

# Relay Selection Methods for Wireless Cooperative Communications

Seunghoon Nam, Mai Vu, and Vahid Tarokh  
 School of Engineering and Applied Sciences  
 Harvard University  
 Cambridge, MA 02138  
 Email: nshid@seas.harvard.edu

**Abstract**—We study the problem of relay selection in a wireless cooperative network. Assuming a single source, a single destination, and  $N$  uniformly distributed candidate relays, we seek to select a set of cooperating relays to minimize the total transmission time of a fixed amount of data. We propose two selection methods: Best expectation, which adaptively selects the relays, and Best- $m$ , which selects an optimally pre-determined number of relays. Each method is implemented with a simple and optimal algorithm. We further provide closed-form, analytical approximations of these algorithms' performance, which help simplify the process of finding the optimal number of cooperating relays. Simulations illustrate the performance of the proposed relay selection methods and show a close match between the analytical approximations and the numerical values. Through some initial studies, we also observe a simple and intriguing connection between the Best- $m$  selection method and the network geometry. Provided that the relays are uniformly distributed, the source can simply cooperate with all the relays within a radius of a fixed proportion of the source-destination distance.

**Index Terms**—Relay selection, cooperative communications, decode-and-forward.

## I. INTRODUCTION

It has been established that cooperation in wireless communications can significantly improve the link quality by exploiting the spatial diversity of multiple terminals [1]–[3]. Cooperative techniques are promising candidates for emerging wireless networks.

Most works so far focus on designing or analyzing relay algorithms that maximize the throughput or minimize the outage probability, given a fixed channel allocation, or minimize the frame error rate of specific cooperative coding schemes [3]–[5]. In these works, either each terminal can act as both source and relay, or/and a pre-determined cooperator is assumed. In a more general scenario, however, the relays may not be pre-determined but have to be chosen from a set of available terminals. In [6], [7], relay selection algorithms are proposed, which search over a set of  $N$  candidate relays with optimization criteria as the outage probability or frame error rate.

In this paper, we propose relay selection methods using a novel criterion. Our goal is to optimize the total transmission time for a given, fixed amount of data. This transmission time consists of the times required for both the source-to-relay link and the relay-to-destination link. Instead of using a fixed channel allocation, we assume the channel allocation is

flexible according to the link qualities. We analyze the effects of fading on our relay selection methods and derive closed-form approximations which provide insight into our schemes.

This paper is organized as follows. In the next section, we present the wireless network model and the transmission protocol. In Section III, we describe the details of our proposed relay selection algorithms. In Section IV, we analyze these relay selection methods and provide some closed-form performance approximation. In Section V, we provide simulation results and compare them with the analytical approximation. We also investigate the effect of the number of candidate relays and the density of the relays. Section VI provides our conclusion and final remarks.

## II. WIRELESS NETWORK MODEL AND TRANSMISSION PROTOCOLS

### A. Network and channel models

We consider a wireless network with  $N + 2$  terminals: a source  $s$ , a destination  $d$  and a set of  $N$  candidate relays  $R = \{1, 2, \dots, N\}$ , as illustrated in Figure 1. We assume that the  $N$  candidate relays are uniformly distributed in a circle of radius  $d_0$  centered at the source node. Among these,  $A$  denotes the set of selected relays.

Assuming flat-fading, let  $h_{sd}$ ,  $h_{si}$  and  $h_{id}$  denote the wireless channel coefficients from the source to the destination, from the source to relay  $i$ , and from relay  $i$  to the destination, respectively. These coefficients capture the effects of both path loss and Rayleigh fading. The path loss is proportional to  $r^\alpha$ , where  $r$  is the distance between the transmitter and the receiver and  $\alpha$  is the path loss exponent, typically ranged from 2 to 5 [8]. The Rayleigh fading component is modeled as a zero-mean complex Gaussian random variable with variance  $1/2$  per real dimension. We assume that the source terminal has instantaneous channel state information (CSI) of  $h_{sd}$  and  $h_{si}$ ,  $\forall i \in R$ , which it can measure directly. However, the source has no instantaneous CSI of  $h_{id}$ ,  $\forall i \in R$ , but only the distribution of these channel coefficients.

### B. Direct Transmission

We assume that the source has a total amount of  $D$  bits of data to transmit to the destination. If the source directly

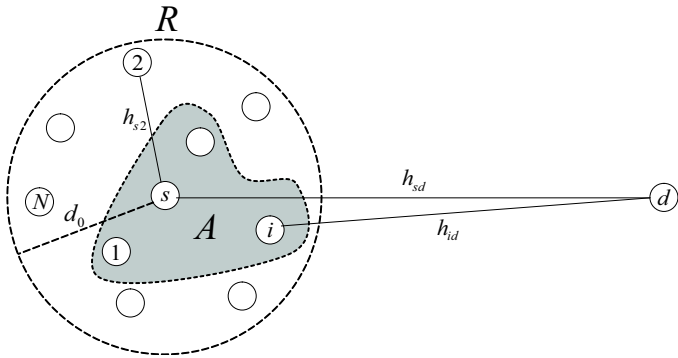


Fig. 1. Wireless Network Model for Relay Selection.

transmits the data to the destination, the transmission time  $T$  may be expressed as

$$T = \frac{D}{C_{sd}W} \quad (1)$$

where  $W$  is the bandwidth and  $C_{sd}$  is the capacity of the channel between the source and the destination. The channel capacity  $C_{sd}$  is given by

$$C_{sd} = \log_2 \left( 1 + \frac{P|h_{sd}|^2}{\sigma^2} \right) \quad (2)$$

where  $\sigma^2$  is the variance of white Gaussian noise added at the receiver and  $P$  is the average power available at the transmitter. Without loss of generality, for the rest of this paper, we will assume that the total amount of data and the bandwidth are normalized as  $\frac{D}{W} = 1$ .

### C. Cooperative Transmission

When the relay terminals can cooperate with the source to transmit the data to the destination, we can think of a cooperative transmission protocol consisting of two phases: listening phase and cooperating phase. In the listening phase, the source transmits the data to the relays. We assume that the destination terminal receives no data from the source terminal during the listening phase, i.e., no side information is acquired at the destination node in the listening phase. The source decides which relay terminals to cooperate with according to an appropriate relay selection criterion. We consider only decode-and-forward transmission at the relay terminals. Thus the time allocated to the listening phase is set to guarantee that all selected relays can correctly decode the transmitted data  $D$  from the source. In the cooperating phase, the source and the selected relays cooperate to transmit data to the destination. We assume that each relay has the same average transmit power  $P$  as the source terminal.

If the source terminal had full CSI of  $h_{sd}$ ,  $h_{si}$  and  $h_{id}$ ,  $\forall i \in R$ , the total transmission time required for this two-phase cooperative transmission would be equal to the time  $T_L$  required for the worst relay in the selected set to decode the data, plus the time  $T_C$  for the cooperative transmission.

This total transmission time may be expressed as

$$\begin{aligned} T &= T_L + T_C \\ &= \frac{D}{\min_{i \in A} C_{si}W} + \frac{D}{C_{sAd}W}, \end{aligned} \quad (3)$$

where  $C_{si}$  is the channel capacity between the source and relay  $i$ , and  $C_{sAd}$  is the channel capacity between the cooperating group (the source and the selected relays) and the destination. These channel capacities are given by

$$C_{si} = \log_2 \left( 1 + \frac{P|h_{si}|^2}{\sigma^2} \right), \quad (4)$$

and

$$C_{sAd} = \log_2 \left( 1 + \frac{P}{\sigma^2} \mathbf{H}_{sAd}^H \mathbf{H}_{sAd} \right), \quad (5)$$

where  $\mathbf{H}_{sAd}$  is the channel matrix between the cooperating group and the destination, described as

$$\mathbf{H}_{sAd} = [ h_{sd} \quad h_{i_1d} \quad h_{i_2d} \quad \cdots \quad h_{i_kd} ]^T. \quad (6)$$

The subscript indices  $i_1, i_2, \dots, i_k$  here denote the relays which belong to the selected set  $A$ . This set  $A$  is determined by a specific relay selection method.

In Section II-A, however, we assumed that the source has no instantaneous CSI of the channels between the relays and the destination, but only the distributions. Thus we cannot compute the instantaneous value of the channel capacity  $C_{sAd}$  in (3) and (5). Below we propose relay selection methods which use an alternative measure on the time  $T_C$  for the cooperating phase.

## III. RELAY SELECTION ALGORITHMS

### A. Selection Criteria

In this section, we propose two alternative criteria to (3) for selecting relays. These criteria do not require the source to have instantaneous CSI of the channels between the relays and the destination, which may be difficult to obtain, but only these channel distributions.

1) *Best Expectation Method*: Here we find the set of relays that minimizes the following equation:

$$\begin{aligned} \tilde{T} &= T_L + E\{T_C\} \\ &= \frac{D}{\min_{i \in A} C_{si}W} + E \left\{ \frac{D}{C_{sAd}W} \right\}, \end{aligned} \quad (7)$$

where the expectation is taken over the distribution of Rayleigh fading. This relay selection method is described as

$$A^* = \arg \min_{ACR} \left( \frac{1}{\min_{i \in A} C_{si}} + E \left\{ \frac{1}{C_{sAd}} \right\} \right) \quad (8)$$

where  $A^*$  is the optimal set of selected relays. We will call this criterion as the *Best Expectation Criterion*.

Since this criterion chooses the optimal set of relay  $A^*$  as a function of the instantaneous source-relay capacities  $C_{si}$ , the number of relays in  $A^*$  depends on the realizations of the channels between the source and the relays. In other words, the Best Expectation Method requires adaptation of the number of cooperative relays to the specific network.

2) *Best-m Method*: A simpler scheme without adapting the number of relays would result if the source can decide the optimal number of cooperative relays without relying on the specific network realization. It can then simply pick an optimal number of  $m$  relays, where  $m$  is chosen to minimize the expected total transmission time. In this method, the relays are selected according to the following criterion:

$$\begin{aligned} A_m &= \arg \min_{|A|=m, A \subset R} \left( \frac{1}{\min_{i \in A} C_{si}} + E \left\{ \frac{1}{C_{sAd}} \right\} \right) \\ &= \arg \min_{|A|=m, A \subset R} \left( \frac{1}{\min_{i \in A} C_{si}} \right) \end{aligned} \quad (9)$$

Denote the transmission time (7) associated with this set of relays  $A_m$  as  $\tilde{T}_m$ . The optimal number of cooperating relays  $m^*$  is then determined as

$$m^* = \arg \min_{m \in R} E \{ \tilde{T}_m \}, \quad (10)$$

where the expectation is taken over the distributions of the source-to-relay channels. We will call this method as the *Best-m method*.

While the Best-m selection method may result in a longer expected total transmission time than the Best expectation method, it is simpler to implement. Next we will describe the algorithms that implement each of these methods.

### B. Best Expectation Selection Algorithm

To find the set  $A^*$  in (8), and exhaustive search would involve over  $2^N$  cases. The computational complexity of the exhaustive search grows exponentially with the number of relay candidates. Below, we propose a simple algorithm to find the optimal set  $A^*$ .

Suppose that the relay terminal  $\bar{i}$  minimizes the source-to-relay capacity  $C_{si}$  among the relays in the set of actively cooperating relays  $A$ . Then, the following equations hold:

$$C_{s\bar{i}} = \min_{i \in A} C_{si} \quad (11)$$

$$A^* = \arg \min_{A \subset R} \left( \frac{1}{C_{s\bar{i}(A)}} + E \left\{ \frac{1}{C_{sAd}} \right\} \right) \quad (12)$$

where the notation  $\bar{i}(A)$  indicates that  $\bar{i}$  is a function of  $A$ .

Since the channel capacity in (5) can be rewritten as

$$C_{sAd} = \log_2 \left( 1 + \frac{P}{\sigma^2} \left( |h_{sd}|^2 + \sum_{i \in A} |h_{id}|^2 \right) \right),$$

it is clear that adding more relays to the set  $A$  increases the total received power at the destination from the cooperating group and hence is always beneficial in minimizing  $E \left\{ \frac{1}{C_{sAd}} \right\}$ . Since we assumed that relay  $\bar{i}$  belongs to  $A$  and minimizes the source-to-relay capacity  $C_{si}$ , another relay  $i$  can only be in  $A$  if and only if  $C_{si} > C_{s\bar{i}}$ . Based on these observations, we propose a simple algorithm to implement the best expectation method as in Algorithm 1.

In this algorithm, the relays in  $A^*$  are those which have the largest source-to-relay channel capacities among the set of candidate relays  $R$ . The number of relays in  $A^*$  therefore

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### Algorithm 1 Best Expectation Algorithm

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- 1: Sort  $|h_{si}|$ ,  $i \in R$  in decreasing order such that  $|h_{s1}| \geq |h_{s2}| \geq \dots \geq |h_{sN}|$ , where  $l_n \in R$  and  $1 \leq n \leq N$ .
  - 2:  $\tilde{T} \leftarrow \infty$ ,  $i \leftarrow 1$
  - 3: Include the relay terminal  $l_1, l_2, \dots, l_i$  into set  $A_i$ .
  - 4:  $T_i \leftarrow \frac{1}{C_{sli}} + E \left\{ \frac{1}{C_{sAd}} \right\}$ , where
 
$$C_{sAd} = \log_2 \left( 1 + \frac{P}{\sigma^2} \left( |h_{sd}|^2 + \sum_{k \in A} |h_{kd}|^2 \right) \right)$$
  - 5: **if**  $T_i < \tilde{T}$  **then**
  - 6:    $\tilde{T} \leftarrow T_i$
  - 7:    $A^* \leftarrow A_i$
  - 8: **end if**
  - 9:  $i \leftarrow i + 1$
  - 10: **if**  $i \leq N$  **then**
  - 11:   go to line 3.
  - 12: **end if**
- 

depends on the specific network realization. This number is determined in the line 4-8 of the Algorithm 1. Thus, the Best Expectation Algorithm may result in a varying number of cooperative relays, depending on the specific network realization.

### C. Best-m Selection Algorithm

For a given number of cooperative relays  $m$ , the Best-m Algorithm for (9) is described in Algorithm 2. This algorithm simply selects  $m$  relays with the best source-to-relay channels. The optimal  $m^*$  can be pre-determined according to (10).  $m^*$  depends on the number of relays  $N$ , the relay distribution and the distributions of the channels  $h_{si}, h_{sd}, h_{id}$ , but is independent of the specific network realization.

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### Algorithm 2 Best-m Algorithm

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- 1: Sort  $|h_{si}|$ ,  $i \in R$  in decreasing order such that  $|h_{s1}| \geq |h_{s2}| \geq \dots \geq |h_{sN}|$ , where  $l_n \in R$  and  $1 \leq n \leq N$ .
  - 2: Include the relay terminal  $l_1, l_2, \dots, l_m$  into set  $A_m$ .
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## IV. ANALYSIS OF RELAY SELECTION METHODS

In this section, we provide mathematical analysis of the effects of fading on the average transmission time of the proposed relay selection methods. Here the channel coefficients are assumed to capture the effect of Rayleigh fading only, and we leave the analysis of the path loss effect to a future work. We analyze each phase, listening and cooperating, separately, and provide closed-form approximations to the expected transmission time in each phase.

### A. Transmission time of the listening phase

The average transmission time  $E \{ T_L(m) \}$  of the listening phase with  $m (< N)$  cooperating relay terminals in the

Best- $m$  method is given by

$$E\{T_L(m)\} = \int_0^\infty \frac{1}{\log_2\left(1 + \frac{P}{\sigma^2}y\right)} g_{N-m+1:N}(y) dy, \quad (13)$$

where  $g_{N-m+1:N}(y)$  is the probability density function (pdf) of the  $(N-m+1)$ th order statistic of  $N$  independent exponential random variables, each with density  $g(y) = e^{-y}$ . Equivalently, by introducing a new random variable

$$t = \left(\log_2\left(1 + \frac{P}{\sigma^2}y\right)\right)^{-1},$$

equation (13) can be written as

$$E\{T_L(m)\} = \int_0^\infty t \cdot h_{m:N}(t) dt. \quad (14)$$

Here  $h_{m:N}(t)$  denotes the pdf of the  $m$ th order statistic of  $N$  independent random variables, of which the pdf  $h(t)$  is given by

$$h(t) = \frac{\sigma^2}{P} t^{-2} e^{\frac{1}{t} - \frac{\sigma^2}{P}(e^{\frac{1}{t}} - 1)}, \quad t > 0, \quad (15)$$

and the cumulative distribution function (cdf)  $H(t)$  is given by

$$H(t) = e^{-\frac{\sigma^2}{P}\left(1 - e^{\frac{1}{t}}\right)}. \quad (16)$$

The average transmission time  $E\{T_L(m)\}$  of the listening phase can be lower bounded by

$$\begin{aligned} E\{T_L(m)\} &\geq H^{-1}\left(\frac{m-1}{N}\right) \\ &= \frac{1}{\log_2\left(1 - \frac{P}{\sigma^2} \log_2\left(\frac{m-1}{N}\right)\right)}, \quad 1 < m < N, \end{aligned} \quad (17)$$

of which the proof may be found in the Appendix. In a similar way that the lower bound is derived, we conjecture an upper bound of  $E\{T_L(m)\}$  without proof:

$$\begin{aligned} E\{T_L(m)\} &\leq H^{-1}\left(\frac{m}{N}\right) \\ &= \frac{1}{\log_2\left(1 - \frac{P}{\sigma^2} \log_2\left(\frac{m}{N}\right)\right)}, \quad 1 < m < N. \end{aligned} \quad (18)$$

The upper bound in (18) is found to be very tight via simulation and can be used as an approximation of  $E\{T_L(m)\}$ , as shown in Figure 2.

### B. Transmission time of the cooperating phase

The average transmission time  $E\{T_C(i)\}$  of the cooperating phase with  $i$  cooperating relay terminals is given by

$$E\{T_C(i)\} = E\left\{\frac{1}{C_{sAd}}\right\} = \int_0^\infty \frac{1}{\log_2\left(1 + \frac{P}{\sigma^2}x\right)} f_i(x) dx \quad (19)$$

where  $x = |h_{sd}|^2 + \sum_{k \in A} |h_{kd}|^2$  and  $f_i(x)$  is the pdf of the random variable  $x$ . With Rayleigh fading alone, the random variable  $x$  follows a chi-square distribution with  $2(i+1)$  degree of freedom with the pdf given by

$$f_i(x) = e^{-x} \frac{x^i}{i!}. \quad (20)$$

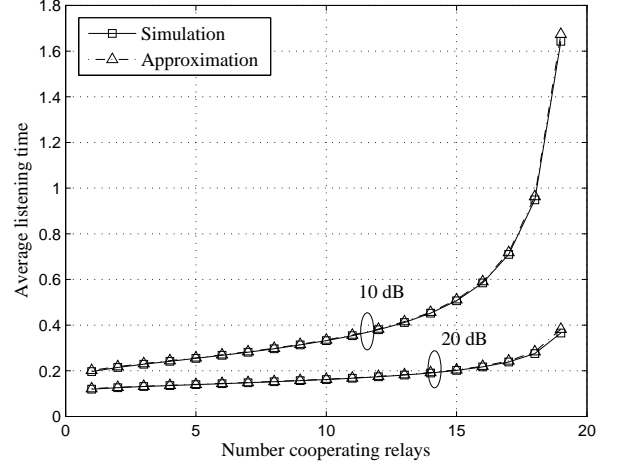


Fig. 2. Average listening time for  $N = 20$ .

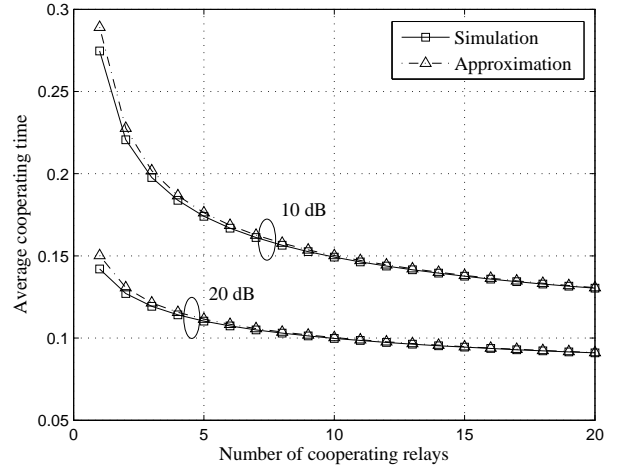


Fig. 3. Average cooperating time for  $N = 20$ .

Unfortunately, a closed form expression of the integral in (19) with the pdf in (20) is not known. We found a good approximation of (19) to be

$$E\{T_C(i)\} \approx \frac{1}{\log_2\left(1 + \frac{P}{\sigma^2}i\right)}. \quad (21)$$

This approximation can be applied to both the *Best Expectation method* and the *Best- $m$  method*.

In Fig. 3, we provide a comparison between this approximation and the simulated values of  $E\{T_C(i)\}$  for various number of relays at  $\frac{P}{\sigma^2} = 10$  dB and 20 dB. The simulations use  $10^5$  samples of chi-square random variable. This example shows a close match for the approximation (21).

Summing (18) and (21), we have a closed-form approximation of the total transmission time for the *Best- $m$  method*. This approximation can be used to pre-determined the optimal number of cooperating relays  $m^*$ , without requiring extensive simulations.

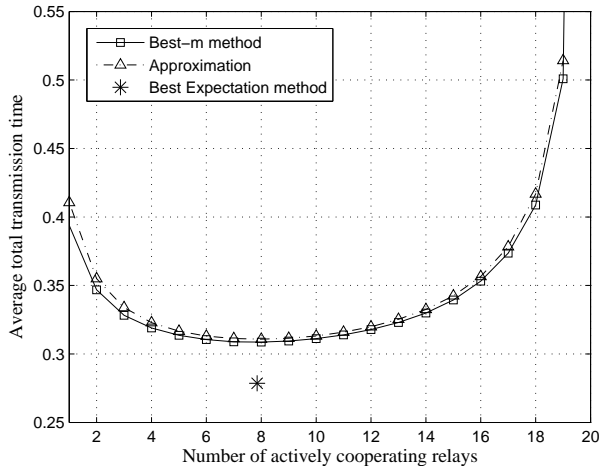


Fig. 4. Average total transmission time and the approximation

## V. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we compare the simulation results of the two relay selection methods and the analytical approximations described in Section IV. We also provide some observations on the number of actively cooperating relays.

Fig. 4 shows the average total transmission time of the *Best Expectation method*, the *Best-m method* and the analytical approximations for  $N = 20$  candidate relay terminals in Rayleigh fading channels (without path loss). The average SNR is set to be 10 dB for the source-to-destination link and for all relay-to-destination links, and 20 dB for all source-to-relay links. For the *Best-m method*, the approximation for the average total transmission time is obtained by summing (18) and (21). The plots show a close match between this approximation and the simulation for various numbers of cooperating relays. We observed that the *Best Expectation method* uses 7.8497 actively cooperating relays in average. For the *Best-m method*, using 8 cooperating relay terminals minimizes the average total transmission time in (10), so the optimal  $m^*$  is 8. The analytical approximation values match well with the simulation and also result in  $m^* = 8$ . In this simulation, the *Best-m method* consumes 11% more transmission time than the *Best Expectation method*, but the *Best-m method* is much simpler to implement once  $m^*$  is predetermined.

In Fig. 5, we study the connection between the *Best-m method* relay selection method and the network geometry in a more realistic channel with both path loss and fading. We set the distance between the source and the destination terminals to be 1 km and vary the relay radius  $d_0$  and the number of candidate relays  $N$ . This simulation uses the path loss model introduced in Section II, with the path loss exponent  $\alpha = 3.5$ . Initial simulation studies indicate that, if we fix the radius  $d_0$  to be constant, the number of actively cooperating relays  $m$  grows almost linearly with the number of candidate relays  $N$ . Thus the active to total relay ratio  $\frac{m}{N}$  is approximately constant. With uniformly distributed relays, this result implies that the cooperating relays are located within a circle of radius  $d_1 (< d_0)$  that is of a constant proportion of

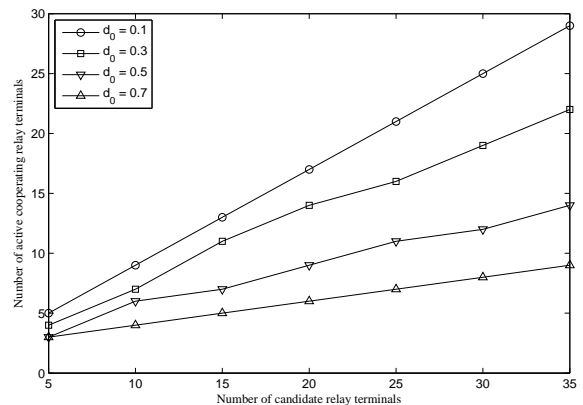


Fig. 5. Number of actively cooperating relays vs. Number of candidate relays for various  $d_0$ .

the total relay radius  $d_0$ . In other words, the ratio of the area of cooperating relays to the total area of all relays,  $\frac{\pi d_1^2}{\pi d_0^2} \simeq \frac{m}{N}$ , is constant in average, regardless of the number of candidate relays  $N$ . Since this ratio appears to depend only on the spatial distribution of the relays (in this case uniform), it appears to be independent of the density of the relays. This result indicates that for uniform relays, the distance from the source terminal to the relay terminal can be a good measure to whether the corresponding relay terminal should be included in the set of actively cooperating relays. The *Best-m method* is closely matched by just selecting the relay terminals within a distance  $d_1$  from the source terminal, provided  $d_1/d_0$  is a constant ratio. This initial result is very encouraging and we plan to study this connection further in our future work.

## VI. CONCLUSION

In this paper, we examined the problem of relay selection for wireless cooperative communications in order to minimize the total transmission time. We presented two relay selection methods: *Best Expectation method* and *Best-m method*, and proposed simple, optimal algorithms to implement them. Furthermore, for channels with Rayleigh fading alone, we provided closed-form analytical approximations of the transmission times consumed by these methods. Simulation results showed a close match between these approximations and the numerical values. The approximations provide a simple way of determining the optimal number of actively cooperating relays in the *Best-m method*. Through initial studies for channels with both fading and path loss, we also observe that the *Best-m method* translates to a simple geometrical method for selecting relays, which uses only the distance between the relay and the source. This source-to-relay distance appears to be a key feature for the source in deciding whether to cooperate with the corresponding relay terminal.

## APPENDIX

For the proof of the lower bound for the transmission time of the listening phase, we utilize the result of convex ordering in [9].

*Definition:* Assume that  $X$  and  $Y$  are arbitrary random variables whose cdfs are given as  $F$  and  $G$ , respectively.  $X$  *c-precedes*  $Y$  ( $X \leq_c Y$ ) if and only if  $G^{-1}F(x)$  is convex.

*Theorem:* If  $(X \leq_c Y)$ , then  $F(E\{X\}) \leq G(E\{Y\})$  and  $F(E\{X_{(m)}\}) \leq G(E\{Y_{(m)}\})$ ,  $m = 1, \dots, N$ , provided the expectations exist, where  $X_{(m)}$  and  $Y_{(m)}$  denote the  $m$ -th order statistics over  $N$  independent samples of  $X$  and  $Y$ , respectively.

The proof of the theorem can be found in [9]. If we consider a random variable  $Y$  with cdf  $G(y) = -1/y$ ,  $(-\infty < y < -1)$ , it is known that the order statistic  $Y_{(m)}$  has the following property [9]:

$$E\{Y_{(m)}\} = \frac{-N}{m-1}, \quad m > 1. \quad (22)$$

For an arbitrary random variable  $X$  with cdf  $F(x)$ , if  $1/F(x)$  is convex, so that  $G^{-1}F(x)$  is convex, the following inequality holds by the theorem:

$$F(E\{X_{(m)}\}) \leq \frac{m-1}{N}, \quad m > 1. \quad (23)$$

We also note that the inequality is reversed if  $G^{-1}F(x)$  is concave.

*Proof of Lower Bound in Eq. (17):* We only need to show that  $1/H(t)$  is convex for  $t > 0$ , where  $H(t)$  is given in (16). Consider the second derivative of  $1/H(t)$ ,

$$\begin{aligned} \frac{d^2}{dt^2} \left[ \frac{1}{H(t)} \right] &= \frac{d}{dt} \left[ \frac{-h(t)}{\{H(t)\}^2} \right] \\ &= \frac{d}{dt} \left[ \frac{-\beta t^{-2} e^{1/t} H(t)}{\{H(t)\}^2} \right] \\ &= \frac{d}{dt} \left[ \frac{-\beta t^{-2} e^{1/t}}{H(t)} \right] \\ &= \frac{[-\beta t^{-2} e^{1/t}]'}{H(t)} - \beta t^{-2} e^{1/t} \left[ \frac{1}{H(t)} \right]' \\ &= \frac{2\beta t^{-3} e^{1/t} + \beta t^{-4} e^{1/t}}{H(t)} + \beta t^{-2} e^{1/t} \frac{h(t)}{\{H(t)\}^2} \\ &= \frac{\beta t^{-4} e^{1/t}}{H(t)} (2t + 1 + \beta e^{1/t}), \end{aligned} \quad (24)$$

where  $\beta = \frac{\sigma^2}{P}$ . Since we have  $e^{1/t} > 0$ ,  $H(t) > 0$ , and  $(2t + 1 + \beta e^{1/t}) > 0$  for  $t > 0$ , we have nonnegative second derivative and the convexity of  $1/H(t)$  is proved. Therefore, from the result of (23) for a concave function  $G^{-1}H(t) = -1/H(t)$ , we have

$$\begin{aligned} E\{T_L(m)\} &\geq H^{-1} \left( \frac{m-1}{N} \right) \\ &= \frac{1}{\log_2 \left( 1 - \frac{P}{\sigma^2} \log_2 \left( \frac{m-1}{N} \right) \right)}, \quad m > 1. \end{aligned} \quad (25)$$

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