A Half-Duplex Relay Coding Scheme Optimized for Energy Efficiency

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***Abstract*—We explore the issue of the network energy efficiency in relay channels. We first propose a half-duplex decode-forward coding scheme. We then optimize the power allocation to minimize the total power consumption while maintaining a desired source rate. We show that this scheme significantly outperforms direct and two-hop transmissions. Moreover, it reduces the relay energy consumption by up to 7dB, which is beneficial for shared relay stations, and smooths out transmit power peaks, which simplifies interference management.**

***Index Terms*—Relay channel; Half-duplex; Coding scheme; Achievable rate; Energy efficiency**

I. Introduction

Relaying is a key feature of future wireless systems. The relay channel has been extensively studied in [1]–[3]. However, while user applications are mainly associated with a fixed minimum rate and while user devices are power-limited, most of coding schemes are optimized for maximum rate [3]–[5]. Power allocation for energy minimization has been proposed in [6]. However, it assumes full-duplex transmissions and no individual power constraints, which does not meet practical constraints for wireless transceivers.

In this paper, we first propose a half-duplex coding scheme performing by time division for the discrete memoryless relay channel. The source splits the message into two parts and transmits in two phases. It sends both message parts during both phases, the relay decodes one part after the first phase and forwards it in the second phase, then the destination decodes both parts only at the end of the second phase by using joint typicality decoding.

Second, we optimize the resource allocation of the proposed scheme for energy efficiency in Gaussian relay channels. We minimize the total power consumption while maintaining a desired source rate and considering individual power constraints. Using KKT conditions, we derive the optimal allocation in closed-form.

Third, we analyze the performance of the proposed scheme as a function of both the source rate and the relay's position. We show that the scheme significantly reduces the required transmit power. It achieves most of its gain by lessening the relay power consumption. This allows, for example, a shared relay to serve more users. This scheme also removes transmit power peaks and additionally achieves higher source rates.

The paper is organized as follows. The new coding scheme in analyzed for general channels in Section II, and for AWGN channels in Section III. We optimize the resource allocation for energy efficiency in Section IV. Section V presents the reference schemes and the performance analysis. Section VI concludes this paper.

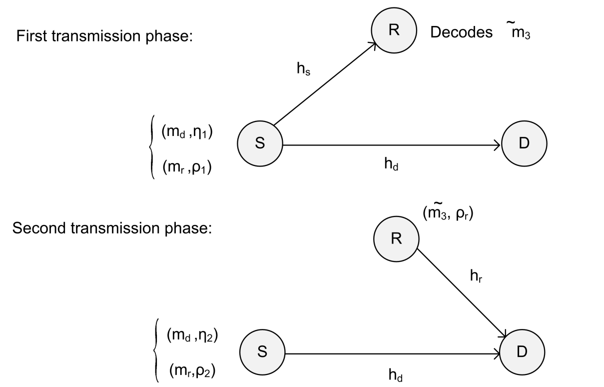

Fig. 1. A Half-Duplex Coding Scheme for Relay Channels

II. A Comprehensive Half-Duplex Decode and Forward Relaying Scheme

A. Channel Model

The relay channel consists of a source input alphabet $\mathcal{X}$, a relay input alphabet $\mathcal{X}_r$, two channel output alphabets $\mathcal{Y}$ and $\mathcal{Y}_r$, and a set of distributions $p(y, y_r|x, x_r)$. We assume this relay channel is memoryless.

We consider a half-duplex channel with time division, such that the transmission is carried out in two phases within each code block of normalized length. During the first phase, of duration $\theta \in [0, 1]$, the source transmits while the relay listens. During the second phase, of duration $\bar{\theta} = (1 - \theta)$, both the source and the relay transmit. Considering this time division, the channel during the first phase is $p(y, y_r|x)$, and during the second phase is $p(y|x, x_r)$.

B. Half-Duplex Relaying Scheme

We consider decode-forward (DF) relaying. A $\left(2^{nR}, n\right)$-code for this relay channel consists of a message set $\mathcal{M} = \{1, 2, \ldots, 2^{nR}\}$, two encoders (at the source and the relay) and two decoders (at the relay and the destination).

To send a message m of rate R to the destination, the source performs message splitting. It divides the initial message into

$$R \le \theta \log_2\left(1+\frac{(\eta_1+\rho_1)\,P_s|h_d|^2}{N_0}\right)+\bar{\theta}\log_2\left(1+\frac{(\eta_2+\rho_2)\,P_s|h_d|^2+\rho_r P_r|h_r|^2+2\sqrt{P_s|h_d|^2P_r|h_r|^2\rho_2\rho_r}}{N_0}\right)=I_1$$
$$R \le \theta \log_2\left(1+\frac{\rho_1 P_s|h_s|^2}{N_0+\eta_1 P_s|h_s|^2}\right)+\theta\log_2\left(1+\frac{\eta_1 P_s|h_d|^2}{N_0}\right)+\bar{\theta}\log_2\left(1+\frac{\eta_2 P_s|h_d|^2}{N_0}\right)=I_2 \quad (6)$$

two parts (m_d, m_r), with rates R_d and R_r respectively, where $R_d + R_r = R$. The message m_d is directly decoded by the destination at the end of the second phase, whereas m_r is intended to be relayed. The coding scheme is depicted in Figure 1. For convenience, we will denote $x^{\theta n} = [x_1, x_2 \dots x_{\theta n}]$ and $x^{\bar{\theta} n} = [x_{\theta n+1}, x_{\theta n+2} \dots x_n]$.

1) Codebook generation: Fix $p(x_r)p(u|x_r)p(x|u,x_r)$. Generate 2^{nR_r} iid sequences $x_r^n(m_r) \sim \prod_{i=1}^n p(x_{r_i})$. Then, for each x_r^n, generate one sequence $u^n(m_r) \sim \prod_{i=1}^n p(u_i|x_{r_i})$. Finally for each $(u(m_r), x_r(m_r))$, generate 2^{nR_d} sequences $x^n(m_d, m_r) \sim \prod_{i=1}^n p(x_i|u_i, x_{r_i})$.

To send a message m, the source encoder maps it to the codeword $x^n(m_d, m_r) \in \mathcal{X}$. During the first phase, the source sends $x^{\theta n}$, while the relay listens. At the end of the first phase, the relay decodes $\tilde{m}_r$ from the received signal and re-encodes it into the codeword $x_r^n(\tilde{m}_r)$. Then, during the second phase, the source sends $x^{\bar{\theta} n}$, while the relay sends $x_r^{\bar{\theta} n}$.

2) Decoding technique: We consider joint-typicality decoding at both the relay and destination, as proposed in [7]. At the end of phase 1, the relay chooses the unique m_r such that

$$\left(Y_r^{\theta n}, U^{\theta n}(m_r)\right) \in \mathcal{A}_\epsilon^{\theta n}. \quad (1)$$

Otherwise, it declares an error. The destination performs joint-typicality decoding using the signal received during both transmission phases. Given the received sequence $\left[Y_1^{\theta n}\; Y_2^{\bar{\theta} n}\right]$, it chooses the unique set (m_d, m_r), such that

$$\left(Y_1^{\theta n}, U^{\theta n}(m_r), X^{\theta n}(m_d, m_r)\right) \in \mathcal{A}_\epsilon^{\theta n} \quad \text{and}$$
$$\left(Y_2^{\bar{\theta} n}, U^{\bar{\theta} n}(m_r), X^{\bar{\theta} n}(m_d, m_r), X_r^{\bar{\theta} n}(m_r)\right) \in \mathcal{A}_\epsilon^{\bar{\theta} n}. \quad (2)$$

Otherwise, it declares an error. This comprehensive coding scheme covers direct and two-hops relaying as special cases, as well as the maximum-rate scheme proposed in [3].

Theorem 1. *All rates satisfying*

$$\begin{aligned} R &\le \theta I(Y_1; UX) + \bar{\theta} I(Y_2; UXX_r) \\ R &\le \theta I(Y_r; U) + \theta I(Y_1; X|U) + \bar{\theta} I(Y_2; X|UX_r) \end{aligned} \quad (3)$$

are achievable for some joint distribution $p(x_r)p(u|x_r)$ $p(x|u, x_r)p(y_r, y|x, x_r)$.

Proof: See the Appendix. ■

III. Relaying Scheme for AWGN Channels

A. AWGN relay channel model

For the rest of this paper, we will consider complex AWGN channels, where h_d, h_s and h_r respectively stand for the gain of the direct link, the source-to-relay link and the relay-to-destination link, as depicted in Figure 1. We assume gain channel information is known globally at all nodes. Consider independent AWGN Z_1, Z_2 and Z_r with variance N_0. The half-duplex AWGN relay channel can be written as follows.

$$\begin{aligned} Y_r &= h_s X_1 + Z_r \\ Y_1 &= h_d X_1 + Z_1 \\ Y_2 &= h_d X_2 + h_r X_r + Z_2 \end{aligned}$$

B. Coding scheme for the Gaussian relay channel

The source and the relay have individual power constraints P_s and P_r within the same bandwidth B. At each phase, each node allocates to each message a portion of its available transmit power P_s. Denote η_1 and η_2 as the portion of source power allocated to m_d in the first and second phase respectively, similarly, we denote ρ_1 and ρ_2 as the portion for m_r. Denote ρ_r as the portion of the relay power P_r used to forward $\tilde{m}_r$ to the destination. We consider transmit power constraint at each node, such that

$$\begin{aligned} P_s^{(c)} = \theta\,(\eta_1+\rho_1)\,P_s + \bar{\theta}\,(\eta_2+\rho_2)\,P_s \le P_s \\ P_r^{(c)} = \bar{\theta}\rho_r P_r \le P_r \end{aligned} \quad (4)$$

Applying the proposed scheme the Gaussian relay channel and denoting $X^n = \left[X_1^{\theta n}\; X_2^{\bar{\theta} n}\right]$, the transmit signal in the two phases can be written as

$$\begin{aligned} X_r &= \sqrt{\rho_r P_r}\, U \\ X_1 &= \sqrt{\rho_1 P_s}\, U + \sqrt{\eta_1 P_s}\, V \\ X_2 &= \sqrt{\rho_2 P_s}\, U + \sqrt{\eta_2 P_s}\, V \end{aligned} \quad (5)$$

where $U^n(m_r) \sim \mathcal{N}(0,1)$ and $V^n(m_d) \sim \mathcal{N}(0,1)$ are independent. Here, the optimal U and X_r are fully correlated to allow a beamforming gain at the destination.

C. Achievable rate region

Corollary 1. *For AWGN relay channels, all rates satisfying* (6)*, at the top of the page, are achievable.*

Proof: These rates are derived by evaluating (3) for Gaussian relay channels, with input in (5). ■

IV. Optimized Scheme for Energy-Efficiency

We consider the total source and relay power consumption during both transmission phases and analyze the following optimization problem:

$$\min\, \theta\,(\eta_1+\rho_1)\,P_s + \bar{\theta}\,(\eta_2+\rho_2)\,P_s + \bar{\theta}\rho_r P_r \quad (7)$$
$$\text{s.t.} \quad \begin{array}{ll} I_1 \ge R & I_2 \ge R \\ P_s^{(c)} \le P_s & P_r^{(c)} \le P_r \end{array}$$

The two rate constraints ensure the achievability of the source rate, given the two individual power constraints. Since the scheme is based on decode-forward, we only need to consider the case $|h_d|^2 \leq |h_r|^2$, for which using a relay is helpful.

Theorem 2. *Given $|h_s|^2 \geq |h_d|^2$, the optimal scheme for network energy efficiency is such that:*

If $R \geq R_{lim}$, then apply sub-scheme A:

- *In Phase 1, the source sends m_r ($\eta_1^\star = 0$)*
- *In Phase 2, the relay sends $\tilde{m}_r$ and the source sends (m_r, m_d)*

If $R \leq R_{lim}$, then apply sub-scheme B:

- *In Phase 1, the source sends m*
- *In Phase 2, the relay sends $\tilde{m}$ and the source sends m*

where $R_{lim} = \theta \log_2 \left(1 + \frac{\rho_1^\star P_s |h_s|^2}{N_0}\right)$ and $\rho_1^\star$ satisfying (8). The optimal power allocations for sub-schemes A and B are given in Propositions 1 and 2 respectively.

Proof: The optimization problem (7) is solved by using Lagrangian techniques. The proof can be found in [8]. ■

Figure 4 illustrates the power consumption of the above two sub-schemes.

A. Sub-Scheme A

This sub-scheme is a special case of the coding scheme proposed in Section III. During phase 1, the source sends only m_r with power $\rho_1 P_s$, instead of both m_d and m_r ($\eta_1^\star = 0$). During phase 2, it sends m_d with power $\eta_2 P_s$ and m_r with power $\rho_2 P_s$, while the relay sends $\tilde{m}_r$ with power $\rho_r P_r$.

Proposition 1. *For a given θ, the optimal power allocation set for sub-scheme A is such that*

$$\eta_2^\star = \left(\frac{2^{R/\bar{\theta}}}{\left(1 + \frac{\rho_1^\star P_s |h_s|^2}{N_0}\right)^{\theta/\bar{\theta}}} - 1 \right) \frac{N_0}{P_s |h_d|^2} \; ; \; \rho_2^\star = \frac{P_r |h_d|^2}{P_s |h_r|^2} \rho_r^\star$$

$$\rho_r^\star = \frac{N_0}{P_r \left(|h_r|^2 + 2|h_d|^2 + \frac{|h_d|^4}{|h_r|^2} \right)} \left(\frac{2^{R/\bar{\theta}}}{\left(1 + \frac{\rho_1^\star P_s |h_d|^2}{N_0}\right)^{\theta/\bar{\theta}}} - \left(1 + \frac{\eta_2^\star P_s |h_d|^2}{N_0}\right) \right)$$

$\rho_1^\star$ is found numerically by solving $g(\rho_1^\star) = 0$ where

$$g(s) = \left(1 - \frac{|h_d|^2}{|h_d|^2 + |h_r|^2}\right) \frac{|h_s|^2}{|h_d|^2} \frac{2^{R/\bar{\theta}}}{\left(1 + \frac{s P_s |h_s|^2}{N_0}\right)^{1/\bar{\theta}}} + \frac{|h_d|^2}{|h_d|^2 + |h_r|^2} \frac{2^{R/\bar{\theta}}}{\left(1 + \frac{s P_s |h_d|^2}{N_0}\right)^{1/\bar{\theta}}} - 1 \tag{8}$$

If $\eta_i^\star$ and $\rho_i^\star$ do not satisfy the power constraints (4), the desired source rate cannot be achieved. Note that $\eta_2^\star$ is positive as long as $R \geq R_{\lim}$ as defined in Theorem 2. When $R \leq R_{\lim}$, we apply sub-scheme B, as discussed next.

B. Sub-Scheme B

This a special case of sub-scheme A. Here, the whole message is relayed: $m_d = 0$ and $m_r = m$. During phase 1, the source sends m with power $\rho_1 P_s$. During phase 2, the relay sends $\tilde{m}$ with power $\rho_r P_r$.

Proposition 2. *For a given θ, the optimal power allocation for sub-scheme B is such that*

$$\eta_1^\star = \eta_2^\star = 0 \; ; \quad \rho_1^\star = \left(2^{R/\theta} - 1\right) \frac{N_0}{P_s |h_s|^2} \; ; \quad \rho_2^\star = \frac{P_r |h_d|^2}{P_s |h_r|^2} \rho_r^\star$$

$$\rho_r^\star = \left(\frac{2^{R/\bar{\theta}}}{\left(1 + \frac{\rho_1^\star P_s |h_d|^2}{N_0}\right)^{\theta/\bar{\theta}}} - 1 \right) \frac{N_0}{P_r \left(|h_r|^2 + 2|h_d|^2 + \frac{|h_d|^4}{|h_r|^2} \right)}$$

V. Performance Analysis

A. Reference Schemes

As reference schemes, we consider both direct transmissions and two-hop transmission. In addition, we also compare the proposed half-duplex scheme to the full-duplex scheme proposed in [6]. The transmitters have access to channel state information in all schemes.

1) Direct Transmission: In this one-phase transmission, the source sends the message m with a minimum power

$$\eta^{(d)} P_s = \frac{\left(2^R - 1\right) N_0}{|h_d|^2}$$

2) Two-Hop Routing: In the classical two-hop routing, the source first sends the whole message to the relay without splitting and with a minimum power $\rho^{(s)} P_s$. If the relay can decode the message, it forwards it to the destination, with a power of $\rho^{(r)} P_r$, where

$$\rho^{(s)} P_s = \frac{\left(2^{R/\theta} - 1\right) N_0}{|h_s|^2} \quad ; \quad \rho^{(r)} P_r = \frac{\left(2^{R/\bar{\theta}} - 1\right) N_0}{|h_r|^2}$$

This two-hop transmission is not equivalent to sub-scheme B, where the source repeats the message in phase 2 and where the destination is listening during both phases and uses the cumulative SNR to decode the message.

3) Full-Duplex Decode-Forward (Full Duplex DF): This power allocation scheme was introduced in [6]. It minimizes the total energy consumption, given a desired source rate. The scheme is based on decode-forward as defined in [1] with block Markov encoding. Theorem 3 of [6] gives the minimum total power consumed by both the source and the relay with this scheme. However, note that total power constraint is assumed rather than individual power constraints.

B. Power Consumption and Source Rate

Figure 2 depicts the total average power required by both the source and the relay to maintain a range of desired source rates. When a source rate is not achievable, an outage occurs, which is depicted by a cut-off in the curve.

We can see that the proposed scheme significantly outperforms both the direct transmission and the two-hop routing,

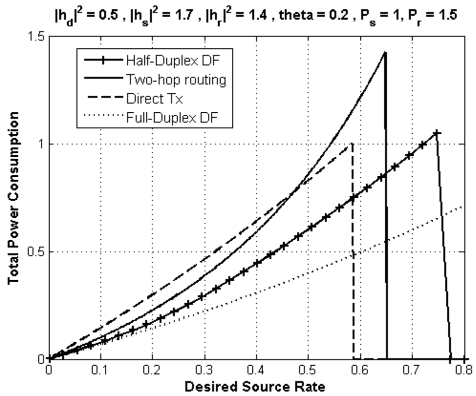

Fig. 2. Total Average Power Consumption

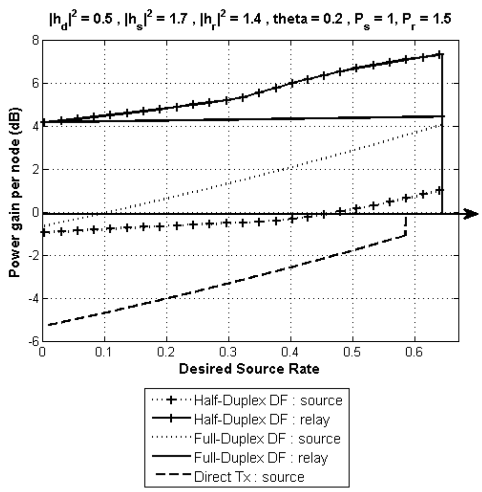

Fig. 3. Power Gain per Node, compared to two-hop transmission

for every source rate. For example, at a source rate of 0.3 normalized b/s/Hz, the power gain is equal to 2dB compared to direct transmission, and 1.2dB compared to two-hop routing. The latter reaches 2.2dB when two-hop routing goes in outage.

Furthermore, thanks to energy savings, the proposed scheme achieves higher data rates than both direct transmission and two-hop routing. However, we propose in [8] another scheme optimal for energy in the half-duplex relay channel which reaches even higher data rates and covers all source rates achievable with decode-forward. Therefore, minimizing the network energy consumption given rate constraints is not equivalent to maximizing the rate given power constraints, as often believed.

C. Power Consumption per Node

In Figure 3, we evaluate the power gains obtained at each transmitter for the different schemes over the two-hop routing. We define the power gain per node in dB realized by any scheme D over the two-hop routing as follows

$$G_i = 10 \log_{10} \left(\frac{P^{(c)}_{\text{two-hop}}}{P^{(c)}_{\text{D}}} \right)$$

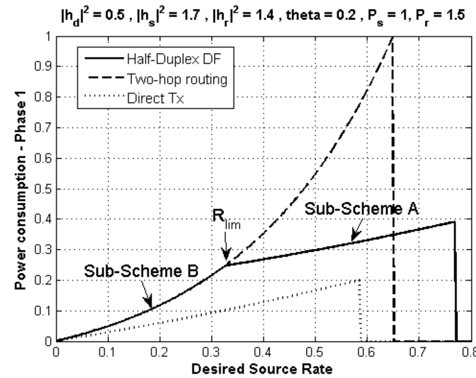

Fig. 4. Average Power Consumption during Phase 1

where $P^{(c)}_{\text{D}}$ is the power consumed by scheme D at node i.

Considering the source (resp. the relay), we plot the power gain realized by both optimized schemes (full and half duplex decode-forward), by comparing with the source (resp. the relay) power consumption in two-hop routing. Results show that the full-duplex scheme of [6] reduces the total energy consumption by lessening the consumption at both transmitters. On the contrary, the proposed half-duplex scheme achieves most of its power gain by significantly reducing the relay consumption. Since the source sends data both on the direct and relaying paths and during both phases, it may consume some more power (-1dB $\leq G_{\text{source}} \leq$ 1dB) than two-hop routing, especially at lower rates. Nevertheless, the relay power gain reaches up to 7dB.

D. Impact on transmit power peaks

Two-hop routing generally suffers from high power peaks. Since the whole message is sent twice by the source and the relay within the same time slot, both transmitters have to increase their transmit power to sustain the desired rate. This is particularly true for the source, during Phase 1 ($\theta < 0.5$). We then plot in Figure 4 the average power that is used by the source only during this phase for the half-duplex scheme, two-hop routing and direct transmission.

First, simulation shows that the source power consumption is higher in the proposed scheme than in direct transmission (but the total energy consumed during both phases is lower). Again, most of energy savings is achieved by reducing the relay consumption. Second, we see that the proposed scheme allocates power in a smooth manner and removes transmit power peaks, which appear in two-hop routing when the source rate increases. Contrary to routing, the proposed scheme spreads energy over both phases and over the direct and the relaying paths.

E. Power consumption and network geometry

Figure 5 presents the total power consumption of the different schemes as a function of the relay's position. We consider a linear geometrical model, where the source, the relay and the destination are aligned. The distance between the source and the destination is normalized, and we let the

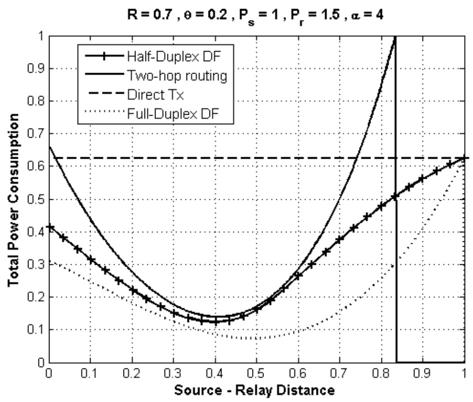

Fig. 5. Power Consumption as a function of the relay's position

relay's position vary in between. The relaying channel gains are modelled by $|h_{i,j}|^2 = \frac{1}{d_{i,j}^{\alpha}}$, where $d_{i,j}$ is the distance between nodes i and j, and where α is the pathloss exponent.

The figure shows that the proposed scheme outperforms both direct transmission and two-hop routing, and again, removes transmit power peaks. Moreover, when the relay is far from the source, the transmission is still successful, whereas routing is in outage.

VI. Conclusion

We designed a half-duplex coding scheme with resource allocation that is optimized for energy efficiency and maintains a desired source rate. We analyzed its performance as a function of both the source rate and the relay's position. Results shows that this scheme achieves significant energy gain, mostly by reducing the relay consumption. Moreover, it removes transmit power peaks and achieves higher data rates.

Appendix

Proof of Theorem 1

We analyze the error events as proposed in [7]. Assume that $(1,1)$ was sent. Three error events can be defined.

If $(1,1)$ is not jointly typical with the received signal, the error event is $E_0 = E_1 \cup E_2$ with

$$E_1 = \left\{ \left(Y_1, U^{\theta n}(1), X^{\theta n}(1,1)\right) \notin \mathcal{A}_{\epsilon}^{\theta n} \right\}$$
$$E_2 = \left\{ \left(Y_2, U^{\bar{\theta} n}(1), X^{\bar{\theta} n}(1,1), X_r^{\bar{\theta} n}(1)\right) \notin \mathcal{A}_{\epsilon}^{\bar{\theta} n} \right\}$$

According to the asymptotic equipartition property (AEP), $P(E_0) \to 0$ as $n \to \infty$.

If m_r is decoded erroneously, the error event is $E_r = E_3 \cap E_4$ where, for some $\hat{m}_r \neq 1$ and any $\hat{m}_d$,

$$E_3 = \left\{ \left(Y_1, U^{\theta n}(\hat{m}_r), X^{\theta n}(\hat{m}_d, \hat{m}_r)\right) \in \mathcal{A}_{\epsilon}^{\theta n} \right\}$$
$$E_4 = \left\{ \left(Y_2, U^{\bar{\theta} n}(\hat{m}_r), X^{\bar{\theta} n}(\hat{m}_d, \hat{m}_r), X_r^{\bar{\theta} n}(\hat{m}_r)\right) \in \mathcal{A}_{\epsilon}^{\bar{\theta} n} \right\}$$

Finally, if m_d is decoded erroneously, given that m_r is correct, the error event is $E_d = E_5 \cap E_6$ such that, for some $\hat{m}_d \neq 1$,

$$E_5 = \left\{ \left(Y_1, U^{\theta n}(1), X^{\theta n}(\hat{m}_d, 1)\right) \in \mathcal{A}_{\epsilon}^{\theta n} \right\}$$
$$E_6 = \left\{ \left(Y_2, U^{\bar{\theta} n}(1), X^{\bar{\theta} n}(\hat{m}_d,), X_r^{\bar{\theta} n}(1)\right) \in \mathcal{A}_{\epsilon}^{\bar{\theta} n} \right\}$$

Let's analyse for example the probability $P(E_d)$ that m_d is decoded erroneously, which can be expressed as

$$P(E_d) = \sum_{i=1}^{2^{nR_d}} P(E_{d,i}) = \sum_{i=1}^{2^{nR_d}} P(E_{5,i}) \times P(E_{6,i})$$

given that

$$\begin{aligned} P(E_{5,i}) &\leq \sum_{(Y_1,U,X) \in \mathcal{A}_{\epsilon}^{\theta n}} p(U,X) p(Y_1|U) \\ &\leq 2^{\theta n(H(Y_1,U,X)+\epsilon)} 2^{-\theta n(H(U,X)-\epsilon)} 2^{-\theta n(H(Y_1|U)-\epsilon)} \\ &\leq 2^{\theta n(H(Y_1|U,X)+2\epsilon)} 2^{-\theta n(H(Y_1|U)-\epsilon)} \\ &\leq 2^{-\theta n(I(Y_1;X|U)-3\epsilon)} \end{aligned}$$

and

$$\begin{aligned} P(E_{6,i}) &\leq \sum_{(Y_2,U,X,X_r) \in \mathcal{A}_{\epsilon}^{\bar{\theta} n}} p(U,X,X_r) p(Y_2|U,X_r) \\ &\leq 2^{\bar{\theta} n(H(Y_2,U,X,X_r)+\epsilon)} 2^{-\bar{\theta} n(H(U,X,X_r)-\epsilon)} 2^{-\bar{\theta} n(H(Y_2|U,X_r)-\epsilon)} \\ &\leq 2^{\bar{\theta} n(H(Y_2|U,X,X_r)+2\epsilon)} 2^{-\bar{\theta} n(H(Y_2|U,X_r)-\epsilon)} \\ &\leq 2^{-\bar{\theta} n(I(Y_2;X|UX_r)-3\epsilon)} \end{aligned}$$

Thus, $P(E_d) \leq 2^{nR_d} 2^{-\theta n(I(Y_1;X|U)-3\epsilon)} 2^{-\bar{\theta} n(I(Y_2;X|UX_r)-3\epsilon)}$ and $P(E_d) \to 0$ if $R_d \leq \theta I(Y_1; X|U) + \bar{\theta} I(Y_2; X|UX_r)$. Similarly, from the error event E_r, we get

$$R_d + R_r \leq \theta I(Y_1; UX) + \bar{\theta} I(Y_2; UXX_r).$$

Finally, error analysis at the relay leads to $R_r \leq \theta I(Y_r; U)$. Consequently, the total source rate $R = R_d + R_r$ should satisfy (3) to be achievable.

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