

The auto-covariance $\mathbf{R}_s$ captures both the channel temporal correlation and the antenna correlation. We now assume that the temporal correlation in a MIMO channel is homogeneous. In other words, all the scalar channels between the N transmit and the M receive antennas have the same temporal correlation function ρ_s. Similar assumptions have also been used in [8, 9]. We can then separate the temporal correlation from the antenna correlation in the channel auto-covariance as

$$\mathbf{R}_s = \rho_s \mathbf{R}_0 \ . \tag{4}$$

The channel temporal correlation ρ_s is a function of the Doppler spread f_d and the delay s. In the classical Jake's model [10] for example, $\rho_s = J_0(2\pi f_d s)$, where J_0 is the zeroth order Bessel function of the first kind. In general, we assume that $-1 \leq \rho_s \leq 1$, and $\rho_s = 1$ only at a zero delay ($s = 0$).

Using this simplified auto-covariance model, the channel estimate and its error covariance become

$$\begin{aligned} \hat{\mathbf{H}}_s &= \rho_s \mathbf{H}_0 + (1-\rho_s)\,\bar{\mathbf{H}} \ , \\ \mathbf{R}_{e,s} &= \left(1-\rho_s^2\right)\mathbf{R}_0 \ . \end{aligned} \tag{5}$$

These functional channel statistics, or the new CSIT, can now be simply characterized as a function of ρ_s and other stationary channel parameters.

A common antenna correlation model assumes a Kronecker structure, representing separable transmit and receive antenna correlations as

$$\mathbf{R}_0 = \mathbf{R}_t^T \otimes \mathbf{R}_r \ , \tag{6}$$

where $\mathbf{R}_t$ of size $N \times N$ and $\mathbf{R}_r$ of size $M \times M$ are the transmit and receive antenna correlations, respectively. This Kronecker correlation model has been experimentally verified for indoor channels of up to 3×3 antennas [11] and for outdoor of up to 8×8 [12]. Using this model, the estimated channel has the functional antenna correlations as

$$\begin{aligned} \mathbf{R}_{t,s} &= \left(1-\rho_s^2\right)^{\frac{1}{2}} \mathbf{R}_t \ , \\ \mathbf{R}_{r,s} &= \left(1-\rho_s^2\right)^{\frac{1}{2}} \mathbf{R}_r \ , \end{aligned} \tag{7}$$

which again follow a Kronecker structure.

In the constructed CSIT models (5) and (7), ρ_s acts as a channel estimate quality dependent on the time delay s. For a zero or short delay, ρ_s is close to one; the estimate depends heavily on the initial channel measurement, and the error covariance is small. As the delay increases, ρ_s decreases in magnitude to zero, reducing the impact of the initial measurement. The estimate then moves toward the channel mean $\bar{\mathbf{H}}$, and the error covariance grows toward the channel covariance $\mathbf{R}_0$. Therefore, the estimate and its error covariance (5) constitute a form of CSIT, ranging between perfect channel knowledge (when $\rho = 1$) and the channel statistics (when $\rho = 0$). By taking into account channel time variation, this framework optimally captures the available channel information and creates a dynamic CSIT model.

4. PRECODING ALGORITHM

We now apply the above CSIT framework to precoding. For this application, we consider channels with a non-zero mean $\bar{\mathbf{H}}$ and

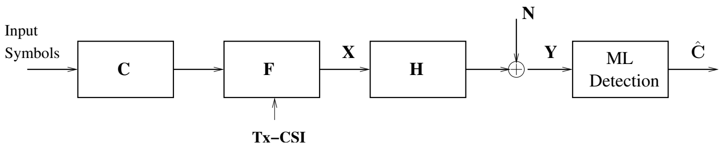

Fig. 1. System architecture with a linear precoder $\mathbf{F}$ and a STBC $\mathbf{C}$.

a full-rank transmit antenna correlation $\mathbf{R}_t$ (6), assuming $\mathbf{R}_r = \mathbf{I}$. We study a system employing a space-time block code $\mathbf{C}$ to capture the channel diversity and a precoder $\mathbf{F}$ to exploit the CSIT, depicted in Figure 1. This structure has often been studied in the literatures for exploiting various forms of CSIT [2, 1, 3].

We focus on a linear precoder, which functions as a multi-mode beamformer. The precoder left singular vectors are the beam directions; the singular values provide the beam power allocation; and the right singular vectors form the input shaping matrix, combining the input symbols from the space-time block code (STBC). To maintain a constant average sum transmit power, the precoder must satisfy the power constraint

$$\mathrm{tr}(\mathbf{F}\mathbf{F}^*) = 1. \tag{8}$$

The receive signal block is then

$$\mathbf{Y} = \mathbf{H}\mathbf{F}\mathbf{C} + \tilde{\mathbf{N}} \,,$$

where $\tilde{\mathbf{N}}$ is the additive complex white Gaussian noise.

4.1. PEP based precoder design

We consider an existing precoder design based on the codeword pair-wise error probability (PEP) criterion [3]. Define the codeword distance product matrix between a pair of codeword $(\mathbf{C}, \hat{\mathbf{C}})$ as

$$\mathbf{A} = \frac{1}{P}(\mathbf{C} - \hat{\mathbf{C}})(\mathbf{C} - \hat{\mathbf{C}})^* \,, \tag{9}$$

where P is the average sum transmit power from all antennas. Using ML detection, the PEP on mis-detecting $\mathbf{C}$ as $\hat{\mathbf{C}}$ can then be upper-bounded by the Chernoff bound as

$$P(\mathbf{C} \to \hat{\mathbf{C}}) \leq \exp\left(-\frac{\gamma}{4}\mathrm{tr}\big(\mathbf{H}\mathbf{F}\mathbf{A}\mathbf{F}^*\mathbf{H}^*\big)\right) \,, \tag{10}$$

where γ is the signal-to-noise ratio (SNR). The precoder is obtained by minimizing this Chernoff bound, averaged over the channel distribution, for a pair of minimum-distance codewords.

Applying the dynamic CSIT model (5), the channel has a functional channel mean $\hat{\mathbf{H}}_s$ and a functional transmit correlation $\mathbf{R}_{t,s}$. For imperfect CSIT ($\rho < 1$), since $\mathbf{R}_t$ is full-rank, the functional correlation $\mathbf{R}_{t,s}$ is non-zero and invertible. Averaging (10) over the functional channel statistics, we obtain the following bound on the average PEP:

$$\bar{P}_e \leq \frac{\exp\big[\mathrm{tr}(\hat{\mathbf{H}}_s\mathbf{W}^{-1}\hat{\mathbf{H}}_s^*)\big]}{\det(\mathbf{W})^M}\det(\mathbf{R}_{t,s})^M\exp\big[-\mathrm{tr}(\hat{\mathbf{H}}_s\mathbf{R}_{t,s}^{-1}\hat{\mathbf{H}}_s^*)\big] \,, \tag{11}$$

where

$$\mathbf{W} = \frac{\gamma}{4}\mathbf{R}_{t,s}\mathbf{F}\mathbf{A}\mathbf{F}^*\mathbf{R}_{t,s} + \mathbf{R}_{t,s} \,. \tag{12}$$

In the case $\mathbf{A} = \mu_0\mathbf{I}$ for some scalar μ_0, minimizing the above bound can be solved analytically using convex optimization theory to arrive at the solution [3]

$$\mathbf{F}\mathbf{F}^* = \left[\frac{1}{2\nu}(M\mathbf{I}_N + \mathbf{\Psi}^{\frac{1}{2}}) - \mathbf{R}_{t,s}^{-1}\right]\frac{4}{\mu_0\gamma} \,, \tag{13}$$

where

$$\mathbf{\Psi} = M^2\mathbf{I}_N + 4\nu\mathbf{R}_{t,s}^{-1}\hat{\mathbf{H}}_s^*\hat{\mathbf{H}}_s\mathbf{R}_{t,s}^{-1} \,,$$

and ν is the Lagrange multiplier satisfying the power constraint (8). Efficient algorithms to solve for ν are outlined in [3]. This reference also includes the precoding solution for the non-scaled-identity $\mathbf{A}$ case.

For perfect CSIT ($\rho = 1$), it can be easily shown that the precoder minimizing the bound in (10) is a rank-one matrix with the defined left and right singular vectors given by the dominant eigenvectors of $\mathbf{H}^*\mathbf{H}$ and $\mathbf{A}$, respectively. Thus, the precoder becomes single-mode beamforming. The average PEP then satisfies

$$\bar{P}_e \leq E\left[\exp\left(-\frac{\gamma}{4}\lambda_{\max}(\mathbf{H}^*\mathbf{H})\lambda_{\max}(\mathbf{A})\right)\right] \,, \tag{14}$$

where the expectation is taken over the distribution of $\lambda_{\max}(\mathbf{H}^*\mathbf{H})$, the largest eigenvalue of $\mathbf{H}^*\mathbf{H}$.

4.2. Diversity analysis

We now analyze the diversity order obtained by systems using the above PEP-based precoder and a STBC, assuming a channel estimate quality ρ. Since diversity is a high-SNR measure, for imperfect CSIT ($\rho < 1$), we examine the bound (11) while taking $\gamma \to \infty$. Noting that only $\mathbf{W}$ depends on γ, and that the Chernoff bound is tight at high SNR, the system diversity order is

$$d = \lim_{\gamma\to\infty} -\frac{\mathrm{tr}(\hat{\mathbf{H}}_s\mathbf{W}^{-1}\hat{\mathbf{H}}_s^*) - M\log\det(\mathbf{W})}{\log\gamma} \,. \tag{15}$$

From (12), $\mathbf{W}^{-1} \to \mathbf{R}_t^{-1}$ as $\gamma \to \infty$; thus, the trace term in (15) vanishes, and the diversity order becomes

$$d = \lim_{\gamma\to\infty}\frac{M\log\det(\mathbf{W})}{\log\gamma} \,. \tag{16}$$

As $\gamma \to \infty$, the solution (13) approaches a precoder with equi-power allocation on the non-zero eigen-modes of $\mathbf{A}$, and the left and right singular vectors given by the eigenvectors of $\mathbf{R}_{t,s}$ and $\mathbf{A}$, respectively [3]. Thus, let $\mathbf{R}_t = \mathbf{U}_t\mathbf{\Lambda}_t\mathbf{U}_t^*$ and $\mathbf{A} = \mathbf{U}_A\mathbf{\Lambda}_A\mathbf{U}_A^*$ be the respective eigenvalue decompositions, then at high SNR, $\mathbf{W}$ (12) approaches

$$\mathbf{W}_{\gamma\text{ limit}} = \frac{\gamma(1-\rho^2)}{4L}\mathbf{U}_t\mathbf{\Lambda}_t^2\mathbf{\Lambda}_A\mathbf{U}_t \,, \tag{17}$$

where L is the number of non-zero eigenvalues of $\mathbf{A}$, determined by the diversity order of the STBC. For $\rho < 1$, the rank of $\mathbf{W}_{\gamma\text{ limit}}$ is L; thus, the system diversity order (16) is ML. Therefore, with imperfect CSIT, the precoder and the channel estimate quality ρ have no impact on the diversity achieved by the system; the transmit diversity is solely determined by the STBC.

For perfect CSIT ($\rho = 1$), it can be shown that the resulting diversity order from (14) is the maximum MN (see [4], Section 5.4.4). The proof is based on loosening the bound in (10), by

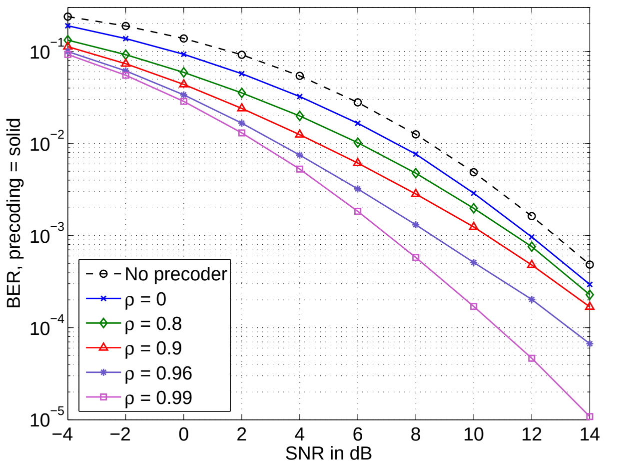

Fig. 2. Performance of the minimum-distance PEP precoder with channel estimate CSIT for a 4×1 system.

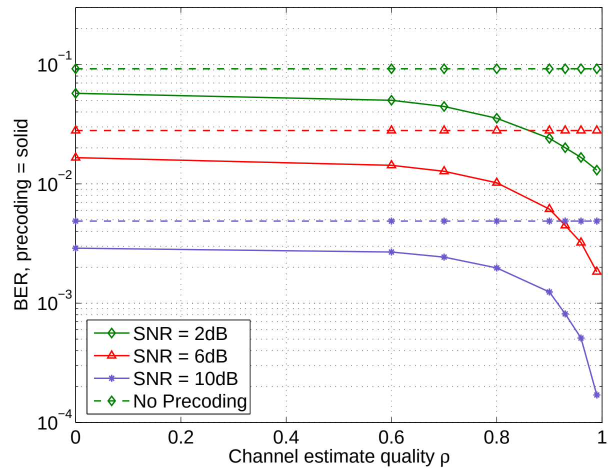

Fig. 3. Performance versus the channel estimate quality.

replacing $\mathbf{A}$ with $\mathbf{I}\lambda_{\max}(\mathbf{A})$ and using no precoder ($\mathbf{F} = \frac{1}{\sqrt{N}}\mathbf{I}$); this relaxed bound has diversity order MN. Thus, the precoder for perfect CSIT achieves full diversity, regardless of the STBC.

The above analyses show that the diversity obtained by the precoder depends on the CSIT. When the CSIT is imperfect, no diversity can be extracted by the precoder; the STBC then plays an essential role in obtaining transmit diversity in the system. Only when the CSIT is perfect that the precoder delivers the full transmit diversity order. In both cases, however, the precoder achieves an SNR gain, which is the essential value of precoding.

5. RESULTS AND DISCUSSION

We provide simulation results for a 4×1 system, using the dynamic CSIT framework. The channel mean $\bar{\mathbf{H}}$ and the transmit correlation $\mathbf{R}_t$ are given in the Appendix. The system employs a rate $\frac{3}{4}$ orthogonal STBC for 4 transmit antennas [13]. The error performance is averaged over multiple initial channel measurements, independently generated from the channel distribution, and multiple channel estimates given each initial measurement.

Figure 2 shows the system performance given different estimate quality ρ values. The performance improves with higher ρ. When $\rho = 0$, the precoding gain is that of statistical CSIT alone; as $\rho \to 1$, the gain increases to the maximum of 6dB with perfect CSIT for a 4×1 system.

Figure 3 presents the performance as a function of the estimate quality ρ. This result shows that the initial channel measurement helps increase the precoding gain only when its correlation with the current channel is reasonably high, $\rho \geq 0.6$ in this case; otherwise, precoding on the channel statistics alone ($\rho = 0$) can extract most of the gain.

The precoder is compared with a beamforming scheme that uses only the initial channel measurement $\mathbf{H}_0$, shown in Figure 4. With perfect CSIT (numerically represented by $\rho = 0.99$), these two schemes coincide and are optimal for a 4×1 channel. However, as ρ decreases, the $\mathbf{H}_0$ beamforming scheme starts loosing diversity at high SNR and eventually performs even worse than without precoding. The precoder exploiting the dynamic CSIT (5), on the other hand, provides gains over no precoding for all

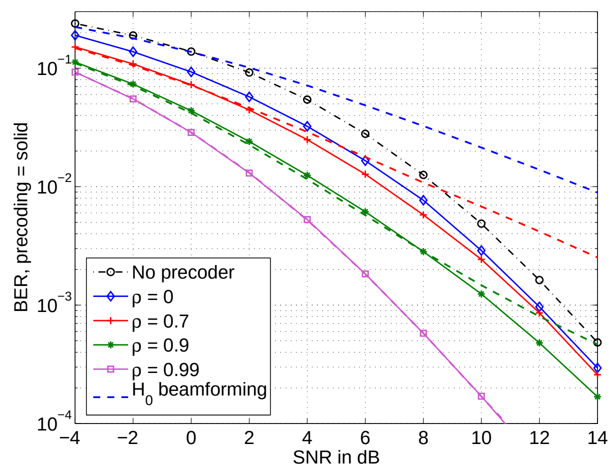

Fig. 4. Performance comparison with a beamforming scheme relying only on outdated channel measurements.

ρ values. This result shows the robustness of the dynamic CSIT framework.

6. CONCLUSION

We have constructed a dynamic CSIT framework, using a potentially outdated channel measurement, the channel statistics (mean and covariance), and the channel temporal correlation factor. The resulting CSIT is in the form of a functional channel mean and a functional covariance. The framework is simple and optimally captures the available channel information, taking into account its time variation via the temporal correlation factor. Depending on this factor, the CSIT ranges from statistical channel information to perfect channel knowledge.

The dynamic CSIT framework allows the application of precoding schemes exploiting both the channel mean and the covariance. Results show that precoding on dynamic CSIT is robust to different channel estimate qualities and can achieve significant gains. Furthermore, the initial channel measurement helps increase the precoding gain if its correlation with the current channel is relatively high ($\rho \geq 0.6$); the gain then improves with a higher correlation factor. When this correlation is weak, precoding on the channel statistics alone obtains most of the available gain.

APPENDIX

The channel mean and transmit antenna correlation matrices used in simulations are (numbers are rounded to two digits after the decimal point)

$$\bar{\mathbf{H}} = \begin{bmatrix} .27 - .17i & -.06 + .18i & .14 + .25i & .33 - .27i \end{bmatrix},$$

$$\mathbf{R}_t = \begin{bmatrix} .86 & .26 - .60i & .25 - .14i & .16 + .10i \\ .26 + .60i & 1.03 & .52 + .03i & -.09 - .08i \\ .25 + .14i & .52 - .03i & .72 & -.50 - .25i \\ .16 - .10i & -.09 + .08i & -.50 + .25i & 1.39 \end{bmatrix}.$$

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